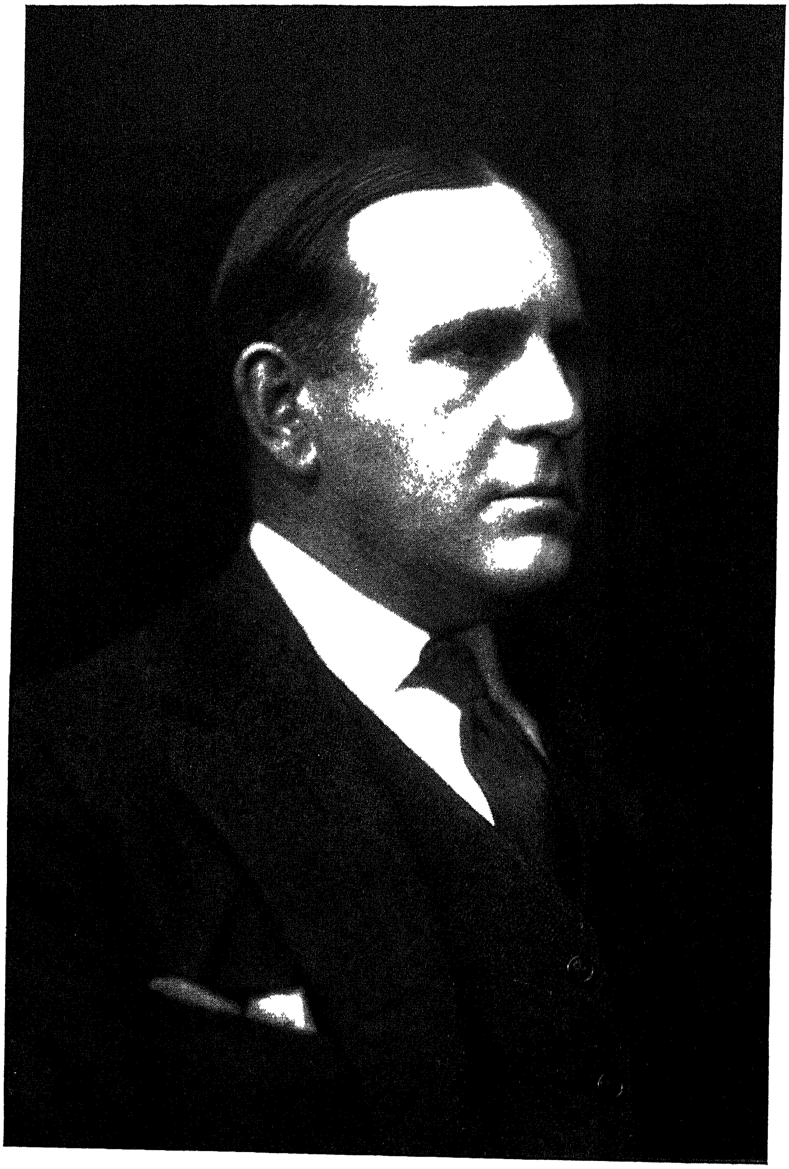

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REGENERATIVE BRAKING OF ELECTRIC VEHICLES

BY R. E. HELLMUND

Abstract of Paper

This paper discusses the problems met in applying regenerative control to electric railways. In the introductory section the purposes, advantages, requirements and disadvantages of regenerative control are pointed out, and then the author takes up the description of various systems. In doing this, special attention is paid to those systems which are well adapted to illustrate some fundamental principle, and to those which are of some practical importance. Other systems are, on account of space limitations, mentioned only briefly, or entirely omitted.

The three-phase system is discussed under the following heads: regeneration near synchronous and above synchronous speeds; line regulation; practical results and safety of operation; influence upon power factor; regeneration at speeds below synchronism.

The phase converter system is next taken up, with a consideration of the requirements of such a system, and practical results attained.

Direct-current systems of regenerative control and their problems are exhaustively discussed, under the following heads: series, shunt, and separately excited generator; classification and description of separately excited systems with generators operated by line voltage; inherently constant excitation; excitation varying with the line voltage; excitation varying with the regenerated current; series-parallel control; capacity of exciter and switches; systems with variable armature voltage; miscellaneous direct-current systems; transmission system and substations.

The next division of the paper deals with alternating-current commutator motor systems, including those with separately excited generators, armature self-excited generators, excitation varying with the regenerated current, and stator self (load)-excited generators, and the subject of commutation is discussed.

The possibilities of regenerative control with a vapor converter system are then analyzed, and the paper ends with a statement of the general conclusions reached.

THE fact that a motor driving an electric vehicle may, under certain conditions, be converted into a generator and that it will as such exert a braking effect upon the vehicle instead of propelling it, was known very soon after the electric propulsion of vehicles was introduced.

This braking effect may be utilized under two conditions: first, when the vehicle is going down hill, for the purpose of avoiding undesirable acceleration, due to gravity, and consequent high speeds; second, after a vehicle has reached a certain speed and it is desirable to reduce this speed to a lower speed or to standstill.

The electric energy generated while braking the vehicle in this manner may be disposed of in two ways: first, it may be used up in resistances mounted on the vehicle, in which case it is customary to speak of dynamic braking; second, it may be returned to the power supply system, in which case it is customary to speak of regenerative braking. A combination of the two cases is, of course, possible and may be found advantageous in certain applications. Since in any of these cases dynamic braking takes place, it would be better to use the terms:

1. Resistance braking.
2. Regenerative braking.
3. Combined resistance and regenerative braking.

This paper deals with regenerative braking in particular.

Common to all three cases of electrodynamic braking, are the following advantages over purely mechanical braking:

1. Reduced wear of mechanical braking devices, which, in the case of electric railways, usually results in reduced maintenance cost for wheels and brake shoes.

2. Greater ease and safety in handling the vehicle during the braking period under certain conditions, which results, especially in handling heavy freight trains, in reduced maintenance of couplings and other parts of the cars.

3. Higher permissible average speed in descending grades in certain applications. The braking effect with air brakes cannot be kept constant, so that speed variations will result, when going down grade. Due to these variations, the average speed is appreciably below the maximum safe speed. With electrodynamic braking without the assistance of air brakes, much closer speed control is possible, so that the average speed can be held much closer to the maximum permissible speed.

4. Increased safety by elimination or at least reduction of tire heating caused by brake shoe friction and resulting in loose tires and accidents.

Regenerative braking has the further marked advantages namely:

5. Power economy which amounts in many cases to a saving of 15 per cent or more in the total power consumption.

6. In connection with subways where generation of heat in the tunnels is very objectionable, regeneration is very advan-

tageous, as it permits the elimination of an appreciable part of the heat now generated by the mechanical braking system.

It may safely be stated that wherever it seems worth while, the above advantages may be secured in the present state of the art, both on running down grades and during retardation, with any of the systems or types of motors which are now used for the operation of electric vehicles, although commercial systems have been developed, up to the present, for only a few cases.

The requirements necessary to accomplish these results are as follows, and are either partly or in their entirety, dependent upon the system, service conditions, etc.

1. Increased current capacity of the main motors and such transforming or converting apparatus as may be located on the vehicle for the purpose of converting power for the main motors. This is evident because the braking current flowing in the motor, etc., naturally raises the r. m. s. or heating currents. This increase may be appreciable when regeneration is used for stopping trains in local passenger traffic, while it may be rather small in the case of freight locomotives descending grades.

Increased current capacity necessitates, in turn, either increased dimensions of all apparatus affected thereby, or increased ventilation, which may at times be attained by increased capacity of existing ventilating equipments, or by introducing additional ventilating equipment. Where the motors are not loaded to their full capacity for propelling purposes, these requirements may, of course, not exist.

2. Adaptation of the motor bearings and other mechanical parts to regenerative operation, where necessary.

3. Additional equipment on the vehicle for the purpose of controlling and safeguarding the regenerative operation and to give, where necessary, the desirable regenerating characteristic to the motors.

4. Additional equipment in the generating or transforming stations for the purpose of controlling the operation in connection with special conditions brought about by regenerative braking.

5. Adjustment of the system wherever necessary due to the increased tendency towards voltage variation introduced by regenerative braking.

6. Modification of train dispatching so as always to have

a load on the line utilizing the regenerated current, in cases where maximum economy in power consumption is of prime importance.

The fulfilment of these requirements as far as they exist in the various systems introduces the following disadvantages of some importance, which have to be compared with the advantages previously enumerated:

1. Increased weight of vehicle equipment, necessitating, sometimes, increased weight of the mechanical vehicle parts.

2. Increased first cost of vehicle equipment, necessitating, sometimes, increased first cost of the vehicle parts.

3. Increased cost of maintenance of the vehicle equipment.

4. Increased complication, and consequent reduced reliability of operation, of the equipment. This will under certain conditions, of course, also impair the safety with which the vehicle may be operated.

5. Increased complication and consequent reduced reliability of the power house, substations and line protecting apparatus. (While some additional complication may be required in the generating and transmitting system, due to the addition of certain controlling apparatus, the weight and cost of the same is usually so low that it is of no practical importance and usually more than outweighed by the gains made in the reduction in size of apparatus made possible by the reduced power requirements).

6. Increased first cost for copper in the transmission system, or for introduction of other means, to compensate, where necessary, for the increased tendency towards voltage variation, whensoever such tendency exists.

The relative importance of the various advantages and disadvantages given above depends, of course, to a large extent upon the individual system applied and upon service conditions. It is evident, for instance, that the importance of power economy depends largely upon the cost of power as part of the total operating expense, and that the advantage of power economy may be entirely eliminated where water power is available in abundance. The latter condition is, of course, as a rule only temporary, because it is to be expected in practically all cases that any such abundance will in time be eliminated by an increased market for electric power.

While the disadvantages, and their importance, vary also in each case, a number of facts apply pretty generally to all systems, and under most service conditions. It is a fact, for instance, that most of the disadvantages enumerated are not of very great practical importance in most cases where conditions are such that the saving in power consumption, made possible by regeneration, is appreciable, say 10 to 15 per cent, or more, or where an equivalent saving or gain is brought about in the maintenance of brake shoes, wheels, increased operating speeds, or the like.

The increased weight of the vehicle equipment and of the vehicle is usually disadvantageous only in so far as it tends to increase the power consumption during acceleration and running. In all but exceptional cases, this increase in power consumption is, however, smaller than the saving made possible by regenerative control, and in many cases it amounts to only a very small percentage of the same. Therefore, the increased weight does not really mean a disadvantage, but merely reduces the advantage of regenerative braking by a certain, and often very small, amount. There are, of course, cases of locomotives where the maximum permissible weight on the drivers is the limiting feature of the design, and any increase of weight may be quite a handicap in such cases.

Similar conditions apply to the increased first cost of the vehicle equipment and the interest and depreciation charges connected therewith. The latter represent again in many cases only a very small percentage of the saving made possible by regenerative control.

In some cases of old equipment where the main motors cannot be readily increased in capacity to take care of the regeneration, or where the mechanical parts, like the bearings, do not operate properly with regeneration, also where extensive changes in the generating and transmission system are necessary, the first cost for introducing regeneration may, of course, make the change prohibitive.

The increased cost of maintenance is in connection with many of the systems not very large, and of no practical importance, as it represents a very small percentage of the saving in power consumption brought about with regenerative control.

The additional complication in the power house or transmission system is also rather small and usually consists, where at all necessary, only in some automatic means for dissipating whatever small amount of energy may at times be regenerated in excess of the energy required for propelling the other vehicles and other loads on the system.

The disadvantages resulting from the increased tendency towards voltage variation which may be introduced by regeneration depends largely upon the individual case. In a system where a large number of individual vehicles is operated at all times, as in surface lines, elevated and subway lines in the center of large cities, there will practically always be a large demand for the regenerative current by other vehicles and the result will be not only that the average power taken from the power house is reduced, but also that the maximum demand, and with it, the maximum voltage drop is reduced. In such cases there is, of course, no disadvantage on this score.

On the other hand, there are certain systems where so few trains or vehicles are on the line that it may happen that, at the time of maximum demand for propelling, none of the vehicles is regenerating and that, consequently, no reduction of the maximum power demand, and therefore, no reduction of maximum drop, is accomplished. At other times it may happen that on a certain line one or more vehicles are heavily regenerating while no power, or only a small amount of power, is being drawn from the line. In such a case the voltage drop will, of course, be negative. In other words, there is a voltage rise over the no-load voltage, bringing about an increased total voltage variation.

In many cases such an increase in voltage variation is of no practical importance. In other cases, where the car lighting system is dependent upon the line voltage, or where the main motors, auxiliary motors or generating apparatus are sensitive to voltage rises, the increased voltage variation may require some compensating means. In cases where the car lighting system is affected, it may be found desirable to make it independent of the line voltage by installing special car lighting apparatus. Usually, however, it will be found best to compensate for the increased tendency towards voltage variation by a slight increase in line copper, etc. In certain exceptional cases this might prove to be an item of no small importance against the introduction of regenerative control, on account of first cost. Since the increased line capacity tends, however, to improve the efficiency of the system, the first cost may, even in such cases, be largely compensated by reduced losses in the line The cases in which the considerations of line regulation are of very great importance are rather rare; and in most cases of the present large properties which, as a matter of course, are the first ones to consider regenerative control, this disadvantage either will not exist or will be of comparatively little importance.

In view of the above, and in view of the fact that great efforts have been made to reduce power consumption by other means, like reduction of equipment and car weight, field control, coasting clocks, etc., it is rather surprising that regenerative control has not been used to a greater extent in the past. The only reason for this could be that the increased complication and consequent reduced reliability were considered as a sufficient disadvantage to outweigh the advantages. As long as the various types of controlling and protecting apparatus, as well as the operating characteristics of certain motors during regeneration, had not been developed to such a high degree of reliability as has been attained in many of the up-to-date apparatus and systems, this was undoubtedly a very sound and sufficient reason. With the present advanced state and knowledge of the art, however, this condition has changed, and although some difficulties have to be overcome with the various systems, there is no doubt but that the requirements for reliability and safety can be successfully met, in spite of some additional complication necessary, to a larger or smaller degree, in the various cases. For this reason the subject of regeneration is well worth serious consideration, and the brief outline of the various possible systems given in this paper may be of interest. As a matter of course, the development along these lines will, in the future, as in the past, follow the path of least resistance and the systems requiring the least complication and interfering the least with the reliability of equipment for propelling purposes will be introduced first.

In describing the various systems of regenerative braking, special attention has been paid to those which are adapted for demonstrating some fundamental principle, or to those which are especially important and promising with regard to practical application; many other systems are by necessity mentioned very briefly, or entirely omitted.

THREE-PHASE SYSTEM

It is a well-known fact that any induction motor if driven by mechanical torque above synchronous speed will be converted into a generator without any change in connections and without requiring any manipulation of the master controller. In other words, a vehicle of a three-phase system when going down-hill will automatically give regeneration as long as the motors are connected to the line, whether such a condition is desired or not—assuming, of course, that the grade is steep enough to accelerate the train and that the accelerating tractive effort is not used up by the mechanical brakes or other losses. The usual practise is to start the vehicle in the normal way and leave the power on, when going down grade; but even if the power has been disconnected and regeneration is desired, power connections can be re-established in the same manner as when the motors are starting, because the introduction and gradual elimination of the rotor rheostat will avoid excessive overloads over wide ranges of speed conditions. Regeneration in this case, therefore, does not introduce any complication whatsoever on the vehicle and in order to obtain satisfactory results, it is only necessary to equip the vehicle with motors and transformer, if the latter be used, of sufficient capacity to take care of the regenerative currents. Wherever there is danger of the regenerative power at times exceeding the power demand, resistance, preferably of the water-resistance type, has to be provided in the power house, with some automatic means to connect it in the circuit whenever a surplus of power is to be taken care of. All alternating-current transmitting, transforming or converting apparatus will act as well under regenerative conditions as otherwise. Only in case of rotating converting apparatus being used in the system, possibly in the form of frequency changers, a certain amount of danger is introduced by the possibility of such apparatus over-speeding when the main power supply is interrupted for some reason or other, and a train going down-hill furnishes power to the converting apparatus. In such a case, the train may, of course, accelerate and increase with its own speed the speed of the converter. This condition can, however, be very easily taken care of by over-speed relays on the converter, the same as is now being done for similar reasons in connection with synchronous converters and the like.

The proper balance of the regenerative load between the different motors of a vehicle in case of slight variation in wheel diameter can be taken care of by the means provided for the same purpose during motoring.

Some additional slight complications may be introduced whenever in some exceptional cases a locomotive is required as part of its regular service to take a train down-hill with tractive effort in excess of that given by the slipping point of the wheels of the locomotive. In such cases it is necessary to use air brakes in the train and to work them in conjunction with the regeneration of the engine. Since it is at present considered very difficult or almost impossible to exert a uniform torque with the air brakes, the use of the air brakes means, as previously mentioned, a necessity for varying the speed of the train about 5 to 10 mi. per hr. (about 8 to 16 km. per hr.) while descending a grade. In order to enable the inherently constant-speed induction motor to follow such speed variations, while at the same time regenerating, it is necessary to introduce a variable resistance into the rotor circuit which in turn permits speed variations from synchronous speed to speeds about 5 to 10 mi. per hr. higher. The resistance may, in this case, be regulated either automatically, which, of course, means some complication, or manually, which, in turn, would require a certain skill on the part of the operator. It is further a fact in this case that a part of the regenerated energy when running above synchronous speed cannot be returned to the line, but has to be dissipated in a resistance, the amount of this energy, in per cent of the available mechanical energy, being approximately in proportion to the actual speed obtained minus the synchronous speed, the difference to be divided by the actual speed. While the loss of energy in itself may not be of great importance, it is at times a serious problem to get a big enough rheostat for successfully dissipating such energy, especially since the necessity for doing so usually arises only in connection with very heavy railway work. In order to demonstrate the possibilities of regenerative braking with speeds varying above synchronism, Fig. 1 is given, showing speed-torque curves of a typical induction motor with different resistances in the secondary; Fig. 2 also gives the approximate percentage of regenerated power which has to be dissipated and wasted in resistances on the vehicle. It will be seen that all curves are in very close agreement with curve A, which is plotted in accordance with the approximate rule given above for the losses.

The conditions just described also apply, of course, if operation at speeds above synchronous speed is desirable or necessary for any other reason. It may, for instance, be desirable in certain cases to take light loads down-hill at over-synchronous speed by using resistance in the secondary; the only limitation in doing this is the limit imposed by the capacity of the secondary rheostat, and the limiting safe speeds from a mechanical point of view.

The increased tendency towards voltage variations with regeneration is of no great practical importance in connection with polyphase systems, because all parts of the system can easily stand a slight voltage increase for the short interval of time during which it usually exists. As previously mentioned, the higher voltage increases the torque of the motors, and, therefore, does not impair the reliability of service. The high voltage may at times unfavorably affect the power factor and the losses in the system, but in so far as it only exists at times of light load, this is of no great consequence. It is, moreover, to be considered in this connection that with the high line voltages possible in alternating-current systems, the line regulation may be kept fairly good without undue amounts of copper in the system. The fact that regeneration with induction motor's does not decrease the wattless currents, but rather increases them, will, to a large extent, eliminate any tendency towards increased voltage variations.

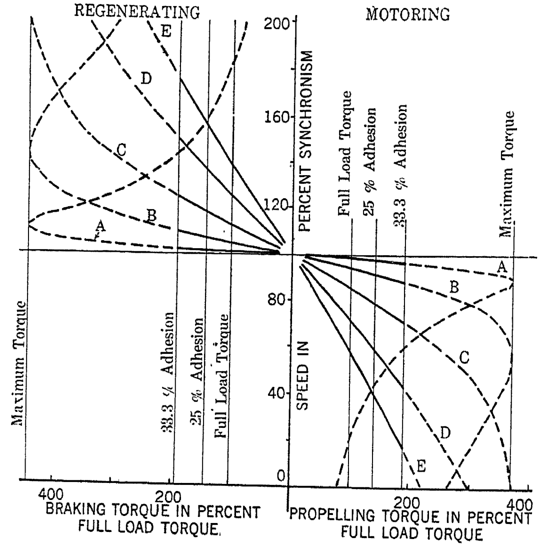


FIG. 1—RAILWAY INDUCTION MOTOR CHARACTERISTICS

Curve *A* = Rotor short-circuited.
" *B* = Rheostat resistance = 3 × rotor resistance.
" *C* = " " = 9 × " "
" *D* = " " = 19 × " "
" *E* = " " = 29 × " "

The great ease and simplicity with which regeneration can be obtained under most conditions with polyphase vehicles for the purpose of braking down-hill is undoubtedly the reason why such regenerative braking was introduced practically as soon as three-phase power was used for railway purposes. The results obtained with regenerative braking on the three-phase electrifications in Europe for the last 12 to 16 years have been exceedingly satisfactory and beneficial, all the advantages of reduced wear and tear of mechanical brakes and other parts, of economy in power consumption, and increased operating speeds on down grades, having been realized to a large extent.

The degree of safety in operating a vehicle or, especially, heavy trains down-hill with regenerating induction motors with short-circuited secondaries is almost perfect, and limited only in a few respects. The vehicle speed is kept practically constant with very wide ranges of load and an interrruption of the braking tractive effort takes place only in case the power supply is interrupted, or possibly if some of the main driving elements of the vehicle are seriously damaged. This is usually at once indicated to the engineer by the meters in front of him, and in so far as the regeneration has definitely kept the vehicle down to safe speeds previous to the loss of power, or the like, it has in practise always been found very easy to get the train or the vehicle

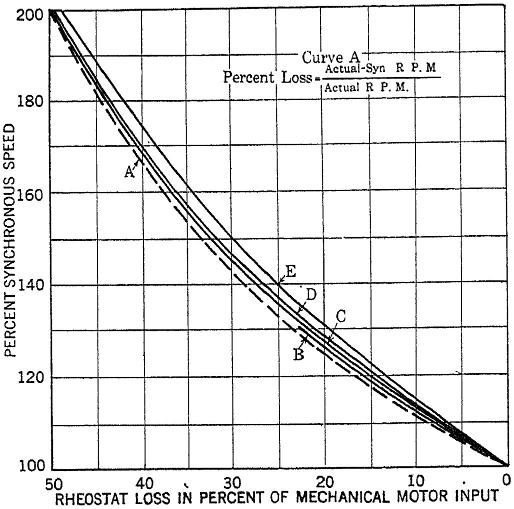


FIG. 2—RHEOSTAT LOSSES, REGENERATION

Curve B = Per cent rheostat loss at 75 % full-load torque.
" C = " " " " " 150 " " " "
" D = " " " " " 225 " " " "
" E = " " " " " 340 " " " "

under the control of the air brakes before unsafe speeds were reached. In the case of especially steep grades or other severe conditions, it is, of course, quite feasible to provide automatic means for putting on the mechanical brakes in case of emergency. The maximum braking torques which can be exerted are usually limited only by the slipping point of the wheels, since it is as a rule easily possible to keep the pull-out torque of the motors a safe amount above this point. While in motoring the pull-out torque may be reduced by excessive voltage drops, heavy regenerative braking will rather tend to bring about a rise in voltage with consequent increase in the maximum braking torque, which is, even with equal voltage, larger than the motor torque (see Fig. 1). Slipping of the wheels, where it is liable to occur, can be avoided by introducing automatic current-limiting or torque-limiting devices which prevent an increase in braking torque above the slipping point, for instance, by introducing a resistance into the motor secondary, thereby limiting the load.

There is one point worth mentioning in connection with regenerating induction motors, which is often construed to be a disadvantage, whereas it actually is not a disadvantage at all, but merely reduces the advantage which may be secured from regeneration. A system with regenerating induction motors is liable to show a very low power factor, due to the fact that induction motors while regenerating actual power take wattless current from the line, unless power factor compensating means are used; in other words, they increase the total wattless current and decrease the total watt current, which has, of course, a bad influence on the power factor. It should not be forgotten, however, in this connection, that the term "power factor" is very apt to be misleading in a case like this. Even if we assume, for instance, that the increase of wattless current as compared with the decrease of watt current is such that the total current remains unchanged, we have, of course, no saving in r. m. s. current drawn from the line, and also, a very much lower power factor. Nevertheless, we have reduced the actual power consumption, which in turn, means a saving in the coal pile of the power house. Moreover, it is a fact that the larger systems usually have synchronous converters, phase advancers, or other means for taking care of the power factor, and a slight increase of this apparatus will usually take care of the additional wattless current introduced by regeneration.

While the three-phase system (and its closely related single-phase system with phase converters on the locomotive) is undoubtedly the system best adapted for effecting the braking near synchronous speeds on down grades for the purpose of avoiding increased speeds, the possibilities for reducing the speed of a vehicle from its normal operating speed by regenerative braking are either very limited in scope with induction motors, or the equipments are, at present, at least, so cumbersome or complicated that their application in practise does not seem to be very likely in the near future.

If the vehicle is arranged so that two or more economical operating speeds may be obtained either by employing two-speed induction motors or by connecting motors in cascade, or both, it is possible to effect regenerative braking in bringing the vehicle speed from the higher speeds down to near the lowest synchronous or economical operating speeds. This can be accomplished by connecting the motors for one of the lower operating speeds and properly manipulating the resistance in the secondary. If done manually, this requires a certain amount of skill on the part of the operator, and if done automatically it means, of course, complication. It has also been observed that under certain conditions such operation is liable to cause over-voltages in the motor secondary. For these reasons, regenerative braking has not been utilized to a very great extent for this purpose.

Regenerative braking with induction motors over a wide range of speeds and down to zero speed may be accomplished by applying a so-called regulating commutating machine, working in conjunction with the secondary of the induction motor. Such a machine is, however, with the present known systems, so large, and, on account of many commutator brushes, subject to such high maintenance cost that its use would, to a large extent, counterbalance the additional saving made possible by braking at lower speeds. It is, however, not at all impossible that as the art advances, developments along this line may make regenerative braking below synchronous speed feasible in practise. Whenever this is the case, the present speed limitations of the induction motor would at the same time be eliminated, or minimized, and acceleration without rheostatic losses, with attendant saving in power, would be made possible at the same time.

PHASE CONVERTER SYSTEM

The conditions prevailing with a phase converter system employing a single-phase line, a phase converter locomotive and induction motors for propelling, are, with regard to regenerative braking, practically the same as with the three-phase system. This means that regenerative braking on down grades can be accomplished with the same ease and safety and without requiring complications of any practical importance, while the use of regeneration for retardation is at present either limited in scope or complicated.

The only slight differences between the phase converter system and the polyphase system are:

1. The system employs on the vehicle a rotary converting device, which, if of the synchronous type, requires for its protection a standard over-speed relay.

2. The use of regenerative control introduces an increased tendency towards unbalancing of the three phases of the converter system, and may require a small extension of the balancing devices—that is, possibly an extra position of the balancing control lever and one or two extra switches. The safety of operation is, however, in no way impaired by this because, even if the balancing switch is not operated correctly, the motors will perform properly and the only result will be an increase in motor heating.

3. Since the vehicle, if equipped with a synchronous phase converter, going down grade represents in itself an independent generator station, this must be taken into account when laying out the overload and short-circuit protection of the system.

In view of the simplicity of the system, it was only natural that regenerative control was utilized as soon as the phase converte system was applied in practise; in fact, the ease with which regeneration can be used in combination with a converter system was one of the factors in favor of the adoption of this system for the Norfolk and Western Railway. The practical value to the railway company of the regenerative characteristic in operating this mountain grade is far beyond expectations and has been a surprise to the railway operators. The economies accomplished in power consumption, reduced wear of tires and brake shoes, etc., are very appreciable, and not only can the trains be handled much more easily and at higher average speeds down grade than with air brakes, but also the reliability of the system as a whole has been greatly improved by the regenerative feature.

DIRECT-CURRENT SYSTEM

The present-day direct-current railway motor lends itself less readily to regeneration than the polyphase induction motor.

The series motor, which has generally been introduced in railway work on account of its superiority for propelling purposes over other types of direct-current motors, cannot as such be readily used as a generator under railway conditions. While it is true that a series motor can be made to act as a generator, if its fields are reversed, it is very unstable. In order to make a series motor regenerate, it would first have to be connected across a resistance to make it pick up and after its voltage has reached a value higher than the line voltage, it could be connected to the line and made to furnish regenerated current. Assuming, however, that the line voltage suddenly drops, as is common in railway practise, the regenerated current would, of course, increase; this, in turn, would increase the field strength, which would again tend to increase the regenerated current. This cumulative effect would quickly and frequently lead to excessive overloads on the entire equipment. Assuming, on the other hand, that the line voltage suddenly rises after the motor has started to regenerate, this will bring about a decrease in regenerated current. This, in turn, decreases the field excitation and produces consequent further decrease in current down to zero; further, since the field and counter-voltage would be zero with zero current, excessive currents in the opposite direction would build up, which are liable to exceed the most severe short circuits, because after the current once reverses, the motor sets up a field and a voltage adding to that of the line. The processes just described happen so quickly, on account of their accumulative nature, that it is practically impossible to build relays or other safeguarding devices which act rapidly enough to be effective.

In order to effect regenerative control with direct current it is, therefore, necessary either to apply altogether straight vehicle shunt or compound motors, which would mean giving up many of the essential advantages of the direct-current series motors for railway purposes, or to convert the series motor temporarily into a machine of different characteristics while regenerating.

Since the field coils of a series motor are wound with only a few heavy turns, they cannot very readily be connected as shunt coils across the line voltage, and it would be necessary, in order to obtain shunt excitation, to connect considerable resistance in series with the field. This resistance would consume a very large part of the regenerated energy, with little or no consequent saving in energy. These conditions can be somewhat improved where several motors are available by connecting all motor fields in series and the armatures in parallel, but even then, only a relatively small part of the regenerated energy remains to be sent into the line. Such a system will, therefore, only be applied in exceptional cases where power consumption is of relatively small importance and the principal purpose is to effect electrodynamic braking with the least amount of complication, in order to secure advantages other than power economy.

A system of that kind is quite safe and reliable in operation A sudden change in line voltage will tend to change the regenerated current, but such a change is counterbalanced by the fact that the excitation changes at the same time and causes a change in the counter e. m. f. which reduces the change in regenerated current. This process is fairly quick-acting because the time element of the field current is rather small on account of the large amount of resistance connected in series with inductance of the field. Excessive regenerated currents are thereby avoided automatically. Only when the motors are highly saturated an increase of field strength is, of course, not possible, and larger regenerative currents may then occur. In view of the small field of practical application for this system, a detail discussion of it may be omitted.

Since generators of the straight series, shunt or compound type cannot be readily made out of the standard up-to-date railway motor and used to good advantage, motors with separate excitation are the next and most promising possibility for successful regeneration, although the addition of a separate source of current for the excitation introduces an undesirable complication.

The various connections, methods of excitation and consequent characteristics of the generators which are possible in connection with separate excitation are very numerous. Therefore only a few typical cases can be described in this paper. Before doing this, some of the more important problems to be met in connection with the separate exciter method of regeneration may be discussed.

First. One of the principal conditions to be met in connection with direct-current generation is, as previously intimated, to avoid excessive armature currents, especially at higher speeds. While with any type of motor excessive overloads are undesirable for any length of time, it is a fact that overloads of short duration are not so very dangerous in connection with induction motors, for instance, because the mechanical parts of freight locomotives on which induction motors are being used are in most cases designed to stand stresses up to the slipping point of the wheel and the wheel slippage acts as a mechanical fuse against much higher stresses; on the other hand, overloads of very short duration are not very serious from a heating point of view. With direct-current motors running as generators, high armature currents, even though of very short duration, introduce an additional difficulty because they are liable to lead to flashing of the motors. It is a well-known fact that flashing of motors is caused by high voltages between the commutator segments. The voltage between segments, in turn, depends upon a great many details of design, but the maximum voltage is largely determined by the armature distortion. With a series motor, in which the field strength and the armature current are always proportional to each other, the armature distortion as a rule does not increase, under normal conditions of operation, with increased load, and has often even a tendency to decrease with heavy loads, on account of saturation. With a separately excited generator, conditions are altogether different. When running at very high speeds with a fixed motor voltage, for instance, the field strength must be, by necessity, very low. This is evident because the voltage induced must be about the same as the impressed voltage, and since the induced voltage is the product of speed and field strength, high speed means, necessarily, low field strength.

Since, on the other hand, the braking torque which the generator can exert is proportional to the product of armature current and field strength, it is evident that at high speeds and consequent weak field, relatively large armature currents are required in order to exert an appreciable braking torque under normal operating conditions. This will in itself tend to increase the armature field distortion materially. This latter condition is, of course, only brought about by the fact that the present-day requirements for retardation are more exacting than for motoring. During motoring it is customary to have a certain maximum rate of acceleration until the motor curve is reached and to tolerate thereafter a decreasing rate of acceleration which is naturally obtained while the motor field, and with it the armature current, decreases and while the speed increases. In certain cases, it may be quite possible to follow the reverse process during braking, that is, start in with small field current and smaller armature current and consequent slow rate of retardation, and increase all the values while the speed decreases. Insofar, however, as the mechanical brakes used at present are able to exert the maximum permissible braking torque at the higher speeds, which, in turn, permits higher schedule speeds with the same maximum speeds, regenerative braking will not be able to compete unless at least similar conditions can be obtained. Therefore, it will be necessary in most cases to work with relatively large armature current, unless provision is made to change the motor voltage during regeneration, thereby permitting the use of stronger fields at high speeds. This solution of the problem could be accomplished in two ways: first, the regenerative voltage could be raised above the line voltage by some means or other, as will be discussed later on and shown to be rather undesirable; second, arrangements could be made to keep the voltage during motoring below the line voltage and regenerate at full line voltage. This could be accomplished by designing the motors to stand full line voltage but to operate, while motoring, with two motors connected in series, with one-half the voltage in each motor. Under special conditions with special motor designs, such a solution may be quite feasible. As a rule, it would be more desirable, however, to use standard motors even though this means the use of heavy armature currents for high speeds, and with it, large field distortion. It is fortunate in this connection that this undesirable feature and its consequent danger are to a large extent eliminated by the fact that when regenerating, the distortions are such that the highest voltages between segments occur at a point a considerable distance away from the toe of the brushes, so that an arc is not as liable to be drawn over from the brush as would be the case if the highest voltages were near the toe of the brush under otherwise equal conditions. It is, therefore, permissible to work with considerably higher maximum voltages and, therefore, greater distortion when regenerating. Practical conditions are nevertheless such that, in view of the high field distortions necessary for normal operation, further increases on account of sudden rises of regenerative currents in the armature are very liable to lead to flashing.

There are two principal causes for sudden increases of regenerated current.

(a) The first of them is the taking of steps in the control. If, for instance, the field excitation should be suddenly raised by a large amount, the regenerated currents would take undue proportions. This, of course, can be avoided by making the control steps small enough by providing a sufficiently large number of them. In order to avoid the expense and complication incident to a large number of steps, it is desirable to have a regenerative machine characteristic which avoids large changes of regenerative current even though the number of steps is small.

(b) The second and much more important cause of sudden changes in regenerative current is as previously pointed out in connection with the discussion of the series generator, sudden changes in the line voltage. While it is not altogether impossible to prevent sudden increases of load due to this cause by having quick acting relays, causing changes in the excitation of the generators to take care of the changed conditions, it is at least very difficult with some systems to get such relay devices and switching operations quick and reliable enough to avoid flashing. For this reason, it is very desirable, and for a reliable system even necessary, to have such regenerative characteristics as inherently to avoid excessive armature currents. In this connection it is not only necessary to have such an inherent characteristic, but it is also important that the time element for effecting changes in the generator conditions be exceedingly small.

Second. Another feature to be considered in connection with regenerative control is the ease with which the motor can be connected to the line after it has been disconnected and when regenerative control is desirable. In order to make such connections with some types of separately excited motors, it is necessary to have the regenerative voltage about equal to the line voltage before such connection can be made. As a matter of course, it is impossible to follow, in operating a vehicle, the power house practise of balancing the voltages by means of a voltmeter before connecting the generators to the line. Therefore, automatic means have to be provided for this purpose. Such automatic means, in turn, require relays which have to be very exact unless the system used is inherently such as not to require a very careful balancing of the voltages before the connections are made. This point is quite important in the choice of the proper system.

Third. Another point of importance in connection with regeneration is that the regenerative load be properly distributed among the various motors of the vehicle. It is evident, for instance, that a slight difference in the air gap of the motors or a difference in the wheel diameters will cause a very uneven distribution of load if the armatures of several motors were to be simply connected in parallel. A good system of regeneration, therefore, has to have means of balancing the load fairly well between the various motors under practical conditions. While such balancing may of course be effected by relays which operate regulating switching devices, it is again preferable to have a system in which the load balancing effect is, at least to some extent, inherently given by the generator characteristics.

Fourth. Another condition to be fulfilled is that the braking effect of the vehicle shall not materially decrease with increased vehicle speed, or materially increase with decreased speed, at least not so suddenly that the relays of the automatic control, or the operator in case of manual control, cannot follow such changes quickly enough. Sudden large changes of braking torque should also be avoided in case of changes in the line voltage conditions.

Let us assume, for instance, that the speed of a train is to be reduced by regeneration, and a certain maximum permissible braking torque has been established. If it happens in this case that the braking torque suddenly increases, either on account of changed line conditions or the generator having a characteristic so that the torque increases while the speed decreases, this will lead either to slippage of the wheels or to an undue rate of retardation resulting in bumping of cars, inconvenience to passengers, etc., all of which should be avoided.

Another case to be considered in this connection is that of a heavy train going down-hill. Let us assume that such a train gets on a steeper grade and consequently speeds up to some extent. If in this case the generator characteristics should be such as to give a decreased torque with increased speed, the train speed may increase unduly on account of the combined effect of the increased grade and the decreased torque before the engineer has a chance to readjust the control or the air brakes, or both.

Unfortunately, the condition that the braking torque should preferably decrease with decreased speed, and vice versa, cannot be combined with a machine characteristic best suited to avoid armature distortion and motor flashing.

The ideal characteristic for avoiding motor flashing is one with which the ratio of armature ampere-turns to the field ampere-turns remains constant with changes in speed. Such a characteristic can be approached in practise, as will be shown later on, and is represented by curve *A* in Fig. 3A. It will be seen that this curve is shown as a straight vertical line down to a certain point.

The armature current curve corresponding to curve *A* is shown by curve *B*. It will be seen that the upper part of this curve shows a rapidly decreasing armature current with increased speed. This is due to the fact that, as previously pointed out, the field strength has to decrease with increased speed, which in turn means that the field ampere-turns have to be decreased. With our present assumption, this means that the armature current must also decrease with increased speed.

Curve *C* shows the torque obtained under the same conditions. Since the torque is the product of field strength and armature current, both of which decrease with the speed in the present case, it is evident that this curve must decrease very rapidly with increased speed, except on the part of it which is turned around by the effect of saturation. Fig. 3A demonstrates very plainly that the characteristic which is ideal for flashing conditions is highly undesirable, for the reasons previously pointed out, in connection with the most desirable torque characteristics.

The next best possible characteristic for flashing which might be considered is one with which the armature current remains practically constant over a wide range of speed, as shown by curve *B* in Fig. 3B. Curve *A* in the same figure shows the corresponding curve for the ratio of armature turns to field ampere-turns.

Curve *C* shows the corresponding torque curve, which still shows a considerable tendency for decreased torque with increased speed. It is evident that such tendency must exist since, as we know, the field strength must decrease with increased speed, which, with constant armature current, means also decreased torque.

The extreme opposite to the characteristic assumed in Fig. 3A is shown by Fig. 3C. The torque curve *C* shows a very rapid increase of torque with increased speed. Such a torque curve affords maximum safety for the vehicle when going down-hill, because any tendency towards increased speed will be counteracted by a rapidly increasing braking torque. For retardation of the vehicle, such a curve would, of course, require a very large number of small steps in the control in order to avoid excessive variations in the braking torque when going from one step to another.

Curve *B* shows the corresponding armature current curve and curve *A* the corresponding curve for the ratio of ampere-turns. It will be seen that the latter curve shows an excessive increase with increased speed and is very undesirable with regard to flashing.

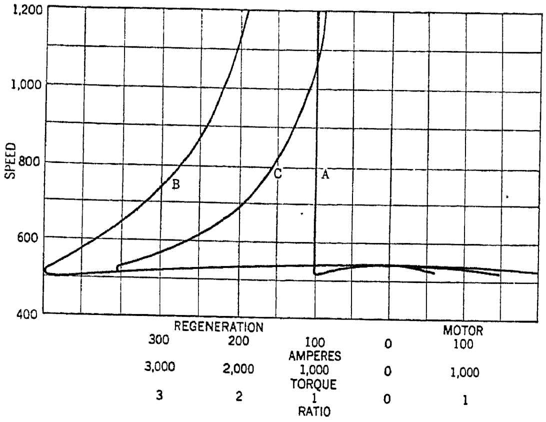


FIG. 3A

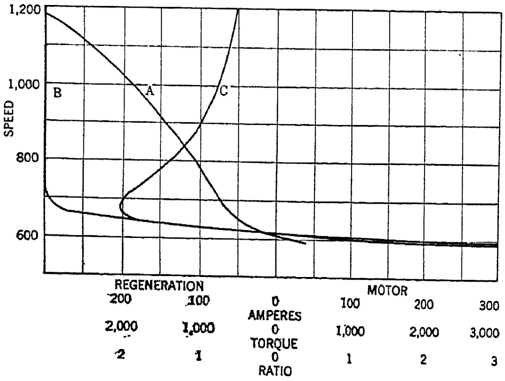


FIG. 3B

A = Ratio of Armature to Field Current
B = Armature Current
C = Torque

The natural conclusion to be drawn from the previous figures is that in practise a compromise between these extremes is the only workable solution. Fig. 3D shows characteristics for the ratio of ampere-turns, current and torque which will be suitable for the average conditions in practise. The steepness of the current curve will, of course, vary, depending upon the service conditions, the inherent sensitiveness of the motor to flashing and a number of other considerations. As a rule it will be desirable to have the curve somewhat steeper for the higher speed notches than for the lower speed notches. It has been pointed out before, that in order to obtain an appreciable torque with the weak field necessary for the higher speeds, large armature currents and high ratios of armature to field ampere-turns are necessary; therefore, it is in this case especially desirable to have a steep current curve, avoiding further increase of armature current. With low speeds, on the other hand, we have, by necessity, strong fields and weak armature currents for similar

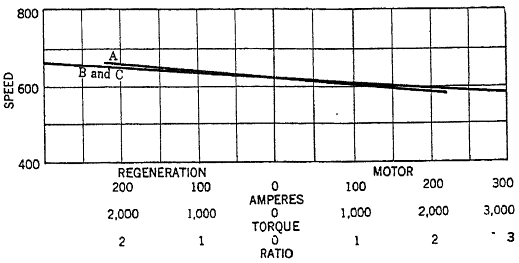


FIG. 3c

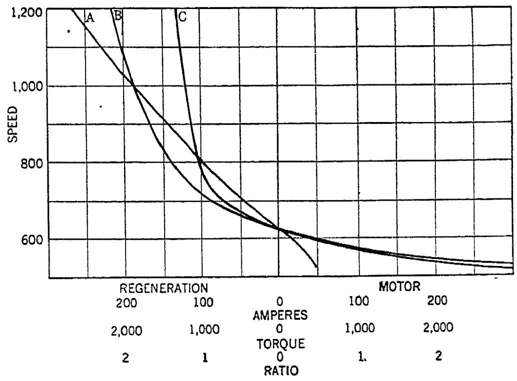


FIG. 3D

A = Ratio of Armature to Field. Current
B = Armature Current
C = Torque

torques, and, therefore, a favorable ratio of armature to field ampere-turns. A certain increase of armature current is, therefore, permissible and a rather flat current curve may be chosen.

While the curves of Fig. 3 show the characteristics with regard to the speed, it will be found that the same conclusions will be reached if current changes caused by fluctuations in line voltage are taken into consideration. The requirements for a satisfactory speed-torque curve will also in these cases limit the use of the most desirable characteristic for avoiding overloads and flashing.

A characteristic showing gradually decreasing torque with decreasing speed should also prove to be rather advantageous where it is necessary in the case of exceptionally heavy trains to operate air brakes in conjunction with the regenerative braking. If, for instance, the air brakes are put on rather heavily, the train speed will decrease because the air brakes are exerting a heavy retarding force. If the motor characteristic is such as to give a decreased torque at the same time, the total increase of torque is counteracted and further decrease in speed will be prevented. The opposite action will take place and it is even more important when the air brakes release a certain amount. If the vehicle speeds up in this case, the inherently increasing regenerative torque will prevent further increases of speed. It is evident, therefore, that a compound characteristic, as previously discussed in the case of the main generators, will permit operation at a more uniform speed, even when using regenerative braking in conjunction with the air brakes, than would be possible with air brakes alone.

Fifth. Still another point to be considered in connection with the various systems is the danger of over-voltages on the motors in case the power supply should be interrupted for some reason or other. The natural tendency with most systems fulfilling the previous requirements is, in case of power interruption, for the voltage of the generators, being no longer limited by the line voltage, to rise until saturation is reached.

Since railway motors when operating at high speeds as generators are always far from being saturated, it is quite possible to obtain voltage rises of two or three times normal voltage. It is evident that such over-voltages are likely to lead to flashing and even insulation breakdowns. While these over-voltages can, of course, be taken care of by relays causing an interruption of circuits or the like, the inherent characteristics of the system should be such as to introduce a time element sufficient to allow over-voltage relays to act before the over-voltage becomes excessive and dangerous.

Unfortunately, it is also a fact that those systems which are the quickest to act in avoiding excessive armature currents are usually the ones which are most liable to set up over-voltages quickly in case of power interruption when running at high speeds. It is, therefore, again necessary to strike the best compromise.

Sixth. It is finally necessary in connection with regenerative control to pay much attention to the proper characteristics of the auxiliary exciting apparatus, in order to have the system free from troubles. The principal danger in this connection is that the auxiliary exciter set, especially the driving part, is subject to flash-overs unless proper precautions are taken in that direction.

With these considerations in mind, we may now discuss the principal possible characteristics of the source used for the separate excitation and the consequent characteristics of the main generators. When referring in this connection to the characteristic of the source of excitation, etc., the inherent characteristic is meant, it being assumed in all cases that variations of excitation can be, and are, accomplished by changes in the circuit connections, changing of resistances, etc. The following cases for the main field exciting voltage may be considered:

1. The excitation is of practically constant voltage.
2. The excitation voltage varies with the line voltage, either in direct proportion or in some other ratio.
3. The excitation voltage decreases with an increase of regenerative load.
4. The excitation voltage changes with both the line voltage and the regenerative load. A number of different combinations are, of course, possible.

The undesirability of an increase of field excitation with increased regenerative current has been pointed out in connection with the series generator. An inherent increase of excitation voltage with decreased line voltage is undesirable, because a decreasing line voltage inherently means increased regenerative load and this load would of course be further increased, were the excitation raised at the same time.

Symbols Used in Connection Diagrams

A, A_1, A_2	Main machine armature or armatures.
B	Booster armature.
C	Control contacts.
D	Relay.
E	Source of field exciting current.
F, F_1, F_2	Main machine field or fields.
G	Ground connection.
H	Field winding of exciter armature.
I	Inductance.
J	Differential field winding on exciter machine.
K	Differential field winding on motor driving exciter machine.
L	Field winding of motor driving exciter machine.
M	Armature of motor driving exciter machine.
N	Armature of machine driving booster or driven by the booster.
O	Shunt field of booster machine.
P	Shunt field of machine driving booster or driven by the booster.
Q	Series field of machine driving booster or driven by the booster.
R, R_1, R_2, R_3	Resistances.
S	Series field of booster.
S_1, S_2	Switches.
T	Trolley.
T_1	Main transformer winding.
T_2	Auxiliary transformer winding.
U	Auxiliary shunt field on exciter machine
V	Auxiliary shunt field on booster.
X	Cross field of main machine.
Y	Cross field of exciter machine.
Z	Reverser.
aa	Working brushes of main machine.
bb	Exciter brushes of main machine.

Practically constant excitation, as obtained, for instance, with a battery (see Fig. 4), does not give very desirable characteristics with regard to limiting the armature currents. If, for instance, the line voltage drops suddenly, the regenerative volttage will be much in excess of the line voltage and the only thing inherently counteracting excessive currents is the small resistance in the circuit and possibly a slight voltage reduction in the generator due to the increased armature field distortion. This, however, is not at all sufficient with the large voltage variations common to railway work. Fig. 6 shows test curves, taken on a standard medium size railway motor, clearly demonstrating this point. It will be seen that with a voltage drop of only about 10 per cent, the regenerative current increases

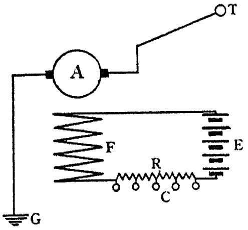


Fig. 4

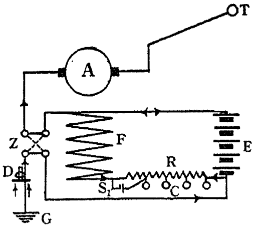


Fig. 5

100 per cent or more, depending upon the load, and the ratio of armature to field current goes up practically in the same ratio. Conditions can of course be improved by introducing a resistance into the circuit.

The only other way in which this shortcoming of the system with constant voltage excitation can be taken care of is by the use of quick-acting controlling devices. The system shown in

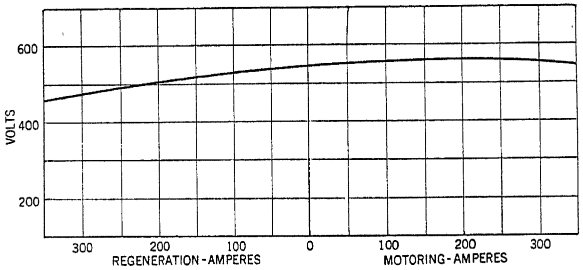


Fig. 6—Speed Constant; Field Constant

Fig. 5 was developed along such lines some years ago and gave quite satisfactory results during an extended testing period. The motor fields were reversed in this case so that the regenerative current assisted in magnetizing the field, while the battery *E* took only the difference between the regenerative current and the exciting current. The regulation of the system was accomplished by a controller device *C*, varying part of the resistance *R*. In addition to this, a quick-acting relay *D* opened the switch S_1 automatically every time the armature current exceeded a certain value, introducing temporarily a part of the resistance into the field circuit. The operation of this relay and switch was found to be somewhat like that of a Tirrill regulator insofar as it very frequently opened and closed the switch S_1, the time of opening usually being only long enough to give the field current a start in the right direction when the armature current exceeded desirable values.

The flat speed characteristic of a system of this kind requires, of course, a relatively large number of notches in the control in order to avoid excessive changes of current and torque while notching up, as is evident from Fig. 7. The necessity for many steps is, however, somewhat reduced by the self-inductive and damping effect of the main motor field, which introduces a time element making the change of the field strength fairly slow, so that the vehicle has a chance to retard somewhat before the new field condition caused by taking a notch in the control has been fully established. This will limit somewhat the current peaks. With the system of Fig. 5, the operation of the relay *D* and the switch S_1 will also oppose any excessive current peaks during the notching up.

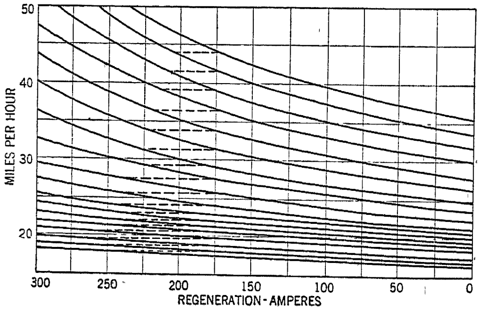


Fig. 7—Constant Volts

The systems with inherently constant excitation require, of course, fairly close adjustment of voltages before the generator can be connected to the line. Certain limited inaccuracies can be taken care of by the system of Fig. 5 if arrangements are made so that the generator voltage is always larger than the line voltage before the circuit is closed. If then the generator voltage is somewhat too large, tending to give excessive regenerated currents, the operation of the relay *D* and the switch S_1 will tend to keep the currents down.

Proper distribution of load between the various motors can, of course, only be taken care of in a system of this kind by individually controlling each motor by means of separate controlling resistances and controlling devices, there being no inherent tendency for balancing the loads.

A system with constant excitation possesses, on the other hand, quite desirable characteristics with regard to some of the other considerations. Any tendency towards increased speed of the vehicle is at once counteracted by a very large increase of torque. Over-voltages can be obtained only after the vehicle speed has increased, and since this requires a certain amount of time, over-voltage relays will have plenty of time to act; as a matter of fact, this system is one of the safest with regard to over-voltages. The use of a battery as a source of excitation is, although fairly satisfactory, not very desirable, in view of the care and maintenance always incident to a battery.

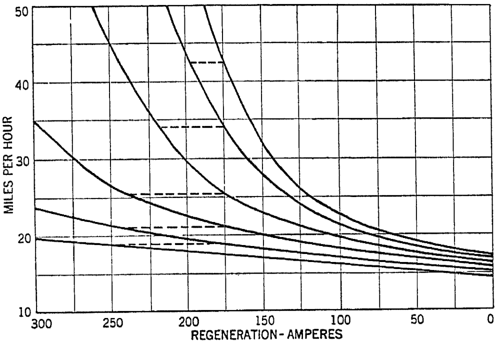


FIG. 8—CONSTANT VOLTS

A system in which the excitation voltage varies in proportion to the line voltage is shown in Fig. 9. A dynamotor merely serves as a transformer for reducing the line voltage to a voltage suitable for the excitation of the motor fields. Therefore this system acts in many respects identically with one having a straight shunt generator. Any increase of line voltage is followed by a proportional increase of excitation voltage. Such an action is ideal, assuming that the change in field follows that of the line voltage quickly enough, and assuming further that the changes in field flux are proportional to the changes in field voltage.

These latter conditions are fairly well taken care of in the system of Fig. 9, as long as the main motors are not saturated, and assuming that the dynamotor field flux responds quickly to the changes of its field current. The latter requirement can easily be fulfilled by exciting the dynamotor with a series field only, as shown, avoiding thereby the damping effect of shunt-connected windings, and by making the dynamotor field core laminated, thereby avoiding the damping effects of a solid core.

If the line voltage, under these assumptions, drops suddenly, the counter e.m.f. in the dynamotor will at first be larger than the line voltage. Therefore, the current taken by the dynamotor, and with it the dynamotor field current, will quickly decrease. Since the laminated field will follow the change of its field current, the voltage induced in the secondary armature winding of the dynamotor will quickly readjust itself to be proportional to the line voltage. Although the inductive and damping effects of the main generator fields introduce a certain time element, very excessive increases of regenerative currents will be inherently avoided when running at higher speeds. For lower speeds for which the main generators may be saturated, their fields of course change less than proportionally to a change of field voltage and current. In these cases the field is, however, very strong and undue field distortion will not take place, even though the armature currents may assume rather large proportions, until the control can be readjusted by relays or otherwise. The effect of changing line voltage with this system is shown by the curves in Fig. 10.

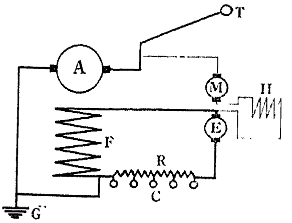


Fig. 9

While, for the above reasons, this system is very satisfactory for taking care of variations in line voltage, it is less so with regard to the steps of the control, the speed-current and speed-torque characteristics being essentially those of a shunt machine, so that in this respect the same conditions apply as in the system of Fig. 5 and as demonstrated in Fig. 7.

Another handicap of this system is that when the motors are connected to the line, excessive currents are liable to be set up unless the voltage balances rather exactly or unless other precautionary steps, like a temporary introduction of a resistance into the circuit, are taken.

Proper distribution of the load is not obtained inherently with this system and has to be taken care of by separate regulating rheostats or the like for each generator. The system is, on the other hand, very satisfactory with regard to the safety of the train, because any tendency towards change in speed materially increases the torque counteracting such change.

Over-voltages are set up rather quickly with this system on account of the fact that the voltage changes of the dynamotor are very quick-acting. The operation of the dynamotor itself is rather safe in this case, since it is of the pure series type, which makes it not very sensitive to flashing.

The system just described is, as previously mentioned, somewhat subject to instantaneous current changes, when the line voltage changes. While, as pointed out, the secondary dynamotor voltage follows the line voltage very quickly, the self-inductive and the damping effect of the main motor fields will prevent the main motor field flux from quickly following the field voltage. To improve this condition it is desirable to change temporarily the secondary voltage of the dynamotor, and with it the field voltage, a greater amount than required for the change in flux, in order to effect the quickest change of the field of the main generator. This can be accomplished by connection as shown in Fig. 11. In this case the exciter armature E of a motor-generator set is excited partly, or entirely, by the motor current of the set, while the motor itself is shunt-excited. Assuming in this case a sudden voltage drop, the damping effect of the shunt field of the small motor will prevent any sudden change of the field strength, while on the other hand the mechanical inertia of the set prevents any immediate change of the speed.

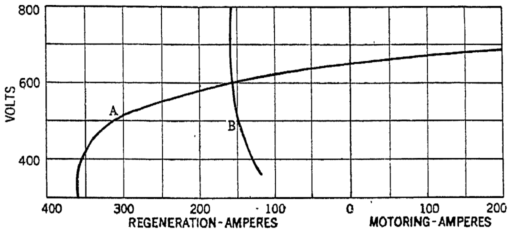


FIG. 10—SPEED CONSTANT, FIELD AMPERES VARYING WITH LINE VOLTAGE

Curve A = Field saturated.
" B = Field unsaturated.

Therefore, the counter e.m.f. of the small motor armature *M* remains unchanged and will be larger than the suddenly reduced line voltage. This will tend to diminish quickly or even reverse the current in the armature *M* and therefore also the current in the field *H* of the little generator. This in turn brings about a material reduction in the voltage of the little generator armature *E*, tending to reduce temporarily the main generator field appreciably. Before this current has, however, changed to the full extent and past desirable limits, the voltage drop of the little generator, the speed of the little set and its field will adjust themselves to the proper condition.

This system is, therefore, very well adapted to take care of even the quickest changes in line voltage. In other respects, the system is very similar to the one previously described, except that the motor of the motor-generator set, being shunt-excited, is subject to flash-overs, so that neutralizing windings or other special features may be required.

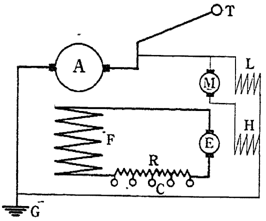


FIG. 11

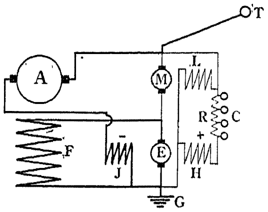


FIG. 12

Since none of the previous systems is satisfactory in all respects, a system with negative compound characteristic during the regeneration period, as shown in Fig. 12, may now be considered. This system also includes a small motor-generator set, the generator *E* of which is, however, provided with a field winding *J* carrying the regenerative current and opposing the other field winding *H*. This general principle of providing the exciter with a field carrying the regenerative current in a demagnetizing direction is applied to the locomotives of the Chicago, Milwaukee and St. Paul Railway. If, with the system of Fig. 12, the regenerative current tends to increase, no matter whether on account of voltage drop on the line, or on account of changes in the control, the increase will cause a decrease in the voltage of the little generator, causing a consequent decrease of the main generator field which counteracts any further increase of regenerative current. The same feature will also be of assistance when connecting the generator to the line; even if it happens in this case that the generator voltage is quite an appreciable amount above the line voltage, excessive currents cannot exist any length of time because, as they tend to set up, they will decrease the main field excitation and reduce the regenerated voltage. The system may be provided with but few steps in the control, because it has a negative compound characteristic with curves bent up in such a way that in going from one step to the other in the control the increase of regenerated current is not very large, even though there are only a few control steps and a few curves, as demonstrated by Fig. 8. It will be seen from Fig. 12 that this system also changes the excitation with the line voltage, as was the case in Fig. 9, and this feature will of course assist in keeping overloads down in case of a change in line voltage, along lines previously discussed.

This system, while counteracting undue increases of regenerative current, is handicapped in so far as there is a certain time element between the increase of the regenerated current, the subsequent decrease of the small generator field and the decrease in field current following thereafter.

This in turn means that short-time overload surges of the armature current are possible until the main field conditions have adjusted themselves, and unless the design of the main motor is such as to be inherently very safe against flashing, flashes are likely to occur. When applied to several motors, the system is further handicapped by the fact that it is either necessary to supply a separate exciter for each motor, or, in order to distribute the load properly between the different motors with a common exciter, special load balancing devices have to be used.

It is evident that with this system any desired speed-torque curve characteristic that will best suit the particular service conditions may be obtained.

While the system is liable to give over-voltages, due to the fact that, upon interruption of power supply, the regenerated current in the field *J* will disappear, permitting the exciting voltage to increase, the previously mentioned time element will permit over-voltage relays to disconnect the motor circuits before the over-voltages are excessive.

The auxiliary machines of the system shown in Fig. 12, especially the motor, being of the shunt type, are not very

desirable, but could, of course be improved by substitu
highly saturated series motor or bymaking other suitable ch

Some of the disadvantages of the previous system m
overcome by a system as shown in Fig. 13. In this cas
possible to use a common exciter to good advantage be
each motor is self-compounding individually, and in cas
of the motors should take more than its share of the lo
will automatically decrease its own field strength, whic
turn, counteracts any further increase of load. In order to o
this effect the upper field portions which carry the regene
armature current are arranged to counteract the lower po
of the field, which are separately excited.

The regulation of the system is accomplished by cont
contacts *C*, eliminating more or less of the exciter series
turns. The operation of the auxiliary machines, which a

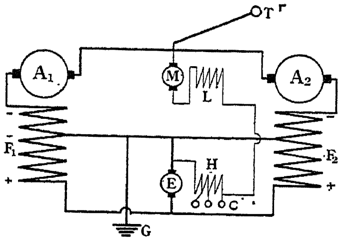


FIG. 13

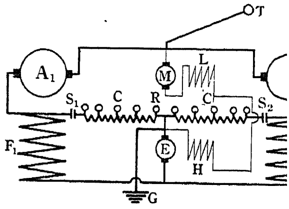


FIG. 14

the series type, should be satisfactory. On the other h
over-voltages are liable to be established very quickly on acc
of the small time element previously mentioned.

One disadvantage of this system is that it requires a sp
motor with a two-part field winding. In some exceptional c
it might happen that a standard field control motor coul
used for this system, but as a rule the ratio of turns betv
the two parts of a standard field control motor is not well ada
for this system. Even with a special design for the motor fi
it is somewhat difficult to obtain the proper compound chara
istic for all notches, unless several main field taps and atten
undesirable complications are introduced. Furthermore,
main motor fields, if of standard size, are liable to overh
since the resultant field is the difference of rather heavy amp
turns in the two opposing field sections.

The system as shown in Fig. 14 fulfills practically all the quirements previously enumerated. In this system, arrangements are made so that the sum of the regenerated current and the field excitation current of each main generator passes through a common resistance. The circuits are further arranged so that the voltage of this resistance subtracts from the exciter generator voltage to give the resultant voltage of the main generator fields. If we assume now that the line voltage suddenly drops, causing an increase in regenerated current, or that the latter increases for any other reason—as, for instance, changes in the control connections—it is evident that the current in the resistance also increases. This being the case, the voltage of the resistance increases also, and since the voltage of the small exciter is either constant or beginning to drop with the line voltage, it is evident that the main field voltage, being the difference between the exciter and resistance voltage, must suddenly decrease and cause the field current, as well as the field, to decrease. We have, therefore, a very quick readjustment and the desirable negative compound characteristic. It may be stated that in general a system giving each machine inherently and individually a negative compound characteristic without too much of a time element is to be preferred over any of the other possibilities mentioned, because such a machine counteracts any undue increases of current no matter whether their cause is decreased line voltage, large step in control, faulty action of control or for any other reason. The desired compound effect and speed-torque characteristic can be varied to any desirable amount in this system by merely varying the resistances to suit conditions. The loss in the resistance is in any case small, since the voltage across the resistance is hardly ever more than the field voltage, that is, only a small percentage of the total voltage.

This system is particularly advantageous with regard to connecting the generators to the line. During some experimental tests, the generators were put on the line with the no-load voltage regulated to 100 per cent larger than the line voltage and the maximum currents obtained were well within permissible limits. The adjustment of relays, therefore, does not require any great degree of refinement.

Since each motor has, as in the previous system, a separate compound effect, the system will tend to distribute the load evenly between the motors and is very satisfactory in this respect.

Over-voltages are taken care of by an over-voltage relay. In case of a loss of power supply, the regenerated voltage will, of course, rise a certain amount, because the regenerated current in the resistance becomes zero, thereby decreasing the voltage across the resistance and increasing the voltage across the main fields. A rapid cumulative voltage rise is, however, prevented because the inertia of the set prevents an instantaneous rise in speed of the motor-generator set and because a damping effect of any desirable amount may be introduced by damping windings on the field of the small generator that will prevent sudden rises of field strength, and therefore, a rise of generator voltage above the initial rise. Therefore, there is sufficient time for an over-voltage relay and the switches interrupting the exciter circuit to operate before the over-voltages have reached dangerous proportions.

The operation of the motor-generator set is safe and flashing avoided on account of the driving motor being of the pure series type. Over-speeding of the set, when running light, is avoided by driving the blower for the main motors by the shaft of the motor-generator set. The fact that the exciter set furnishes an easy means for driving a ventilating fan introduces a convenient possibility for adding forced ventilation to existing equipments now operating without forced ventilation. The addition of the motor-generator set with a fan furnishes in such a case not only the necessary excitation, but also the additional ventilation for the main motors which usually is necessary to counteract the increased losses in the motors incident to the introduction of regeneration.

Where service conditions make it desirable, the connections shown in Fig. 14 may also be used without any alteration during motoring, and give in this case a very convenient method of field control. The only difference is that in this case, the resistances carry the difference between the field exciting current and the motor load current. An increase in motor current will therefore, mean decreased current in the resistance, a decreased drop across the resistance, and increased field strength. In other words, we have a positive compound characteristic as desirable for motor operation. The system may also be arranged to use this compound characteristic up to a certain point and subsequently simply interrupt the resistance circuits by the switches S_1 and S_2, after which the motors will work as straight series motors, with the exciter armature E merely acting as a small booster adding a few volts to the line voltage.

The advantage of using one and the same connection for both motoring and regeneration is that it gives exceedingly simple control, permitting at the same time field control during motoring. On the other hand, it has the disadvantage of requiring a somewhat larger exciter, since the latter is in operation a much greater part of the time. It may also, at present, at least, be advisable to select systems which operate the motors altogether independently of the features introduced by regeneration, so that the failure of any of the regenerative parts does not in any way affect the reliability of the vehicle when motoring, and simply means that the air brakes have to be used temporarily. For this reason the system of Fig. 14 has so far been applied only in combination with the standard motor connections.

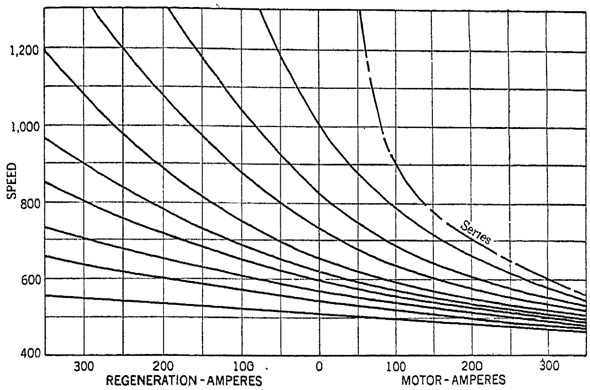


FIG. 15

The system of Fig. 14 may, of course, be modified in a number of ways. The resistance *R* may, for instance, be replaced by a constant resistance, and the regulation may be accomplished by a resistance in series with the armature *E* (see Fig. 17). This will change the regenerative characteristic for the higher speeds as shown in Fig. 16, as compared with Fig. 15, which applies for the system of Fig. 14. Regulation may also be accomplished with a constant resistance *R* and means for regulating the strength of the field *L* (see Fig. 18).

The system of Fig. 14 could, of course, be further improved by introducing some means for temporarily forcing the change in the main generator fields in order to shorten still more the time element introduced by the self-induction damping effect of the main generator fields; while this time element is rather small in the system shown in Fig. 14, and is of no practical importance in most cases, it may in case of very sensitive motors be desirable to reduce it further. The system of Fig. 17 shows one possible way of doing this. The resistances *R* fulfill the same purpose as the resistance *R* in Fig. 14. The two resistances are so dimensioned in Fig. 17, however, that they in themselves would give a very steep compound curve, much steeper than desirable for the operation of the vehicle. This would mean that in case of any change in line voltage or other cause for increased regenerated currents, the resistances will bring about a material decrease of the main generator voltage, and, therefore, a quick decrease of the main generator fields. The materially reduced field voltage will not, however, continue to exist because the field *K* on the motor of the exciter set carries the regenerated current and opposes the field *L*. An increase of regenerated current will, therefore, reduce the total field and cause the set to speed up, thereby increasing the voltage of the exciter armature *E*. This increase counteracts the decrease in field voltage brought about by the resistances *R*. On account of the inertia of the little set, however, it cannot speed up instantaneously, and therefore, there is a certain time element introduced in the counteracting effect. The combined effect of the resistances *R* and the field *K* may be arranged to give a fairly flat current curve and yet sudden increases of regenerative current are prevented because of the

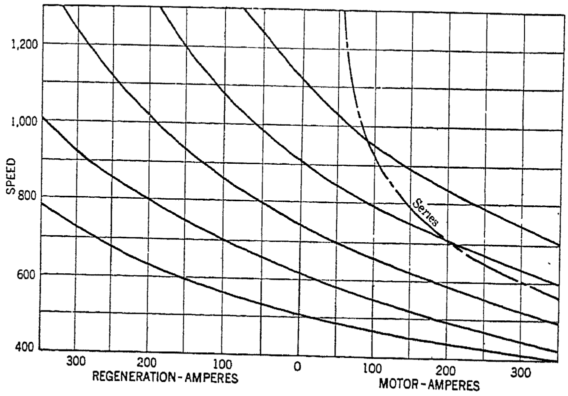


FIG. 16

smaller time element in the effect of the resistance R and the large time element in the effect of the field K.

A rather interesting case is shown in Fig. 18. The driving motor of the exciter set is shunt-excited and, therefore, essentially constant speed. The exciter armature is series-excited and its voltage will, therefore, increase materially with increased load, and vice versa. If we assume now, for instance, a sudden increase of regenerated current, the field voltage of the main generators will be decreased on account of the increased voltage drop across the resistances R_1 and R_2. This will mean that the current in the field H also decreases and with it the voltage of the armature E, which in turn tends further to decrease the current of the main field. It can be shown mathematically that it is possible, in this case, to obtain a characteristic with constant ratio of main field current to armature current, as shown in Fig. 3A. While, as pointed out in connection with Fig. 3A, such a curve is as a rule not desirable, there may be cases of motors exceptionally sensitive with regard to flashing where such a characteristic would make regeneration possible.

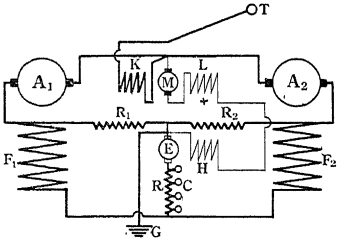


FIG. 17

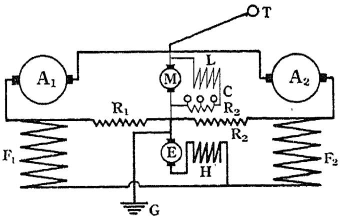


FIG. 18

All systems as shown so far are, of course, limited to a certain range of speed over which regeneration can be accomplished. The maximum possible speeds are usually only limited by the maximum operating speed, of the vehicle but the minimum speeds are given by the maximum possible saturation of the motors. It is evident that regeneration cannot be effected unless the generated voltage is slightly higher than the line voltage. If, therefore, the speed is such that even with fully saturated main fields, a voltage above the line voltage cannot be induced, regeneration will be impossible. Since most standard railway motors are fairly well saturated at the hour rating, it is, therefore, usually not possible to effect regeneration for speeds much below the standard hour rating speed.

The range of regeneration below this speed can only be extended by reducing the voltage across each motor. This can be accomplished in the same manner as during acceleration, by series-parallel control, or still further, by parallel, series-parallel and series control.

The fundamental principles illustrated in Fig. 14 have been very successfully applied to series-parallel control during regeneration, whereby the range for regeneration has been extended from the maximum speed down to about 35 to 40 per cent of the hour rating speed. This range is fully sufficient to cover all desirable operating speeds on down-grade in most known applications. In braking, for the purpose of reducing the vehicle speed, this range permits the regeneration of as much as 85 to 95 per cent of the stored energy which can be regained by regeneration, leaving only a very small percentage to be handled by the air brakes.

The control of the series-parallel system referred to is arranged so that parallel connections are obtained automatically for all speeds for which regeneration in parallel connection is possible. For speeds below this, permitting regeneration in series connections, series connections are at once established automatically. Further arrangements are made for very low speeds which do not permit regeneration at all so that the regenerating circuits are not established. When regeneration is started in parallel connection, and the vehicle speed falls below speeds satisfactory for parallel connection, the transition to series connection is accomplished without loss of braking torque during the transition. The scope of this paper does not permit a detailed description of all the control features. It may merely be stated that the system has been developed and successfully operated with multiple-unit cars and automatic control for the regenerative braking, together with automatic control for acceleration.

The system also has been worked out for manual control with automatic maximum torque arrangements, in connection with locomotives, and one locomotive has been in successful operation on the Lake Erie and Northern Railway for the last seven months without any difficulties of main motor or auxiliary motor flashing or control failures of any kind, although the operating voltage is 1500 volts, with an occasional maximum of 2000 volts obtained during regeneration, the line being 15 miles (24 km.) long, with only one substation in the center during the first four months of operation.

The master controller handle is arranged so that acceleration is effected by moving the handle from a certain middle position in one direction, and so that braking is effected by simply moving the same handle in the opposite direction from the same middle position. The motor connections are arranged so that they are independent of the regenerative features of the control. The manipulation of the locomotive by this single master controller is, of course, exceedingly simple and naturally very much to the liking of the operator. Every regenerative braking position is a running position, so that electric braking on down-grades is possible in fine gradations over a wide range of speeds.

In order to establish the reliability of the system, tests were made in which the line voltage was suddenly changed from 900 volts to 1800 volts, and vice versa. Other tests were made by interrupting and re-establishing the power supply for various periods of interruption, and the inherent characteristic of the system, together with the control features, handles all these cases with ease.

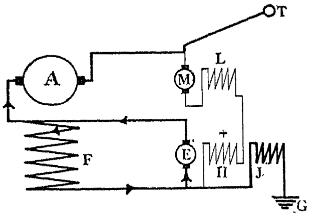


FIG. 19

Outside of the essential features and requirements to be met by an ideal system of control, a great many minor features have to be considered. Among other things, it is of course desirable to keep the exciter as small as possible in order to keep first cost and weight down to a minimum. In this connecttion, three different principles of excitation may be considered, namely:

1. The exciter carrying the sum of the exciting current and the regenerated current, as shown in Fig. 19. This gives, of course, a very large exciter with large commutators and numerous brushes. The principal advantage of this system, which has been applied on the locomotives of the Chicago, Milwaukee and St. Paul Railroad, seems to be that it avoids the necessity of connecting the motors to the line as generators. The arrangements in the above case are such that the motors are connected to the line in the regular way, first as motors, and the exciter is subsequently connected across the field and raises its excitation until the motor voltage overcomes the line voltage and regeneration takes place. This requires the manipulation of two separate handles and does not permit series-parallel control without interrupting the braking torque.

2. The exciter set carrying the field current only. This gives an exciter of smaller current capacity. This principle is used in the systems of Figs. 4, 9, 11, 12, 13, 14, 17 and 18.

3. The exciter set carrying the difference between the field current and the regenerative current. This gives, of course, a further opportunity for reducing the average current in the exciter and its r.m.s. capacity. It so happens, however, that with the direct-current systems previously discussed, especially for higher speeds, the maximum regenerative current is usually so large and the field current so small that the maximum currents to be carried by the exciter are about as large as in the previous case. For this reason the saving in exciter size is not very great. This system has other disadvantages—among others, the fact that the exciter works at times as a motor and at times as a generator, which in turn, makes it impossible to use a series connected machine for driving it. This, as previously pointed out, is liable to lead to flashing difficulties in the driving machine. In combination with battery excitation, as shown in Fig. 5, on the other hand, this principle is very desirable, since by proper design it is possible to charge and discharge the battery alternately and keep it charged the proper amount.

Another point of practical importance in consideration of the various systems is not only the number of steps required for control, but also the size of the current which has to be handled by the controllers. It will be seen, for instance, that in the systems shown in Figs. 4, 5, 9 and 11, the entire main field current, which is relatively large, has to be handled by the contacts. This, of course, leads to a heavier and more expensive control as well as increased maintenance cost, as compared with the systems shown in some of the other figures, for instance, Figs. 12, 13, and 18, where only the small shunt current of the exciter set is handled by the contacts.

A great many other similar systems possible for regeneration might be described and considered in detail, but it is thought that the fundamental principles have been sufficiently demonstrated by the previous examples. It may suffice to mention as further possibilities the use of exciter machines driven by one of the axles, or the main motor gears, as well as the use of one of the main motors as exciter machine for the other motors during regeneration. Numerous combinations of this sort, which are more or less promising for practical application, are possible.

While the foregoing discussions were limited to systems with one or two voltages across the motor, that is, the line voltage and half of the line voltage obtained by series connection of the motors, it was repeatedly intimated that extended voltage variations will introduce further possibilities for regeneration. If the motor voltage could be varied at will, for instance, it would permit regenerative braking down to zero speed.

The system of Fig. 20 shows a possibility for varying the motor voltage. In this case, a booster armature *B* is connected in series with the armature *A* and arranged so that its voltage subtracts from the line voltage, thus permitting the voltage of the main motor armature *A* to be anything between zero and the line voltage. Since the negative booster will, in this case, act as a motor, the mechanical energy developed by it has to be taken up by a driven generator armature *N*, which returns the energy to the line. The field *F* is excited by a small exciter armature *E* coupled to the same set.

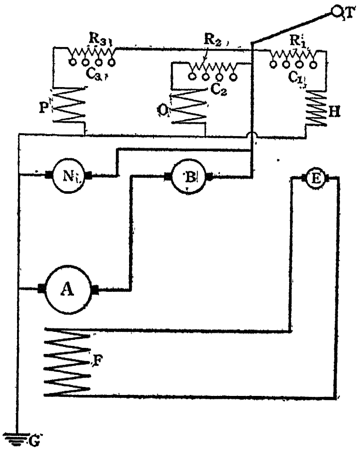


FIG. 20

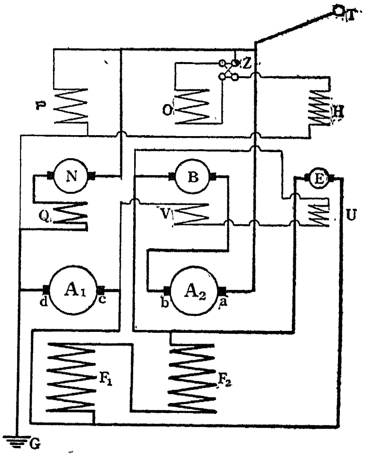


FIG. 21

Since with the previous systems and series-parallel control it is possible to return by far the larger part of the stored energy to the line and since it is, on the other hand, not safe to dispense with the mechanical brakes altogether, the additional gain in regenerative power made possible by a system as shown in Fig. 20 would hardly warrant the addition of a booster to the equipment, especially since the losses of a relatively large booster set are likely to exceed the gain made by regenerating at the lower speeds.

A variable voltage system has, however, other advantages which make its consideration worth while. The existence of variable voltage permits acceleration without rheostatic losses, introducing thereby further economy in power consumption. Further, it is possible to eliminate from the vehicle a considerable number of switches now used in the main circuits for cutting out resistance and for effecting series-parallel control and to use in their stead very small switching devices in the fields of the booster set. The resistance grids proper may also be eliminated. These latter features largely compensate for the extra complication and weight of the booster set.

Instead of only reducing the line voltage by the booster, it is, of course, possible to reverse the shunt field of the booster machine and make the booster add voltage to the line voltage. It has been pointed out previously that the use of higher voltages for regeneration permits the use of stronger field currents and smaller armature currents during regeneration, which, in turn, means a smaller r.m.s. current in the motor armatures and, under otherwise equal conditions, somewhat smaller main motors. This latter advantage, however, can only be utilized if the motors are designed for higher voltages across the commutator, which, as a rule, is not very easy with some of the standard railway motor voltages. Under special conditions, however, advantage may be taken of this point.

In spite of the above advantages, the system of Fig. 20 represents hardly a practical solution, because with this system the booster set has to be quite large and heavy. It will be seen that the booster armature B carries the full motor current all the time and that its voltage varies between zero and the full motor voltage. The current capacity is, therefore, equal to that of the main motors, and while the average voltages are less than the line voltage, the maximum magnetic sections and field coils must at least be such as to carry temporarily the maximum fluxes and exciting currents. The total capacity of the booster must, therefore, be only slightly less than the total of the main motors, and somewhat similar conditions apply to the armature N.

There are, however, a number of possibilities for materially reducing the booster size. It is, for instance, quite possible to keep the booster in circuit only while the motor voltage is different from the line voltage during the first part of acceleration and the last part of retardation, and to connect the motors directly to the line the rest of the time. This will materially reduce the r.m.s. current in the booster set.

Another possibility is to adopt series-parallel control of the motors and take full advantage of field control, which will further materially decrease the r.m.s. currents furnished by the booster set. The introduction of series-parallel control, on the other hand, eliminates one of the main advantages of booster control, namely, simplicity in the control and small number of switches.

Another possibility for reducing the booster size is shown in Fig. 21, which reproduces a system which has been tried out in Paris (for complete description, see *Electric Railway Journal*, 1914).

With this system two motors, each of which is designed for a voltage equal to the line voltage, are connected in series with the booster *B*, which is again connected to another machine of equal size, *N*, and a small exciter armature *E*. The booster armature *B* is connected between the two motor armatures. In starting, the booster armature opposes the line voltage and the motor voltages are very low. Subsequently, the booster voltage is decreased until it is zero and the voltage of the two motors is equal to the line voltage. Then, the booster voltage is reversed and added to the line voltage until the total motor voltage equals twice the line voltage. Since the booster is, however, connected between the two armatures, the voltage to ground is in no case more than the line voltage and the voltage across any of the motor armatures is never more than the line voltage. Starting out, for instance, at the brush *a* with the line voltage, the armature A_2 will by its counter e.m.f. bring the voltage at the brush *b* down to ground voltage. The booster *B* will bring it up to line voltage again at brush *c* and the armature A_1 will by its counter e.m.f. bring it down to the ground voltage again at brush *d*. The armature *B* carries, however, in this case, for the same total motor capacity, only half as much current as in the case of Fig. 20, where if two motors were used the armature *B* would have to carry double motor current. The booster set of Fig. 21 is, therefore, only about half as large in capacity as in Fig. 20.

Fig. 22 shows another possibility for reducing the size of the set. In the previous figures, the booster set may be compared with a voltage transformer, with two separate circuits. With the connections in Fig. 22, the booster set may be compared with an auto-transformer, and it has the same advantages over the other systems which an auto-transformer has over a two-coil transformer. If we assume, for instance, in Fig. 22, a starting condition with nearly zero voltage on motor armature *A*, we will find that nearly all the current is furnished by the armature *N*, while the current in the armature *B* is nearly zero. At half line voltage the motor current will be furnished about one-half by the armature *N* and one-half by the armature *B*. At full motor voltage, armature *B* will carry nearly the full current, while armature *N* has practically no current. While, therefore, each of the two armatures still has to be designed for a maximum voltage equal to the line voltage, it will be seen,

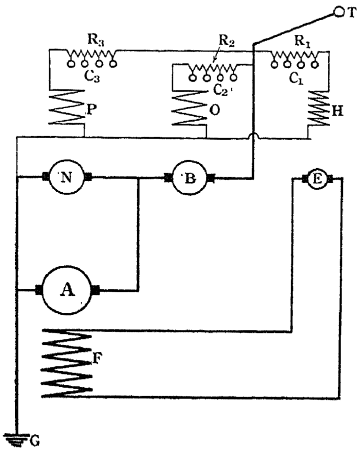


FIG. 22

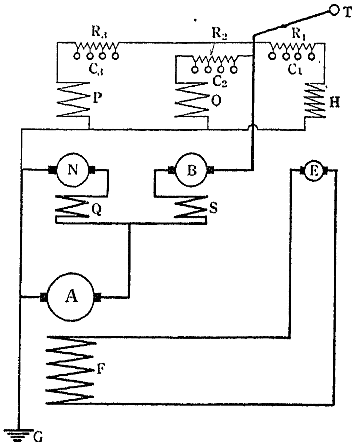


FIG. 23

for instance, that the current in the booster armature *B* varies between zero and the full motor current, instead of being the full motor current continuously, as in the system of Fig. 20. Consequently, the r.m.s. capacity of the booster, as well as that of the other machine *N*, would be very much smaller.

By properly combining two or more of the possible fundamental principles for reducing the booster size, it is quite practicable to make the set so small that the booster system may work out to be quite advantageous with regard to both weight and cost considerations.

By combining various possible methods of excitation with the various methods of booster connections, a very large number of possible combinations can be devised, a great many of which are very promising for practical application. Further possibilities are introduced by influencing the motor characteristic by properly compounding not only the exciter but also one or both of the booster machines. Fig. 23 shows, for instance, series fields *Q* and *S* for these machines, which are so arranged that an increasing regenerative current in the armature *A* will decrease the voltage of *B* and increase the voltage of *N*, thereby increasing the motor voltage and counteracting a further increase of regenerative current.

The possibility of governing the main generator voltage to a large extent by the amount of the regenerative current, as shown in Fig. 23, introduces even a possibility of obtaining a fairly stable condition with a series generator, as shown in Fig. 24. We assume, for instance, that when regeneration is desired the main generator field *F* is reversed. The field *P* may now be connected to induce in the armature *N* a slight voltage in line with that of the line voltage, and the field *O* induces in the armature *B* a voltage slightly in excess of the line voltage. The small voltage in the armature *N* will now cause the main generators to pick up as series generators. As soon, however, as the regenerated current becomes an appreciable amount, the field *R*, being arranged to oppose the winding *P*, will reverse the voltage in the armature *N* so that it opposes the voltage of the armature *A* and further increases of regenerated current are prevented. The field *S* is arranged to decrease at the same time the voltage of the booster armature *B*. This same effect of the windings *Q* and *S* will prevent excessive regenerated currents in case of changes of the line voltage. A drop in the line voltage, for instance, will tend to increase the regenerated current. Such a tendency will, however, be at once counteracted by changes in the distribution of voltage between *N* and *B*, as described before. It is also a fact that slight increases in regenerated current with the connections as shown in Fig. 24 are not so very objectionable, because for a given connection of the field, the ratio of armature to field ampere-turns is always constant by necessity in a series generator.

A booster control used for both acceleration and retardation has, of course, the disadvantage that the operation of the entire vehicle is dependent upon the additional booster set, and for this reason railway men may be somewhat reluctant at the present time to introduce such a control, and will possibly prefer systems requiring an exciter set only and arranged so that the motor operation is not dependent upon the operation of the exciter.

Another disadvantage of the booster systems is that both machines of the booster set are provided with shunt windings, making the machines rather subject to flash-overs and requiring special precautions in the design, and possibly the addition of neutralizing cross-field windings.

The systems discussed so far are all worked out with the idea of using standard or practically standard railway motors. When this condition is not imposed, further possibilities are made available. Satisfactory regenerative systems might, for instance, be devised by the use of compound-wound main motors acting as negative compound generators during regeneration, although, as previously mentioned, the introduction of any shunt winding on the main motors is undesirable, not only because it introduces complications, but also because a shunt winding with many turns is subject to high-voltage surges and consequent breakdowns, and moreover, because a shunt winding while closed increases materially the damping effect upon the fields and the flashing tendencies of the motors. Such difficulties can, however, be taken care of by the proper control and neutralizing cross-field windings on the motor.

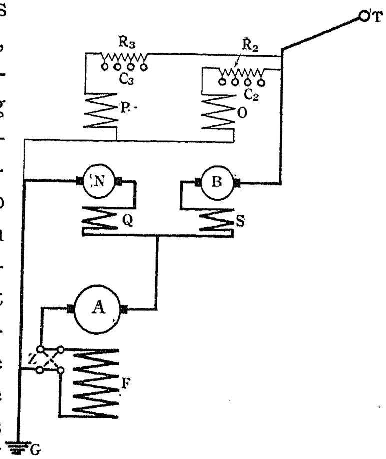


FIG. 24

Other possibilities consist in the introduction of auxiliary exciting brushes which are used during regeneration for the purpose of setting up exciting currents in the armature.

The result of regeneration upon direct-current lines may at times be rather disadvantageous with regard to voltage variations, as previously discussed. Especially with low-voltage systems with low density of traffic, voltage rises of 10 to 15 per cent above the maximum voltages otherwise obtained are not at all unlikely in a great many cases. Unless such voltage rises are premissible for all apparatus on the line, the only remedy is, in such cases, increased line and feeder copper, or some equivalent means for taking care of this condition.

With direct-current systems, regeneration may be also disadvantageous in certain cases of existing substations. While with up-to-date converting and generating machines which are all equipped with commutating poles, reverse load can, as a rule, be easily taken care of, this is at times not the case with non-commutating pole apparatus which is dependent upon a certain shifting of the brushes out of neutral to commutate successfully. With such apparatus when the current reverses, the brush shift is in the wrong direction and rather bad commutating conditions will obtain, which in turn may lead to flashing.

Alternating-Current Commutator Motor System

Regeneration with alternating-current commutator motors offers in certain respects less difficulty, in other respects more difficulty, than regeneration with direct-current commutator motors.

Since alternating-current commutator motors are always provided with cross-field windings preventing armature distortion, and since they always have, moreover, relatively low commutator voltage, the flashing difficulty is largely eliminated. For the same reason the danger of over-voltages is minimized, and operation at voltages appreciably above the voltages used for motoring is usually permissible.

The alternating-current system also has the advantage that the existence of a transformer on the vehicle makes the variation of the motor voltage very easy. This, in turn, means that weakened fields at high speeds are not a necessity and that regeneration at all speeds down to standstill can be accomplished with any desired field strength, the only limitation being imposed by commutation considerations, much the same as during motoring. This permits the obtaining of any desired torque with any desired speed, with the armature and field currents adjusted to obtain minimum r.m.s. currents in either member. In general, therefore, it will be possible to effect regeneration with a lower r.m.s current in the armature than in the case of the direct-current systems with limited voltage range.

On the other hand, additional problems are introduced, due to the fact that the characteristic of the regenerating voltage must not only be right with regard to its size but it must also be correct in phase. Another difficulty is to obtain proper commutating characteristics for all speeds during regeneration, this latter problem being, in general, identical with the commutating problem of single-phase motors during acceleration.

The proper excitation of the generators during regeneration may be obtained, in connection with alternating-current commutator motors, in three ways:

1. By the use of a separate exciter as in the case of the direct-current exciter systems.

2. By the application of exciter brushes on the main motors.

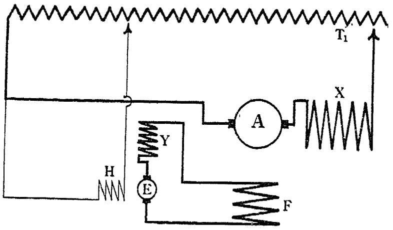


FIG. 25

3. By reversing the main fields of the motors and by operating the motors as load-excited (series) generators.

A system of the first kind is shown in Fig. 25. In order to induce a regenerated voltage of the proper phase, the main field of the generators must be about in phase with the line voltage. Since the field is purely inductive, the exciter voltage impressed upon the field must be shifted 90 deg. against the line voltage. This condition can be fulfilled by connecting the exciter as shown with its fields connected to the line voltage; the exciter field, being almost purely inductive, will have a phase 90 deg. shifted against the line voltage, and will also induce in the exciter armature a voltage shifted 90 deg. against the line voltage, as required.

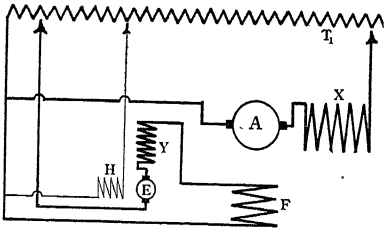


FIG. 26

On account of the various ohmic and inductive drops of the system, a slight phase adjustment is necessary and may be accomplished by adding a small part of the line transformer voltage to the excitation of the main field, as shown in Fig. 26. By properly adjusting the phase of the excitation voltage, power factor compensation can be accomplished to a certain degree.

One of the principal handicaps of the systems just described is that the exciter has to be rather large, because on account of the inductive effect of the main generator field, a large amount of kilovolt-amperes is required for its excitation.

A practical application of a system in which the exciting current was furnished by an alternating-current generator was made on a locomotive built several years ago for the Midi Railway in France. This locomotive was equipped with only two large motors, one of which was used as exciting generator for the other motor during regeneration. The system worked very satisfactorily but was later abandoned by the railway company in favor of dynamic resistance braking, because abundant water power was available and because, therefore, no advantage could be secured by regeneration which could not be secured by a simpler arrangement for dynamic resistance braking. Another handicap in this particular case, with only two motors on the locomotive, one of which was serving as an exciter, was that only half of the total motor capacity was available for braking purposes. On locomotives with a greater number of motors this condition changes, of course, and a similar system with one motor out of a large number of motors serving as an exciter would give very satisfactory results.

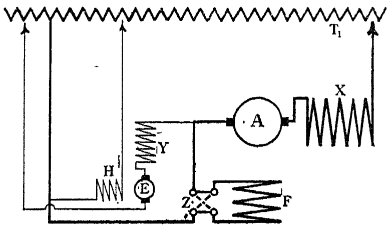


Fig. 27

Whenever all motors are to be used for regeneration, the handicap of too large a separate exciter might be eliminated to a great extent by applying a system in which the exciter furnishes only the difference between the load current and the exciter current, as shown in Fig. 27. As previously pointed out, it is possible in an alternating-current system, on account of the existing possibility of voltage regulation, to maintain any desired ratio of field to armature current. By choosing this ratio as near as possible to 1:1, the average current furnished by the exciter can be kept very small, and although its voltage has to correspond to the necessary field voltage, the machine can be kept fairly small in size.

There is danger, however, with the system of Fig. 27, that in case of a power interruption or even during normal operation the main motor might pick up as a direct-current series generator short-circuited by the secondary transformer winding.

By separating the field conductively from the armature and connecting it in series with the stator cross field, as may be done, for instance, in the case of a repulsion connection, the possibility of direct currents may be eliminated. It is still possible, however, for low-frequency regenerated currents to be set up under certain conditions, unless special means for their elimination are devised.

A regenerative system of the second kind, with the main motors acting as armature (self)-excited generators, is shown in Fig. 28. The operation of this system is as follows:

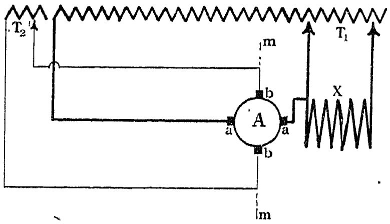


Fig. 28

Assume that with the motors rotating at a certain speed the cross-field circuit *X* is connected to the line. It is evident that a cross field will be set up, the size of which is essentially governed by the number of cross-field turns and the voltage impressed upon the cross field *X*. The phase of the cross field is shifted 90 deg. against the line voltage. This cross field will naturally induce by rotation a voltage between the exciter brushes *b b*, which is proportional to and in phase with the cross field. If we now close the exciter circuit, this voltage will set up a magnetizing current which, in turn, excites the field along the axis *m m*. Since the circuit is practically purely inductive, the magnetizing current as well as the field will be shifted 90 deg. against the voltage induced between the brushes *b b* and, therefore, shifted 180 deg. against the line voltage. This field will, therefore, induce a voltage in the armature between the working brushes *a a* which is opposite to the transformer voltage. The value of the generated e.m.f. can easily be adjusted by adjusting the voltage impressed upon the cross-field circuit and making it sufficiently in excess of the impressed effective armature voltage so that regenerated currents are furnished to the transformer. The small transformer winding T_2 in the armature exciting circuit is provided for the purpose of correctly adjusting the phase of the exciting field, which is somewhat influenced by the ohmic drops, etc., in the exciting circuits.

Such a system is advantageous insofar as it eliminates the necessity of auxiliary rotating apparatus for regeneration. It also permits of power factor compensation during regeneration, and possibly during motoring, within certain limits.

The system is, however, handicapped due to the fact that the armatures and also the commutators for the main motors have to be increased in capacity because they carry the exciting currents in addition to the load currents. This, in view of the space limitations existing for the main motors in connection with railway work, is quite a serious handicap. Another disadvantage of the system is the necessity of additional brushes around the commutator. With the large number of brushes inherent with single-phase motors, even of the stator-excited type, the addition of extra sets of exciter brushes becomes in many practical applications a rather serious problem.

Both of the previous systems have essentially a shunt characteristic but the current curves are not as flat as they would be with a direct-current shunt machine, on account of the reactive drops present in alternating-current machines and in the rest of the system in addition to the ohmic drops. Since, moreover, the low voltage of alternating-current motors, and the absence of armature distortion, make flashing unlikely, the curves obtained with these systems are satisfactory in many cases. The excitation is in both cases proportional to the line voltage, so that variations in the latter are always compensated by proportional variation in the field strength.

When desirable, steeper curves can be obtained for the separate exciter system as shown in Fig. 29, and for the self-exciting system as shown in Fig. 30.

The small transformer T_2, as shown in Fig. 29, being connected in series with the load current, is arranged so that its secondary voltage subtracts from the exciter voltage. An increase in load, therefore, means decreased exciter voltage and decreased generator voltage. In other words, we obtain a negative compound characteristic which in its purpose and effect is similar to such a characteristic when obtained in connection with direct-current motors.

The working principle of Fig. 30 is as follows:

The stator field connected in series with the load current is arranged to oppose the magnetizing effect of the armature magnetizing circuit. If, therefore, the load increases, the negative magnetizing effect of the stator field increases. On the other hand, it must be considered that the voltage induced by rotation between the exciter brushes has remained unchanged. The demagnetizing effect of the stator ampere-turns will, therefore, tend to set up increased magnetizing turns in the armature exciting circuit. This in turn, will increase the part of the exciting voltage taken up by the inductance *I*, leaving only a smaller part of the total voltage generated by rotation to be consumed by the self-induction of the main field in the armature.

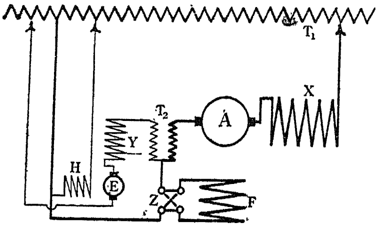


Fig. 29

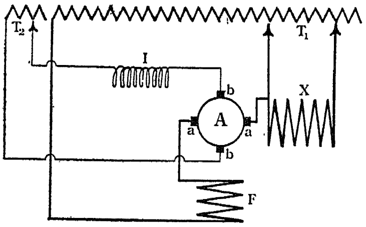


Fig. 30

This, in turn, means that the main field must be smaller than before and that, therefore, lower working voltage is induced.

In this manner it is possible to obtain any desired amount of compound characteristic.

The third possibility for obtaining regeneration by reversing the main field of an alternating-current series motor, for instance, is rather interesting insofar as such an alternating-current series generator acts in a way entirely different from a series direct-current generator. A suddenly decreased line voltage will not increase the regenerated current of the line frequency as in the case of direct current, but will tend to decrease it. Similarly, an increased speed will decrease this regenerated current. These changed conditions are brought about by the inductive drops of the machine and their phase relations to the load current, especially the large inductive drop of the main field. While this characteristic of the alternating-current series generator would make its use for regeneration possible, other difficulties have to be overcome for a practical solution along these lines. The system is handicapped by the strong tendency of the generators to pick up as direct-current generators with the secondary transformer winding serving practically as a short circuit. Even if direct currents are made impossible as in the case of a repulsion motor, there is a strong tendency for low-frequency currents to be set up, causing trouble. In view of these and other difficulties, this method of regeneration has so far not been seriously considered for railway application in this country, and a detailed discussion of its theory is, therefore, hardly within the scope of this paper. Undoubtedly, it has, however, certain possibilities for the future.

The difficulty of connecting alternating-current generators to the line is materially reduced as compared to direct-current motors because the inductive effects of an alternating-current system materially reduce the tendency towards current peaks. The introduction of a certain amount of compound characteristic by the means previously described will, of course, further assist in keeping current peaks down.

The various considerations and methods possible to take care of proper commutation during regeneration are very numerous, so that a complete analysis of them would easily take as much space as the present paper. It is, therefore, considered advisable to dispose of this subject by merely stating that satisfactory commutation can be obtained during regeneration, although certain complications in the control may have to be introduced for this purpose, under certain conditions.

The considerations in connection with regenerative control referring to the transmission, transforming and generating systems are with single-phase commutator motors no different from these previously discussed in connection with the three-phase and the phase-converter system; in other words, there is usually no particular difficulty and the changed conditions can be taken care of by very few and simple safety devices.

VAPOR CONVERTER SYSTEM

The use of vapor converters in connection with electric railways, while not of present commercial importance, may be so in the future, and, therefore, may be mentioned briefly in this connection.

When a vapor converter is used in a substation, conditions are much the same as in ordinary direct-current systems. One difference is, however, that the regenerated power coming back at times to the substation in the case of rotating converting apparatus can readily be returned to the alternating-current generating system and used up by other loads of this system, while with the present practical forms of vapor converters this is not possible. It is, therefore, necessary to dissipate a larger percentage of regenerated power in the substations by resistances. On the other hand, the vapor converter substation is free from certain dangers caused by voltage rises in substations with rotary apparatus as previously discussed.

When the vapor converter is used on the vehicle, the problem of regeneration seems, at least at the present time, rather difficult. While certain suggestions have been made for rectifying devices, transforming direct-current power into alternating-current power, their commercial development seems to be far off. The only possible way for returning regenerated direct-current power to an alternating-current line seems to be, therefore, the addition of a small synchronous converter or motor-generator set to be installed on the vehicle and used during regeneration. Such a solution of the problem appears, of course, to be cumbersome, heavy and complicated.

CONCLUSIONS

In view of the numerous possibilities for successful regeneration, only a few of which have been considered in this paper, it is to be expected that regenerative braking will soon assume great commercial importance. It should, therefore, be exhaustively considered in any new electrification, and the time is approaching when its possible application should be investigated for the larger existing non-regenerative installations. In view of the practical advantages of regeneration for heavy railroad work, it is even quite likely that regeneration will in time become one of the factors which will win some railroads, now operated by steam, over to the electric cause.

DISCUSSION ON "REGENERATIVE BRAKING OF ELECTRIC VEHICLES" (HELLMUND), PITTSBURGH, PA., JAN. 12, 1917.

Robert Lundell: My experience goes back as far as 1895, when I first commenced to experiment quietly with small regenerative motors for vehicle purposes. The motors I started with were compound wound, and I confined my experiments to series-parallel control, of course, because at that time the adjustable-speed motor was not good enough to give a sufficiently wide range of speed, so that, my first motors were sometimes furnished with double commutators—that is, each motor was furnished with double commutators and double armature windings with two motors in combination, giving practically a speed ratio on the motor curve of 1 to 8. That is to say, the motor was capable of speeding up to twice its normal speed by weakening the field. Then the commutators were put in series-parallel, and finally four commutators were put in parallel, giving an effective speed range on the motor curve of approximately 1 to 8. These were experiments made in the construction of a street railway equipment, which I believe I designed about the year 1899. The equipment—which was a full-size railway equipment of two 35-h.p. motors—had certain curious features, for instance, the field structure was laminated throughout. The field was compound wound, and, of course, I had found that it was necessary when starting, using the motors as motors, to start with a very strong shunt field. This was first weakened to about half-field strength, then came the series-parallel connection which weakened the field again, and finally in the parallel connection the fields were weakened again after, of course, first being started strong.

It was found necessary to obtain on the lower speed-ranges, a characteristic similar to the ordinary shunt characteristic, so as to drive home to the line, when regenerating, the greatest amount of amperes at the lowest possible speed; in other words, the compound or the differential characteristic did not appear to any great extent until the series-parallel connection was reached; that is to say, when I had four commutators in series, the field characteristic was practically that of a shunt-wound motor. In the series-parallel connection the fields were changed, so that, when motoring they had a fairly strong compound-motor characteristic, and of course differential as generators, or, negative compound. In the final parallel position it was necessary to have the motors run well and divide the load properly, and I therefore gave them considerable compounding with approximating series characteristic at top speed. We found it worked very well, and this equipment was tested at Newcastle in the year 1902.

Before I left New York, I dug out a report by the well known engineer Mr. H. F. Parshall, made in August, 1902, which says: "It appears from these tests that under the particular condition at Newcastle the total energy consumed by your equipment was 25 per cent less than that for a standard equipment working under the same conditions."

After testing this equipment which was compound wound, having shunt coils wound with fine flat strands, I quickly made up my mind it was not the best way to solve the problem. I believed that the right way to solve it was to use a separate low-voltage exciter in connection with an ordinary series motor and I patented such a contrivance in the year 1900.

I will describe this arrangement, shown in Fig. 1. There is a separate or independent excitation from a small motor-generator superimposed on the excitation from the ordinary series coil. In motoring this apparatus acts as a diverter of the field current, causing the equipment to speed up, as you move the controller from the first to the last notch in the series position, which in turn means that the field has been weakened to about half of the normal strength. Now, we are ready to throw the motors over into the parallel position, and the motor-generator is again caused to give the full voltage. I tried, as far as possible, to avoid any resistance in series with the main motor circuit, so that, while I admire a good many of the combinations suggested by Mr. Hellmund, I think his arrangement No. 14 seems a trifle too inefficient to be seriously discussed to-day. We have, as I see it there, without studying the diagram very closely, the regenerated current going through the entire regulating resistance. Now, in order to speed up the equipment, it is necessary to have a sufficient amount of regulating resistance, at least equal to the resistance of the field windings, otherwise we could not possibly speed up to twice the normal speed. This in turn means an extra resistance in the main circuit causing considerable loss when we have heavy currents to deal with.

I was looking for some comparative figures in this paper by Mr. Hellmund as to the amount of current which he thinks it might be possible to regenerate in a large equipment, but I did not find any, and so I brought along another book which decribes another equipment which I was partly responsible for. This equipment was tested in 1907 at Solingen, in Germany, and we did a good deal better. It says:

"The average net input of the respective cars for the three runs of the first mentioned test was as follows: 'Standard input = 3.37, Time = 986 seconds, Regenerative input = 2.21 Time = 994 seconds.' Showing a net saving of 34.4 per cent. Taking the grand average of the total runs, that is, seven of the regenerative and six of the standard, the net saving works out as 30 per cent."

Comfort A. Adams: What kind of service was that?

Robert Lundell: Street car service. These were street car equipments. I never designed anything larger than that. Most all of the motors described or hinted at by Mr. Hellmund deal with the old fashioned type of apparatus, including, of course, the interpole machines.

I have recently had in mind an entirely different type of motor as standard for regenerative work, and I wish to put it up for your consideration and criticism. In Fig. 2 each circle represents simply an evenly distributed drum winding, so that the inner circle means an ordinary bi-polar armature in the diagram, having a drum winding. The outer circle represents the field. If we look at the latter it would look like an induction-motor field or a polyphase field. It is proportioned in such a way that the stator field slightly overpowers the rotor field, say in the ratio, as I found, of 1.06 to 1. That is about the right ratio for good commutation under ordinary severe commutating conditions. Sending through a line current, we find the motor as arranged has no torque, but is a complete compensated machine. But, if we take hold of two points in this winding, at 90 electrical degrees to the axes of armature magnetization and connect these to an independent source of current supply, such as the armature of a little motor-generator set, and then put in resistance, we can do almost anything we like with it. We can also arrange the excitation in such a way that this little motor-generator set is furnished with both shunt excitation and a series coil in series with the main motor current. Then if we wipe out the shunt excitation, the characteristic of the main motor will be that of a series motor. If we short-circuit the series coil, and use the shunt coil, the characteristic of the main machine is that of a shunt motor or separately-excited motor. With the combination of both coils we have the compound motor when motoring, and the differential generator when regenerating. This motor possesses a good many fine characteristics. In the first place, we have complete compensation for the armature reaction. The compensating field combines with the excitation field in such a way that we have at all speeds and all loads a perfect commutating field. We do not have to worry much about flash-over, because the time constants of the field and the armature are nearly alike and because the distribution of the voltage around the commutator is far more even than on an

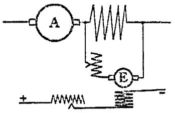


FIG. 1

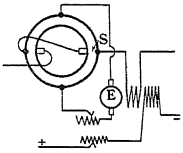


FIG. 2

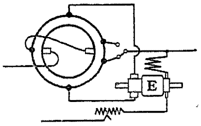


FIG. 3

interpole machine, which is liable to give trouble somewhere on the commutator. In connection with the motor-generator, if we do not like to have this machine arranged for constant speed or furnished with any shunt coils, there is another combination which is extremely useful. This (see Fig. 3) is a modification of the first arrangement, using an exciter which is a plain series machine. The method of regulation is simply this: As the main machine is thrown across the line the motor-generator set, which has only a series coil for excitation, is running at its maximum speed. As the main field is weakened, the motor-generator commences to slow down, thereby giving a lower voltage for excitation, so that, as the large machine runs up to maximum speed, the little exciter runs down in speed; in other words, it is running as fast as possible just at the moment you start, and when the big motor is well under way, the little set, as you might say, is loafing, thereby saving wear and tear etc.

I built a motor with a 22-in. armature and 12-in. core, which was originally built as a 230-volt motor, with a speed variation of 1 to 4. We ran that motor on 500 volts and even on 1000 volts, without a sign of sparking at the brushes, the average voltage at 1000 volts being about 35 volts per bar.

E. F. W. Alexanderson: Nothing more need be said about the early realization of the possibilities of using direct-current series motors for regenerative braking. It is however not until recently that regenerative braking has found practical application in large scale, that is on the Chicago, Milwaukee and St. Paul Railway. I believe that one of the reasons why it has not been done earlier is that the fundamental requirements which make possible a practical electric braking system with direct current have not been generally understood.

When I undertook ten years ago some experimental work along these lines certain conclusions and basic principles were arrived at and these conclusions have later been confirmed through extensive tests on the experimental track and have been further borne out by the thoroughly successful operation of the regenerative braking system on the Chicago, Milwaukee and St. Paul Railroad.

The two requirements which must be realized for a practical braking system are that the regenerating dynamos should have such volt-ampere characteristics as to have electrical stability and such speed-torque characteristics as to realize mechanical stability. It is as a rule difficult to give an accurate and comprehensive analysis of conditions that lead to stability or instability. While any child knows that a pencil cannot be balanced on its point because this involves an unstable static equilibrium, it is often difficult to explain what constitutes dynamic instability. Briefly, it can be said that dynamic instability is a condition where a change in one direction brings into action forces that cause a still further change in the same direction. Thus is commenced a series of increments and this series may be a converging or diverging series. The latter condition of a diverging series of increments constitutes dynamic instability and usually leads to self-sustaining oscillations. This condition is sometimes classified as a negative characteristic, and as an example may be mentioned the electric arc or the series generator. As a condition of mechanical instability may be mentioned a case where a train is descending a hill and the brakes have such characteristics that the retarding force decreases as the speed increases, and vice-versa. This condition may lead to a simple runaway or it may lead to violent oscillations if two braking locomotives of such characteristics are connected together in the same train.

The series motor has a stable electrical characteristic for reasons that are too well known to mention, and any electric generator with a drooping voltage characteristic is also well known to have electrical stability, whereas a differential compounded generator is inherently unstable and cannot be operated in multiple with another compound generator without means for neutralizing the instability. If a series motor is to be used as a generator it must obviously be a generator of the type which is known to be electrically stable. The relations defining stability already exist in the motor connection, and therefore the same relations with reference to increments of currents must be reproduced in the generator connection. The change of the absolute value of the current may be said to be only equivalent to the change of the integration constant from positive to negative, whereas the differential equation remains the same, giving a slope of the characteristic curves in the same direction. One way of arriving at these results is to superimpose upon the normal motor currents, a current in the field from a source of separate excitation which has a drooping or flexible characteristic so that only the absolute values of the currents are changed while the increment relations are maintained. As an illustration of this I wish to show the characteristic curves obtained from test which were originally made to verify this theory. These curves show that the generator characteristics are of the same shape as the motor characteristics, only they are located in a different place which means negative values of currents instead of positive, whereas, the relation of increments is the same. From the analogy with the well known characteristics of the series motor, it is thus proven that electrical stability has been established. The characteristics shown in Fig. 4 were produced by the use of a separate exciter having a drooping characteristic as indicated in Fig. 19 of Mr. Hellmund's paper. Any other method for arriving at these characteristics will obviously give similar results; for instance, a constant-voltage exciter may be combined with a resistance as shown on Fig. 5. In fact this connection shown in Fig. 5 was devised at the same time and proven to have the characteristic curves shown in Mr. Hellmund's paper in Fig. 15. This connection is apparently in principle the same as the one described by Mr. Hellmund as being used by the Lake Erie and Northern Railway.

Several arrangements outside of those referred to may be used for attaining the condition for electrical stability but in each case it must be proven that the speed-torque characteristics in the regenerating connection also possess mechanical stability because as a matter of fact electrical stability may exist under conditions that give mechanical instability. To illustrate this point reference may again be made to a speed-ampere diagram of the usual kind. In Fig. 6 is drawn an arbitrary set of regeneration characteristics which possess electrical stability over the whole range. Yet as will be shown mechanical stability exists only over a portion of each curve. If on such a diagram are drawn curves which represent the condition bordering between mechanical stability and instability, it is found that these curves become straight lines through the origin. The condition defined is the one where a change in speed does not cause either an increase or decrease of braking effort. Thus each of these curves should represent a condition of constant torque at varying speeds. With a constant trolley-voltage, it is obvious that definite torque corresponds to currents which are proportional to the speed and thus the characteristic curves defined must be straight lines through the origin. (See Fig. 3.) The point on each regenerative characteristic which is tangent to such a line through the origin is the limit beyond which the curve is characterized by mechanical instability although it is electrically stable. In order to find what range of operating speeds or currents are possible with mechanical stability it is therefore only necessary to draw the tangents through

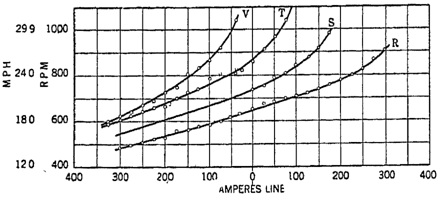


FIG. 4

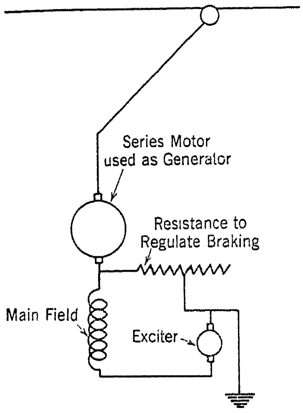

Series Motor used as Generator
Resistance to Regulate Braking
Main Field
Exciter

FIG. 5

the origin for all the characteristic curves and thereby find the locus for relations between currents and speeds which may be used with mechanical stability. The correctness of this theory has been proven in practise, as it was found that mechanical oscillations were created if the regenerative characteristics were such that the theoretical range of mechanical stability was exceeded.

Summing up the conclusions through two simple analogies, we may say that if the regenerative braking system behaves like a shunt motor with the brushes shifted backwards, then it is electrically unstable, and if it behaves like an induction motor beyond the breakdown then it is mechanically unstable. In other words it should behave electrically like a series motor and while it is too much to ask that it should also behave mechanically like a series motor we must be satisfied if it behaves like an induction motor will on the breakdown torque.

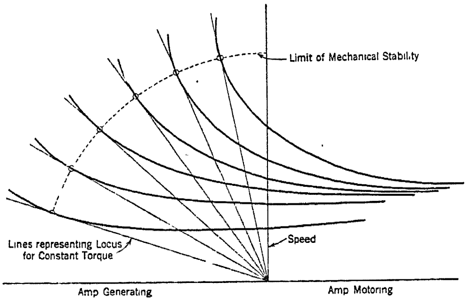

Limit of Mechanical Stability
Lines representing Locus for Constant Torque
Speed
Amp Generating
Amp Motoring

FIG. 6

The previous analysis shows that the series motor when used for regeneration has in essential respects the same characteristics as the plain motor, and therefore with a properly designed regenerating connection there is no distinct difference between the motor and generator operation except a matter of degree and it is thus possible to gradually change over from motoring to regenerating as the change of the road demands it.

These general conclusions which I have attempted to state have been verified through extensive tests in the factory and on the experimental track and have been further borne out by the thoroughly successful operation of the regenerative system on the Chicago, Milwaukee and St. Paul Railroad.

A. J. Hall: While certain possibilities of regeneration have been known for a number of years, the three-phase system is the only one, until recently, that has really been placed upon a commercial basis. This condition has been due to the apparently necessary complications in the control to produce a successful d-c. regenerative equipment.

A successful regenerative control must be as reliable as present-day equipments without regeneration. There are two classes of service, main-line locomotive and car equipments which must be considered separately when dealing with this problem.

Regeneration on car equipments, especially for multiple-unit equipments, will be used chiefly to stop trains. The control equipments should be arranged for automatic acceleration and regeneration and work automatically in conjunction with the air-brake system to effect a smooth and reliable stop. The requirements for street car service are similar, although less exacting.

Regeneration on locomotives will be used primarily to descend grades at certain speeds, and may also be used to bring a train to a stop. The following are some of the points which must be considered in connection with regenerative locomotives. The master controller should be arranged for step-by-step acceleration and regeneration, and the manipulation should be simple, preferably along the same lines as the manipulation of the air-brakes.

It should be possible to change over from acceleration to regeneration without opening the main motor circuits, making a surgeless transition. The transition from one speed combination to another, as from parallel to series on direct current while regenerating, should be made without a complete loss of tractive effort.

It should not be necessary to continually regulate the master controller while descending grades. When descending a grade with a load greater than the regenerative capacity of the locomotive, the control equipment should automatically take care of itself, exerting its full retarding effort, so that the motorman's entire attention may be applied to the manipulation of the air-brakes which will be used sufficiently to compensate for any excess load. The locomotive air-brakes should be arranged so that if applied while regenerating, they will be effective only on the trailing load.

The operation of the control equipment, as used on the Norfolk & Western Railway Company's a-c. locomotives, fulfills more nearly these conditions than any other regenerative-braking equipments so far built.

The regenerative-control equipment on the Chicago, Milwaukee & St. Paul locomotives is a wonderful step forward in the art of d-c. regenerative braking, although it does not fulfill all of the ideal requirements mentioned.

A small d-c. locomotive incorporating most of these requirements has been in successful operation for the past nine months on the Lake Erie & Northern R. R. It is probable that in the near future, a d-c. type of locomotive will be available which will incorporate practically all these ideal requirements for regenerative braking.

R. E. Ferris: The efficiency and reliability of railway motors have been improved to a point where there does not seem to

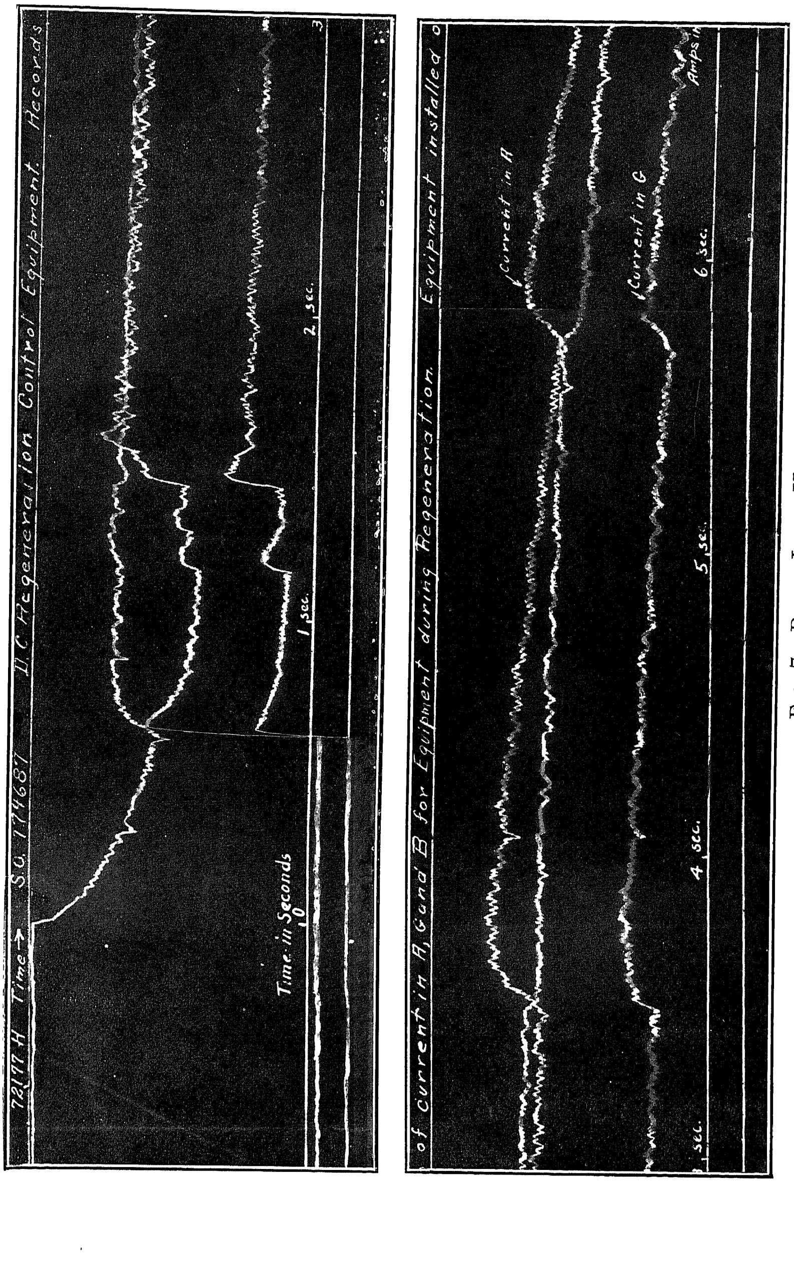
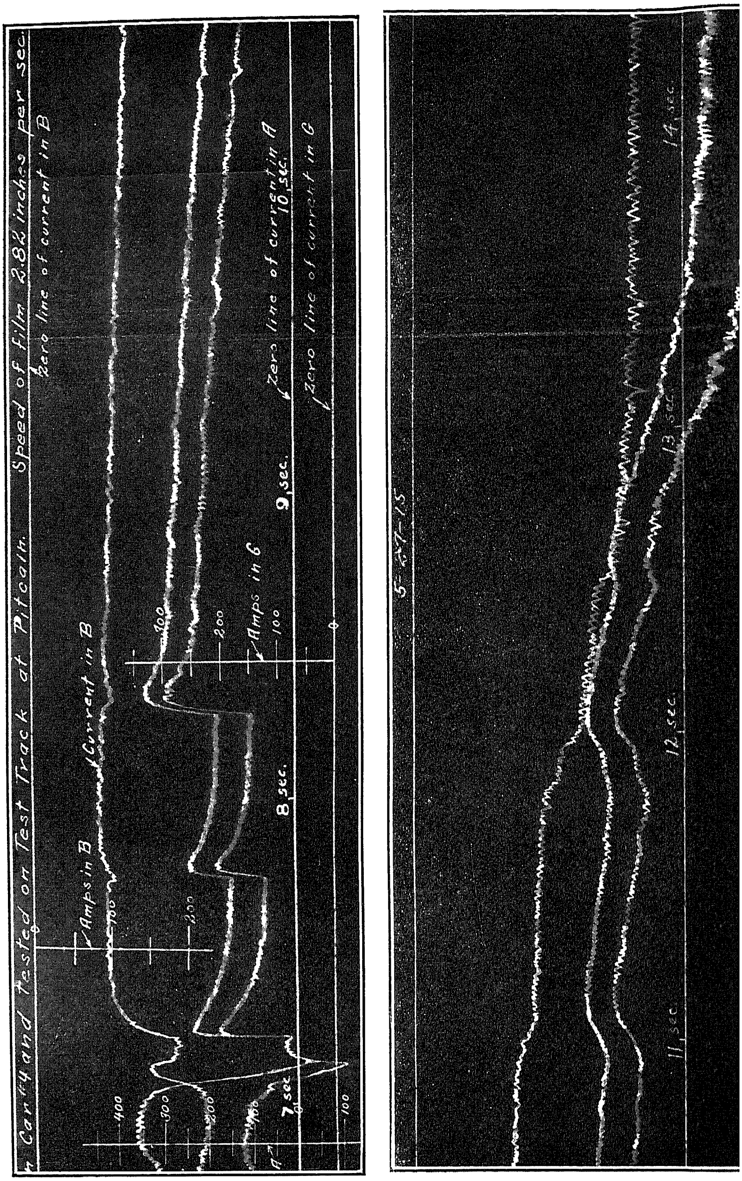


FIG. 7—PARTS I AND II

[FERRIS]

PLATE II.
A. I. E. E.
VOL. XXXVI, 1917



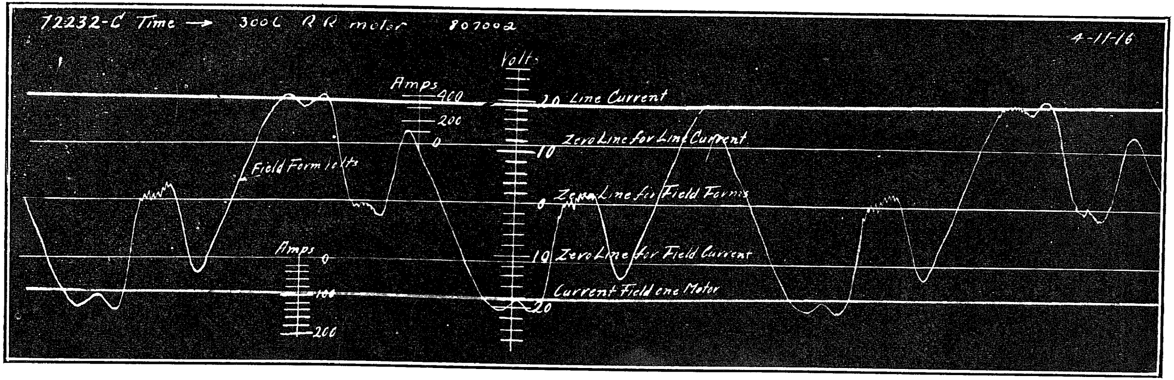

72232-C Time → 300C R R motor 807002
4-11-16
Volts
Amps 400
200
0
20 Line Current
Zero Line for Line Current
10
Field Form volts
0 Zero Line for Field Forms
Amps 0
100
200
10 Zero Line for Field Current
Current Field one Motor
20

FIG. 8

[FERRIS]

be much hope left for radical changes along this line. The weight has been reduced apparently to almost a minimum, and application has been thoroughly analyzed. The most obvious remaining method, therefore, of increasing the overall efficiency of the railway system would seem to be the return of stored energy of the car or train to the line. The problem involved in regeneration, especially for passenger service, is not merely to bring the car or train to a stop, lower the speed, or hold on a grade, but to accomplish this smoothly.

In order to determine how the currents varied in the different circuits of a motor when retarding a car smoothly, oscillograms were taken on a car equipped for regeneration with a scheme giving the same characteristics as that shown in Fig. 14 of Mr. Hellmund's paper. After adjusting resistance properly, the car retarded very smoothly on regeneration, and even at the point of transition from parallel to series control no diminution of retarding torque was noticed. A typical oscillogram is shown in Fig. 7 in which *A* is total current returned to line, *B* is field current of one motor of the two motor equipment, and *G* is current in one armature. It will be noticed that the field current builds up before the line switches are closed, at which point field current decreases at once, due to the reaction of the armature current. In fact, the field and armature current mutually react on each other over the entire period. Some of the small ripples, even, in the armature current affect the field current. This mutual reaction of field and armature current gives, of course, a very stable system.

The effect of cutting out main-line resistance is shown at points *D* for parallel, and points *E* for series. The retarding torque was calculated during regeneration by means of the saturation curve of the motor and the field and armature current shown on the oscillogram, and found to be fairly uniform over the entire period down to saturation of the main motors.

In retarding a car or train by means of regeneration, one of the fundamental principals involves a change of field strength. It is therefore pertinent to inquire how far the ratio of armature ampere turns to field ampere turns may be carried on an ordinary commutating-pole railway motor.

A standard stock motor of 115-h.p. nominal rating, 600 volts, 675 rev. per min. was placed on test, and with connections as shown in Fig. 14 of Mr. Hellmund's paper, the field strength was varied. The highest point to which the ratio of armature amperes to field amperes was carried was 4.8. Speed at this point was 1590 rev. per min. with reasonable commutation. With this ratio of current, the windings of this motor give a ratio of total armature ampere turns to field ampere turns of 2.65. With the ratio of armature current to field current set at 3.7, the voltage applied to the motor while regenerating under this condition was suddenly changed from 560 to 830, with no other result than a slight spit at the brush.

An oscillogram of the field form of a motor when thrown on regeneration is shown in Fig. 8. This oscillogram plainly shows a large reversal of flux under a portion of the pole, but the motor under the condition shown in the oscillogram had almost sparkless commutation.

W. V. Turner: Lest some people think that regeneration will do away with air-brake systems, it should be said that regenerative braking is needed for reasons of economy, but the air brake will always be needed for reasons of safety. The air brake is needed as a reserve to back up the regenerative brake. With the latter, all the control is vested in one or two units, where the failure of one means a 50 or 100 per cent failure of the regenerative system. With the air-brake system, however, the air-brake control is vested in from 10 to 150 units, according to the number of operative brakes. The failure of one unit there is of comparative insignificance.

I may point out here that the regenerative brake is virtually a straight air-brake such as Mr. Westinghouse invented, in the first place, and which was abandoned because of the fact that if the connection between the cars became ruptured there was no brake existing, and that is, of course, true with regenerative braking. Therefore you see the necessity, as Mr. Hellmund has very well pointed out, of having the regenerative brake work in connection with the air brake, or vice versa.

The greatest need for freight carrying cars today is an empty and load brake. With regeneration, the need in many cases is reduced for grade service. The reason for this is that the combined use of the two braking systems will supply the retarding effort necessary for proper control—a control which is lacking on a great many grades today. Such as for instance, on the Norfolk & Western grades or the mountain grades of the Chicago, Milwaukee & St. Paul; but it is to be expected that with the air brake a regenerative brake will be used at the same time, for reasons which I will point out a little later.

This consideration, however, does not influence the vital necessity for the empty and load brake in level road service. In this connection it may be well to state that not only is the endeavor made to haul long and heavy trains, but to reduce the percentage of weight per load, and this makes it practically impossible to control trains on grades without the empty and load brake. In other words, the load as compared with the weight of the car, has increased to such an extent that the single capacity brake is now fairly safe, except at over-load speeds, in other words, the capacity of grades is greatly reduced.

Of course, it should be understood in this connection that only a slight portion of the possible economy of air brakes is now being realized. What I mean to say by that is, that it is quite possible to get 200 or 300 per cent more retarding force out of air-brakes than is now being obtained with loaded cars.

In general it will require as many locomotives to take a train down a grade by means of regenerative braking as it will to take the same train up that grade. Of course, in descending the grade, all internal train resistances, due to curvature, journals, etc., are acting in favor of train control, while in ascending, these resistances are opposing the locomotive effort. The greater the grade the less influence this internal resistance will have upon the truth of the general statement first made. An internal resistance of 4 lb. per ton is 20 per cent of the resistance due to a 1 per cent grade, but only 5 per cent of that due to a 4 per cent grade. This means then that helper locomotives cannot be cut off from a train upon reaching the summit of a grade unless the descent is milder than the ascent, or unless air-brakes are to be used in conjunction with regenerative, that is to say, the two together make the braking system.

In order the better to control slack action and favor draft rigging, helper locomotives used as pushers back in the train for the ascent should be shifted to the head end for the descent. In placing helpers at intermediate points in trains rather than at the ends, a safe rule and the one customarily observed is to avoid neutral points as to slack at these heavy locomotive units. That is, a helper must be in the rear a distance or number of cars, greater than the tonnage capable of being handled by the head locomotive or locomotives. On the other hand, in descending the grade the position of the helper in the train should be ahead of the point where the tonnage rating of the leading engine or engines runs out. In this way, the helper locomotive will have command of its own slack, but it will not be subject to slack changes created by the other locomotive to the detriment of the integrity of the train. The point here made is that the same helper position in the train cannot be utilized with operating safety for both the ascent and the descent of a grade.

While mentioning helper locomotives, though it is not directly of concern to regenerative braking, it is well to consider the need for means to avoid break-in-twos when the power goes off the line in ascending a grade. With a steam locomotive the power never goes off the line, as it does with an electric locomotive, with the result that the helper behind can always keep the slack well bunched in the train. With an electric locomotive, however, the loss of power means that the train will start to drift off down hill. The head locomotive, operating the air-brakes in order to stop the train, anchors the head end while the rear end keeps on going. This is a result borne out in experience. It is essential that, therefore, in event of power leaving the line, the air-brakes be set automatically on each locomotive in the train in order to avoid this trouble of breaking in two.

There should also be an automatic interlock between the regenerative brake and the air-brake. It will often-times be desirable or necessary to use the air-brake in conjunction with regeneration. In such cases, the air brake on the locomotive must be automatically cut out and released, else the adhesion between the drivers and rail will be over taxed and slid flat spots will result. Where traffic is largely in one direction, or for other reasons it will be found inconvenient to use the same number of locomotives in a train for descending a grade that is used for the ascent. In this event, the air-brakes must be used to supply the additional retarding effort required to control the train.

I would like to point out in this connection that for such work on a 2 per cent grade it will require, say, three locomotivs to pull the train up, and one can take it down with the present steam locomotive; but since it only furnishes one-third of the total power, you must use the air-brake for the other two-thirds, or the train will run away. Another thing, the speed must be kept down to the capacity of the driver, otherwise if anything should happen to the circuit the train would run away. These things must be considered in connection with regenerative braking, if it is to made a success.

H. M. Hobart: In 1903, tramcars equipped with the Raworth system of regenerative control were put in service at Devonport in England, and in the immediately following years several dozen cars equipped with the Raworth system were employed in a number of towns in England.[†]

It is interesting to observe that in England we first find an approach to fair success in reducing this important invention to practise. The proposal was not favorably regarded in America. It is true that the first application to which I have alluded, was of an American invention, but the second, that of the Raworth system, was the invention of British engineers. England was thus the first country in which sufficient discernment was shown and sufficient encouragement was extended to permit of carrying through the pioneer work required for the successful application of regenerative control to railways and vehicles. It is always the pioneer work which is beset with the greatest difficulty.

No one could wish to minimize the credit due to those concerned in these practical applications of the principle in America many years later, but I think all of you who have had to do with discouraging pioneer work which requires so much determination and perserverance, will be inclined to agree with me, that a vast amount of patience and courage and perserverance was required in those years to which Mr. Lundell refers, and in the later years during which Messrs. Raworth put many equipments into successful operation.

I may also mention the fact that amongst the earliest workers in this field was Mr. Sprague* who advocated the system in

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[†] See paper entitled "Regenerative Control of Electric Tramcars and Locomotives" by Alfred Raworth at p. 374 of *Jour. Inst. Elec. Engrs.* (1907) Vol. 38. Also an article at p. 526 of the *Electrician* for July 17, 1903 entitled "The Raworth Regenerative Control for Tramway Motors". Also a paper by J. J. Hall entitled "Raworth's Automatic Regenerative Control" at p. 68 of Vol. 54 of *The Electrician.*

1885 in a paper read at Boston before the Society of Arts and carried out experimental demonstrations in 1886 and 1887. All these workers contributed in large measure to pave the way for the splendid success on a large scale which is now recorded in America.

On the basis of the demonstrated economies attainable with regenerative control, one cannot but regret the enormous economic waste which has been occasioned by the delay of a quarter of a century in taking advantage of this simple feature. Although progress is almost invariably greatly delayed by scepticism and conservatism, examples of such long-maintained opposition to a sound improvement are fortunately rare in the engineering profession.

F. R. Phillips: Among the multitudinous disadvantages of regenerative braking, there is one outstanding feature among them which appeals to me, and that was mentioned by Mr. Hellmund, viz; increased weight. It will perhaps be of interest to you to know that electric railway engineers for the last two years, at least, have made great strides toward reduction in the weight of car equipment to the extent recently of as much as 45 per cent, so that I wish to say to those who are interested in the development of regenerative braking, to bear in mind this particular feature, because we figure very closely, and our estimates are that the addition of one ton of weight to a street car would mean the additional expenditure of something like $70 a year, in energy consumption alone.

Comfort A. Adams: The point which has just been made in regard to increased weight is one which Mr. Hellmund treated from only one side. In the case of regenerative braking, it is to be noted that insofar as that braking is perfect, any power expended in accelerating the extra weight, is restored when the car is decelerated.

C. E. Fortescue: Mr. Hellmund mentions in his paper certain disadvantages in connection with regeneration. Two of these disadvantages do not appear to me to exist in most cases. For example, increased first cost of copper. Where the condition of traffic justifies electrification there will generally in a given section be as many trains going down grade as going up grade, and regeneration in such a case will reduce rather than increase the copper losses. As for the matter of regulation, a distribution system designed to give good regulation with power consumption at the trolley will give good regulation with power regenerated at the trolley, whether this power is consumed near its source or at the power house.

In connection with the phase-converter system and to a large extent, in connection with all constant-speed systems, Mr. Hellmund's curves show that with rheostat control, maximum torque may be maintained at all speeds up to within a small percentage of synchronous speed. These motors when regener-

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*See p. 1076 of the *Electric Railway Journal* for June 5, 1915.

ating may also be controlled at speeds considerably in excess of synchronous speed. The series motor is not suitable for regeneration, since it is not self-exciting and as a generator is essentially unstable. To adapt it for regenerative purposes, it must be temporarily changed into a shunt machine. The constant-speed motor therefore has acceleration characteristics as good as the series motor and from the point of view of regeneration, it is superior.

A vehicle going slowly up grade will have to make very high speed down grade to attain the same average speed as one having a uniform speed both up and down grade. On the other hand, the vehicle having a uniform speed will require more power for its propulsion up grade than the variable-speed machine, but the total energy consumption will be the same in each case, except for the losses due to friction and windage, which are greater in the variable-speed machine.

One of the principal advantages of electrification is that the source of energy is centralized and the various demands upon it are averaged so that it is capable of supplying a large demand at one point because there is a corresponding smaller demand or regeneration of energy at another point. This advantage is used to the maximum extent with a locomotive having constant-speed characteristics.

In a steam locomotive the rate at which energy may be used is dependent on the rate of generation of steam, the limit of which is defined by the capacity of the boiler, and it is therefore essentially a machine having a constant output. A machine of this type is at a disadvantage on a grade since its speed has to drop with increased tractive effort and constant horse power. It is pretty generally realized that the maximum limit has practically been reached in steam locomotive design.

A constant-speed motor on the other hand has an increase in power output with increase in tractive effort within certain limits, and is therefore capable of maintaining high speed on grades. The sole limit in the operation is the heating of the machine and therefore, if generation is to be used down grade, the increase in energy thereby dissipated in the machine has to be taken into account.

Railway engineers frequently base their prejudice in favor of the series motor on its ability to make up time. This contention is not justified, I believe, by the actual facts. Series motors are designed to give a certain speed on a level track, and this speed is not much in excess of the average requirements. The speed of a locomotive with such an equipment up grade will not be as great as one having a constant-speed equipment, and down grade the latter is capable of as high a speed as the former. The advantage in ability to make up time will therefore be in favor of the locomotive having the constant-speed equipment.

Where the traffic is heavy the power demand from the trains in a given section will be averaged at the substation supplying that section, so that the large power demand from trains proceeding up grade will be offset by the power regenerated from trains going down grade. Where the traffic is not so great the load on the substations will have greater fluctuations, but with apparatus having high thermal capacity, these fluctuations should not cause undue heating.

It appears, therefore, that in order to obtain all the possible advantages from electrification, the characteristics of the motors should be such as to take full advantage of the centralization of the power house, both in respect to speed requirements in climbing grades and ability to regenerate on the down grade.

These requirements in my opinion indicate that the motor having constant-speed characteristics is the ideal motor for heavy traction and long haulage electrification. The results obtained in the Norfolk & Western electrification on the Elkhorn grade seem to justify these conclusions.

K. A. Simmon: The constant-speed argument that is advanced in this paper leads me to a question that was asked of some ten or twelve engineers today, with the result that an incorrect answer was obtained from all.

Assume a given cycle of operation with up grade for a distance of one mile and a down grade for a distance of one mile. Assume that the locomotive goes up the grade at fifteen miles per hour. How fast would the locomotive have to come down the grade in order to produce a schedule of thirty miles an hour?

It is a comparatively simple question, and follows very pertinently the application of constant-speed equipment. The answer received from most engineers is that the down-hill speed must be 45 miles per hr. Inspection shows that it requires an infinite speed down grade to get an average speed of thirty miles an hour.

Mr. Hellmund's paper, I believe, will do much to educate a lot of us to use the terms dynamic braking, resistance braking and regenerative braking more correctly. We have referred in the past in a good many cases to magnetic braking and resistance braking as regenerative braking. In fact it is not at all uncommon to refer to all sorts of magnetic braking and dynamic braking systems as regenerative braking.

There are several points which have been made this evening that should have special emphasis. Among these are the following: First, that the problems confronted in city and subway service are entirely different from the regeneration problems confronted on long steep grades or under mountainous conditions; second, that while shunt characteristics may not be desirable for direct-current regenerative equipments, similar characteristics existing in three-phase motors are not to be considered as a handicap, as is evidenced by the phenomenal operation of the Norfolk & Western equipment. The smooth operation obtained on these locomotives is largely due to the smooth control obtained by the liquid rheostat which unfortunately cannot be employed for direct-current operation; third, direct-current regenerative equipment should preferably be based on standard motors, at least for the present; fourth, regeneration may lead to the more extensive use of adjustable constant-speed motors; and fifth, it appears to be desirable to interlock the regenerative control equipment with the air-brake equipment.

F. D. Newbury: You have heard considerable discussion this evening concerning the locomotive. Other elements of the problem must be considered. There are the line to the substation, the machinery in the substation, the transmission line and the generating system. In considering the application of regenerative braking particularly to existing systems, these other elements must not be overlooked. The characteristics of the substation machinery must be investigated, particularly if the substation happens to contain synchronous converters. Reversal of power means a change in the internal or external operating conditions of the machines that may lead to new phenomena that may not be altogether satisfactory. This in general will not be serious but should be investigated in any particular case.

W. B. Potter: Mr. Hellmund has indicated a vital feature to the stability and smoothness of operation, in pointing out the difference in the characteristic of a series-wound machine when acting as a motor and as a generator; the characteristics which make it stable as a motor having an opposite effect when being driven as a generator. Of the many ways by which a machine, suitable as a motor for railway service, can be given the stability essential to successful regeneration, a number have been described but the interest will presumably centre upon the two methods now in operation,—the Lake Erie and Northern and the Chicago, Milwaukee & St. Paul.

Speaking with more knowledge as to the latter, the results of a year's service have unquestionably demonstrated the success of direct-current regeneration and its value as improving the train service on all down grades, which would otherwise require use of the air brakes. On down grades, ranging from 2 per cent to 0.6 per cent, and through a wide range in speed, the trains are controlled by regeneration, the air brakes being used only for the purpose of stopping. The variations in speed incident to repeated applications and release of the air brakes, hot brake shoes and delays incident to inspection are noticeably absent under regenerative handling, and the mountain division under electric operation has become a division on which lost time is made up, rather than added to.

Under some conditions regeneration may prove profitable on other than grade service, but as the value in this connection will be influenced by the saving in energy and lower maintenance of mechanical equipment as offset by presumable increase in cost and maintenance of electrical equipment, the extent to which it may be used will be largely influenced by the skill and ingenuity in devising simple methods of accomplishing the result.

Mr. Hellmund's conclusions as to the present status of regenerative braking might well be amended by emphasizing that it has already assumed great commercial importance, and that it is one of the factors now having a favorable influence toward the electrification of mountain service and ruling grades.

N. W. Storer: In common with a great many other engineers, I have worked for a long time with the ideal in my mind of returning to the line the lost energy which is spent in heating up brake shoes, and ruining the braking equipment. There is one thing I have had in mind through it all, and that is we must with direct-current motors utilize the standard series-wound motor. Its advantages in hauling trains are so numerous that we cannot afford to give them up. That is one reason why the earlier experiments were unsuccessful. They departed from this motor. I do not believe it is possible to have really successful regenerative braking on any kind of a system with shunt motors. They are inherently too sensitive on a variable voltage system, such as any railway is bound to be.

I have had some question also in my mind as to the desirability of a constant-speed machine for handling trains on long hauls, or short hauls, either. The instability of the series-wound machine during regeneration which has been mentioned is, of course, a factor which must enter provided that instability exists. There are a great many schemes, however, that will make the series motor a perfectly successful and stable machine.

I do not want to differ too much from the statements which have been made heretofore, as to the desirability of having the torque or tractive effort increased as the speed increases, but I will say I do not believe that it is necessary. I think the equipment will be more suitable if it is allowed to have an electrical stability which is obtained by a slight decrease in the torque with increase in speed, such as normally will obtain with the series-wound machine properly separately excited.

For the reason that we can have, as has been pointed out so plainly tonight, perfect co-operation between the air-brake and regenerative braking, we do not care whether the speed increases slightly or not. We have the air-brake to fall back on, and in my opinion the successful braking equipment of the future will consist of a combination which will effect the most perfect operation by automatic control of the air-brakes and the regeneration. Let the speed increase to a certain amount. You can stop it right there by the application of the air-brakes. If you get on a heavy grade, and you find that the regenerative brake will not hold the train, let the air-brake help. The flat characteristic is not well adapted to that, without additional excitation variation. If you have a steep curve. it will take care of quite a wide range of speed without any adjustment whatsoever. If the train reaches the limiting speed, you can protect your motor fram flashing by permitting the braking power to decrease.

It has been pointed out very clearly that the motors are unsafe if they are allowed to have too great a ratio between armature and field. That is absolutely true. Why then permit them to have such ratio? Why not keep it down, and keep the ratio within proper limits, even though it does mean decrease in maximum power at high speed. You can apply the air-brakes to slow the train down to the point where the regeneration can hold the whole train.

I have one point in mind which has been of the greatest encouragement to me throughout the years in which I have been interested in regenerative braking, and that is a knowledge of the amount of energy which is stored in the train. This is shown in Fig. 9, in terms of watt hours per ton plotted in terms

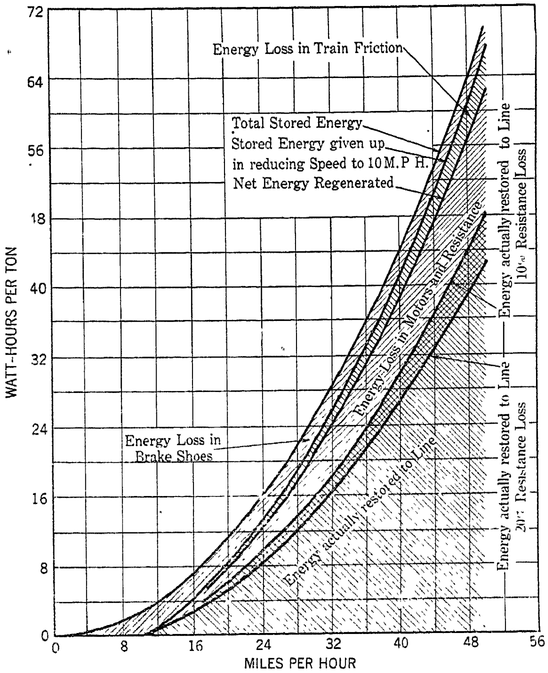

FIG. 9—REGENERATIVE CONTROL-CURVE SHOWING DISTRIBUTION OF ENERGY IN REDUCING SPEED TO 10 MILES PER HR.

of miles per hour, and represents an amount that is well worth a great effort to save.

Now, the total amount of energy that is going to be returned is simply a function of the speed and the number of stops, that is the speed at which retardation begins and the number of stops which are made in given time. For instance, if you stop from a speed of 24 miles per hour, an amount of energy, about 9.5 watt-hours per ton can be returned out of a total of 16. If there are three stops per mile made, that amounts to about 29 watt-hours per ton mile. Three stops per mile are about the number of stops which are made in the subway in New York, in local trains, and the average power consumption amounts to something like 90 watt-hours per ton mile. Now, if we stop from a speed of about 24 miles per hour, which is about the average, you can see at once the amount of power that will be returned per mile. It is in the neighborhood of 30 per cent. From that we must deduct a small amount for the increased weight of the equipment, which Mr. Philips is worrying about.

In any regenerating scheme, the net saving in power always takes into account the additional power which is required by the addition of weight to the equipment. For that reason, we seldom considered that more than 20 per cent, at the outside, can be guaranteed on any equipment, but it depends simply on the number of stops per mile that are made and the speed at which regeneration begins.

R. E. Hellmund: Regarding the special motor with a neutralizing winding, as proposed by Mr. Lundell, I have reasons to believe that its adoption would hardly be advisable at the present time. While it is true that the neutralizing effect will prevent the usual field distortion and flashing caused thereby, we are liable to run into other difficulties. If the main field, for instance, should be without damping effect, which Mr. Lundell apparently assumes when he says that the time constant of armature and field are alike, it will build up very rapidly after a temporary power interruption. Such quick field changes induce high voltages in the coils under commutation, and make the motor very sensitive with regard to flashing.* If, on the other hand, a damping effect exists, as, I believe, is actually the case with Mr. Lundell's arrangement of Fig. 2, we get an appreciable temporary field distortion after the circuit is closed. This is because the machine must be over-compensated for the sake of good commutation; such overcompensation causes field distortion, which is opposite in direction to that obtained with a standard motor. As pointed out in the paper, the direction of distortion is favorable while regenerating with a standard motor; with an overcompensated motor, it is, therefore, unfavorable. It is thus evident that a motor of the compensated or neutralized type is by no means foolproof with regard to flashing, but that the designer must, as in case of the standard motor, give due consideration to the flashing problem, both in designing the motor as well as in choosing the control system. Aside from this, a neutralized motor for high voltages will not only be more subject to insulation breakdown, but it will also be more costly than the standard type of motor, on account of the distributed field winding. While Mr. Lundell's arrangement may have sufficient merits, to find practical application in the future, I believe that the extra cost and the necessity of a special motor type would, at present, retard the introduction of regenerative control rather than further it.

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*See *Electric Journal*, July 1915, page 298 and seg. Article on Commutation and Flashing of Railway Motors.

Mr. Storer took some exception to the statements made in the paper regarding the most desirable speed-torque characteristics. This merely indicates that we electrical engineers are liable to differ on the requirements of heavy traction work, and that we need the cooperation and advice of the railroad men. Unfortunately, none of them has given us the benefit of their opinion in the discussion.

Personally, I believe very strongly in constant-speed operation, especially for heavy trunk-line work. If it is safe to run over a certain piece of track at a certain maximum speed, maximum traffic efficiency can only be obtained by accelerating up to that speed as quickly as possible, running at the maximum speed as long as possible, and then decelerate. Therefore, I do not think that a motor with varying speed characteristics, like the series motor, is really wanted from a traffic point of view, although it may have advantages in other respects. The principal advantage of the series motor, which has been so successful for many years, is, in my opinion, that such a motor is simple and rugged, and that it protects itself and the rest of the electrical equipment against undue overloads, by falling off in speed with increased torque. This, however, does not mean that such characteristic is either necessary, or even desirable, from a traffic point of view, especially if other means for protecting the electrical equipment against overloads are available. It seems, from Mr. Fortescue's discussion, that he fully concurs with me on these points.

As a matter of course, the permissible maximum running speed is not the same for all track and traffic conditions, and a possibility for adjusting the constant running speed to a number of different values is, therefore, undoubtedly desirable.

For these reasons, I believe that the energy devoted in the past by many electrical engineers in trying to give an inherently constant-speed motor like the induction motor a varying-speed characteristic, should, in the future, rather be directed towards the development of both a-c. and d-c. equipments giving adjustable constant-speed characteristics, for both motoring and regenerating.

R. E. Hellmund (communicated after adjournment): It might be inferred from Mr. Lundell's remarks regarding the arrangement of Fig. 14 that the losses in the stabilizing resistance *R* are of great practical importance. As already pointed out in my paper, this is not the case. The resistance is less than half the field resistance and, although it carries the sum of the armature and field current, the losses will be about the same as the field losses, which are very small in an up-to-date railway motor. The losses caused by the resistance are something like 2 to 5 per cent during the regenerative periods, which means that they are in the neighborhood of 1 per cent of the total power consumption. Thus, the power saving accomplished with regenerative control may be reduced from, say, 21 per cent to

20 per cent, on account of these resistances. I am confident that a large majority of operating men will gladly make this comparatively small sacrifice, enabling them to use a stable system with the standard type of railway motor, in preference to using a stator structure with distributed field winding, as proposed by Mr. Lundell.

Mr. Alexanderson's remarks bring out in a somewhat different, but very interesting manner, two of the principal conclusions presented in my paper, viz., the necessity of electrical and mechanical stability as relating to volt-ampere and speed-torque curves. While these two conditions are of prime importance, their fulfillment is not sufficient to give successful operation of a d-c. machine unless the problem of flashing is given careful attention in all its phases. This can hardly be emphasized too strongly since it is one of the principal limitations of the d-c. machine as compared with induction motors. While it is true that the induction motor loses its mechanical stability after reaching the pull-out point, as mentioned by Mr. Alexanderson, it is also a fact that, with the induction motors used for heavy traction work, there is no difficulty in keeping this point for all variations of line voltage above the slipping point of the locomotive wheels. The limitation introduced by the pull-out torque is, therefore, of practically no importance. With the d-c. motor it is also easy to maintain mechanical stability up to the slipping point of the wheels, but in doing this it is, for higher speeds, usually very difficult to maintain the flashing conditions of the motor within safe limits, for reasons pointed out in my paper, especially during transient conditions. This point has been frequently overlooked, which fact I believe has, more than anything else, prevented regenerative braking from being a complete success.

Mr. Turner points out in his discussion the importance of proper joint operation of air-brakes and regenerative braking, a point which also has been touched upon briefly in my paper. As mentioned in the paper, some slight complications may be introduced where it is necessary to operate induction motors together with the air-brakes. Upon further consideration it appears to me, however, that this problem may solve itself very simply in actual practise. It should be kept in mind that the irregular operation of the air-brakes, causing a speed fluctuation of five to ten miles, with a train which is handled with the air-brakes alone, is fundamentally caused by a variation in the braking efforts exerted by the air-brakes. Let us assume now that an induction motor with no resistance in the secondary operates together with the air-brakes, so that the motors are exerting their maximum braking torque. If, in this case, the braking effort of the air-brakes increases, the speed of the train will decrease slightly, which, in turn, means a materially decreased braking effort exerted by the motors. As the braking effort of the air-brake subsequently decreases, the speed of the train increases, and with it the braking effort of the motors. We see, therefore, that the motors change their participation in the braking automatically, so as to take care of the unavoidable fluctuations in the air-brake, and apparently this takes place without any action of the engineer or switching devices. The only undesirable thing that might happen, is that the braking effort of the air-brakes reduces the train speeds so much that the motors pass through their no-load point to a motoring torque. No particular harm is done if this happens once in a while. If it should, however, happen to any considerable extent with the flat curve obtained from an induction motor without secondary resistance, there is no doubt but that it can be avoided by a small permanent resistance being inserted during regeneration; this would make the motor curve slightly steeper, and reduce the amount of change in torque with change in speed. These and similar questions can best be determined by practical experience; there is no doubt, however, but that one or the other method would take care of this condition satisfactorily.

Mr. Hobart expresses regret in his discussion that the simple feature of regeneration has not been used for the past twenty-five years, and it might be inferred from his statements that this could have been done just as well as not, and would have avoided an enormous economic waste. If this is what Mr. Hobart means, I cannot but feel that he does not fully realize all the problems which had to be solved, and still have to be overcome, in connection with d-c. motors, to make regeneration commercial for all classes of service. Whatever successful regenerative operation has been accomplished in the past, with battery driven vehicles, for instance, would contribute but little towards the solution of the difficulties encountered in heavy traction high-voltage work. Even the successful operation on a number of small tramway propositions does not solve these problems, nor a great many other conditions which have to be met in other cases, as, for instance, elevated and subway service. The very important points brought out in the discussion by Messrs. Ferris, Turner, Hall, Newbury, Simmon, Alexanderson and Storer, in addition to the points discussed in the paper, give fair evidence of the large number of problems to be solved. Not only is it a fact that a great many of these points were not fully realized until during recent years but, as pointed out in my paper, it is also to be considered that the detail apparatus needed for taking care of the various conditions, were not sufficiently perfected to give fully reliable operation in the past. While I am very optimistic as to the future of regenerative control, I do not believe that the general introduction of the principle is merely a question of doing it, but will require further engineering developments in addition to those accomplished at present. I fully agree with Mr. Potter in his statements that the extent to which it may be used will be largely influenced by the skill and ingenuity in devising simple methods of accomplishing the results.

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INTERNAL TEMPERATURES OF A-C. GENERATORS

BY RALPH KELLY

ABSTRACT OF PAPER

The paper deals with the internal temperatures of a number of typical large a-c. generators; this temperature being measured by a thermo-couple placed between armature coils in the same slot and in the center of the core. The difference between these measured internal temperatures and the corresponding surface temperatures in the tests given varies from 0 deg. to 35.5 deg. This wide divergence is explained by the aid of the tests and of calculations. A method of calculation for internal temperatures is given which is based on simple heat laws and on data obtained from many tests. By means of the tests and of parallel calculations, the effect on the internal temperature of changes in frequency, core length, thickness and quality of insulation, armature current density and core densities, is explained. The capacity of the end windings to dissipate heat from the center of the core is also discussed. In conclusion, it is stated that although the wide divergence between surface and internal temperatures of different classes of generators is regular and can be justified, yet it is plain that there is no one average figure that can be used for that difference for all classes of a-c. generators.

THERE have been many attempts, more or less unsuccessful, to establish average figures representing the difference between internal and surface temperatures of different classes of a-c. generator stators. When test results from a large number of generators are reviewed, the engineer's first conclusion is apt to be that there is no relation or consistency between temperatures measured by thermometer and by thermo-couple. One generator may have a difference between inside and outside temperatures of only a few degrees while in others a difference of fifty degrees and more may be found.

If, however, these test results are checked by calculations based on fairly simple laws of heat flow, it will be found that these widely varying results can be satisfactorily explained and shown to be consistent. Such a comparison will also explain why attempts to establish average figures are fore-doomed to failure. Frequency, voltage, quality of insulation, copper and core densities, shape of slots, and core length, all have a bearing on the drop in temperature from the inside to the outside of the stator. With so many factors affecting the temperature drops there is no average figure that can be used for the difference between internal and surface temperatures for all classes of a-c. generators.

In the following pages such a comparison of typical test results and the corresponding calculated temperatures is given. The test figures have been selected for this purpose from some fifty odd tests that have been made under full-load current and voltage conditions on generators ranging in size from 1000 kv-a. to 20,000 kv-a. and in speed from 100 rev. per min. (in vertical waterwheel generators) to 1500, 1800 and 3600 rev. per min. (in turbo-generators). In one series of tests measured temperatures are available under different load conditions and at all important parts of the slot and core. In the other only the maximum internal slot temperature and the surface temperature readings are available. In such cases, however, if the calculated and tested temperatures agree reasonably, the calculated temperature at the intermediate points in the heat path may be considered reliable. We may confidently use these test temperatures and these calculated temperatures, checked by test, to show the magnitude of the effects of the different factors on the internal temperatures.

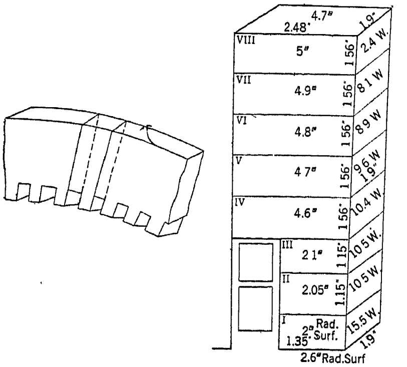

Fig. 1—Segment of Central Package with unit Package Selected Shown in Detail at Right

In obtaining the calculated results, a part of the armature laminations included between two adjacent air ducts, and one tooth pitch in length, is selected for analysis. This portion of the armature with the parts of the armature coils embedded in it will be termed in the paper "a unit package," Fig. 1. As the losses in all unit packages are practically identical, the one in the center of the core is chosen for analysis because the internal temperature rise is usually the highest in that unit package. The losses are calculated under load conditions in the armature conductors and in the different sections of the armature laminations included in this unit package. The method of temperature

TABLE I

	Test 1		Test 2		Test 3		Test 4		Test 5		Test 6	
Kv-a	7,660		3,830		4,570		7,500		2,750		20,000	
Amperes	1,100		550		550		578		255 (2ϕ)		1,750	
Volts	4,000		4,000		4,800		7,500		5,400		6,600	
Per cent inductive P. F.	0		0		0		0		0		0	
Load tooth density in C. G.S lines sq. in.	114,000		105,500		126,300		118,100		120,500		114,500	
Load core density in C. G. S. lines sq. in.	66,000		60,600		73,200		69,000		97,800		67,700	
Arma. copper density, amps. per sq. in.	2,180		1,090		1,090		1,605		2,730		1,845	
Cycles	60		60		60		25		62.5		50	
Rev. per min.	400		400		400		375		375		375	
Core length in inches	23½		23½		23½		45		16½		41½	
Velocity in air gap cu. ft. per min.	8,250		8,250		8,250		6,460		5,700		8,600	
Velocity in air duct cu. ft. per min	3,300		3,300		3,300		2,580		3,800		3,440	
Velocity in core cu. ft. per min.	2,000		2,000		2,000		1,290		2,140		2,230	
Vel. rear of core cu. ft. per min.	1,500		1,500		1,500		950		1,600		1,670	
	Tested values	Calc. values	Tested values	Calc. values	Tested values	Calc. values	Tested values	Calc. values	Tested values	Calc. values	Tested values	Calc. values
Calculated hot spot temp.		**72.5**		**36.5**		**53**		**35**		**80**		**89**
Test thermo-couple temp.	**73.3**		**39.4**		**59**		**36**		**82**		**93**	
Calculated rear of core temp.		**45**		**28.5**		**49**		**22.3**		**65**		**48**
Test rear of core temp. by thermometer	**40**		**30.5**		**47.5**		**23**		**56**		**43.5**	
Max. Temp. Sec. I.	61.7	61.3	44.7	46.3	66.7	68.5		33.1		77.2		70.8
" " " II.		61.4		46.3		69		32.5		77.7		70.3
" " " III.		58.2		45		66.5		30.8		76.7		67.3
" " " IV.	54.5	54	43.6	41.8	62.5	62.1		27.7		74.4		62.2
" " " V.		51.1		39.5		58.3		25.7		71.1		58.5
" " " VI.	54.5	48.7	44.4	37.5	62	54.8		24.4		66.8		55.7
" " " VII.		46.9		35.8		51.8		23.6				53 5
" " " VIII.	47.9	45.7	38.1	34	55.5	50		23.1				52
" " " IX.								22.9				
Thermometer rise arma. copper	**44**		**17**		**26.5**		**36**		**47.5**		**57.5**	

All temperatures listed are temperature rises above inlet air.

calculation used is essentially a cut and try method. A probable armature copper temperature in the central unit package is first assumed; the flow of heat to the air ducts, armature tooth, and to the coil ends is then calculated. If the assumed copper temperature is correct, the heat flow from the copper to the tooth determines the temperature drop across the insulation and the temperature of the armature tooth. The laminations of the unit package are divided into sections to facilitate the calculation of the heat flow through the core. With the tooth temperature as a basis the heat flow from section I to the air ducts, to the air gap, and to section II is determined. This procedure is followed from section to section until the rear of the core is reached when if the proper initial temperature has been assumed the total loss will have been dissipated to the cooling air. If the loss dissipated is greater or smaller than the actual loss in the unit package, a new trial temperature must be assumed and the calculation repeated.

Table I gives the calculated and test temperatures with other data for four generators that are similar in design except for frequency and core length. In test 1, 2 and 3 the temperature measurements were made by thermo-couple at several points of the unit package for three conditions of load. In tests 4, 5, and 6 only the internal copper temperature and the surface temperatures were measured. The close agreement of the calculated and tested temperatures in these last three tests justifies the use of the calculated temperatures for the intermediate parts of the unit package in discussing the effect of different factors on the internal temperatures. While the tested internal copper temperatures vary from 36 deg. to 93 deg. the average difference between the tested temperatures and the corresponding calculated temperatures is less than 3 deg.

METHOD OF CALCULATION

The object of the calculation is to determine the maximum measurable temperature to which the armature insulation is subjected, which in practise is usually the temperature measured by thermocouple placed between coils in the same slot and in the center of the core. This temperature is calculated by the use of simple heat laws and of empirical data based on a large number of experimental tests. The capacity of air cooled surfaces to dissipate heat, the correct velocity of air passing over the air-exposed surfaces of a generator, the temperature rise of that cooling air, and the heat conductivity of the heat conducting paths of a generator, must be determined in order to calculate this temperature. The calculation applies only to radially ventilated salient pole generators with enclosing end bells. For other types of generators suitable changes, based on appropriate test data, must be made in the calculation.

The temperature rise of an air-cooled surface varies with the air velocity and may be determined when the relation between the air velocity and the heat dissipated per unit surface per degree rise are known. It is assumed that the heat dissipated from a surface at any given velocity increases directly with the temperature rise, which permits the use of the equation

$$W = \frac{T}{K} \tag{1}$$

W = watts radiated per sq. in. of surface.
T = temperature rise of surface above cooling air.
K = a constant depending on the air velocity.

The data for the constant K (Fig. 3) have been determined from tests where air at different velocities has been passed over heated surfaces.

In Table I the air velocities through the armature core are tabulated with the corresponding values of K. In long core machines with enclosing end bells insuring practically uniform velocities in the air ducts, the following values generally apply:

(a) The velocity of air in the air gap is 75 per cent of the peripheral speed of the rotor.

(b) The velocity of air in the air duct at the tooth is 30 per cent of the peripheral speed.

(c) The velocity of air at the back of the core is 75 per cent of the air-duct core velocity.

These values are by no means constant but should be varied with different methods of ventilation as well as different proportions of the armature core. They are based on measurements taken on a number of generators and checked, in common with the other constants involved, by the agreement between calculated and test temperatures.

The losses generated in all unit packages are practically identical. For convenience in calculation this unit package is divided into sections as shown in Fig. 1; the dimensions shown in Fig. 1 are taken from the generator in Test 1. The iron and copper losses under load conditions are calculated for each section. An allowance is included for increased saturation due to the influence of the armature reactance, for eddy current losses in the conductors, and when operating at high power factors, for distortional losses. Recognizing that the density in the core is not uniform, the distribution of the loss across the core depth is assumed to vary in such a manner that the loss in the section next to the bottom of the slot is greater than the loss at the rear of the core by the ratio $\frac{\text{pole pitch} + \text{core depth}}{\text{pole pitch}}$. In long core machines there is a negligible flow of heat from the central unit package to the coil ends, so that the watts generated in that unit package must be dissipated through the medium of its own surfaces exposed to the cooling air. In short core machines the heat flowing to the coil ends from the central package should be calculated and deducted.

The cooling air in passing through the machine has its temperature raised by the losses that are dissipated to it. The amount by which the temperature of the cooling air is raised before it reaches the first section of the unit package may be calculated from the fact that 1 kw. of energy dissipated to 100 cu. ft. (2.8 cu. m.) of air at an initial temperature of 25 deg. cent. will raise the temperature of that air 18 deg. cent. in one minute. The temperature rise of cooling air from section to section may be calculated similarly. Due to the rise of cooling air through the air duct, a correction is made for the flow of heat toward the slot from the back of the core by taking a proportionate number of watts from the rear section and arbitrarily adding them to the losses in the first section.

In figuring the heat flow through the package, the following constants are used for heat conductivity through the different materials.

Lengthwise in iron laminations 1.1 watts per cu. in. per deg. cent.

Crosswise in iron laminations 0.045 watts per cu. in. per deg. cent.

Insulation,* 0.0025 to 0.005 watts per cu. in. per deg. cent.

Copper, 8 watts per cu. in. per deg. cent.

The heat conductivity of the insulation not only depends on the materials used but depends greatly on the compactness of

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*"Heat Path in Electrical Machinery", Harold D. Symonds and Miles Walker, *Journal I. E. E.*, Vol. XLVIII., 1912.

the coil. In a loosely wound coil with minute air spaces, a low value of conductivity must be used.

As the first step in the calculation of heat flow, the temperature rise of the copper above the inlet air is assumed. The heat generated in the armature conductors included in the central package must be transferred to the air duct, to the coil ends, or to the armature iron. The temperature drop through the insulation for a given watt flow per 1 in. (25.4 mm.) length of coil is expressed by

$$T = \frac{\text{single wall thickness of insulation} \times \text{watts conducted.}}{\text{mean periphery of coil insulation} \times \text{conductivity of insulation.}} \tag{2}$$

Using equations (**1**) and (**2**) the loss transferred from the copper to the air at the air duct may be determined by trial. The temperature difference between the copper and the iron to permit the transfer to the iron of the remaining loss is calculated from (**2**). This energy is divided equally among the three sections of the tooth.

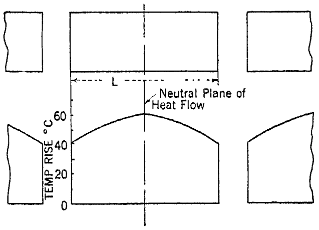

FIG. 2—TEMPERATURE RISE ACROSS LAMINATIONS

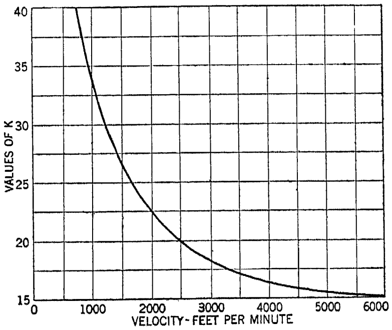

FIG. 3—HEAT DISSIPATION FROM AIR COOLED SURFACES

In determining the temperature difference between the copper and tooth it must be considered that the temperature of the iron of a unit package varies along its length. In any unit package the heat that is radiated to the air duct must be conducted across the laminations. To permit this flow there must be a temperature difference between the laminations at the center of the unit package and at the air duct. The center of the unit package (Fig. 2) is the neutral plane of heat flow and consequently the hottest part of that package. The heat flow from the copper to the iron is then variable. This flow varies along the length of the unit package, being small, if any at the center and increasing from point to point to the air duct. The temperature rise of the laminations in the unit package is assumed to be a straight line. From this assumption it develops that the temperature difference previously calculated between the copper and iron is the temperature difference between the copper and the laminations midway between the neutral plane and the air duct. The heat dissipated to the air duct from section I is calculated by trial from (**1**) and from

$$T = \frac{W}{0.045} \times \frac{L}{8}$$

T = temperature difference between the plane of average temperature and the air duct iron.
W = watts flow per sq. inch from section I to air duct.
L = unit package length.

This last calculation also determines the temperature of the neutral plane. In figuring the air-duct dissipating surface, one-half the surface of the air-duct spacers is considered as effective radiating surface in addition to the surface of the laminations at the air duct. From the temperature of the neutral plane and equation (**1**) the heat flow from section I to the air gap can be calculated.

The difference between the total watts in section I and the watts dissipated to the air ducts and gap must flow to section II. It often occurs that more than the total watts are dissipated from section I to the cooling air in which case there is a flow from section II to make up the deficiency. This heat flow from section I to II or vice versa causes a temperature difference from which the maximum temperature of section II is determined. The total watts in section II consist of the watts from section I the watts from the armature copper, and its own loss. The same procedure is followed as in section I, except that all of the loss dissipated to the cooling air must be by means of the air ducts alone.

This procedure is followed from section to section until the calculation for the last section is completed when, if the correct copper temperature has been assumed, all of the loss will have been dissipated. If there is any energy loss remaining that has not been dissipated, a higher copper temperature should be assumed; if there is not enough loss to satisfy the condition, a lower temperature should be assumed and the calculation repeated.*

The effect of various generator proportions and characteristics upon internal and surface temperatures and on their relative values, based upon actual test data and data obtained by the use of this method of calculation, will be discussed in detail in the following paragraphs.

EFFECT OF FREQUENCY

In Table 1, the 50-cycle generator, test 6, has practically the same core length and armature densities as the 25-cycle generator, test 4, yet its internal copper temperature rise measured by thermocouple is 57 deg. cent. higher than that of the 25-cycle generator.

The maximum temperature rise of the copper above the inlet air is the maximum temperature rise of the armature tooth in the center of the core plus the temperature drop through the armature insulation at that place. The maximum temperature rise of the armature tooth depends to a small extent on the heat flowing to the tooth from the copper and largely on the iron loss in the teeth and core.

The difference in the internal temperature rises experienced in generators of different frequencies is caused mainly by the difference in the core loss. In the 50-cycle generator the losses are considerably greater than in the 25-cycle generator, even though the core densities in the 50-cycle generator are slightly smaller. The temperature difference in the two machines due to the increased iron loss caused by the higher frequency is accounted for very nearly by the temperature rises in section I of the tooth. These rises are 70.8 deg. cent. for the 50-cycle generator and 33.1 deg. cent. for the 25-cycle generator, as compared with the corresponding measured copper temperatures of 93 deg. cent. and 36 deg. cent. It is noted that the difference between the armature-tooth temperature rises for the two generators is somewhat smaller than the difference of the thermocouple temperature rises; this being due to a greater temperature drop through the armature insulation in the 50-cycle generator caused by a greater flow of heat from the copper to the iron.

The 60-cycle generator, test 1, may also be compared with the 25-cycle generator, test 4, to show the effect of frequency on internal temperature rises. The comparison is not as direct as in the previous case, for, although as before the armature densities are practically the same in both cases, the core length of the 60-cycle generator is roughly one-half that of the 25-cycle generator. Yet, in spite of this advantage, the 60-cycle generator armature-tooth temperature rise is much higher than the same temperature rise of the 25-cycle generator, although the difference is not as great as in the previous case. The reasons for the difference in temperature of the 50-cycle and the 60-cycle generators will be discussed later.

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*This form of temperature analysis was devised by Mr. B. G. Lamme a number of years ago, and was more completely worked out and applied by his assistant, Mr. F. T. Hague, in 1913.

If the 50-cycle generator, test 6, were operated at 60 cycles with the same armature densities, the additional losses caused by the higher frequency would increase the temperature rise measured by thermocouple to a still greater value than 93 deg. cent. Thus, in comparing the two generators, tests 4 and 6, on the basis of the extreme commercial frequencies of 25 and 60 cycles, the differences in internal temperature would be even greater than on 25 and 50-cycle basis.

From the test results in column 4 it is evident that in large 25-cycle waterwheel generators the internal temperature will be approximately the same as the copper thermometer temperature and will always be relatively low. This is confirmed by several tests on 25-cycle generators; for example:

Rating	Phase	Rev. per min.	Volts	Surface temperature rise	Internal temperature rise
12000 kv-a.......	3	116	11000	47	47
12000 kv-a.......	3	94	11000	27.5	33
4500 kv-a.......	1	500	11000	45.5	55.5
7640 h.p.......	3	375	13000	31.5	35.5
6250 kv-a.......	3	1500	6600	22	31.

For 50 or 60-cycle maximum rated generators there is always a large difference between thermometer and thermocouple readings. The thermometer readings in many cases are not appreciably greater than for 25 cycles but the thermocouple readings show a large difference over those for the lower frequency.

EFFECT OF CORE LENGTH

A change in core length affects the internal temperature in two ways; it changes the volume of armature material to be ventilated and to a lesser extent it changes the proportion of heat transmitted to the coil ends. Of the two effects the change of armature material to be ventilated with a change of core length is the most important. With a given pole arrangement the inlet area, between the poles, for the armature ventilating air is fixed. An increase in core length, with other conditions remaining the same, results in a proportional increase in the armature losses. The volume of cooling air through the fixed inlet area remains practically constant with a given fan pressure so that there is less cooling air per kw. loss than before and consequently the temperature rise increases. In the case where a given generator is increased in core length and it is required that the temperature rise be kept constant, it is often possible to change the ventilating fans to increase the air pressure, which results in an increase in air volume. This increase is usually not commensurate with the increase in armature loss, so that this loss must be reduced by using more material. This makes it evident that there is a definite core length in radially ventilated generators for each number of poles and each rotor diameter that it is inefficient to exceed.

The end coils which are directly exposed to the cooling air have the capacity to dissipate heat; this capacity varying with the temperature difference between copper and air, the air velocity, and with the quality and thickness of the insulation. With the heat dissipating capacity of the coil ends constant, it follows that with an increase in core length the heat flow decreases from the center of the embedded armature coil to the coil ends. For long-core generators, *i.e.*, generators with a core length greater than 30 in. (76.2 cm.), it has been found by checking a number of tests, that with a normal winding practically no heat flows from the center of the coil to the coil ends. On the other hand in short-core generators an appreciable part of the "buried" copper loss will be dissipated from the coil ends.

The two generators, tests 1 and 6, afford an excellent example of the increase in temperature rise with an increase in core length; the generator in test 1 having the greater frequency, but other conditions remaining practically the same. The temperature increase of the long-core generator, test 6, of 19.7 deg. cent. more than the generator, test 1, is caused principally by the poorer ventilation of the long-core generator due to its greater core length. The coil ends but slightly affect the temperature difference in this case, for in one generator only 10 per cent of the "buried" copper loss of the central unit package is dissipated to the coil ends while in the long-core generator practically none of that loss is conducted from its central unit package to the coil ends. This temperature difference is all the more noticeable for the " short-core " machine has a higher frequency so that were the generators the same frequency, other characteristics the same as at present, the temperature difference would be still greater.

*EFFECT OF THICKNESS, COMPACTNESS AND QUALITY OF INSULATION

In long-core a-c. generators the flow of heat from the armature copper to the cooling air is through the insulation to the laminations, so the temperature drop through that insulation is an important factor in determining the internal copper temperature. This temperature drop varies inversely as the heat conductivity, which in turn depends on the compactness and quality of insulation. The numerical value of the heat conductivity of armature insulation varies over a broad range, being different even in samples made up of the same insulating materials; this difference being due to the effect of air pockets in the insulation. Tests made by H. D. Symonds and Miles Walker, previously referred to, illustrate the great effect of air pockets on the value of heat conductivity of insulation. In these tests the heat conductivity of insulation composed of empire cloth and mica wound tightly was almost double that of the same material wound loosely. These tests, checked by the calculations of generators, indicate that the heat conductivity of armature insulation varies within the approximate limits of 0.0025 to 0.005 watts per cu. in. per deg. cent. The lower value is for hand wrapped unimpregnated insulation and the higher value for compact machine wrapped insulation.

The temperature drop through armature insulation also varies with the thickness of the insulation. This means that the higher the voltage of a generator for a given heat flow the greater will be the temperature drop across the insulation providing the same type of insulation is used.

In the generators under discussion the armature insulation for the one in tests 1-2-3 consists of mica and fibrous material; the entire coil being hand wrapped and thoroughly impregnated to eliminate as completely as possible minute air spaces. The conductivity constant used for this material is 0.004 watts per cu. in. per deg. cent. which, in test 1, results in an average tempera-

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*"*Rational temperature guarantees for large a-c. generators*", TRANS. A. I. E. E., 1916, Vol. XXXV, Part II, p. 1489.

ture drop in the central unit of 22 deg. between the copper and iron.

For the generator in test 5, the armature coil is made up of stranded conductors insulated from each other with asbestos and mica, the whole then insulated with a wrapper of thin paper covered with splittings of mica. This wrapper is applied while heated and under tension, so that when completed the coil is a relatively solid mass with a high percentage of mica. Due to the use of superior insulation, the insulation thickness is the same as in the previous case, even though the voltage is somewhat greater. In this case with approximately the same heat flow per square inch as for the generator, test 1, the temperature drop through the insulation is only 16 deg.

If these generators had been wound for 13,200 volts, the insulation thickness would be approximately doubled. The temperature drop through the insulation would increase approximately in the same proportion to the increase in insulation thickness, which in turn would increase the internal temperatures.

Effect of Current Density

The current density has an important and direct bearing on the internal temperature since it largely determines the heat loss that must be transmitted through the insulation. The total copper loss is the I^2R loss plus the eddy current loss. Neglecting the eddy current loss the drop through the insulation in any given generator is very nearly proportional to the square of the armature current or to the square of the current density. A low current density reduces the drop not only by reducing the loss but also by increasing the area of the heat path, since a larger slot is required for the increased copper section. With a given limiting temperature drop through the insulation, the use of machine wrapped mica insulation, with its better heat conductivity, justifies the use of higher armature copper densities.

The test temperatures in tests 1 and 2 clearly illustrate the effect of the armature current density on the temperature drop through the insulation in the same generator. In test 1 with a current density of 2180 amperes per sq. in., the armature copper temperature rise is 11.6 deg. cent. higher than the maximum temperature of section I. In test 2 with a current density of 1090 amperes per sq. in. the armature copper temperature rise is 5.3 deg. cent. lower than the maximum temperature of section I.

EFFECT OF TOOTH AND CORE DENSITIES

The tooth and core densities have a similar effect on the internal temperature as the generator frequency, since both affect the copper temperature through the core loss.

Tests 2 and 3 bring out very clearly the effect of increasing the saturation of the armature iron, other conditions remaining the same. The test conditions for test 4 were exactly the same as for test 3 with the exception that the armature voltage was increased 20 per cent which proportionately increased the tooth and core densities. The result of this increase was to raise the internal copper temperatures from 39.4 deg. to 59 deg. cent. even though the copper loss remained the same. This difference in copper temperatures is very nearly the same as the difference in the maximum temperatures of the armature iron; indicated in the table by the test temperature of section I. The difference in copper temperatures in the two tests is slightly less than the difference in the iron temperatures, this being due to a slightly increased heat flow in test 4 from the iron to the copper. Both tests and calculations show that a variation of the iron density alone produces a change in the internal temperature of the copper almost proportional to the change in iron loss.

EFFECT OF THE END WINDINGS ON THE INTERNAL TEMPERATURES

It has been previously pointed out that the end coils usually have the capacity to dissipate heat from the embedded part of the armature coil. The test temperatures in tests 1, 2 and 3 show roughly the effect of this flow under different conditions of load for the same generator. In test 1, with a comparatively large copper loss, the temperature of the armature copper is 11.8 deg. cent. higher than the maximum temperature of section I indicating a comparatively large flow of the buried copper loss from the copper to the iron. In test 2, with roughly one-quarter of the copper loss in test 1 and a slightly lower iron loss, the armature copper temperature is 5.3 deg. cent. lower than the *maximum* temperature of section I, indicating practically no heat flow between the copper and the iron. In test 3, with the same copper loss as in test 2, but a greater core loss, the armature temperature is 7.7 deg. cent. lower than the maximum temperature of section I, indicating that there is a small heat flow from the iron to the end coils in addition to that of the buried copper loss.

The calculation of the heat flows from the central unit package to the end windings in these three tests not only confirms the test results, but gives more exactly the actual conditions of that heat flow. In test 1, 10 per cent of the buried copper loss of the central unit package was conducted to the end coils. In test 2 all of that loss was conducted to the end coils besides which 0.4 watt iron loss was conducted from the central unit package through the insulation to the end coils. In test 3, all of the buried copper loss was again conducted to the end coils, and the iron loss conducted from the central unit package was increased to 1.4 watts. This is equivalent to approximately 25 per cent of the buried copper loss in the central unit package.

The increase of end winding temperature of 19.6 deg. cent. in test 3 over test 2 shows that in fairly long-core machines it requires the cumulative effect of but a very small watt flow from each unit package to raise the temperature of the end windings a large amount. With long-core high-current-density machines with a large temperature difference between the center of the machine and the cooling air at the end coils, a very small percentage watt flow per inch length of coil will satisfy that difference and this flow may be neglected when analyzing the heat flow of the central unit package.

RELATION OF INTERNAL TEMPERATURES TO SURFACE TEMPERATURES

It has been shown previously in this paper that the maximum tooth temperature of a generator is very closely related to the internal copper temperature. An inspection of the detailed test temperatures in tests 1, 2 and 3 will show that the core surface temperature measured by thermometer varies almost in proportion to the tooth temperature.

A thermometer placed on the armature end coils rises after shut down when the flow of the ventilating air is stopped so that the recorded test temperature rise of the end coils will be between the surface temperature of the coils during running conditions and the internal copper temperature of the end coils. This test temperature in any machine wil be nearest the operating surface temperature or the internal temperature of the end coil copper, dependent on the rate of cooling of the generator after shut down, on the methods of placing the thermometers on the coils and on the degree of sluggishness of a thermometer in recording temperature changes.

The variations incurred in measuring the end coil temperatures make the relation between the internal copper temperature and the end coil temperature less direct than in the case of the iron temperatures. Nevertheless, an inspection of tests 1, 2 and 3 shows that a change in the internal copper temperature results in a corresponding change in the end coil temperature as measured by thermometer.

CONCLUSION

The results of the tests in Table 1 show that there is a wide divergence between internal and surface temperatures in a-c. generator stators. In the preceding pages it has also been shown that the amount of divergence of the surface temperature from the maximum internal temperature depends mainly on frequency, core length, and armature voltage. Of the generators listed the greatest difference between surface and internal temperature is found in the 50-cycle generator, test 6; this temperature difference being 35.5 deg. cent. For a generator of the same characteristics except designed for 60 cycles and 13,200 volts, it is safe to say that the temperature difference between the surface and internal temperatures would be approximately 50 deg. cent. In the 25-cycle generator, test 4, which is similar to the 50-cycle generator, test 6, with the exception of the frequency, the internal and surface temperatures are the same. The results of these two tests show clearly there is no one value that applies to the temperature difference between surface and interna temperatures for all classes of a-c. generators. Furthermore, it is evident from these tests that surface temperatures measured by thermometer give no indication of the maximum temperature to which the insulation is subjected, unless all the factors that effect that temperature are given due consideration.

DISCUSSION ON "INTERNAL TEMPERATURES OF A-C. GENERATORS" (KELLY), NEW YORK, FEBRUARY 14, 1917.

Henry G. Reist: This paper shows very clearly the method of analysis for the location of the losses that has become necessary to produce the large turbo-generators which are now being built. In these machines the losses are very great, and as the author points out, the distances to the end of the core and to the back of the core are so large that very little heat can be taken off from those parts, and the heat must be transferred to the cooling air near the point where it is generated.

This is usually done by having radial or axial air ducts. In the case of axial ducts, they must be placed very close to the coils, because as is clearly shown the heat generated in the teeth and in the slots is very much greater than in any other part of the machine. This heat cannot be conducted to the back of the core on account of the surprisingly large temperature drop in the iron, good a heat conductor as it is.

I noticed in the generators in the Table I that test 1 and test 3 have apparently a higher heat drop from the inside of the coil to the outside than test 5, and that the difference was due to the method of applying the insulation.

It will be noted that the other generators have a lower potential and a larger output than No. 5. They probably have larger conductors, and possibly have greater eddy currents. If this was not taken into account in making the calculations, it may possibly account for part of the apparent difference. I do not think that it makes any difference whether the insulation is hand applied or machine wrapped, so long as it is applied with equal compactness, which may readily be accomplished by removing the air and by properly pressing and cementing the coils.

Comfort A. Adams: It is only within the past very few years that any quantitative recognition has been taken of this drop of temperature between the hot spot and the surface where the thermometer temperature is obtained. When a large turbo-alternator would burn out with surface thermometer temperatures of 90 deg. cent. it was hard to make the operating engineer believe that the type of insulation employed would safely withstand 125 deg. cent. continuously. It is a fact, however, that in some cases of long-core machines with poorer ventilation than is now generally provided, this temperature drop is sometimes 60 deg. or 70 deg. cent.

Thus it is that the designing engineer has been forced to develop some method of approximately predetermining this temperature drop. This is quite consoling to the college professor who has been emphasizing the importance of such analysis for many years.

Even were the method more tedious than that proposed by Mr. Kelly, it would be well worth while, if the results obtained are reasonably reliable, since the ability to predetermine the approximate hot-spot difference in an entirely new machine may save much time and money, cutting and trying. This whole thermal problem is at once one of the most important and one of the most complicated problems to handle in connection with the design of electrical machinery. The accuracy with which temperature rise is predetermined by ordinary methods, is almost negligible.

If I were to offer any criticism of the paper, it would be to express some doubt as to the assumptions made as to the air velocities in the gap and ducts.

First, it is necessary to define what you mean by the velocity of air in the gap relative to the stator surface. There are two velocities, peripheral due to the rotation of the rotor, and axial usually due to a forced draft supplied either by fans on the armature shaft, or by external fans. Moreover the velocity varies at different points radially across the gap.

Second, in what degree does the heat dissipating power of the stator surface depend upon the air velocity.

These quantities are not only difficult to define, they are more difficult to measure. However, it is always possible to choose a velocity and a coefficient of the velocity term in the expression for heat dissipation per unit of surface, which will (in a method such as that here given) yield a result consistent with that determined experimentally. It is obvious also that a lower velocity with a higher coefficient (of the velocity term above mentioned) will yield the same result, or vice versa. But although any one of these combinations may yield correct results for a particular machine or possibly for a particular type of machine, it is pretty sure to lead one astray when applied to a machine of different proportions unless it is the one correct combination.

I should like therefore to ask Mr. Kelly as to the basis of his assumptions with regard to the air velocities in gap and ducts.

W. F. Dawson: I suggest that the temperature coil in addition to filling its primary function of measuring the maximum internal temperature of the apparatus can also be made very useful in detecting excess losses, particularly those in armature conductors. Coils placed between the top and bottom armature coils in the slot portion give a reasonably accurate measure of the maximum internal temperature of the coil; while coils placed at the bottom or side of the slot give a lower temperature unless the tooth loss is comparatively high. The difference in the readings of these two temperature coils gives an indication (though not an accurate value) of the temperature drop across the insulation. As designers have a fairly accurate knowledge of the temperature gradient of standard insulating materials with known losses, excess losses such as those due to Foucault currents will be promptly indicated by an excess drop across the insulation.

Another method of detecting excess losses by means of temperature coils is available to designers, namely: The checking of the actual rate of initial temperature increase due to normal load with the estimated rate of increase. The estimated rate of increase will depend upon the average current density and upon the amount the thermal capacity of the copper conductor is increased by the insulation and other surrounding material. This increase in the thermal capacity of the copper conductor averages about 100 per cent and thus a current density of 1100 amperes per sq. in. will give about one-half deg. cent. rise per minute instead of the theoretical 1 deg. rise. The rate of temperature increase varies with the square of the current density.

I would point out particularly, in respect to high-speed apparatus, like turbo-generators, that we have by no means shown up all our losses when we take the I^2R for copper losses (from known resistance) and core losses, not even when we have added to these the known parasitic losses in the copper and the core. The windage losses are very important, and that makes computations of this sort difficult. Our experience indicates that the windage loss of two-pole turbo-alternators is approximately equal to the core loss and yet we do not consider that excessive. The core loss in turn is two to two and one-half times the total armature copper loss, so it can readily be seen that the core loss and windage play an important part in the losses and heating of the machine, much more so in general than the copper losses.

All of the speakers and the author of the paper have pointed out the importance of having sufficient cross-section through the air gap to feed the radial ventilating ducts. That is particularly important at the present time when we are reaching out for greater outputs, sometimes reducing the air gap, and constricting actual axial cross-section in other ways. Unless care is exercised the machine may easily be too long to have the center properly cooled by air fed from the ends through the air gap. This limitation however, can be overcome to a certain extent in various ways, for example, by leading axial canals from the end or pressure chamber axially along outside of the stator punchings, directing the air into the ventilating ducts opposite the center of the machine, thence through the air gap and out through other radial air ducts. This plan has been adopted on certain long machines and the general result has been very satisfactory.

H. M. Hobart: I share Prof. Adams' opinion that the problem of ascertaining internal temperatures is one of great difficulty. I do not think that by any method now available, we can determine the internal temperatures of machines with any close degree of accuracy. The degree of accuracy to which we can do it, unless we go to great expense, is roughly of an order in which at least ten per cent error will be involved. It is not a matter of a few per cent; it is out of the question to attain to such accuracy as that; it is more apt to be a matter of 10 per cent or 20 per cent.

I want to call attention to the undesirability of pinning too much faith to the results obtained with internal temperature detectors. They were quite rightly welcomed a few years ago as a valuable addition to our means of informing ourselves about the internal temperatures of electrical machinery. I think there is a danger of accepting the results obtained by such temperature indicators as too much superior to other means of ascertaining temperature. Cases will frequently arise where resistance measurements will disclose higher temperatures than those obtained by internal detectors, and also cases do arise every day where the thermometer will give higher temperatures than resistance measurements, for instance, in long shallow field spools.

I am of the opinion that one should use all these means to become acquainted with the temperature of a generator on the occasion of a test, and it is by no means a foregone conclusion which method will show the higher result. In one type of machine it will generally be a particular one of the three methods, in other types of machines it will be another one of the three methods, and it is dangerous to reach conclusions without making use of all available means.

My opinion is that the superiority of the embedded detectors, as compared with the determination of the temperature by resistance, is that the latter is based on the assumption that we have knowledge of the cold resistance at an exactly known temperature. This knowledge is generally not of the exact character credited to it. The temperature which should be coupled with the cold reading, the temperature the coil has at the time of the cold reading, is often quite different from that which we assume it to be. The error is apt to be especially great in the case of large machines.

But embedded resistance detectors are calibrated at a definitely known temperature at which they have a definite resistance. They are true thermometers, and they can generally be depended upon to be quite reliable. They also have the advantage that we can keep track of the temperature through the progress of the heat test. These are their chief advantages, rather than that they necessarily disclose any greater approximation to the hottest-spot temperature attained.

In listening to Prof. Gray's very clear description of the paper, my attention was drawn to his allusion to the dependence of the core losses on the frequency. There is some discussion of it in the paper. He spoke of the core loss as being more or less proportional to the square of the frequency for a given density. I should like to call attention to the fact that an analysis of any large number of machines will show a much closer approximation if the core loss is taken as the product of the first power of the frequency and the first power of the density.

I have often made such studies and have taken the opportunity on several occasions to draw attention to the results, but the old traditions persist. Too much credit is given to the curves obtained on laboratory samples, so far as relates to the function of the frequency or density which the core losses show on such tests, and we shut our eyes to the fact that it is utterly otherwise in the actual machine.

Comfort A. Adams: I agree with Mr. Hobart on one point, namely that the ordinary calculations of core losses carried out in the conventional manner are apt to be very wide of the mark. This is not due to too much theory but to too little.

Some of the reasons why our core-loss calculations are wild are as follows:

(a) We assume a uniform average peripheral density in the core back of the slots.

(b) We neglect the difference between the alternating flux in the back layers and the nearly revolving flux variations in the front layers. In fact we know as yet comparatively little about this difference.

(c) We assume some crudely approximate equivalent average value of the flux density in the teeth.

(d) We neglect the effect of the transverse peripheral or leakage flux in the teeth.

(e) We usually do not attempt to compute the wave losses in the pole faces or in the tips of teeth facing teeth.

(f) We neglect the tooth frequency losses in the teeth. These sometimes extend a short distance into the core back of the teeth.

(g) We neglect the tooth frequency pulsations throughout the whole magnetic circuit due to pulsations of reluctance as the teeth pass the poles or opposing teeth.

(h) Any of these high tooth frequency pulsations are accompanied by additional losses due to the skin effect in the laminations.

(i) Finally there are the commonly recognized uncomputable losses due to imperfect insulation between laminations, filing, burring, etc., also losses in the heavy end plates, supporting castings, bolts, etc.

Of all these, the tooth frequency losses with the extra loss due to the skin effect, are probably the largest.

As an illustration take the first large steam-turbine-driven induction generator tried several years ago at the Interborough power station. The slot openings were relatively large and the air gap short, and the tooth frequency losses so great that the machine overheated at no load, and had to be redesigned in this respect.

The fact is that in our ordinary core-loss calculations we really compute only from one-third to one-half of the total core losses and apply a *factor of ignorance* of from two to three (sometimes it should be even more).

In the Electrical Engineering Research Division at the M. I. T., we have had a research assistant struggling with this problem for over two years. In a few years more we hope to have developed a method which will require a slightly smaller factor of ignorance.

Alexander Gray: When first I started to teach the principles of electrical design to students I was tempted to use Mr. Hobart's formula that

$$\text{Iron loss} = K \times \text{frequency} \times \text{flux density}$$

The formula is simple but it happens to be a providential fact and not according to fundamental principles that under normal operating conditions the value of K is approximately constant for machines of different frequency.

If the law held generally, then the curve of core loss plotted against flux density would be a straight line, but the experimental curve bends over and the loss is more nearly proportional to the square of the flux density.

We teach the student that

$$\begin{aligned}\text{Iron loss} &= k_1 B^{1.6} f = k_2 B^2 f^2 \\ &= k B^n f^n \text{ for a standard line of machines}\end{aligned}$$

The actual value may be found from a series of experimental curves of iron loss against flux density B for different values of frequency f.

With regard to the paper itself, it has already been pointed out that the assumptions made might be considered unreasonable. Thus the air velocity past the teeth is not necessarily twice that past the iron behind the teeth, nor can we readily divide the iron losses into tooth loss and loss in the remainder of the core; the stray loss also due to eddy currents in the conductors is hard to predetermine. By some designers it is claimed that air blown parallel to the laminations is not as effective as is air blown across the ends of the laminations, but the experimental data on this subject are contradictory. Other tests again have indicated that the barrel surface of a core is not as effective as is the surface of the cylinder ends. In spite of all this however the author is within his rights in making a reasonable set of assumptions and by applying a cut and try method on machines on which data are available to obtain a set of constants that may then be applied in the predetermination of the characteristics of another machine of a similar type. Designers are doing just this thing every day. I therefore agree with the author that it is possible to come much closer to the value of the hot-spot temperature by reasonable assumption and by calculation, than by any kind of guess work.

Hot-spot temperatures is one of the subjects that have been thrust upon us. The early Niagara machines had high internal temperatures for twenty years before the fact was recognized, because they were insulated with mica. The trouble became a live one about eight years ago when high-speed long-core machines with cotton insulation began to break down due to charring of the insulation in the centre of the core. Methods for predetermining the hot-spot temperatures then became necessary and one of the most rational is that suggested by the author of this paper.

H. M. Hobart: If Prof. Gray will again look up the publication to which he refers*, he will see that I specifically disclaimed the interpretation he gives. My point is, if we take the *working* density of a 25-cycle machine, and multiply by the periodicity, *i.e.*, 25, and take the working density of a 50-cycle machine, and multiply by the periodicity, *i.e.*, 50, these products will be criteria of the working core loss per pound; thus giving us a means, a purely empirical means, of getting at the core losses. I do not assert that the core loss is proportional to the product of the frequency by the density, but merely wish to point out that this product provides an empirical means of arriving at the core loss. It is simply a case where we must resort to empiricism. Prof. Adams states that there are fifteen factors affecting the core loss and that at present we only understand two or three of them. We cannot reasonably expect to obtain the aggregate result due to fifteen causes by estimating the result which would be occasioned by the two or three understood causes and then multiplying by a factor to take into account the 12 or 13 causes whose laws have not yet been discovered. If by any happy chance we find two quantities which we can multiply together, whose product for a group of one hundred machines is found to be a function of the core loss, we ought to be glad to have such a useful empirical rule. We should not point to it with any particular pride, but it is useful in the hard tasks of daily life in making machines to do what we want them to do.

A. M. Gray: If I take a 25-cycle machine, with ten kilowatts core loss, and sell it as a 50-cycle machine, do I get 20 kilowatts core loss or 40?

H. M. Hobart: It is rare that you can sell a 25-cycle machine for a 50-cycle machine and get good results. If with all the data available along the best lines, Mr. Gray designs a 25-cycle machine, using the best approved practise, with regard to working density, and multiplies the density by 25, he will find it bears the same relation, roughly, to the core loss that he will find if he designs the best 60-cycle machine he can design when he multiplies its density by 60.

A. M. Gray: You are dealing particularly with the density?

H. M. Hobart: I am limited to that.

A. M. Gray: I would point out the difficulty the teachers have. If we ask a student to predetermine the core-loss curve, we do not care much for the actual curve, but like to get it in the right shape.

H. M. Hobart: I know from experience that the teachers' task is quite difficult.

W. F. Dawson: I think possibly there is a misconception in reference to this matter of core losses. There certainly is a difference between the results achieved with the old style armature iron and the modern alloy steel. With the old style iron, the eddy-current losses, which increased as the square of the frequency, or thereabouts, predominated, whereas with the alloy steel the resistance is so high that the eddy-current losses are almost negligible at any regularly generated frequencies. Recently I was consulting certain curve sheets, and found that the core losses for that alloy steel varied as the first power of the frequency. I do feel that that would not be the case with the old style iron having a resistance, say, of one-quarter or less, that of the alloy steels.

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*See pp. 112 to 118 of Parshall and Hobarts "Electric Machine Design" (John Wiley & Sons.)

A. M. Gray: Was that a rotating machine or a transformer?

W. F. Dawson: That was a transformer.

A. M. Gray: The trouble is that the other thirteen you are losing, are all eddy currents.

W. F. Dawson: That would probably not be so if you have large air gaps.

A. M. Gray: I agree perfectly with respect to transformers, but I have not found that the alloy steel makes very much difference in not only the shape but the magnitude of the core-loss curve, in any machine except the turbo-generator.

Ralph Kelley: Mr. Reist brings out the point that the type of ventilation used for a generator, whether radial or axial, must be effective in that it cools directly the teeth and the armature coils. Thus, where axial vents are used, the vents must be placed in the teeth or directly in back of the slots to avoid an excessive temperature drop between the copper and the duct. The problem of ventilating a long-core machine evolves itself into the problem of forcing a sufficiently large amount of cool air to the center of the core to dissipate the losses generated at that part of the machine. As stated in the paper, there is a certain core length for each type of machine beyond which it is impractical to radially ventilate the machine, and axial ventilation must be resorted to for a core length greater than this value. The method of calculation given is of service in determining that limiting core length, and particularly, for axial ventilated machines, is of service in determining the number and position of the vents.

It is true that the generator in test 1 and 3 has larger eddy-current losses than the generator in test 5, but allowance for this increase was made in the calculation, so that the difference in temperature drop across the insulation represents largely the difference in heat conductivity between the two types of insulation. The same difference appears in other calculations and tests of machines insulated with hand-wrapped and machine-wrapped insulation, where it was reasonable to expect that they had the same amount of eddy-current loss in the armature copper. The present hand-wrapped coil under the best condition of pressing and impregnation is not as compact a coil as the machine-wrapped coil where the wrapper is applied under tension while subjected to heat.

In answer to Prof. Adams' question, the velocities in the air ducts were determined by anemometer measurements of the air velocity of the air ducts at the back of the core. This measurement is necessarily approximate, and as Prof. Adams points out, may be in error, and still give correct results for one machine or a group of similar machines when combined with a like error in the heat conductivities. However, for a large number of different types of machines, any consistently wrong value of air velocity or heat conductivity would not give correct results for all the machines. It is important to remember that the air velocities are by no means fixed percentages of the peripheral speed, and should be varied to suit the different types of machines and ventilation.

Mr. Dawson brings out a very good point in his statement that temperature detectors, properly placed, are of use in locating and approximating excess losses in a generator. There is no doubt that the use of temperature detectors, combined with proper temperature analysis are useful in locating excessive local temperatures caused by the parasitical losses.

Mr. Hobart is very pessimistic about the possibilities under present-day conditions, of even approximating the real internal temperatures of large a-c. machines. Readings obtained from temperature detectors in themselves do not give the actual internal temperatures, but when these results are carefully combined and analyzed, the true internal temperatures can be very closely approximated.

I do not agree with the statement that it is a matter of everyday occurrence for the thermometer readings to be higher than those obtained by internal temperature detectors. From a large number of tests in which temperature measurements were made by the three regular methods, in only one machine were the thermometer temperatures higher than those measured by thermocouple placed between coils in the center of the core. This high thermometer temperature was measured on the coil ends, and, on investigation, it was found that the coil ends received practically no ventilation. Consequently, the loss developed in the end coils had to be conducted to the iron, and dissipated to the air duct. This heat flow raised the temperature of the end coils higher than that part of the coil embedded in the core.

In but two or three of the number of machines in which tests are available, is the temperature rise by resistance higher than that measured by thermocouple. These machines are all 25-cycle machines with low temperature rises by thermocouple and poor end-coil ventilation.

I am inclined to criticize Prof. Adams' discussion and list of inaccuracies and omissions in the present manner of calculating iron loss as being misleading to the engineer who is not actively engaged in design work. That these inaccuracies and omissions are real is unquestioned, but, with the exception of the pole-face loss, these factors are in the nature of a refinement in the calculation and affect the result by only a small percentage as compared with those caused by variations in manufacture. When it is considered that machines of the same electrical design rarely have the same tested core loss, due to manufacturing variations, too much weight should not be given to the greater accuracy to be achieved by greater refinements in calculation. The tooth pulsation loss, however, is of prime importance, and often amounts to comparatively large values in high-speed machinery. Further investigation regarding the characteristics of that loss would be of great value to the designing engineer.

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REACTORS IN HYDROELECTRIC STATIONS

BY J. ALLEN JOHNSON

Abstract of Paper

The advantages and limitations in the use of current limiting reactors in hydroelectric stations are not so well recognized or understood as they are in steam turbine stations, but it is now coming to be realized that their use may be quite as justifiable and necessary in the former as in the latter.

The two beneficial results of reactance, protection and localization, are distinct: the former being associated with the square, and the latter with the first power of the reactance.

Of two detrimental results of reactance, or limitations to its use, one, that of voltage drop, is well understood but the other, the reduction of synchronous stability is not so well understood.

The installation of reactance usually results in a decrease in the stability of synchronism. Such stability may be expressed in terms of the angular phase displacement between two groups, due to a sudden load disturbance. Practically, complete instability or asynchronism occurs in a large station when the phase angle exceeds 90 deg. The asynchronizing effect of a sudden load change is approximately twice that of a gradual change of the same magnitude.

The origination of a power surge by sudden loss of load and the accompanying hunting oscillation is described. Increased reactance increases the amplitude of both the power surge and the phase-angle oscillation and also increases the period. A certain amount of reactance will result in complete asynchronism immediately following the load disturbance, and a smaller amount may cause troublesome and persistent hunting by forced harmonic oscillation of the turbine governors in combination with certain hydraulic conditions, such as long penstocks.

It is shown that these phenomena form a practical limitation to the use of reactance, in that the reactances necessary to create the conditions described are of the same magnitude as those often indicated for protective and localizing purposes. The actual occurrence of a surge is described.

THE THEORY and practise regarding the use of current-limiting reactance in steam turbine stations are now fairly well established and the advantages and limitations of reactors are well understood. Their use in hydroelectric stations, however, is not so general, nor are the limitations imposed by such service nearly so well understood as are those of the steam station.

The reasons for this apparent slowness on the part of the large hydroelectric stations to realize the benefits of reactance are that the number of such stations is comparatively few, they do not in general lend themselves so readily to reactor protection, their growth to mammoth proportions has been somewhat slower, and so long as the principal function of reactance was believed to be the protection of the generating equipment, the necessity for it in the hydroelectric station was not apparent.

Recently, however, it has been realized that the chief function of reactance is not the protection of the generators but the so-called "protection of service" or what the writer prefers to call the isolating or localization of disturbances. With this new viewpoint it has rapidly become apparent that reactance is just as necessary to the hydroelectric stations and just as valuable to them as to the steam turbine stations.

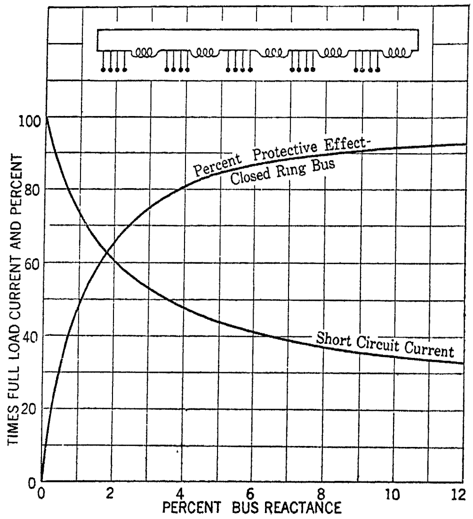

FIG. 1—PROTECTIVE EFFECT OF BUS REACTANCE

Generator reactance—20%
20 generators in groups of 4
Bus reactance 4%
Bus reactance based on rating of one generator
Fault at any point on bus

$$\text{Per cent protective effect} = 100\left(1 - \frac{x_0^2}{(x_0 + x_2)^2}\right)$$

x_0 — resultant reactance of generators alone
x_2 — resultant reactance external to generators

The load of a large hydroelectric station usually divides into three groups:

First, the existing local industries which adopt electric power when it becomes available.

Second, industries which build up about the power station, due to the availability of power at a low price.

Third, long distance load supplied to cities within transmission distance.

The requirements of each of these three groups, as well as the likelihood of their causing interruptions differ materially, making it desirable for their mutual benefit that they be operated from separate bus sections, with the consequent sacrifice of simplicity, capacity and flexibility which such subdivision always introduces. Without, therefore, pursuing this phase of the matter further it is at once evident that the hydroelectric station has use for reactance, as by its means the whole station may be operated in multiple while at the same time the several sections may be protected from each other and each section from the individual lines which it feeds. Troubles may be localized or isolated practically where they originate, without communicating their evil effects to other points.

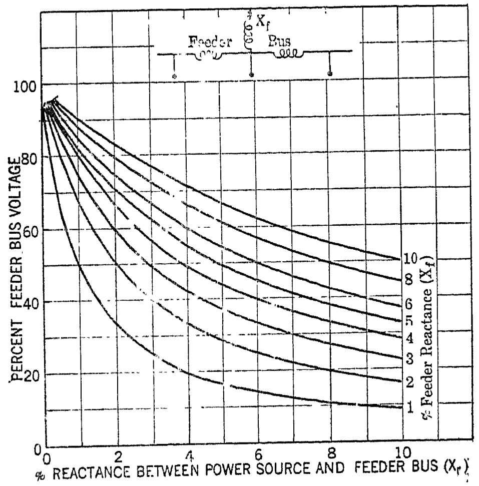

FIG. 2—ISOLATION EFFECT OF FEEDER REACTANCE

All reactance based on same kv-a. rating

$$\text{Per cent feeder bus voltage} = \frac{x_f}{x_r + x_f} \times 100$$

The use of reactance is also often required, especially in older stations, for the protection of apparatus other than the generators, such as oil switches and wiring which have been outgrown by the expansion of the plant but which cannot readily be strengthened or replaced.

The protective and localizing functions of reactance are quite distinct. The former, since all the evil effects of heavy current,—mechanical forces, heating, etc.—are proportional to the square of the current, is measured in terms involving the square of the total reactance; and the latter is measured in terms of the first power of the reactance involved. (See Figs. 1, 2 and 3.)

Unfortunarely the installation of reactance gives rise to increased voltage drops and phase displacements under load conditions which place a limit upon the amount of reactance that can be used. Each installation is in this respect a problem by itself. (See Fig. 4.)

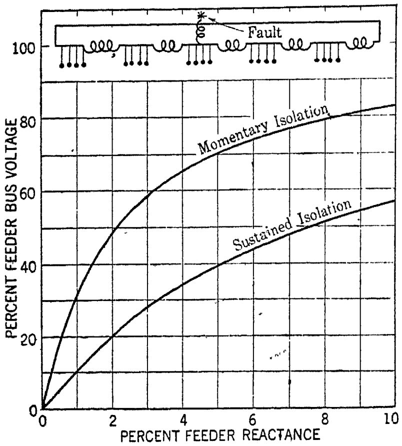

FIG. 3—MOMENTARY AND SUSTAINED ISOLATION BY FEEDER REACTANCE

Momentary generator reactance—20%
Synchronous generator reactance—50%
Bus reactance—4%
20 generators in groups of 4
All reactances based on rating of one generator

Owing to the different natures of the three load groups above referred to, and to the diverse character of the apparatus and transmission lines required to supply them, it will usually be found that the effective localization of disturbances requires the installation of reactance in two locations; in the bus between sections and in the individual outgoing feeders.

In connection with the bus reactances a new limitation now appears which apparently has not heretofore been recognized, due perhaps to its unimportance in the steam turbine station. I refer to the decrease in the synchronous stability of the station with the installation of reactance in the bus.

There seems to be a somewhat general impression that the addition of reactance increases the synchronous stability. Such, however, is by no means necessarily or even usually, the case. It is a more or less well known fact, which however can be readily demonstrated (See Appendix A) that the synchronizing power between two generators is a maximum with reference to the circuit constants when $x/r = \sqrt{3}$, or $x = 1.732\, r$.

Now consider the constants of a typical hydraulic turbine-driven 25-cycle generator of say 10,000-kv-a. capacity at 12,000 volts. Its armature resistance, is perhaps 0.1 ohm, whereas its reactance, perhaps 20 per cent, would be about 3 ohms per phase. That is we have here $x = 30\, r$. Even allowing for the additional resistance of connections, it is obvious that the reactance is already far beyond the point of maximum synchronizing power, and that any addition, either in generator leads or busses, acts to *reduce* rather than to increase the synchronizing power and stability. Between stations widely separated by transmission lines of considerable resistance this condition may of course be reversed.

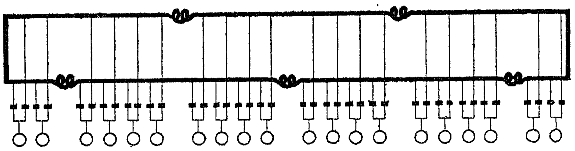

FIG. 4—ARRANGEMENT TO CONTROL BUS REACTANCE CURRENT

By means of selector switches on generators, power transfer through bus reactances may be limited.

Let us now consider a system consisting of generators connected in groups, with bus reactances between, which would represent a typical large hydroelectric station. In such a system the resistances would be small compared to the reactances, and hence will be neglected in the following consideration. (See Appendix B). Let us assume this system in full operation, each group of generators supplying its own feeders, when there comes a sudden complete interruption of the load on one group. For convenience let us call this group A and the remainder of the system group B. For simplicity let the power factor of the interrupted load be such that the terminal voltage plus armature reactance drop numerically equals terminal voltage. Also assume that the turbine gates of group A remain open. It is apparent that to determine what happens we must consider the transient conditions.

At the instant of interruption the conditions are as shown in Fig. 5.

E_0 is the terminal voltage of group A and $-E_0$ of group B before interruption.

E is the terminal voltage of A at the instant after interruption.

$E_1 = I_0 X_1$ is the drop due to load current I_0 through X_1, the internal reactance of group A. The interrupted load is

$$P_0 = E_0 I_0 \cos \theta$$

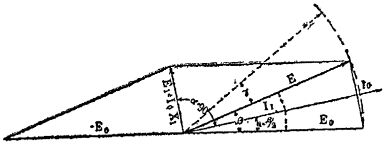

Fig. 5—Vector Diagram of Stability Conditions

At the instant after interruption the terminal voltage of group A leads that of group B by angle β thus producing the voltage E_1 which is available to circulate a current through $x = x_1 + x_2 + x_3$ the total reactance of groups A and B and the bus, in series. The magnitude of this initial circulating current is,—

$$I_1 = \frac{E_1}{x_1 + x_2 + x_3} = \frac{E_1}{x} = I_0 \frac{x_1}{x} \qquad (1)$$

and as it lags 90 degrees behind E_1 it is in the same phase position as I_0 and has a component in phase with E and in phase oppostion with $-E_0$. It is synchronizing current, accelerating group B and retarding group A.

The initial synchronizing power flowing from group A to group B then is

$$P_1 = E\, I_1 \cos \theta = E\, I_0 \frac{x_1}{x} \cos \theta = P_0 \frac{x_1}{x} \qquad (2)$$

or the synchronizing power immediately following the interruption is $\frac{x_1}{x}$ of the interrupted load.

The conditions then are as follows: The interrupted power P_0 is still pouring into the generators of group A from the turbines. Of this, $P_1 = P_0 \frac{x_1}{x}$ is leaving the generators and accelerating group B, but the remainder $P_2 = P_0 - P_1$ is being applied to accelerate group A. As the ratio of the inertia of group A to that of group B, is less than that of P_2 to P_1, group, A accelerates faster than group B. This results in an increase, which we will call γ, in the phase displacement of group A with respect to B and causes an increase in the synchronizing power, according to the equation,—

$$p_1 = \frac{E^2 \sin(\beta + \gamma)}{x} \tag{3}$$

As p_1 increases, p_2 $(= P_0 - p_1)$ decreases but still has a positive value until the displacement reaches a point where $p_1 = P_0$. At this point all the power input from the turbines to group A is being passed on to group B and the acceleration of group A with respect to group B ceases. But the acceleration now being zero, the speed with respect to group B is a maximum. From this point on p_2 becomes negative, that is, the energy stored in the flywheel effect of group A is taken out and passed on to group B causing A to retard. Its velocity, however, with respect to B being still positive, the phase displacement continues to increase, with a resultant increase of the synchronizing power, p_1, which is now supplied from the stored energy of A plus the turbine input P_0.

This increase in p_1 and γ continues until the speed of group A with respect to group B is brought back to zero, that is until both groups are again momentarily running at the same speed, when γ is at its maximum value.

The maximum values reached by p_1 and γ are difficult to exactly calculate. While a rigid mathematical determination of their values might be of academic interest, an approximate solution is of more practical value. Attacking the problem therefore in a practical way approximate values may be arrived at with various degrees of accuracy in the following ways.

First, if we assume that the maximum value of p_1 is the same as it would have been, had it been reached by a straight line instead of a sine curve; that is, if p_1 and $(\beta + \gamma)$ are united by a straight line instead of a sine function, we can apply the usual law of the suddenly applied force, that maximum deflection is twice that due to the same force slowly applied, or maximum reactive force twice the suddenly applied force.

The suddenly applied force here is P_2, (strictly the torque produced by P_2). The maximum reactive force then is $2\,P_2$ which added to the initial force P_1 gives the total resultant force (synchronizing power) at maximum deflection. The maximum deflection or angular displacement then is approximately

$$(\beta + \gamma) \text{ max.} = \sin^{-1} \frac{P_1 + 2\,P_2}{\frac{E^2}{x}} = \sin^{-1} \frac{(P_1 + 2P_2)\,x}{E^2} \qquad (4)$$

Second, if we neglect the slight change in speed so that we can put torque proportional to power we can immediately equate the work done on group A by the turbine with that done by group A on group B during the time that γ is increasing, as at the instant when γ is a maximum the speeds of the two groups are again the same and all the energy put into group A has been taken out again. (This is strictly true only if group B be of infinite inertia, so that its speed cannot change, but is relatively true in any case.) Since the speed is assumed constant we have torque $T = P_0$ times a constant K or $T = K\,P_0$, whence Work done on group A is

$$W_e = T\,\gamma_m = K\,P_0\,\gamma_m \qquad (5)$$

Work done by group A is

$$W_i = K \int_{\beta}^{(\beta + \gamma m)} p_1\, d\,\gamma$$

$$= K \int_{\beta}^{(\beta + \gamma m)} \frac{E^2}{x} \sin(\beta + \gamma)\, d\,\gamma$$

$$= \frac{K\,E^2}{x} [\text{vers}\,(\beta + \gamma_m) - \text{vers}\,\beta]$$

$$= \frac{K\,E^2}{x} [\cos\beta - \cos(\beta + \gamma_m) \qquad (6)$$

Equating

$$W_e = W_i = K\,P_0\,\gamma_m = \frac{K\,E^2}{x}\,[\cos\beta - \cos(\beta + \gamma_m)]$$

$$\frac{P_0\,x}{E^2}\,\gamma_m + \cos(\beta + \gamma_m) = \cos\beta \qquad (7)$$

The values of P_0, x, E and β being known, this equation can easily be solved for γ_m, the maximum value of γ, by trial and error.

Third, the result can be attained by arithmetic integration, which practically must be employed when hydraulic conditions enter into the calculations, as in the case where there are long penstocks. In an actual case the first method gave $(\beta + \gamma_m) = 48.3$ deg., the second method 46 deg. and the third 46.05 deg.

The synchronizing power p_1 reaches its maximum value when $(\beta + \gamma)$ becomes equal to $\frac{\pi}{2}$ radians or 90 deg. Therefore if $(\beta + \gamma)$ exceeds the value $\frac{\pi}{2}$, p_1 begins to decrease, the condition becomes completely unstable and the two systems will fall out of step. It is therefore essential, if this condition is to be avoided, that the reactance be so chosen with reference to the other conditions that $(\beta + \gamma_m)$ shall always be less than $\frac{\pi}{2}$ radians.

A mathematical expression for relative stability may be derived in terms of the angular electrical phase displacement of two systems when subjected to a certain definite load disturbance. Since sudden loss of full load of one group is possible, and may be considered the worst possible case, it appears desirable to use this condition as the criterion of stability. For any given case the maximum, or 100 per cent, stability will exist with zero external reactance; and zero stability, or complete asynchronism will occur when the reactance is such that the maximum phase displacement due to sudden loss of full load becomes equal to $\frac{\pi}{2}$ radians or 90 deg. (assuming r negligible compared to x). The expression for relative stability then becomes

$$\text{Per cent stability} = 100\left(1 - \frac{\gamma_m}{\frac{\pi}{2} - \beta}\right)$$

It remains to be shown that instability of synchronism is a practical limitation. As a typical illustration I have assumed a plant consisting of 20 units of 10,000 kw. each, connected in five groups of four generators each, separated by bus reactances. Figs. 6 and 7 show the stability conditions when connected, respectively, to a closed ring, an open ring, and a reactance bus, expressed in terms of the maximum angle of phase difference due to sudden total loss of load of one group. Fig. 8 shows the comparison of protective effect and stability of the reactance bus scheme, both expressed in per cent. These curves are calculated according to the first method previously stated, and are therefore approximate. They show clearly that the values of reactance which will cause instability are about the same as those often found desirable for protective purposes.

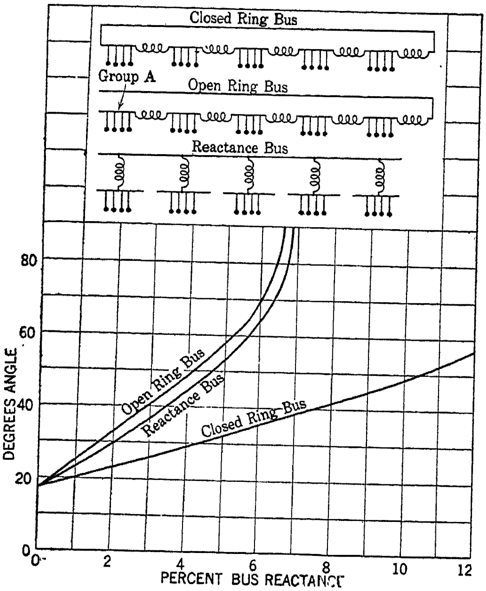

FIG. 6—MAXIMUM ANGULAR PHASE DISPLACEMENT ON SUDDEN LOSS OF LOAD OF ONE GROUP

Generator reactance—20%
20 generators in groups of 4
Bus reactance—4%
Bus reactance based on rating of one generator
Load power factor 100%

Even though the amount of external reactance installed is not sufficient to cause complete instability, serious results may, nevertheless, result. The period of the oscillation, *t*, depends upon the moment of inertia of the rotating machinery, the speed of rotation and other constants, and for small oscillations is given approximately by the formula

$$t = 2\pi\sqrt{\frac{W r^2 S Z}{K g n E^2 p}} \qquad (8)$$

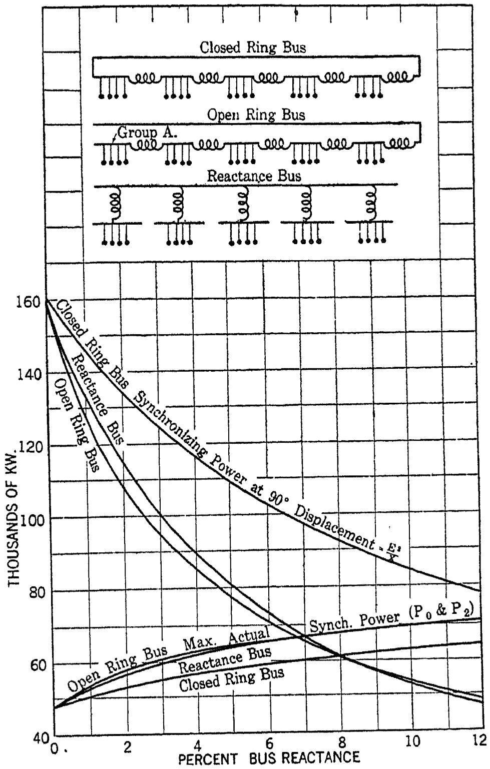

FIG. 7—MAXIMUM POSSIBLE AND MAXIMUM ACTUAL SYNCHRONIZING POWER ON LOSS OF FULL LOAD OF ONE GROUP

Generator reactance—20%
20 generators in groups of 4
Bus reactance based on rating of one generator

Where W = weight of revolving parts of generators of one group.
r = radius of gyration of generators.
S = speed of rotation in rev. per min.
Z = impedance of circuit.
n = number of phases.

E = volts per phase to neutral.
p = number of field poles
g = acceleration of gravity.
K = a constant (= 3.5 when W, r and g are in English measure).

In the specific case under consideration.

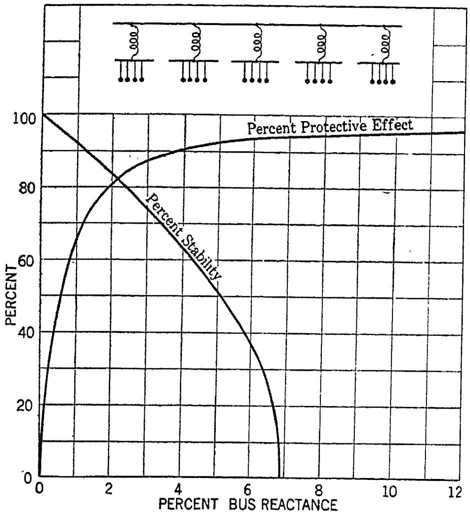

FIG. 8—COMPARISON OF PROTECTIVE EFFECT AND STABILITY OF REACTANCE BUS SCHEME

Generator reactance—20%
20 generators in groups of 4
All reactance based on capacity of one generator
Explanation—if reactance exceeds 6.9%, loss of total load on one group will result in complete desynchronizing of the group affected

$$\text{Per cent protective effect} = 100\left(1 - \frac{x_0^2}{(x_0 + x_2)^2}\right) \quad \text{(See Fig. 1)}$$

$$\text{Per cent stability} = 100\left(1 - \frac{\gamma_m}{\frac{\pi}{2} - \beta}\right)$$

Wr^2 = 5,500,000 lbs. ft.^{2} per generator, or 22,000,000 lbs. ft.^{2} per group.
S = 187.5 rev. per min.
Z = x (since r is negligible compared to x)
n = 3
E = 6930 (12,000 volts between terminals)
p = 16.
K = 3.5.

The periods of the oscillation for the various values of bus reactance considered are then shown in Fig. 9. The initial values are those determined by the instantaneous reactance and the final values by the synchronous reactance. From Figs. 6 and 9 it is seen that the addition of reactance increases both the period and the amplitude of the hunting oscillation. Both of these effects tend to aggravate the accompanying gate movements, so that under suitable conditions very considerable turbine gate movements may result from and accompany the hunting oscillation. As turbine governors operate on the centrifugal principle, the most rapid gate movement occurs when the speed is at its maximum departure from normal, which is at the instant when the phase displacement and synchronizing power are passing through their average values, that is, when the angle of oscillation, γ is passing through zero.

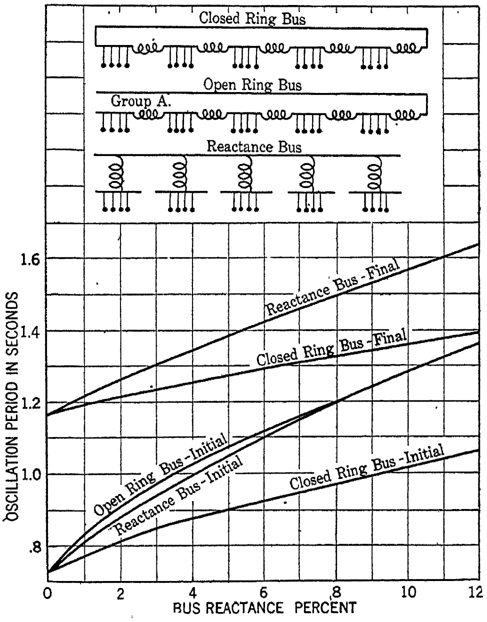

Fig. 9—Period of Hunting Oscillation

Generator—12,000-volts—187.5-rev. per min.—25-cycles—3-phase
20 generators in groups of 4—Wr^2—5,500,000 lbs. ft.^{2} each
Bus reactance based on rating of one generator
Generator reactance—momentary 20%—synchronous 50%

In plants with long penstocks, when the turbine gates move, the first effect is the opposite of that intended. That is, as the gates start to close, the sudden retardation of the penstock velocity creates additional pressure, which momentarily increases the power delivered to the turbine, thus causing acceleration of the turbine where retardation was intended.

In the particular case chosen for illustration, the turbines are assumed supplied through penstocks 322 feet (98.1 m.) long and 63 sq. ft. (5.85 sq. m.) in area under an effective net head of 189 feet (57.6 m.). (These values are chosen for convenience in calculation. They are not intended to represent exactly any actual case.)

Assuming an efficiency of 80 per cent the discharge will be

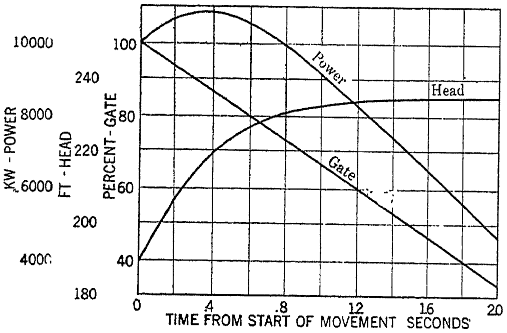

FIG. 10—POWER AND HEAD DURING CLOSING OF WATER-WHEEL GATES

Time for complete stroke 3 seconds
Length of penstock—322 feet
Area of penstock—63 sq. ft.
Net head—189 ft.

$$\frac{10{,}000 \times 550}{0.746 \times 62.5 \times 189 \times .80} = 782 \text{ cubic feet (22.13 cu. m.) per}$$

second and the penstock velocity will be

$$\frac{782}{63} = 12.4 \text{ feet per second. (3.78 m. per sec.)}$$

With the governors so adjusted as to make a complete stroke in 3 seconds, the mean effective increase in head while the gates are closing will be

$$h_a = \frac{L \times V}{g \times t} = \frac{322 \times 12.4}{32.2 \times 3} = 41.3 \text{ feet (12.58 m.)}$$

The increase in head increases the velocity through the gates and the power for each position of the gate is increased above what it would be at a constant head. Fig. 10 shows the values of head and power for the above case during the first two seconds of the closing stroke. These curves show that for 0.77 of a second the power is highe·· than the initial value, although during this time the gate has closed to 74.3 per cent of its full open position. Thus it is seen that the oscillation of the gate, a quarter cycle out of phase with the synchronizing power oscillation serves to supply positive and negative power to the oscillating system, just at the proper times to keep it in oscillation. If the power supplied in this manner exceeds the losses due to

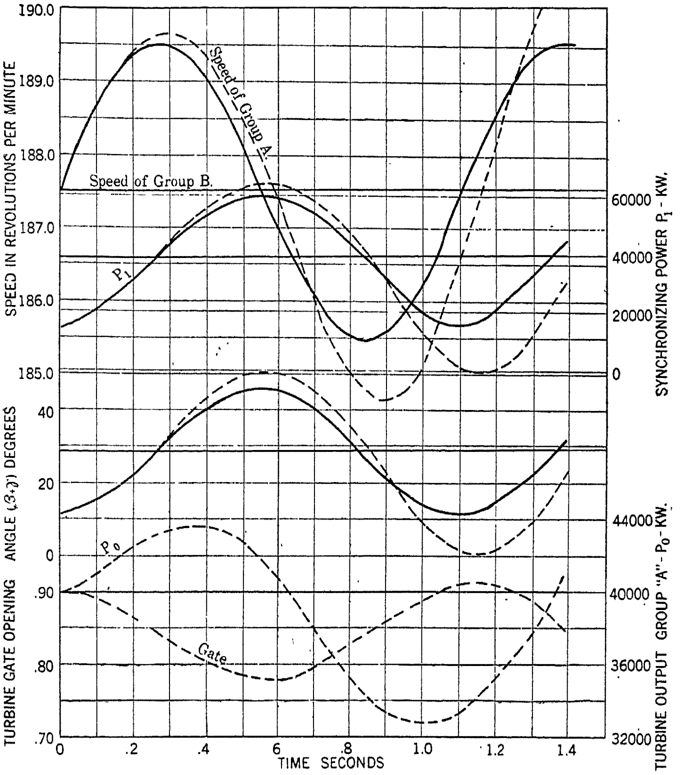

FIG. 11—POWER SURGES OR HUNTING BETWEEN WATER-WHEEL DRIVEN GENERATORS DUE TO REACTANCE

Full lines—wheels without penstocks
Dotted lines—wheels with penstocks

the oscillation, the hunting will increase until the supply and the losses become equal Fig. 11 illustrates the conditions graphically for the above specific case as worked out by arithmetic integration, showing clearly the time phase relations between the various actions.

The addition of reactance, therefore, in such a location that it may be encountered by the synchronizing current between generators or between groups of generators, produces two results as affecting the synchronous stability of the system. First, it increases the amplitude of the hunting oscillation. This makes it more likely that the two systems will twist out of the synchronism on the first quarter cycle of the oscillation. Second, it increases the period of the hunting oscillation. The combined effect of the increased amplitude and the increased period causes a wider range of turbine gate movement. The resulting oscillation of the gates may cause very appreciable power impulses, in time with the hunting oscillation, and in such phase relation thereto as to maintain the oscillation indefinitely or possibly to increase it to the point where asynchronism finally results.

In a certain case where there were 16 generators connected in groups of four in an open-ring system with 12 per cent reactance between groups, capacity of generators about 9000 kw., 12,000 volts, a surge was started by the sudden rejection from the end bus section of a load of about 20,000 kw., and continued until the circuit breakers were opened, separating the groups. The surging did not confine itself to the section in trouble and the immediately adjoining section but was communicated with practically unreduced violence to all the sections. Wattmeters connected in the bus reactance circuits showed more than full scale —12,000 kw.—at each oscillation, and this power surged back and forth between generator groups about 60 times a minute until the switches were opened. Overload relays in the circuits set for one second time limit failed to trip the switches owing to the rapid reversals of the surges. These surges were accompanied by oscillations of the turbine gates over a range of about 10 per cent of full stroke.

The amount of reactance required to create such conditions of instability as herein described is of the same order of magnitude as that required for protective and localizing purposes. Such being the case it follows that no installation of current limiting reactance can safely be undertaken, at least in a hydroelectric plant, without a thorough investigation of the effect of such installation upon the synchronous stability of the station.

APPENDIX A

PROOF THAT MAXIMUM SYNCHRONIZING POWER OCCURS WHEN $\frac{x}{r} = \sqrt{3}$.

Let E be the voltage of one generator or group.

Then $-E\,(a + j\,b)$ will be the voltage of the other generator (motor) or group, $\frac{b}{a}$ being the tangent of the phase angle between them.

The resultant or vector sum of these two voltages E_1, is that which causes current to flow and has the value

$$E_1 = E - E\,(a + j\,b) = E\,(1 - a - j\,b) \tag{9}$$

The current which flows then is,

$$I = \frac{E\,(1 - a - j\,b)}{r - j\,x}$$

$$= \frac{E\,(r + j\,x)\,(1 - a - j\,b)}{r^2 + x^2}$$

$$= \frac{E}{r^2 + x^2}\,[r\,(1 - a) + b\,x + J\,\{\,(1 - a)\,x - b\,r\}] \tag{10}$$

The synchronizing power then is,

$$P_s = I\,[-\,E\,(a + j\,b)] = -\,\frac{E^2}{r^2 + x^2}\,[a\,r\,(1 - a) + a\,b\,x + b\,\{(1 - a)\,x - b\,r\}]$$

$$= -\,\frac{E^2}{r^2 + x^2}\,[a\,r - a^2\,r + a\,b\,x + b\,x - a\,b\,x - b^2\,r]$$

$$= -\,\frac{E^2}{r^2 + x^2}\,[a\,r + b\,x - r\,(a^2 + b^2)]$$

but $a^2 + b^2 = 1$ and $b = \sqrt{1 - a^2}$

$$\text{Whence } P_s = -\,\frac{E^2}{r^2 + x^2}\,[a\,r + x\,\sqrt{1 - a^2} - r] \tag{11}$$

Differentiating with respect to a and equating to 0,

$$\frac{d\,P_s}{d\,a} = -\,\frac{E^2}{r^2 + x^2}\left[\,r\, + \frac{\frac{x}{2}\,(-\,2\,a)}{\sqrt{1 - a^2}}\right] = 0$$

$$r\,\sqrt{1 - a^2} = x\,a$$

$$r^2\,(1 - a^2)\, = \,x^2\,a^2$$

$$r^2 - r^2\,a^2\, = \,x^2\,a^2$$

$$a^2\, = \,\frac{r^2}{r^2 + x^2}$$

$$a\, = \,\frac{r}{\sqrt{r^2 + x^2}} \tag{12}$$

Substituing this value of a in equation (**11**) gives the equation for the maximum value of P_s with respect to the phase angle:

$$P_s\,(\max)\, = -\,\frac{E^2}{r^2 + x^2}\left[\frac{r^2}{\sqrt{r^2 + x^2}} + x\sqrt{1 - \frac{r^2}{r^2+x^2}} - r\,\right]$$

$$= -\,\frac{E^2}{r^2 + x^2}\left[\frac{r^2}{\sqrt{r^2 + x^2}} + \frac{x^2}{\sqrt{r^2 + x^2}} - r\,\right]$$

$$= -\,\frac{E^2}{r^2+x^2}\,[\sqrt{r^2 + x^2} - r]$$

$$= -\,E^2\left[\frac{1}{\sqrt{x^2 + r^2}} - \frac{r}{x^2 + r^2}\right]$$

$$= -E^2\left[\frac{1}{r\sqrt{\frac{x^2}{r^2} + 1}} - \frac{1}{r\left(\frac{x^2}{r^2} + 1\right)}\right] \tag{13}$$

Differentiating with respect to x/r and equating to 0,

$$\frac{d\,P_s}{d\left(\frac{x}{r}\right)} = \frac{-\frac{1}{2}\cdot 2\cdot \frac{x}{r}}{r\left(\frac{x^2}{r^2}+1\right)^{3/2}} - \frac{-2\,\frac{x}{r}}{r\left(\frac{x^2}{r^2}+1\right)^{2}} = 0$$

$$= -\frac{\frac{x}{r}}{r\left(\frac{x^2}{r^2}+1\right)^{3/2}} + \frac{\frac{2x}{r}}{r\left(\frac{x^2}{r^2}+1\right)^{2}} = 0$$

$$\frac{2}{\sqrt{\frac{x^2}{r^2}+1}} = 1$$

$$\sqrt{\frac{x^2}{r^2}+1} = 2$$

$$\frac{x^2}{r^2}+1 = 4$$

$$\frac{x^2}{r^2} = 3$$

$$\frac{x}{r} = \sqrt{3} \qquad \textbf{(14)}$$

The writer wishes to acknowledge indebtedness in connection with the above to Mr. H. R. Woodrow.

APPENDIX B

EFFECT OF NEGLECTING RESISTANCE OF GENERATORS AND CONNECTIONS

The equation for maximum synchronizing power is,

$$P_s\,(\max) = -E^2\left(\frac{1}{r\sqrt{\frac{x^2}{r^2}+1}} - \frac{1}{r\left(\frac{x^2}{r^2}+1\right)}\right) \qquad \textbf{(13)}$$

from which values of P_s (max) can be determined for assumed values of x and r

If we put $r = 0$ this equation becomes indeterminate.

The general equation for P_s is,

$$P_s = - \frac{E}{r^2 + x^2} \left(a\, r + x \sqrt{1 - a^2} - r\right) \tag{11}$$

putting $r = 0$

$$P_s = - \frac{E^2}{x^2} \left(x \sqrt{1 - a^2}\right) = - \frac{E^2}{x} \sqrt{1 - a^2} \tag{15}$$

differentiating with respect to a and equating to 0.

$$\frac{d\, P_s}{d\, a} = - \frac{E^2}{x} \left(\frac{a}{\sqrt{1 - a^2}}\right) = 0$$

whence $a = 0$

Substituting in (**15**)

$$P_s\,(\max) = - \frac{E^2}{x} \sqrt{1 - 0^2} = - \frac{E^2}{x} \tag{16}$$

Also since when $a = 0$ the phase angle $= \frac{\pi}{2}$ radians or 90 degrees it follows that when r is zero the maximum synchronizing power occurs at a phase angle of 90 degrees.

The following table serves to illustrate the comparatively small error in the maximum synchronizing power due to neglecting the resistance in a case where there are two groups of four 10,000-kv-a. generators of 20 per cent individual reactance, 12,000 volts, 3 phase, 187.5 rev. per min., for various values of bus reactance between the two groups.

Bus reactance per cent.	Total reactance		Total resis.	Max. syn. power resis. considered	Max. syn. power res. neglected	— Per cent error
	Per cent	Ohms				
0	10	1.44	0.042	97,300 kw.	100,000 kw.	2.77
2	12	1.73	0.048	81,200 "	83,200 "	2.47
4	14	2.02	0.054	69,400 "	71,300 "	2.73
6	16	2.30	0.060	61,300 "	62,600 "	2.12
8	18	2.59	0.066	54,300 "	55,600 "	2.40
10	20	2.88	0.072	48,800 "	50,000 "	2.46
12	22	3.17	0.078	44,400 "	45,500 "	2.40
15	25	3.60	0.087	39,000 "	40,000 "	2.57
20	30	4.32	0.102	32,700 "	33,300 "	1.84

It is therefore evident that no appreciable error is made by neglecting the resistance in calculating the values for the first half cycle of the oscillation. The resistance would of course have to be considered in calculating the attenuation during subsequent cycles.

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EFFECT OF CURRENT LIMITING REACTORS ON TURBO-GENERATOR SYSTEMS UNDER CONDITIONS OF SHORT CIRCUIT

BY P. B. JUHNKE

Abstract of Paper

An account of a cable breakdown near one of the generating stations of the Commonwealth Edison Company's system. The breakdown resulted in a second breakdown on same line within the station, which prevented the oil switch from automatically disconnecting the fault. The system is amply protected by reactors for generators and between bus sections. Their effect on the stability of synchronous apparatus on sections protected by them is very marked. The protection to generating station apparatus was complete. Comparison of maximum stresses encountered in the short circuit as it occurred with what they might have been without reactors present are made, and the conclusion is drawn that without them the damage resulting would have been considerable and the service interruption far more general and serious.

WHILE current-limiting reactors are by no means of recent date, having long passed the experimental stage, comparatively little data are available showing in what manner they perform their intended functions under conditions of short circuit on systems protected by them. This paper contains an account of a cable breakdown near the generating station, which resulted in a second failure on the same line within the station, between the current transformer and the automatic oil switch, thereby preventing immediate operation of the latter. As the system in question is one of the largest in the country, amply protected by reactors on the generators and between sections of the bus, an analysis of the effect of the short circuit on the system might not be without interest.

Events Before and During Trouble

The system in question is that of the Commonwealth Edison Company. The breakdown in question occurred at the Quarry St. station on line No. 223, December 22, 1916, 7:31 p.m. Its first indications were the tremendous roar of the generators, the violent swinging of all tie-line and transmission-line ammeters, the automatic opening of the switches on the 5000-kw. frequency changer, which tied the 25 and 60-cycle systems, and the automatic opening of lines 212, 213, and 226. After a period of about 10 to 15 seconds, the automatic opening of line switch No. 223 cleared the trouble.

When the short circuit had cleared itself, about 80,000 kw. of system load had been automatically disconnected by the operation of the protective devices in the sub-station. This resulted in a rise of the system frequency, which caused three of the four Quarry generators to trip out on the steam end.

An investigation revealed the fact that the connector between oil-switch leads and current transformers on *B* and *C* phases of line No. 223 as shown in Fig. 1, had burned off and that one of the leads had established an arc to a ground bus and the other to a bolt embedded in concrete. The arc had damaged the relay wiring on *B* and *C* phases of line No. 223, making these two relays inoperative, but leaving *A* phase relay intact. A test on the cable of line No. 223 indicated that the line had burned open on *B* and *C* phases outside of the station, and that *A* phase was grounded. The cable fault was located in the conduit on station grounds, about 600 ft. from the oil switch. About two feet of the cable was found to have been destroyed.

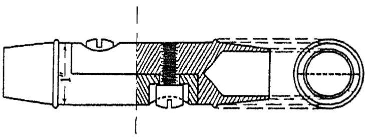


FIG. 1—CURRENT TRANSFORMER CONNECTOR

When repairs were made it was found that this burnout had occurred within a very few feet of the location of a previous burn-out on line No. 1222, which was in a diagonally adjacent duct. Nothing serious, however, was indicated by the latter burn-out. The switch had opened promptly and the cable gave no evidence of any unusually severe trouble.

When repairs on line No. 223 were made and the close proximity between this burn-out and the burn-out on line No. 1222 was established, it was thought best to dig up the conduit, break out the ducts and expose the cables to determine if any other cables had been injured. This was impossible at the moment, as the ground was covered with storage coal, and it was several days before the coal could be removed for the purpose.

On exposing the cable it was found that an adjacent duct wall had been broken and that the lead had been burned from one

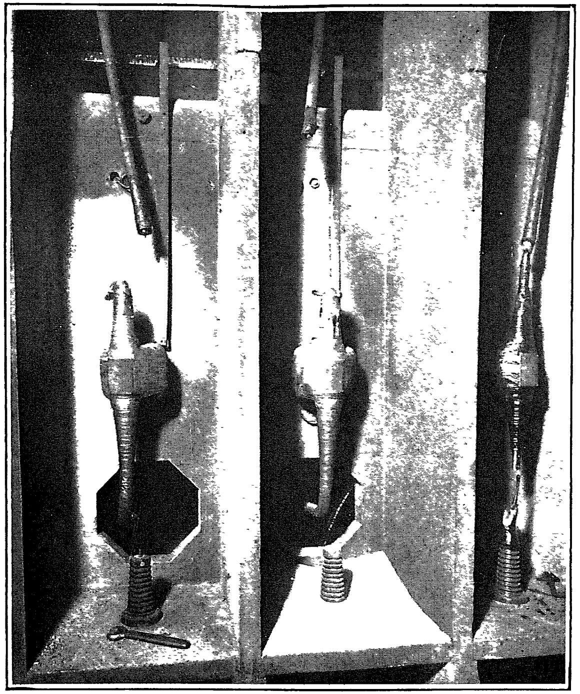


FIG. 2—BURNED OFF LINE LEADS

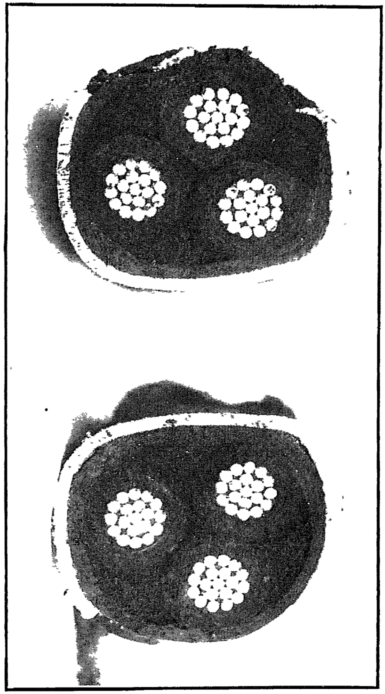
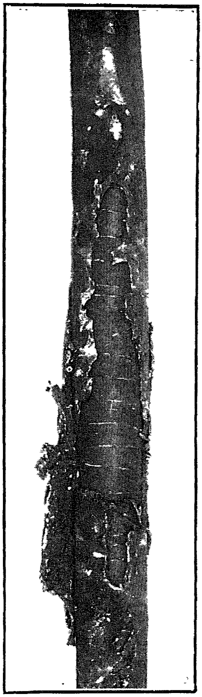


FIG. 4—SECTION OF CABLE FROM LINE 1217 AFTER REMOVAL



[JUHNKE]

FIG. 5—CABLE OF LINE 1217 AT POINT OF BURN-OUT

side of line No. 1217 in this same duct. The force of the explosion had pushed the cable of this line forcibly against one side of the duct, so that the exterior of the cable assumed very closely the form of the inside of the tile ducts. This is shown in Fig. 4, which is reproduced from a photograph of a section of this cable after removal. The external injury to this cable at the location of the burn-out is shown in Fig. 5, as it was found eight days after the trouble on line No. 223 occurred, during which time it had been in continuous operation. This line did not break down, but the cable was removed and replaced.

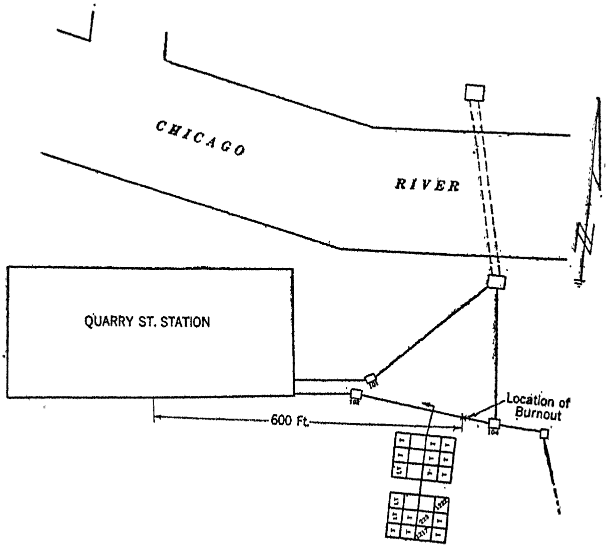


FIG. 3—PLAN SHOWING CONDUIT LOCATION AND POINTS OF BURNOUTS—QUARRY STATION PROPERTY

ANALYSIS OF SEQUENCE OF EVENTS

An analysis of the trouble led to the conclusion that the initial fault on line No. 223 occurred between *B* and *C* phases in the cable. With the lead sheath at the location of the burn-out destroyed by the previous burn-out on line No. 1222, this was the most likely failure and probably due to moisture penetrating the insulation.

It is believed that when the fault outside was established, the great rush of current, largely due to the nearness of the fault to the station bus, caused the joint between the lead from the oil switch and the current transformer to break apart on both *B* and *C* phases. It is probable that when the arc to ground was established, the arc to the short-circuited cable was maintained until *A* phase broke down either to ground or to one of the other phases, which caused the *A* phase relay to open the oil switch on line No. 223, thereby clearing the trouble from the bus.

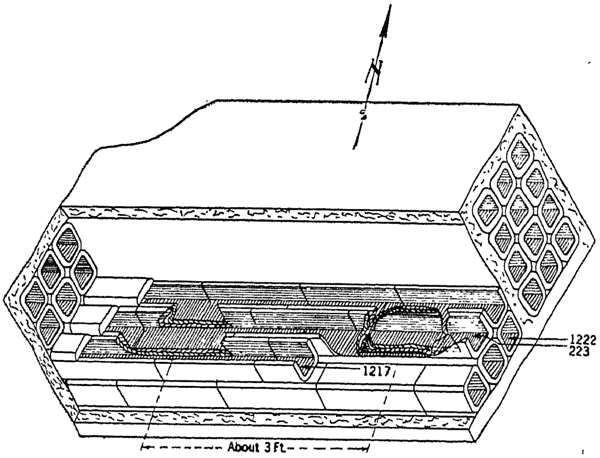


FIG. 6—SECTIONAL SKETCH OF EXPOSED CONDUIT

DESCRIPTION OF SYSTEM

The total capacity of the 25-cycle, 9000-volt system here concerned consisted of 17 generators and a 5000-kw. frequency changer of a combined capacity of 251,000 kw., each of the generators having, including its external reactance, a total reactance of approximately 8 per cent. The load carried was approximately 200,000 kw. This system, at the time of the trouble consisted of four more or less distinct parts, with generating capacity and load at time of trouble, as follows:

Station or section	Generating capacity	Number of machines	Load
Northwest....................	50,000 kw.	2	50,000 kw.
Fisk A. Section................	60,000 "	5	48,000 "
Fisk B. Section................	80,000 "	6	55,000 "
Quarry St.....................	61,000 "	5	49,000 "
	251,000 kw.	18	202,000 kw.

Fisk *A* and *B* sections are the two parts into which the Fisk St. station 25-cycle bus is divided, each of these two sections being separately in parallel with Quarry St. over a tie line protected by a balanced relay, and consisting of three 250,000-cir.

mil, three-conductor, 9000-volt cables of an average length of 2400 ft. In each phase of this tie line is a 20 per cent series reactor, this value being based on Y-voltage drop at the rated full load of 10,000 kv-a. per line. These three sections are thus operated on the open-ring principle. Northwest is connected with Fisk section *B* over four tie lines of a total cross section of 1,000,000 cir. mils and average length of eight miles, these tie lines carrying stations along their route. The relays on these tie lines are of the bellows type, with a setting on each line as follows: Minute, 9375 kv-a.; 2 seconds, 11,700 kv-a., and inst., 14,000 kv-a. Fig. 7 is a diagrammatic representation of the system as it existed at the time the trouble occurred.

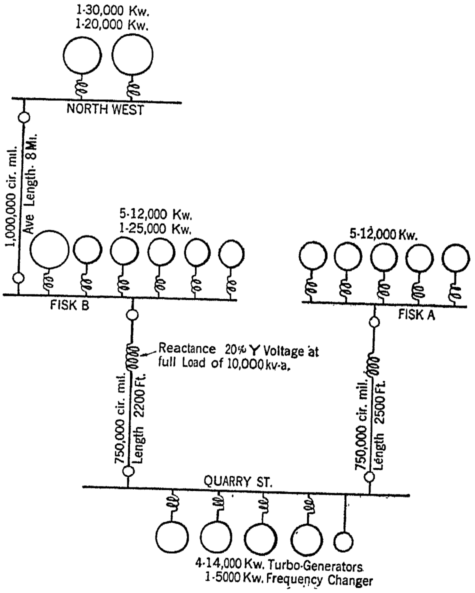


Fig. 7—Diagram of System as Operated at Time Trouble Occurred

At this time the synchronous apparatus supplied by the different sections was as follows:

Sections	Apparatus in substations	
	Synch. conv.	Synch. motors
Northwest	9 substations	2 substations
Fisk A Section	20 " "	3 " "
Fisk B Section	18 " "	1 " "
Quarry St.	15 " "	1 " "

Effect of Short Circuit on System

At the time the trouble occurred, nearly all the synchronous apparatus feeding from Quarry St. lines was automatically disconnected, about 70 per cent through operation of a-c. overload relays opening oil switches, apparently due to back feed into the short circuit; about 28 per cent due to reverse-current or low-voltage release relays tripping d-c. circuit breakers. This ratio of apparatus disconnected by operation of oil switches to that disconnected by operation of d-c. breakers is exceedingly small in the stations supplied by either of the Fisk St. sections, while none of the apparatus on Northwest lines was disconnected on the a-c. side. The only machine which was automatically disconnected on the a-c. side on Fisk *B* section tripped out because it was operating in parallel on its d-c. end with a machine twice its size, feeding from Quarry St., due to the overload caused by the reversal of the latter, on which only the d-c. breakers opened. Fig. 8 is a diagram of generating capacity, load carried, and load interrupted when trouble occurred.

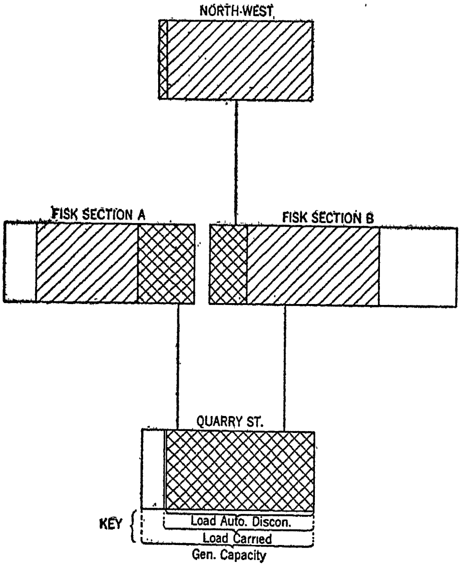


FIG. 8—DIAGRAM OF GENERATOR CAPACITY—LOAD CARRIED AND LOAD AUTOMATICALLY DISCONNECTED

This diagram clearly demonstrates the beneficial effect of current-limiting section or bus reactors on the stability of apparatus on the sections protected by them. While very nearly 100 per cent of the capacity and load supplied by the section on which the trouble occurred was affected, only 36 per cent of the load on Fisk section *A* and 27 per cent on Fisk section *B* was interrupted. Of the total Northwest load only about 6 per cent was affected. The smallness of this latter figure undoubtedly is due to the considerable amount of tie line resistance between Northwest and Fisk section *B*. The rather considerable loss in load on Fisk *A* and *B* sections was mostly due to d-c. circuit breakers opening in some of the substations supplied by *A* and *B* sections, which in a large measure was caused by overloads from the interruptions to substations which had been supplied by Quarry St. lines.

PROBABLE VALUES OF SHORT CIRCUIT

Calculations to determine the dimensions of the short-circuit current and stresses due to them can only be approximate. The calculations presented herewith are largely based on test data contained in a paper submitted to the Institute in June, 1911, by Messrs. Schuchardt and Schweitzer. Certain assumptions, however, can be made with a fair degree of probability, which will at least permit of determining the maximum values of the current into the fault. These are:

1. That the fault in the cable was of negligible resistance (variations in the value of this quantity, if same is taken at 0.025 ohm, do not materially affect the result).

2. That the behavior of the Quarry St. machines is comparable to that of the Fisk St. machine, tests on which were reported in the paper by Messrs. Schuchardt and Schweitzer.

3. That the synchronous apparatus connected to Quarry St. at the time the trouble occurred, returned energy at a power factor of about 10 per cent for the 230-volt synchronous converters and motors, and 5 per cent for the railway converters.

Based on these assumptions, the maximum short-circuit current is made up of the following components:

A. Current supplied by the Quarry St. generators.

B. Current supplied by the Fisk St. generators, over tie lines *A* and *B*.

C. Current supplied by synchronous apparatus directly connected to Quarry St. lines.

This is determined by taking the short-circuit currents of the three parts and combining them vectorially. This gives the combined impedance, to which then is added the impedance of the faulty line, giving the resultant short-circuit impedance.

The calculation is carried out in detail for the Quarry St. machines as follows:

Maximum short circuit between phases on four 14,000-kw. turbo-generators = 4 × 900 × 11.9 = 42,840 amperes, at a generator open-circuit voltage of 9000 volts.

Total maximum effective short-circuit current = 49,240 amperes.

Generator open-circuit voltage corresponding to excitation with load carried before trouble occurred = 10,200 volts.

$$\text{Impedance of armature circuit} = \frac{10,200}{49,240} = 0.206 \text{ ohm.}$$

Power factor taken as 7 per cent, which is the probable figure given in the paper previously mentioned.

Resistance of armature circuit = 0.07 × 0.206 = 0.014 ohm.

Reactance of armature circuit = 0.205 ohm.

Maximum effective short-circuit current = 45,800 amperes.

Power factor = 7 per cent.

In like manner the combined Fisk A and Fisk B section current, limited almost entirely by the sectionalizing reactors, can be shown to be............................5,800 amperes.

Power factor = 2.8 per cent.

SHORT-CIRCUIT CURRENT OF SYNCHRONOUS APPARATUS

This calculation assumes approximately 5 × full-load current for railway machines and 20 × full-load current for the 230-volt converters and the frequency changers.

Total capacity of railway machines connected to Quarry St....................	42,000 kw.
Total capacity of 230-volt converters and frequency changers connected to Quarry St..................................	17,100 "
Maximum effective short-circuit current of railway machines at power factor 24.7 per cent..............................	11,500 amperes.
Maximum effective short-circuit current of 230-volt converters and frequency changers at power factor 54.5 per cent...	11,800 "

Total maximum effective short-circuit current supplied by synchronous apparatus at power factor 40.6 per cent..............	23,000	amperes
Total maximum effective short-circuit current of Quarry St. generators, Fisk A and B Section generators, and synchronous apparatus connected to Quarry St........	73,300	"
Power factor = 17 per cent.		
Combined armature impedance = ...	0.14	ohm.
" " resistance = ...	0.024	"
" " reactance = ...	0.138	"
Impedance of line No. 223 to fault, neglecting resistance of arc = ...	0.062	"
Resistance of line No. 223 to fault = ...	0.06	"
Reactance " " " " " = ...	0.018	"
Total resistance of armature and line = ...	0.084	"
" reactance " " " " = ...	0.156	"
" impedance " " " " = ...	0.176	"
Power factor of circuit = 48 per cent.		
Maximum effective short-circuit current into fault, added vectorially = ...	58,000	amperes
Maximum effective value of energy in arc at a resistance of 0.025 ohm = ...	74,100	kw.

It might be of interest to compare this with the value of current that would have obtained under similar conditions had the system been without reactors in the generators and between sections of the bus. In this event the total short-circuit current, computed in same manner as with the reactors in circuit, and neglecting transients would have been approximately 106,000 amperes, and the power factor 83 per cent. This would have multiplied the stresses incidental to the short circuit as it occurred by 3.3 and would have given stresses of approximately 800 lbs. per foot of parallel conductors below oil switch. Such stresses more than likely would have damaged the bus structure, would have immensely increased the task of oil switches and made the service interruption far more extensive and serious.

BEHAVIOR OF SUBSTATIONS.

While synchronous apparatus in a number of substations was automatically disconnected through operation of either the a-c. or d-c. relays, the interruptions in the substations so affected were of remarkably short duration and the recovery was exceedingly rapid. As nearly as can be determined, the service interrupted from substations connected to the section in trouble amounted to about 50,000 kw. for five minutes, on one of the two sections protected by sectionalizing reactors to about 20,000 kw. for three minutes, and on the other to about 10,000 kw. for two minutes. The behavior of sub-station protective devices during the trouble is a subject full of interest, the complete analysis of which, however, has not yet been completed.

CONCLUSION.

Considering results in their entirety, the influence of the reactors must be pronounced as exceedingly beneficial. Not a single piece of generating station apparatus was disabled, as was almost invariably the case before the installation of current limiting reactors. Even on the section directly affected everything was ready for immediate resumption of service as soon as the trouble was cleared. The value of the reactors seems to have been happily chosen; no evil results attended the operation of the system during and after the trouble; no oil switches failed; and, most important of all, comparatively little load was interrupted on the sections protected by the sectionalizing reactors.

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PROTECTION OF TRANSFORMER NEUTRALS AGAINST DESTRUCTIVE TRANSIENT DISTURBANCES

BY MAX H. COLLBOHM

Abstract of Paper

The paper points out the danger to which the neutral point of transformers, connected to long distance transmission lines, are exposed through the building up of excessive potentials at this point under conditions of atmospheric lightning. This is due to the fact that the neutral point in a bank of Y-connected transformers acts as a reflection point for all the waves that are produced by induced lightning, which waves, at the place of origin, have the same amplitude and sign, and, therefore, travel with equal speed, considering that the impedance of line and transformer is the same for each phase. The three waves meeting at the neutral point build up to three times their individual potential at that place, thereby producing excessive potential stresses, and, according to experience, destruction of bushings and windings. The author recommends the installation of lightning arresters at the neutral point to provide a discharge path for the excess potential, which, according to measurement with a spark gap, may reach values of 350,000 volts, and higher.

IT IS a well established practise to protect the low-tension windings of a transformer through a spark gap, against the excessive potentials induced upon it through capacity effects between high-tension and low-tension windings under abnormal voltage conditions on the high-voltage side. There also exists, however, a distinct danger to the high-voltage side which is not generally recognized and which cannot be guarded against by the ordinary types of arresters of American manufacture, such as the electrolytic and multigap types, installed in the usual way. This danger is evident from the following consideration:

If during a lightning storm a discharge takes place from a cloud above a transmission line, the electrostatic charge, previously induced by this cloud in equal quantities and like polarity on all three wires of a three-phase line, is suddenly set free and travels in both directions along the line. Inasmuch as for all practical purposes the three wires of the same line offer the same impedance, and as this is also true of the main power transformers on either end of the line, the speed of propagation of the released electrostatic charge, or more correctly that portion of it which has not been discharged over the insulators or through corona, is exactly alike in all three wires. Consequently the neutral point of the transformer, if connected in Y, constitutes the reflection point for all these traveling waves. If the potential of this traveling wave after having reached the station is below the value for which the arresters have been set it will pass through the transformers, and as the charges from all three wires meet at the neutral point they build up to three times their individual potential at that place. Although the potential difference to ground of each individual charge may be below the safe electrostatic stress of the transformer winding and terminal bushing, the combined effect of the triple voltage is likely to produce disastrous results.

The writer was confronted with this condition in the operation of the Peninsular Power Co's. system at Iron Mountain, Mich., where the 66,000-volt step-down transformers in the Iron River substation showed repeatedly momentary arcing from the neutral transformer terminals to the case, a distance of three feet (0.91m.) In one instance the arc formed between the neutral lead under oil and the transformer iron resulting in the burning off of said lead.

In order to provide a discharge path for this dangerous potential a multigap arrester will be installed at the neutral point and adjusted to discharge at 42,000 volts. This high-voltage setting for the arrester became necessary for the reason that the system operates with the transformer neutral at the generating station grounded over a metallic resistance to avoid service interruption in case of accidental grounds on one of the line wires and which under such conditions, otherwise harmless, would produce a potential difference of 38,000 volts between transformer neutral and ground. A multigap arrester was chosen in place of an electrolytic arrester in order to avoid the inconvenience of providing for the necessary special arrangement and constant attendance for the regular charging of the latter.

On account of the urgent need for protection of the substation transformer neutral and the present inability of the manufacturers to effect short time deliveries, a makeshift arrangement has been used to serve until the multigap arrester is received. This arrangement consists of two zinc balls 5 in. (127 mm.) in diameter, set 6 in. (152.4 mm.) apart, each supported by an iron rod and connected to the neutral point and ground, respectively, thus forming a relief gap for the transient excess potential. The sphere spark gap was chosen on account of its greater sensitiveness under sudden impulses as compared with the needle gap. The spheres have been made of zinc on account of the arc extinguishing tendency of this metal.

This arrangement has worked very well during recent lightning storms, when discharges over this spark gap have repeatedly been observed. This proves that protection against the above-mentioned transient disturbances can be obtained by a shunted spark gap. It is of interest to note that the spheres were originally set 12 in. (304.8) mm. apart by error and even with this wide setting the previously mentioned discharges have been observed, indicating the surprising potential difference of about 350,000 volts between neutral and ground.

It should be noted that the trouble occurred only on the transformer neutral in the substation, while no disturbances of this nature were observed in the generating station where the metallic ground rheostat between transformer neutral and ground served to drain off these transient charges.

The danger from this source is only present in Y-connected transformers where the neutral acts as a reflection point. If delta connection is used on the high-tension side, the incoming charge divides at each corner of the delta, one-half going into one transformer and the other half going into the other transformer. The middle point of each transformer will in this case be the reflection point, but the accumulation of potential at this place will, therefore, not be greater than that of the original charge on each line wire at the transformer terminal. This statement, however, must not be taken to dispute the fact that potentials higher than that possessed by the orignal charge at the transformer may exist within the transformer winding, due to the piling up of potential through reactance under high frequency or steep wave fronts. However, as experience proves that the electrostatic charges actually pass through the transformer and build up at the neutral point to destructive potentials, it appears that their wave front has become less steep, due to the impedance of the transmission line between the point of lightning discharge and the transformer station. From this consideration it would seem that lines of greater lengths should be more subject to this danger of potential rise at the neutral than those of shorter lengths where the impedance may not be great enough to flatten out the transient wave front sufficiently to permit of its passing through the windings. In this latter case, however, danger is present from abnormal potential rises within the transformer due to reactance as mentioned before.

From the foregoing it is evident that protection must be provided for the neutral point of transformer banks, especially on long lines. Additional protection can be obtained through the installation of condenser arresters, manufactured at present in Europe only, on the line side of the transformer leads, as they tend to absorb all transient charges having steep wave fronts or high frequency and thereby prevent, or at least reduce, the danger from abnormal voltage rises within the windings.

DISCUSSION ON "REACTORS IN HYDROELECTRIC STATIONS" (JOHNSON), "THE EFFECT OF CURRENT-LIMITING REACTORS IN STEAM TURBINE STATIONS UNDER SHORT-CIRCUIT CONDITIONS" (JUHNKE), "PROTECTION OF TRANSFORMER NEUTRALS AGAINST DESTRUCTIVE TRANSIENT DISTURBANCES" (COLLBOHM), NEW YORK, FEBRUARY 14, 1917.

F. H. Kierstead: Mr. Johnson has brought out the bad effects of reactors between groups of generators upon the synchronizing power of generators. Mr. Juhnke has shown the advantages of having reactors placed between groups of generators. It seems to me that generators not protected with reactors will fall out of step more readily when the system is short-circuited than if the generators were protected. In other words, I mean to point out that the most serious effects would obtain in the case where there was no protection. Furthermore, the greater the reactance, the less will be the trouble. Therefore, a value of reactance sufficient to protect the circuit should be used irrespective of its effect upon the synchronizing power of the generator.

Harry R. Woodrow: Mr. Kierstead mentioned that it would do no harm to let the generators fall out of step in cases of short circuit. It was my understanding, in reading over Mr. Johnson's paper that the conditions of stability which were referred to were not under short-circuit conditions but were in cases where, under normal operating conditions, a heavy load is thrown off from a section of the bus.

Mr. Johnson's paper shows very clearly the predominating importance of reactance coils in giving protection to service which factor is also corroborated by the results of actual short-circuit conditions presented in the paper of Mr. Juhnke.

The question of rating the percent reactance, in cases of bus reactance coils, is very important as it may lead to some misunderstanding regarding the stability or amount of protection obtained.

At the present time there is no standard form of rating the percent bus reactance, as in some cases this rating is based on the full-load current of one generator, in others on the full-load current of the bus section, and in still others on the current carrying capacity of the reactance. As an example, in figuring this in Mr. Johnson's paper, the 6 per cent bus reactance (which is based on the full-load current of one generator) gives 44 per cent reactance when based on the full-load current of the bus section, including internal reactance of the group of generators and the bus reactance between sections.

The question of stability is determined by the amount of reactance in the total bus section. In the steam stations the unstable conditions do not exist to the same degree as in the water power stations, which difference is due to the better governing conditions of the steam station. This condition is also aggravated by the fact that the individual loads are usually much larger on the water power stations and a dropping of a

20,000-kilowatt load is not unusual with these stations whereas it would be of very rare occurrence in the case of the New York Edison Company.

In figuring on the amount of bus reactance desired the question of distribution of feeders must also be considered. Although this point is not particularly covered in these two papers, it has been considered, in considerable detail, in previous papers before the Institute.

The method of determining the results given in Mr. Johnson's papers is very unique, in that it gives in concrete form practical results. I have checked up the results given in his method from an entirely different viewpoint, in which case I used the fundamental equation of the alternating-current system and found my result to be practically the same as his.

The results of the short-circuit conditions described by Mr. Juhnke as occurring on the system of the Commonwealth Edison Company are certainly very interesting and show very clearly the benefits derived from the installation of reactance coils. I would like to inquire if the feeders Nos. 212, 213 and 226, which are marked as supplying transmission lines, are tied in with any other station?

P. B. Juhnke: No, they carry independent loads.

H. R. Woodrow: Are the ties between Northwest and Fisk Street and Quarry Street and the two sections in Fisk Street protected by relays, and if so, did these relays operate to sectionalize these several sections?

P. B. Juhnke: Yes, they are protected by relays, and they did not open.

H. R. Woodrow: Are the tie buses between Fisk and Quarry Streets protected with balanced relays on both ends?

P. B. Juhnke: On both ends.

H. R. Woodrow: What was the setting of the relay on the feeder that was damaged?

P. B. Juhnke: About 400 amperes minimum, and about 750 amperes instantaneous.

H. R. Woodrow: Were the current-transformer leads burned off or were they blown off.

P. B. Juhnke: They were actually blown off, and then they were burned as a secondary result.

H. R. Woodrow: That would account for the relays not operating?

P. B. Juhnke: Yes.

H. R. Woodrow: On the question of the Y-delta transformers, it seemed to me that the amount of energy which would go through the transformer on the surge must be a very small amount. It would seem to me that a second ground of high resistance would take care of any abnormal voltage conditions that would arise, as the power which could go through these transformers on the high-frequency surge would be of small magnitude. I would like to ask if that phase of the problem had been given consideration, if so what the effect was.

A. R. Cheyney: The use of protective reactance has proceeded carefully and conservatively among American plants of large size, and careful presentation of information showing the behavior of same in service affords the best possible means for keeping engineers within right lines. A break-down in insulation which renders inoperative the switching mechanism on the feeder or machine in question, or a breakdown between the station bus and the automatic protection of the circuit breaker is always most carefully guarded against. Nevertheless, as in this instance, it is the unexpected that sometimes happens.

Desired protection of any large inter-connected high-tension system is only effected when a breakdown at any point is prevented from seriously interfering with voltage or frequency of machinery and apparatus in parts of the system outside of a pre-arranged radius. In the ideal protection afforded by reactances we would thus have a locality of low voltage existing until cut out by automatic protection, and also around this locality a region where the voltage is somewhat disturbed although perhaps not sufficient to interfere with the operation of substation machinery, and still further beyond an area practically unaffected either as to voltage or as to frequency. Of course, with growing loads, and also from hour to hour with the load of every station changing very considerably, it is extremely difficult to lay out any scheme of reactors which are to be kept constantly in circuit that will be equally effective at all times, and the case which we are now considering is an instance where practically on peak-load conditions, both as to the yearly load and the time of day, the protection was most urgently needed. Some form of reactance must be provided that can vary with the load. In this case, the main part of the reactance is installed in the generator leads with the additional protection of heavy reactors in the tie lines between Quarry Street station and the two Fisk Street stations, *A* and *B*. It is also possible to secure this variable degree of reactive protection by installing smaller reactors in the individual feeders leading from the generator station: Also, as in the present instance, there seems no choice but to install additional reactors between sections of the main-station busbars, subdividing same into two or three sections, as considered best. In addition to the above segregating function of the reactance, it is also and more commonly used to protect machines, lines and switching apparatus from electrical and mechanical stresses, which, without this protection might otherwise be seriously damaged in cases of severe short circuits.

The present instance would seem almost conclusive evidence as to the necessity for installing reactance in the bus bar, if we are to prevent the centering of the whole station generating capacity at any point along the busbars or connections. In order to make this effective, it may be found desirable to reduce the reactance in the generator leads, and it must always be borne in mind that reactance has a very serious defect of dropping the voltage instead of holding it up, if not installed in the best possible location and in proper degree. It is obvious that in the case of a comparatively large number of feeder reactors, each of relatively small size, we have the highest degree of insurance against interruption of substation service and at the same time under conditions of feeder short circuits outside the station, a fair degree of assurance that the trouble will be maintained within comparatively narrow limits.

The remarks on reactors in a generating station apply in a lesser degree to the substations and especially so if these are of large size. In the case noted by the author, where there are fed from the Quarry Street station some 16 substations, practically all of the synchronous-converter type and of an average capacity of 3000 kw. per substation, protection in the form of reactance would be unnecessary. It is to be noted that the reactors on the two tie lines between Quarry Street and Fisk Street so well fulfilled their mission that the trouble was practically confined to Quarry Street and its substations. Sufficient evidence that the Northwest station was unaffected is shown by the fact that the tie lines from Fisk Street, with a setting of only 14,000 kv-a. instantaneous were not opened during the trouble. There would seem to be, however, a chance of looking at these tie lines in another light, and by setting their breakers for very heavy overloads and by reducing the amount of reactance in same to allow of one station assisting another by a much larger extent than was here possible without disturbing the load carried by the station unaffected by the short circuit. We would naturally suppose that with busbar reactors and the addition of feeder reactors, and the possibility of interchange of power in greater quantities between Fisk and Quarry Streets, that even with the present location of insulation failure, it would have been possible to have held up the voltage sufficiently to have avoided throwing off on reversal so many of the a-c. feeders from Quarry Street at the substation ends. This, however, is but a surmise.

It will be a welcome addition to the literature of central station service to have the author at a later date give more details as to circuit-breaker settings and also as to performance of station auxiliaries at Quarry station, particularly those motor driven. Also, in view of the fact that the technical press has recently brought to our attention the fact that the Boston Edison Company is being criticised before the Massachusetts State Commission for providing storage battery protection on the Edison d-c. system, it will be gratifying to hear just how much the storage batteries in Chicago assisted in maintaining the voltage when Quarry Street service was interrupted. The writer has always been rather strongly of the opinion that the storage battery was a very important part of the Edison system and every occurrence, such as has been exemplified here, only tends to still further confirm this opinion.

As to the actual physical damage done by the short circuit to cables and transformer terminals, considering the time involved, we should have expected more evidence of destructive power of the arc, judging from experience with smaller generating capacities involved where practically as great cable damage has been effected.

One other point is brought to mind by the author's paper and that is the delay involved at times in restoring service when the throttle valves on the turbines trip as was here instanced, when three of the four machines at Quarry Street had the steam supply shut off on account of overspeed. It has been the writer's experience that with certain classes of turbines at least, it becomes necessary to wait until the turbine speed is reduced by load or otherwise to slightly below normal before these throttle valves could be opened again. In view of the extremely short length of time, from two to five minutes, before full service was restored, it will be of assistance to know more in detail as to how this matter was handled. The shortness of the interruption is indeed sufficient testimony of the excellent organization and training of the Chicago operating force.

Louis F. Blume: Mr. Collbohm's paper, with reference to the phenomenon observed in the discharge of the neutral of a Y-connected transformer bank to ground, as the result of lightning disturbance, is interesting because the phenomenon as reported in the paper is quite unusual. Breakdown through high-voltage windings to ground in transformers which are insulated in accordance with the Standardization Rules of the A. I. E. E., that is an insulation equal to twice the normal, plus 1000, has been extremely rare.

The simplest and surest protection from such voltages as the author indicates, is to ground the neutral point; the next best method, perhaps, is to insert an arrester between neutral and the ground.

I have been unable to follow the author's reasoning in his explanation of the phenomenon observed, by which he concludes that the voltages which can be derived in the Y neutral, when the neutral is isolated, are considerably larger than can be derived in an isolated-delta bank of transformers. I do not believe the reasons given are at all conclusive; as an equally plausible line of reasoning can be given which, although also not conclusive, would indicate that the stresses may be derived by steep wave fronts in the interior of delta windings, equally as great as the voltages derived at the Y neutral. Suppose for example, a traveling rectangular wave on a transmission line enters or impinges upon the terminals of a transformer winding connected in a Y bank, and assume that the surge impedance of the transformer bank is very much higher than the surge impedance of the transmission line. It would naturally be expected that the wave at the very beginning of the transformer winding would be reflected, and that the maximum voltage that could possibly be generated at the transformer terminal would be twice the voltage of the incoming wave. The majority of the energy would be returned to the line, and only a fraction would enter the transformer. Assuming that a fraction of the wave at twice the potential of the incoming wave enters the transformer winding, at a very much reduced velocity, it is possible to imagine, that wave would travel through and reach the neutral point. Reflection at the neutral point will then take place just as at the open end of a line, causing a second doubling of the voltage, and therefore, four times the voltage of the original impinging wave.

On the other hand in a delta-delta transformer bank, assuming that the surge impedance is comparatively high, the wave dividing between the two transformer banks will have no effect on the potential of the wave which enters the winding; and the middle of each phase acting in the same manner the neutral would be raised to four times the voltage of the original wave.

The actual phenomenon of a rectangular wave entering a transformer winding, is much more complicated than has been described, since a wave on entering a transformer winding is very much distorted, and whether it is capable of arriving at the neutral point, and building up high voltages, as explained above, is by no means certain.

John B. Taylor: I am glad to see that Mr. Johnson states very definitely that the maximum synchronizing power results for a reactance which has some definite value, because I have been repeatedly confronted with the argument which he says is a general understanding, that the more you increase the reactance, the more stability you will have. Of course, that is obviously absurd for, if you carry the reactance to very high value, sooner or later you reach such high voltage that the system voltage would get about one ampere through it. Nevertheless, almost invariably the suggestion is made to put in more reactance, if there is any trouble with parallel operation.

In regard to the oscillation of the system, it seems to me here we are getting back to the old troubles that for a time we have almost forgotten, that is the question of parallel operation of reciprocating engines where governors, natural periods, and such things frequently give considerable trouble. One of the common remedies was to put a dashpot on the governor, which would tend to slow things down and prevent this building up. I would like to ask, in the case of this waterwheel trouble, whether any such remedy was considered or applied.

As I understand it, the trouble in the case of the fault in the cable described by Mr. Juhnke was probably due to heavy current, combined with the mechanical stress. The leads at the current transformers broke, and immediately broke the secondary leads, which accounted for the failure of the switches to operate, and for a period of some fifteen seconds this short-circuit current was flowing until the arc had spread to the third phase and then the switch dropped. I want to ask what difference was found on the cable between the station and the fault, and also on reactors and other devices. In the case of a current, in round figures, of 50,000 amperes, taking this cable at 250,000 cir. mils, and assuming a normal load current of 250 amperes, the 50,000 amperes is 200 times normal, and the heating effect in the cable would be 40,000 times the normal. It would seem doubtful that you could generate heat in a cable at 40 times the normal rating for fifteen seconds, without charring it up pretty well throughout its whole length.

With reference to Mr. Collbohm's paper Mr. Blume has already questioned the explanation, and that is about as far as I can go, as I have no better reason than he has for doubting the fact that 300,000 volts may have been observed in this particular case, but I do object decidedly to the line of explanation which is offered. There is no good reason for calling the neutral point of that transformer, the reflection point. The reflections occur at discontinuities. The biggest discontinuity that is encountered in that system is between the line and the terminal of the transformer, and that is probably the natural point for the voltage to pull up. It also seems to me that the reason whereby three conductors connected in parallel coming to a point would give three times the charge, and three times the voltage, is not a sound physical basis. I think a little parallel case will make this obvious.

Suppose we have four banks of Y-connected transformers, each on a different line. Now, the neutrals might be tied together. Are we going to have twelve times the voltage at that neutral point in that case? We might have any number we choose, up to several dozen conductors, meeting at one point, and just because we make the circuits multiple we would expect the figures to go up in to the millions. The notion that there is a charge on a point is quite beyond me.

R. F. Schuchardt: Mr. Taylor has just mentioned conditions which could be corrected by "more reactance". This may lead to the conclusion that we have gone reactance mad, but Mr. Juhnke's paper clearly shows that this is not the case and Mr. Johnson also recognizes that certain limitations must be kept in mind. We must remember that reactors are only a part of the total protective scheme—a very important part, it is true, but not the entire scheme. Protective relays, capacity of oil switches, strength of windings, etc., must all be taken into consideration in determining the amount of reactance to be installed in various parts of the system.

Mr. Cheney suggested that feeder reactors might be of more use than the generator or busbar reactors. Such feeder reactors are doubtless desirable in many cases but their installation in existing stations is usually an exceedingly difficult matter because of structural difficulties, while it is usually much easier to find the necessary space for generator and for busbar reactors. Furthermore the Chicago experience indicated that what was needed most were generator reactors and bus sectionalizing reactors.

Referring to the breakdown described in Mr. Juhnke's paper, we find that the evidence left after the trouble does not completely indicate the exact happenings. Mr. Taylor touched on a point which we could not definitely determine; that is, how long the current actually flowed through the cable into the fault. This may have been anything from one second to ten or fifteen seconds. The failure at the current transformers consisted in the shearing of the upper connector, first in the outer or *C* phase and immediately thereafter in the *B* phase. The amount of material actually burned off at the break is probably less than one-half inch in each case. The photographs are misleading on this point since the conductor was bent forward when the picture was taken and gives the appearance of some six to ten inches actually burned off. This break being on the switch side of the current transformers, of course prevented further excess current flowing through the relays on these two phases. The *A*-phase relay may have chattered, as these relays have occasionally done in the past on very heavy current, and thus had its action delayed, or the current in this phase may not have risen to the amount of the relay setting for several seconds. The evidence, as I said, is not clear, but we do know—and this is the important thing—that in this occurrence, of a kind which probably happens only once in a thousand times, the reactors which were installed for protection furnished the protection that was expected of them.

H. R. Summerhayes: In Mr. Johnson's paper he takes as the standard for stability the condition when there is no external reactance in circuit, that is, he simply has his generator reactance, and calls that 100 per cent stability and then compares that as a basis with conditions when certain percentages of bus reactance are introduced. Comparing it in that way leads to the conclusion that the bus reactances make very considerable difference in the amplitude of oscillation. It would seem to be fairer to take the condition which is demonstrated in the appendix $x = r\sqrt{3}$ as the maximum synchronizing power—take that as the 100 per cent—and then compare the actual condition of the generators in parallel without any reactance, except the generator reactance. That would be some other percentage. Then the condition of the bus reactances would be still another percentage. That, it would seem to me, would be a fairer comparison and would show that the bus reactances do not increase the period by such a great percentage as is apparent from the curves. It would appear to me that the cure for the trouble would be found in the governors rather than in decreasing the percentage of reactance.

C. A. Adams: I think that Mr. Taylor's difficulty is due to the difference between "*synchronizing power*", and the "*process of synchronizing*".

The synchronizing power of two machines may be defined as the restoring torque or power, due to a given small angle of displacement of their rotors from the position of equilibrium. It may be thought of as the stiffness of the electromagnetic coupling between the two machines. For small resistances in circuit and for moderate angles of displacement, the synchronizing power is approximately proportional to the reciprocal of the total equivalent reactance in circuit. But the greater this stiffness of coupling, (that is the lower the reactance), the greater the accuracy that must be employed when synchronizing, in order to avoid excessive circulating power between the two machines and the consequent stresses and surges. Thus a high synchronizing power may mean difficult synchronizing. Hence extra reactance is sometimes desirable during the process of synchronizing.

J. Allen Johnson: Mr. Woodrow was quite correct in that my arguments regarding instability refer to sudden changes of load rather than to short circuits. Sudden large changes of load are much more apt to occur in hydroelectric plants, where the feeders are usually few and of relatively large capacity, than they are in steam stations, where there are many feeders of small capacity.

It is interesting to know that Mr. Woodrow has confirmed my results by a different and more purely mathematical method. In my analysis of the problem I tried to stick as closely as possible to the fundamental principles of physics without going deeply into electrical mathematics, in the belief that the resulting formulas would be more generally comprehensible and useful. The results are not held to be final, as the process is complicated by the damping arrangements which the generators may have and by the changes which take place in their effective reactance. It is to be hoped that more light may be thrown on these matters. The point which I wished especially to bring out is that here is a limitation, a practical limitation, in the use of current-limiting reactance which has not been sufficiently recognized and which cannot safely be overlooked.

I was also very glad to have Mr. Woodrow bring out the importance of the feeder reactance and emphasize the fact that the benefit derived from feeder reactance in isolating disturbances is closely dependent upon the amount of generator and bus reactance, the greater the value of the latter the less being the benefit obtained from a given value of feeder reactance. This points to the advantage of concentrating as much of the total reactance as possible in the feeders.

In regard to Mr. Summerhayes' suggestion as to the use of a value of reactance equal to the square root of three, times the resistance, which gives the maximum synchronizing power, as the standard or representing 100 per cent stability, I would say that in a general consideration that would perhaps be desirable but in a specific practical case the engineer is confronted by a certain amount of reactance in his generators which he cannot change, and what he really wants to know is how far he will be going in the direction of complete instability by putting in external reactance. For this reason it seems to me preferable to base the expression for stability on the existing inherent reactance.

Regarding Mr. Summerhayes' suggestion as to correcting the hunting tendency at the turbine governors—that is a matter that has been given some consideration and it might perhaps be done by using inertia governors in hydraulic plants instead of centrifugal governors, which would change the phase relation of the turbine-gate oscillation with reference to the hunting oscillation, with probably beneficial results.

P. B. Juhnke: Concerning Mr. Cheney's inquiry in regard to the amount of battery capacity floating on the system at the time of the trouble, I wish to state that this amounted to 23,000 kw. In the substations feeding directly from the Quarry St. station the floating battery capacity totaled only about 9000 kw. During the period the short circuit existed, the entire battery capacity undoubtedly came into play in varying degrees in the different substations, due to the general drop in generating station pressure.

Mr. Blume mentioned the operation of the generator emergency valves and pointed out delays that possibly might result therefrom. I wish to say that the only operation necessary to again make these machines operative was to raise and block these valves, an operation requiring about 10 or 15 seconds, the machines having remained connected to the bus on the generator side.

With reference to the behavior of substation apparatus, I stated that this is a subject which lends itself to considerable study. While the relays and circuit breakers functioned as they were designed to do, the question may be raised as to whether their designed functions are absolutely the most desirable. The d-c. circuit breakers on all rotaries have an instantaneous setting of about 225 per cent of full load. The oil switches on the Edison 220-volt synchronous converters have a three-point setting of 220 per cent, 330 per cent and 440 per cent respectively, of full load; those on the railway converters an instantaneous setting of approximately 300 per cent.

In regard to the probable value of the short-circuit current, I might mention here that the values given in the paper do not take into account transient quantities, but are expressed in terms of mean effective values.

The maximum values of current mentioned here, the correctness of which Mr. Taylor doubted in the absence of evidence of damage due to excessive heating, could not have been maintained for any longer period than one or two cycles, for during this interval the break of the terminals with the swinging away of the oil switch leads is presumed to have taken place. This, in turn, introduced considerable resistance in the circuit to the fault, thereby alone very materially reducing the maximum value. The resulting drop in bus voltage also would have had its effect, so that the maintained short-circuit current could have been but a mere fractional part of the maximum initial value. Moreover, the current to ground established by the swinging of the two broken switch leads to ground constituted a path parallel to the faulty cable, the characteristics of which, however, are beyond calculation. This, I believe, answers the principal questions which have been raised in the discussion.

V. Karapetoff (communicated after adjournment). The theory given in the appendices to Mr. Johnsons' paper can be considerably simplified by using trigonometric expressions for power and impedance. The matter is worth mentioning because in technical literature one finds rather frequently mathematical discussions that are unnecessarily involved, because algebraic expressions are used in place of trigonometric. Referring to Fig. 1, let E_1 and E_2 be the equal voltages of the two alternators displaced by an angle δ. The difference of these voltages, E_c, produces a circulating current I_c. This current lags behind E_c by an angle ϕ such that $\tan \phi = x/r$. From the geometry of the figure, $E_c = 2E \sin \frac{1}{2} \delta$, where E is the absolute value of E_1 or E_2. The current $I_c = E_c/z$ where z is the impedance of the circuit. Therefore, Mr. Johnson's expression for synchronizing power in Appendix A, becomes

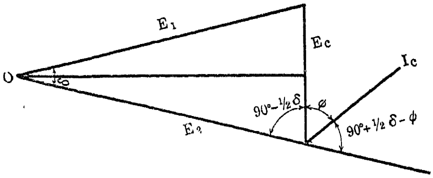

FIG. 1

$$P_s = E_2 \, I_c \cos \left(90 + \frac{1}{2} \delta - \phi\right)$$

$$= 2E^2 \sin \frac{1}{2} \delta \sin \left(\phi - \frac{1}{2} \delta\right) \div z \qquad (17)$$

Replacing in this expression the product of the sines by a sum, and writing $r/\cos \phi$ in place of z, we get

$$P_s = (E^2/r) \left[\cos (\delta - \phi) - \cos \phi\right] \cos \phi \qquad (18)$$

which corresponds to Mr. Johnson's equation (**11**) with the minus sign omitted.

With given r and x, angle ϕ is constant, and the expression in the brackets becomes a maximum when $\cos(\delta - \phi) = 1$, that is, when

$$\delta = \phi \qquad (\mathbf{19})$$

This corresponds to equation (**12**) in the paper. It will be seen from the diagram that in this case I_c is in phase with E_1. Thus,

$$P_s\,(\max) = (E^2/r)\,(1 - \cos\phi)\,\cos\phi \qquad (\mathbf{19})$$

Let now r be given, but x or ϕ be variable. Expression (**19**) becomes a maximum when $(1 - \cos\phi) = \cos\phi$, or

$$\cos\phi = 0.5 \qquad (\mathbf{20})$$

This is according to the well-known theorem that when the sum of two variables is constant, their product is a maximum when the variables are equal to each other. Sometimes this is expressed by saying that of all rectangles having the same perimeter the square has the greatest area. The same result is obtained by equating to zero the first derivative of expression (**19**). Thus, from equation (**20**) we get

$$\tan\phi = x/r = \sin\phi/\cos\phi = (\sqrt{1 - 0.5^2})/0.5 = \sqrt{3} \qquad (\mathbf{21})$$

which agrees with result (**14**) in the paper.

Let now r be variable, and x constant. Expression (**19**) can be written in the form

$$P_s\,(\max) = (E^2/x)\,(1 - \cos\phi)\,\sin\phi \qquad (\mathbf{22})$$

If $r = 0$, $\phi = 90°$, and equation (**22**) becomes

$$P_s\,(\max) = E^2/x \qquad (\mathbf{23})$$

This corresponds to Mr. Johnson's equation (**16**). When r is small, so that ϕ is nearly 90 deg., it is convenient to replace ϕ by its complementary angle

$$\varphi = 90\text{ deg} - \phi \qquad (\mathbf{24})$$

so that equation (**22**) becomes

$$P_s\,(\max) = (E^2/x)\,(1 - \sin\varphi)\,\cos\varphi$$

$$= E^2/x\left(\cos\varphi - \frac{1}{2}\sin 2\,\varphi\right) \qquad (\mathbf{25})$$

The expression in the parentheses is a measure for per cent error committed by neglecting the resistance, that is, by assuming, $\varphi = 0$. For example, if $\varphi = 3$ deg., $\cos\varphi - \frac{1}{2}\sin\ 2\ \varphi$

= 0.99863 – 0.05226 = 0.94637; the error amounts to about 5.4 per cent.

A. L. Harrington (communicated after adjournment): I am acquainted with a certain 60,000-volt bank of transformers, whose connections agree closely with those mentioned by Mr. Collbohm, but on which very different results have been observed.

These transformers are 60,000-volt Y connected with ungrounded neutral and part of the year they transmit power from a small hydroelectric plant and the remainder of the year, during the low-water season, they act entirely as substation transformers, and thus agree with Mr. Collbohm's condition. These transformers are tapped to a line which extends 35 miles to the south, the line being provided with but a small amount of overhead ground wire, and 25 miles to the north provided with a ground wire. At this point the line runs east about 60 miles with ground wire and northwest 75 miles with ground wire. There is also about 10 miles of line going directly west from the station. Mr. Collbohm does not state the length of his transmission line so we are unable to compare closely.

These transformers are provided with choke coils and with electrolytic arresters. They have been operating about eight years, the first three being at 33,000 volts, delta connected, and the balance of the time 60,000 volts, Y.

The clearance of the neutral bus to ground is the height of the insulator and from 15 to 16 in.

While very severe lightning occurs in this territory, no arcs from neutral bus to ground or other disturbances that Mr. Collbohm mentioned have ever been noticed.

M. H. Collbohm: Mr. Woodrow's suggestion of using a rheostat of high resistance is impracticable, and perhaps even dangerous. If built of a permanently reliable design, its cost would be ten to twenty times as high as that of a multi-gap arrester. Its high resistance might cause an excessive voltage drop under heavy charges, thereby unduly raising the neutral potential and defeating its purpose. It will, furthermore, permit a triple harmonic exciting current to flow over the ground connection, which might produce disturbances in the telephone lines.

Mr. Blume is in error in stating that an electrostatic charge meeting the inductance of a transformer is not only reflected back with double amplitude, but also sets up a wave of double amplitude going through the transformer to the neutral point. The doubling up of the wave crest in the reflected wave is conditional upon a complete reflection permitting no charge whatever to pass through the transformer to the neutral. If the reflection should be incomplete, the reflected wave will not have double amplitude. The wave passing through the transformer would, therefore, have a reduced amplitude, the more so the nearer the reflected wave approaches double amplitude. A wave with double amplitude passing through the transformer would, therefore, seem to be impossible. Again, the arrival of the wave at the neutral would not double its amplitude for the reason that the neutral is not a free end, but the three waves from the three different phases meeting at the neutral will superimpose and build up to three times their individual amplitude. Mr. Blume is also in error in stating that an electrostatic charge traveling along a single wire will maintain its amplitude after being split up into two charges, each entering a different circuit, as is the case in delta connected transformer relative to an electrostatic charge entering over the line. In such a case the amplitude of the original charge will be reduced to one-half for each of the two separate charges in the two branch circuits. In case of total reflection through the inductance of the transformer, these two waves will each build up to the original amplitude of the charge in the line, in which case no charge will flow through the transformer. For the same reason as stated above in case of Y-connected transformers, no charge of double amplitude seems possible to flow through delta-connected transformers. This, in connection with the above, supports the writer's argument that delta-connected transformers are not subject to triple-potential stresses through superimposed electrostatic charges entering over the lines as in the case of Y-connected transformers.

Relative to the points raised by Mr. Taylor, it may not be strictly correct to call the neutral point a reflection point, however, this name was chosen for lack of a better one available that would be suggestive of potential accumulation through the effect of traveling waves meeting and superimposing at that place. Mr. Taylor doubts the fact that three independent electrostatic charges traveling over separate wires and meeting at a junction point will build up to three times their individual amplitude. To illustrate the apparent absurdity of this argument, he suggests the case of four transformer banks with common neutral each feeding into a separate three-phase line, under which condition the neutral would be subjected to twelve times the potential possessed by the individual traveling wave at the neutral. The writer strongly believes that such an arrangement, if it ever did exist, would prove the writer's contention. Fortunately, however, transmission systems are not connected that way and, therefore, not subjected to danger from this particular cause. The nearest approach to this dangerous condition, as found by the writer, existed in a transmission system operating a 26,000-volt and a 33,000-volt line, running over the same route and connected to an auto transformer. This auto transformer has been punctured six times during lightning storms, and every time the puncture occurred at the neutral point. This persistent puncture at the neutral point would seem to give substantial evidence in support of the writer's contention.

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CORONA AND RECTIFICATION IN HYDROGEN

BY J. W. DAVIS AND C. S. BREESE

ABSTRACT OF PAPER

The results of an investigation of the corona discharge between co-axial cylinders in an atmosphere of hydrogen are given in this paper.

Both direct and alternating electromotive forces were used. The characteristic behavior of the corona is given by means of curves, photographs and oscillograms.

The corona discharge in hydrogen was found to differ from that in air in the following particulars:

The discharge from a negative wire was found to differ widely from a positive wire in the magnitude of the voltage necessary to start the discharge in the shape of the volt-ampere characteristic and also in the stability of the discharge.

Corona in hydrogen between concentric cylinders is shown to be a practicable method for rectifying high potential alternating currents.

The apparent evidence of ionization, potential gradients at the surface of the tube and the general character of the visual phenomena are discussed.

A brief statement of conclusions is given.

WHEN a sufficiently high potential difference is impressed between two parallel wires, or a wire and concentric cylinder, separated by air or some other gas, this gas which for low potential gradients is a very good insulator breaks down and becomes a partial conductor. The phenomena connected with this character of conduction through gases are known collectively by the name corona. The failure of the gaseous dielectric separating the metallic conductors is made evident by a flow of current from one conductor to the other, by a power loss and, in practically all cases, by the appearance of light at either one or both conductor surfaces. In some cases light appears in the intervening space.

Since the present theories as to the mechanism of corona formation do not satisfactorily account for all of the observed phenomena it was decided to carry out further investigations in the hope that when enough data were accumulated some theory based on fundamental principles and explaining the observed phenomena might be evolved. With this purpose in mind it was attempted to simplify the conditions of corona formation.

A wire and concentric cylinder were used in order to make the field radial and to get away from the secondary effects due to the high intensity electric field surrounding a second wire. Hydrogen was used as the dielectric in order to minimize the effects due to changes in the chemical constitution of the gas. When air is used as the dielectric the formation of ozone may produce marked changes in the voltage necessary for corona formation. Continuous potential was used in order to separate the effects accompanying a discharge from a positive wire to a negative tube from those which are characteristic of the discharge from a negative wire to a positive tube.

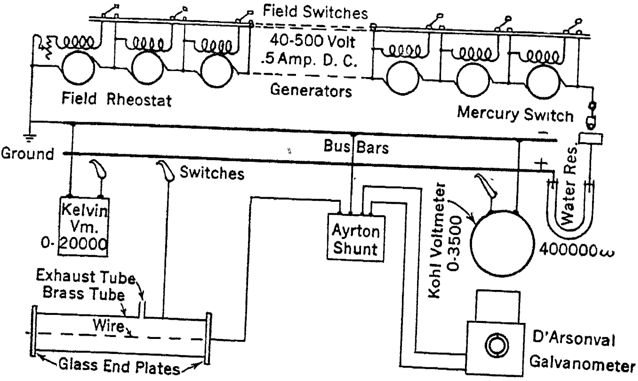

FIG. 1—DIAGRAM OF CONNECTIONS

APPARATUS

The continuous voltage used in these investigations was obtained by means of a battery of forty 500-volt 250-watt continuous-current shunt-wound generators connected in series.

These machines are divided into two sets of ten machines each and one set of twenty machines, each set being driven by a belt-connected continuous-current shunt motor. The generators are mounted on insulating bases and the shafts of the separate machines are connected by insulating couplings. In the newer part of the installation one terminal of each machine is permanently connected to the frame of that generator in order definitely to limit the strain on the machine insulation to the voltage generated in one armature.

The field of each machine is connected directly across the armature terminals, a single-pole knife switch being included in the circuit in order that the machine may either be made to generate or to run idle at will. These switches were operated by means of a hard rubber rod approximately eighteen inches in length, since they may be 20,000 volts above earth potential. The generators were run somewhat below rated speed in order to limit, to a safe value, the voltage generated with no external resistance in the field circuit.

The diagrams of connections for the direct-current and alternating-current tests are given in Fig. 1 and Fig. 2 respectively. These are self-explanatory.

Instruments. Voltages were measured with a vertical type Kelvin electrostatic voltmeter having ranges of 20,000, 10,000 and 5000 volts and a Kohl electrostatic voltmeter having a range of 3500 volts. These instruments were calibrated from time to time by means of an attracted disk electrometer.* The maximum error in these voltage measurements for a given voltmeter scale reading was less than two per cent. The probable error is less than one per cent.

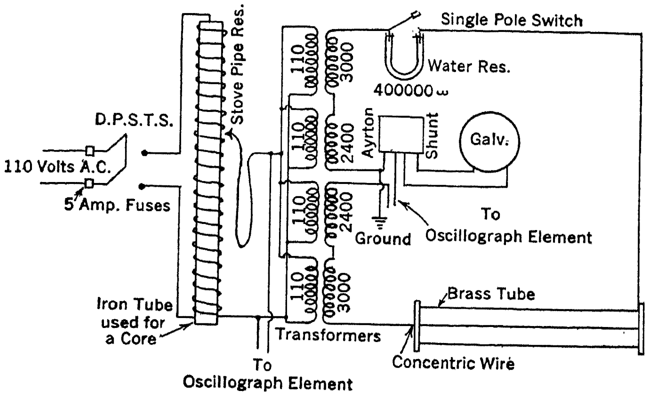

FIG. 2.—CONNECTION DIAGRAM FOR HALF-WAVE RECTIFICATION

Currents were measured by means of a d'Arsonval galvanometer and Ayrton universal shunt. These were calibrated as a unit by connecting them in series with a high resistance and a dry cell, the voltage of which has been accurately determined by means of a potentiometer. The voltage of the dry cell was de-

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The theory of the attracted disc electrometer. (TRANS. A. I. E. E., Vol. XXXIII, Part II, 1914, p. 1635.)

termined both on open circuit and with the smallest series resistance used in calibrating the galvonometer connected between its terminals as a load. The change in terminal voltage due to the current flowing was, in every case, found to be entirely negligible.

PREPARATION, PURIFICATION AND TEST OF HYDROGEN

The hydrogen used in these investigations was prepared by the action of water on hydrone, in a glass containing vessel. Hydrone is the trade name for a sodium lead alloy. This alloy was used in order to reduce the violence of the reaction between the sodium and the water. The hydrogen prepared in this manner was collected over water in a copper vessel.

When it was desired to fill a corona tube with hydrogen the gas was forced out slowly by water pressure. It was passed through drying bottles containing concentrated sulphuric acid and calcium chloride respectively. It was then passed through a hard glass tube containing red hot calcium or calcium hydride. This acted as an effective reagent for removing any traces of oxygen or nitrogen.

The purifying tube was filled with metallic calcium but when this is brought to a red heat in an atmosphere of hydrogen a rapid combination takes place. The hydride formed in this manner must be kept in an atmosphere of hydrogen as it combines with air at room temperature.

From the purifying tube the hydrogen was passed through glass tubing to the corona tube. When a corona tube was to be filled with hydrogen it was exhausted and filled with hydrogen three times in order to remove any traces of air. In every case a Geissler tube was connected to the system while the corona tube was being filled. The spectrum of the gas in this tube was observed at pressures ranging from atmospheric down to the hard X-ray stage. If lines belonging to any spectrum other than that of hydrogen were observed at any pressure the charge was discarded and the corona tube was refilled.

Corona Tube. The results reported in this paper were all obtained with copper wires strung along the axis of a cylindrical brass tube.

The corona tubes were connected to vacuum pumps, air drying bottles, hydrogen apparatus and a mercury manometer by means of a small side tube on each corona tube. One side of the mercury manometer was kept at zero pressure. In this way the manometer readings were made to give the absolute pressure of the gas and were not affected by changes in the barometric pressure. The ends of the brass tubes were closed by means of glass plates through which holes had been ground. Metal bushings having a small hole along the axis were fitted into these holes and secured with red sealing wax. The glass plates were fastened to the ends of the tubes in such a manner that the hole through the metal bushing coincided with the axis of the tube.

The wire on which the corona discharge was to be observed was passed through these bushings. One end was fastened to a spring enclosed in a bottomless glass bottle which was sealed to the end plate so as to surround the bushing. The stopper was removed from this bottle when a new wire was being placed in the tube. When the wire was in place the stopper was sealed tight with half and half wax.* After the wire had been stretched tight the other end was fastened in the metal bushing with red sealing wax. In this way the wire was maintained tight and accurately centered.

The metal bushings were necessary as it was found that with hydrogen a very decided effect was produced on both the corona starting voltage and the volt-ampere characteristics, when corona was allowed to form in the vicinity of these end plates. This effect was due to the breaking down of the gas between the glass and the wire where the wire passed through the glass end plates, as it was not always possible completely to fill this space with wax and at the same time keep the wire within the tube free from the wax, which would start a local discharge.

The wires used in these investigations were carefully cleaned and straightened and then calipered with a micrometer caliper before being placed in the tube. These wires varied in diameter from 0.12 mm. to 1.59 mm. The larger part of the data was taken with the wire in a brass tube 4.45 cm. in diameter but the results were checked with a 7-cm. tube in order to determine whether the size of the tube would affect the nature of the phenomena. No changes due to the size of the tube were noticeable as far the general character of the results and the critical theoretical electric intensity at the surface of the wire are concerned.

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*Half and half wax is prepared by melting together approximately equal quantities of bees wax and rosin. The heat should be applied by means of boiling water.

CORONA IN HYDROGEN

Throughout the investigation with hydrogen it was found that the breaking down of the dielectric and the appearance of light at the surface of the wire were simultaneous. The voltage necessary to cause these phenomena will be called the critical voltage. The intensity of the electric field at the surface of the wire, under these conditions, computed on the assumption that the field between the wire and the tube is completely determined by the charges on the wire and tube, will be called the critical intensity. The above assumption neglects the possibility of distortion of field due to space charge before the corona starts.

Corona between a positive wire and negative tube will be spoken of as positive corona, while corona between a negative wire and positive tube will be spoken of as negative corona.

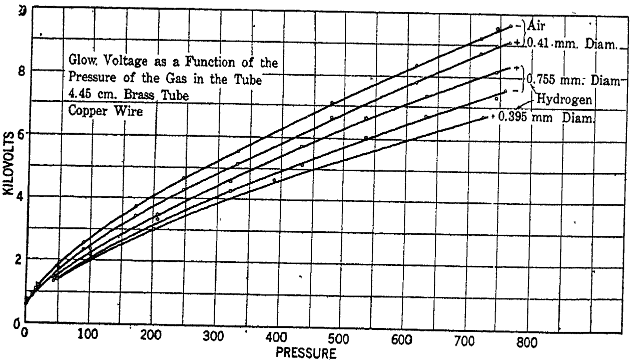


FIG. 3

The + and − signs on the curve sheets refer to the polarity of the wire.

In Fig. 3 are given curves showing the variation of critical voltage with gas pressure for air and hydrogen with two sizes of wire. The curves for air were taken from Farwell's paper (TRANS. A. I. E. E., Vol. XXXIII, Part II, 1914, p. 1648.). The curves for air and hydrogen were taken with tubes of the same diameter and therefore may be justly compared. From a casual observation of these curves it is apparent that the law of variation of critical voltage with pressure is essentially the same in the two cases. The curves for hydrogen agree fairly well with Peek's formula when this is put in the form

$$E = E_0 p + C \sqrt{p}$$

where E = critical intensity.
p = pressure in per cent of atmospheric pressure.
E_0 and C = constants.

This shows that as far as variation of pressure is concerned, with a given size of wire, the critical intensity for corona in hydrogen obeys Peek's law.

The data so far obtained do not show conclusively that Peek's formula

$$E = E_0 p \left(1 + \frac{b}{\sqrt{pr}} \right)$$

holds for hydrogen when the variation of E with changes of the radius of the wire is considered. In this formula

E = critical intensity.
p = pressure in per cent of atmospheric pressure.
r = radius of wire.
E_0 and b = constants.

The general nature of the results so far obtained however seems to indicate that this formula may hold.

From the curves of Fig. 3 it may be seen that there are two decided differences between the critical voltages for air and hydrogen.

1. With air the positive critical voltage is lower than the negative, while with hydrogen the negative critical voltage is less than the positive. This is true in the case of hydrogen for all sizes of wire and all pressures covered in this investigation.

2. For the same size of wire and the same pressure the critical voltage in hydrogen is much lower than the critical voltage in air.

Characteristic Curves. Characteristic curves for positive corona in hydrogen for two sizes of wire and various pressures are given in Fig. 4 and Fig. 5. These represent the general form of the characteristics for all sizes of wire and all pressures covered in these investigations. The most remarkable points about these characteristics are:

a. The marked difference between the critical voltage and the voltage at which corona ceases.

b. The difference between the points taken with increasing and those taken with decreasing current.

This persistence of corona at voltages less than that necessary to start the discharge has not been observed with air and continuous potentials. (See Bennett TRANS. A. I. E. E., Vol. XXXII, Part II, 1913, p. 1796, for alternating potentials.) If there is any such difference for air its magnitude is certainly very much less than with hydrogen.

This difference between the critical voltage and the voltage at which corona is maintained may be explained by the change in the electric intensity (volts per cm.) at the surface of the wire after the corona has been formed. This change of intensity is caused by the space charge due to the positive and negative ions in the space between the wire and the tube. That such a distortion of the electric field exists is well known. The difference between the critical voltage and the voltage at which the discharge ceases is not detected unless all sources of irregular ionization such as rough spots on the wire, high intensity where the wire passes through the glass, etc., are eliminated.

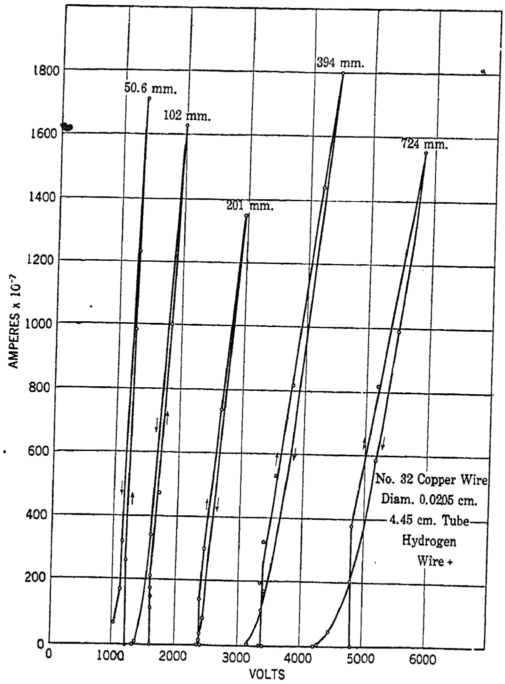


FIG. 4

The negative corona characteristic curves for a No. 36 B. & S. gage copper wire, 0.121 mm. in diameter, in a brass tube the inside diameter of which was 4.45 cm. are given in Fig. 6. It will be noticed that the shape of these characteristics depends on the gas pressure to a very decided extent.

With this size of wire corona seemed to start in each case with a number of very small bright points on the wire. Usually, however, the negative corona started with an unstable flickering glow along the wire. With an increase of voltage the current gradually increased until a point was reached where the small bright spots combined into one negative bead of the kind described by Farwell (TRANS. A.I.E.E., Vol. XXXIII, Part II, 1914, p. 1631.) This change in formation was accompanied by a large increase in current and a drop in the potential difference between the wire and the tube. This drop in potential difference across the tube was, in a certain sense, due to the resistance in series with the corona tube. The increased current taken by the tube

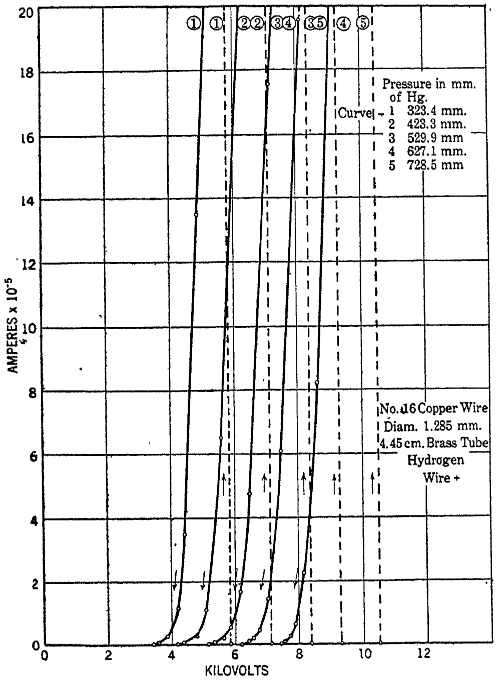


FIG. 5

causes a higher drop through the series resistance, which stabilizes the corona discharge and prevents the indefinite increase of current which might result if there were no resistance in the circuit. The gaps in the curves represent unstable conditions. An increase in the generated voltage caused an increase in the corona current which was accompanied by either an increase or

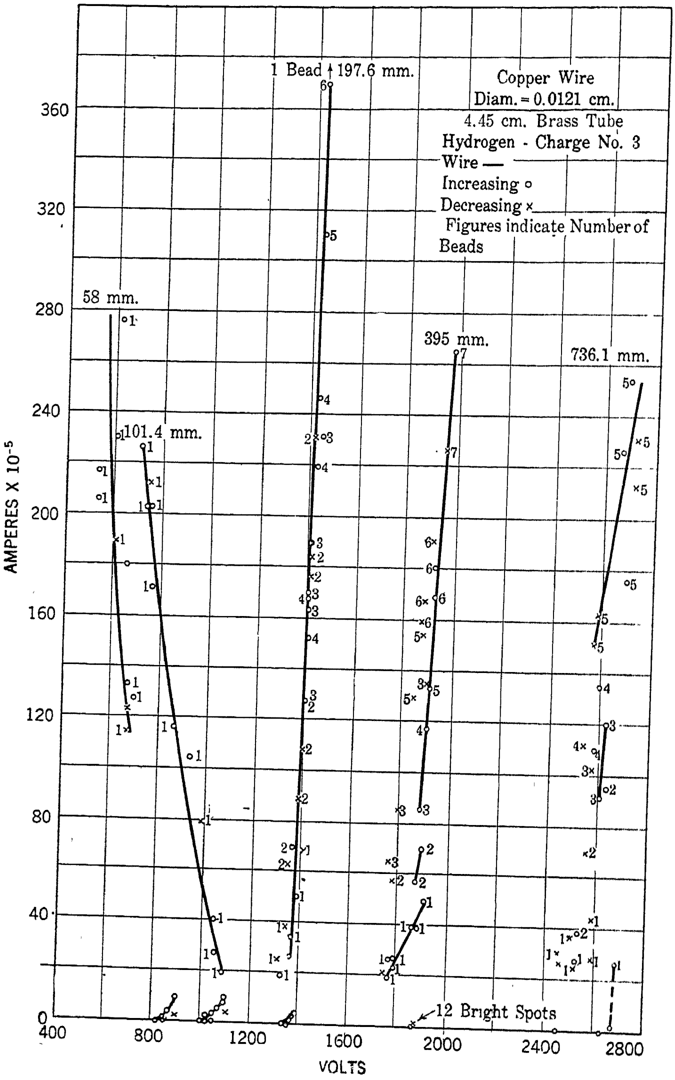


FIG. 6

a decrease of the voltage between the wire and the tube depending on the size of wire and the gas pressure.

It may be noticed that for the lower pressures the corona current-voltage characteristics are similar to the usual arc characteristic. The discharge, however, was not the arc discharge. The arc formed if the machine voltage was sufficiently increased. This was accompanied by a further drop in the voltage between the wire and tube and an entire change in the general appearance of the discharge.

Difference Between Air and Hydrogen. The characteristic curves in Fig. 7 show clearly the difference between the corona in air and in hydrogen. The two curves on the left hand side of the sheet refer to hydrogen while the curves on the right were obtained with air. The data for these curves were obtained with wires of approximately the same diameter. The data for corona in air were taken from Farwell's paper. These curves show two differences between air and hydrogen. In air the positive corona starts at a lower voltage than the negative while the reverse is true in case the dielectric is hydrogen. The other and more important difference is the entire dissimilarity of the positive and negative curves with hydrogen. This difference has not been found with air.

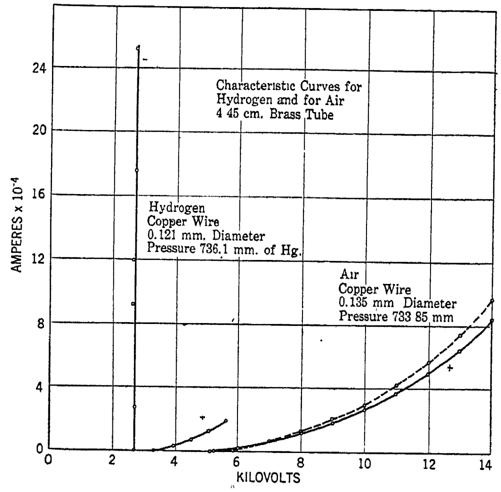


FIG. 7

It must be remembered that, although the characteristic curves for the wire positive appear to be the same for air and hydrogen, there is a decided difference even in this case as, with hydrogen, the corona was maintained at voltages lower than that necessary to start the discharge.

RECTIFICATION

After looking at the above curves it is at once evident that corona in hydrogen between a wire and a tube is an effective means of accomplishing rectification. This is true since large currents may be obtained from a negative wire with voltages which will give little or no current from a positive wire. This rectification may be accomplished with the wire cold and is not a case of rectification by means of a hot cathode. The principle of electron emission from a hot cathode might be used, however, in connection with the corona rectification.

Following up the question of rectification the corona tube was connected to an alternating source of voltage as shown in Fig. 2. A sensitive oscillograph element was connected directly in series with the corona tube and another element was connected across the primaries of the transformers. The oscillograms shown in Figs. 8, 9 and 10 were taken with the connection as shown in this diagram.

Fig. 8 gives the current and voltage curves when an alternating voltage is impressed across a corona tube. The curve having both positive and negative lobes is the voltage. The non-symmetrical character of this curve is due to the drop in the resistance in series with the primaries of the transformers. The curve lying mainly below the axis represents the current flowing between the wire and tube. The part of this curve slightly above and slightly below the axis is of the same shape and order of magnitude as the charging current of the condenser formed by the wire and cylinder. This was determined by an oscillogram taken at a voltage slightly less than that necessary to form corona.

The voltage at which negative corona starts as shown in this oscillogram is approximately twice the voltages across the tube at the instant that the discharge ceases.

The rectification with corona in hydrogen is practically perfect as is shown by the oscillograms in Figs. 8 and 9. Fig. 10 gives the voltage and current with corona in air at 100 mm. pressure. It is of interest to compare Figs. 8, 9 and 10 with those given by Bennett for air at low pressures (TRANS. A. I. E. E., Vol. XXXII, Part II, 1913, p. 1787.).

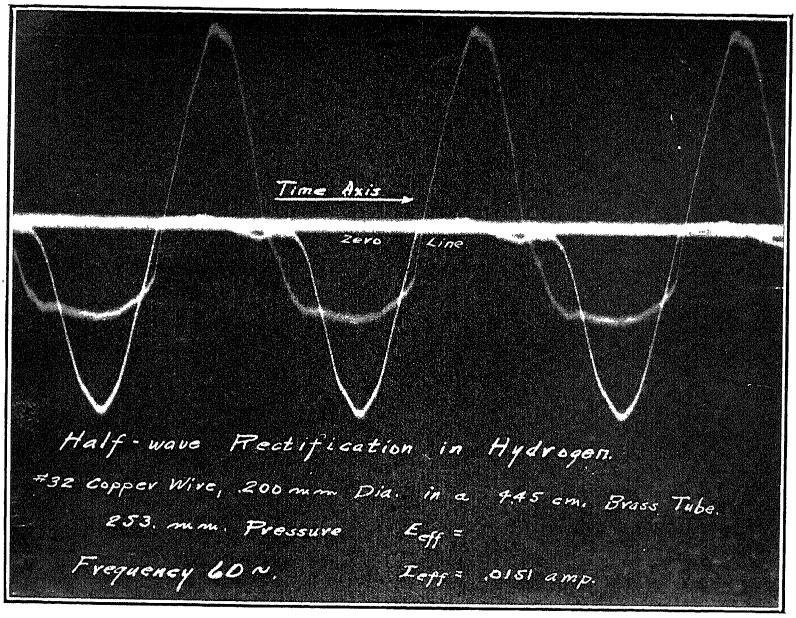

Time Axis
Zero Line.
Half-wave Rectification in Hydrogen.
#32 Copper Wire, .200 mm. Dia. in a 4.45 cm. Brass Tube.
253. mm. Pressure E_{eff} =
Frequency 60 ~.
I_{eff} = .0151 amp.

FIG. 8

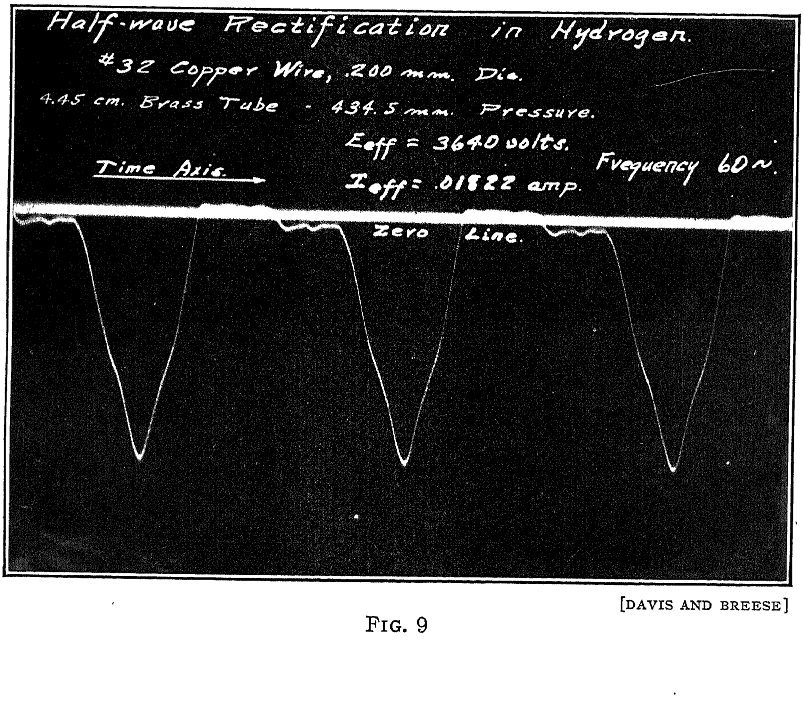

Half-wave Rectification in Hydrogen.
#32 Copper Wire, .200 mm. Dia.
4.45 cm. Brass Tube - 434.5 mm. Pressure.
E_{eff} = 3640 volts.
I_{eff} = .01822 amp.
Frequency 60 ~.
Time Axis.
Zero Line.

FIG. 9

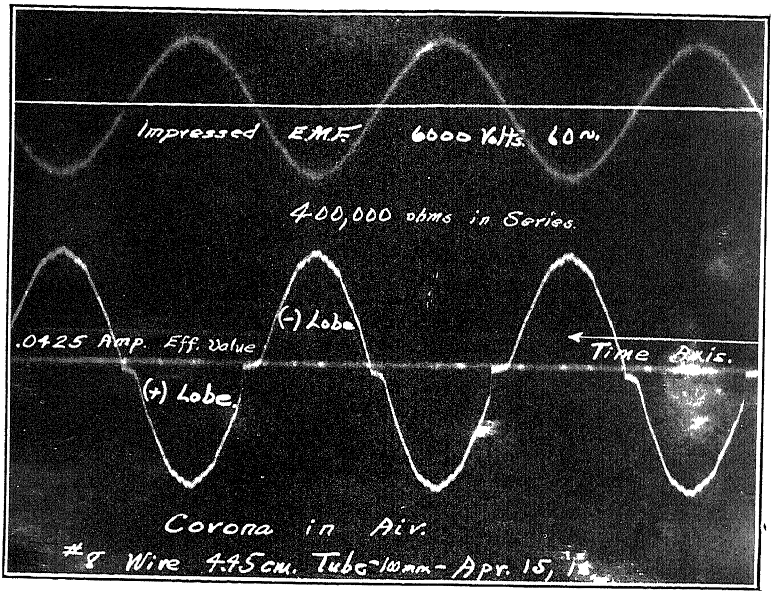
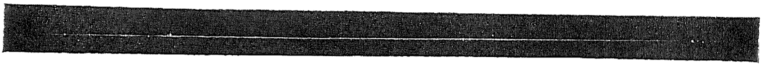
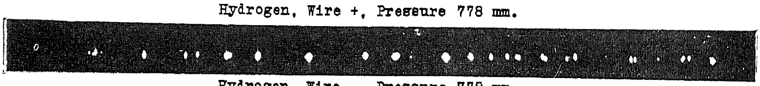


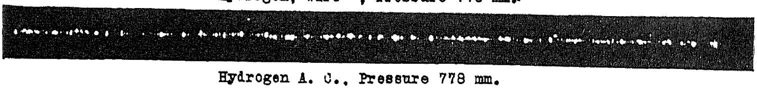
Fig. 10



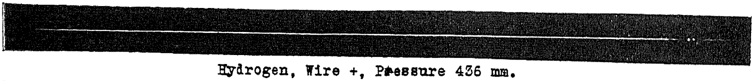
Hydrogen, Wire +, Pressure 778 mm.



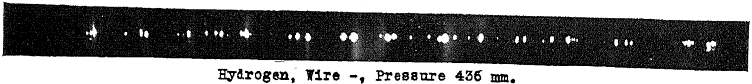
Hydrogen, Wire -, Pressure 778 mm.



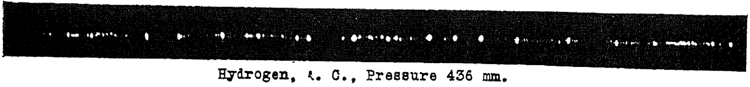
Hydrogen A. C., Pressure 778 mm.



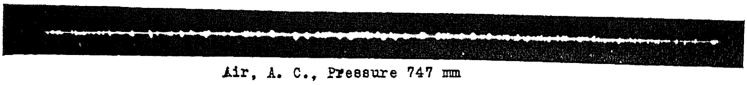
Hydrogen, Wire +, Pressure 436 mm.



Hydrogen, Wire -, Pressure 436 mm.



Hydrogen, A. C., Pressure 436 mm.



Air, A. C., Pressure 747 mm

[Davis and Breese]

Fig. 11—Corona in 4.45-cm. Brass Tube with 0.200-mm. Copper Wire

The irregularities in the part of the curve near the zero line seem to be due to the capacity effect and are in no sense characteristic of the corona discharge.

The whole question of rectification is at present being made the subject of further investigation.

General Appearance

In general appearance the corona in hydrogen is similar to that in air, but upon critical examination it is seen that there are many points of difference. Some of these similarities and differences are shown in Fig. 11.

With the wire positive and sizes of wire smaller than No. 26 B. & S. (0.405 mm. diam.) the wire was surrounded by a thin luminous layer which was essentially uniform. This is shown in Fig. 11. As the size of the wire was increased this glow became more and more irregular. At first there were spots somewhat brighter than others. For a wire as large as No. 10 B. & S. (2.59 mm. diam.) the discharge took the form of a large number of brushes similar to the point discharge in air. These brushes were closely spaced along the length of the wire with a fairly uniform glow between them. It was impossible to obtain a satisfactory photograph of this. This discharge was blue in color. If the voltage was increased to a high value a brilliant red spark passed between the wire and tube. This was followed by a pale blue arc having incandescent blue spots at both ends.

The typical form of discharge with the wire negative is also shown in Fig. 11. It is seen that the discharge consisted of bright beads on the wire. As the potential was continuously increased, the first evidence of a breakdown of the gas was the appearance of an intermittent flickering glow. In every case the first noticeable current and visible corona occurred simultaneously. If bright spots were formed on the surface these increased in number and the current gradually increased until a stage was reached where the small bright spots collapsed into one brilliant bead. This change was accompanied by a rapid increase of current and a drop in the voltage across the tube. (See Fig. 6.) A further increase in voltage caused more small bright spots to form and also caused an increase in the brilliancy of the beads. The bright spots collapsed into another bead and in this manner the number of beads and the current increased as the generator voltage was made larger. If the generator voltage was now decreased the behavior of the corona was somewhat different. Suppose that there were nine beads on the wire. These beads persisted and became less and less brilliant until they finally collapsed into one or two beads. Thus it is seen that with increasing voltage the beads appear one after another, while with decreasing voltage they will continue until several disappear at one time, thus concentrating the current in a smaller number of more brilliant beads.

With the wire negative there was no evidence of a uniform glow for any size of wire or any pressure covered in this investigation unless an arc was in series with the tube. This is not true with air.

Opposite to each bead for practically all pressures and all sizes of wire used in these experiments there were a number of bright spots on the tube. The number of spots varied from ten or twelve up to a hundred or more depending upon the current flowing from the bead. If there were only a few spots they would be bunched together, but when the current was large the spots formed a band with a width of approximately one centimeter completely surrounding the tube. These spots varied in color from pale yellow to milky blue, depending on the pressure, current and size of wire. With low pressures and large currents these spots appeared as brilliant green fans with yellow spots where they touched the tube. The effect was then truly beautiful. This green color may be due to the brass tube, and thus may not be characteristic of hydrogen. The presence of these spots seems to indicate an ionizing gradient at the surface of the tube.

With an alternating voltage impressed across the tube the corona may take the form shown in Fig. 11. In the illustration for 436 mm. this is essentially the same as the negative corona, since the conditions under which this picture was taken were such that there was perfect rectification and no positive corona was present. If any positive corona was present it would be made evident by a uniform glow between the negative beads. The corona in air with alternating voltage is also shown in Fig. 11. The difference is at once apparent. If the alternating voltage on the hydrogen filled tube is increased sufficiently, positive corona will form and finally the positive arc will follow. The arcing during the half cycle when the wire is positive can be determined immediately, both by the fact that the red positive spark is easily visible shooting across the blue corona and negative discharge, and also by the fact that the rectification is much less perfect.

The predominating color of the various discharges through hydrogen was blue, ranging from a light silver blue to sky blue. The one marked exception to this was the brilliant red of the disruptive discharge from a positive wire. This brilliant red spark was followed by a pale blue·arc.

When a continuous voltage was impressed on the tube and a short arc (1mm. or less) was introduced into the circuit the following changes in the character of the discharge were observed.

With a No. 26 B. & S. gage (0.405 mm. diam.) wire in the tube and the wire positive, the introduction of a short arc caused the uniform glow to break up into straight blue streamers extending from the wire to the tube with brilliant light blue spots at each end. This discharge resembled a radial shower of blue light. Lengthening the arc caused the streamers to disappear and the entire tube was filled with a blue glow. With a No. 36 B. & S. gage (0.121 mm. diam.) wire, the radial streamers did not appear when the arc was introduced in the circuit, but the tube was filled with a blue glow.

With the wire negative the introduction of an arc in series with the tube caused very little change in the appearance of the corona at the surface of the wire, but caused a disappearance of the bright spots on the surface of the tube.

With the wire negative and hydrogen in the space between the wire and the cylinder an increase in the gas pressure as great as 3 cm. of mercury has been observed on closing the circuit. This pressure increase was due to ionization of the gas and not to increased temperature as it immediately disappeared when the current was shut off.

Conclusions

1. The critical intensity for corona in hydrogen follows Peek's law fairly closely as far as changes due to variations in the gas pressure are concerned.

2. The variation of the critical intensity with the radius of the wire is not as yet conclusively proved either to follow or not to follow Peek's law but the general character of the results so far obtained seem to indicate that the law may hold.

3. Hydrogen is different from air in that corona is maintained at voltages considerably lower than the critical voltage while this has not been found to be the case with air.

4. The characteristic curves with the wire positive in hydrogen differ from those with air in that the voltage necessary to maintain a given current depends on whether that value of current has been approached by increasing or decreasing the machine voltage.

5. The characteristic for the wire negative in hydrogen is entirely different from the characteristic obtained with a negative wire in air.

6. Complete rectification for alternating current is obtained between concentric cylinders when the intervening space is filled with hydrogen and an alternating potential difference sufficiently high to produce negative corona is impressed between the cylinders.

7. This rectification continues after the negative arc has been established, under the proper conditions of pressure, size of wire and size of tube. Direct currents as high as 0.12 ampere have been obtained with alternating voltages of approximately 8000 volts.

The investigations outlined in this paper were carried out in the Laboratory of Physics at the University of Illinois under the direction of Dr. Jacob Kunz, Associate Professor of Physics. To him, to Professor E. B. Paine of the Department of Electrical Engineering and to Professor A. P. Carman of the Department of Physics the authors wish to acknowledge their indebtedness for suggestions and aid which made this work possible.

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THE ELECTRIC STRENGTH OF AIR.—VII

BY J. B. WHITEHEAD AND W. S. BROWN

ABSTRACT OF PAPER

In view of the variation among the values obtained by different observers, this paper aims to make a careful determination of corona-forming voltages for alternating and for positive and negative continuous voltages in the same apparatus and under the same conditions.

Within the range of wire diameters used, corona appears at a lower value when the wire is positive than when it is negative, although the two curves converge for increasing diameters. The maximum excess of negative over positive, as observed, was 6.3 per cent.

The values with alternating voltage coincide with those of negative continuous voltage. Positive continuous voltage therefore forms corona at the lowest value.

The observations on the negative corona give values higher than any heretofore obtained.

Other experiments are described, giving qualitative indications of the correctness of Townsend's theory of ionization by collision.

THE voltage at which corona appears on a round wire in air depends on the diameter of the wire, the potential gradient at the surface and the density of the air. The relation as based on the results of these papers and those of Peek[1] is of the form,

$$E = A\,\delta + B\sqrt{\frac{\delta}{d}} \qquad (1)$$

in which E is the potential gradient at the surface of the wire of radius r or diameter d, δ is the so-called "density factor", its value being unity at a pressure of 76 cm. and a temperature of 25 deg. cent., and A and B are constants. In the experiments on which formula (**1**) is based alternating voltages were used, and it was shown that the maximum value of voltage or gradient determines the appearance of corona.

If the value of E as obtained from the empirical formulas of Whitehead[2] and Peek and from the results of some of the other investigators of the alternating-current corona be plotted as a function of the radius of the wire, we find that the value of E for any one wire shows noticeable discrepancies, *i.e.*, the constants A and B have different values. The differences in the observed values are probably due to the difficulty in measuring the ratio of maximum to effective values of the high voltage, to the unequal surface condition of the wires and to failure to properly correct for atmospheric conditions. The observations of Whitehead gave $A = 32$ and $B = 13.4$. The work of Peek[1], somewhat later, showed a similar relation between E and d, giving $A = 29.8$ and $B = 12.7$. These values in his subsequent papers have been brought up to 31.0 and 13.5 respectively, which are in close agreement with the result of Whitehead.

The most important work on the direct-current corona has been done by Watson[3], Shaffers[4], and Farwell[5]. Watson made observations with wires ranging from 0.70 mm. to 12.76 mm. in diameter, using as a source of power an influence machine of special design. For the case of a wire and cylinder he found that the polarity of the wire had a marked influence on the appearance of corona and on the value of the critical intensity. Schaffers, using wires from 0.0006 to 0.70 cm. in diameter in tubes of various sizes, found that for the larger wires the positive corona appeared at a lower voltage than the negative, while for the smaller sizes the reverse was true. The curves of critical intensity crossed at a point corresponding to a wire 0.01 cm. in radius.

Farwell, using a series of 500-volt generators as a source of high potential, investigated the influence of the polarity of the wire, of temperature, pressure and humidity on the corona-forming voltages on copper wires ranging from 0.009 to 0.258 cm. in diameter in a tube 4.45 cm. in diameter. For a given size wire, he found that corona appeared at a much lower voltage when the wire was positive. His observations can be represented by formula (**1**) and the value of the constants as given in his A. I. E. E. paper are for the positive wire, $A = 31.6$, $B = 8.47$, for the negative wire, $A = 38.0$, $B = 8.06$. Here again, as in the alternating-current case, the results of different investigators show considerable divergence.

It was in view of these differences among the values of the constants of formula (**1**) as determined by different observers that the present investigation was undertaken. The different conditions under which the several investigators worked should account in a large measure for the different values obtained for both alternating and continuous voltage. The aim of the present experiments has been to compare alternating and continuous values of corona voltage under identical conditions and under the best possible conditions for accuracy.

Some later experiments outlined in Section V had as their purpose the testing of some of the assumptions of the present day theories as to the start and ultimate nature of corona.

Since the experiments of this paper were completed Peek[10] has published results of a comparison of alternating and continuous corona voltages. He finds that they are identical and independent of the polarity of the wire in the case of continuous voltage. The results described in this paper are noticeably at variance with those of Peek.

II. Description of Apparatus

For Alternating Voltages. The diagram of connections for the production of and measurements on the alternating-current corona is given in Fig. 1. The motor-generator G supplies the

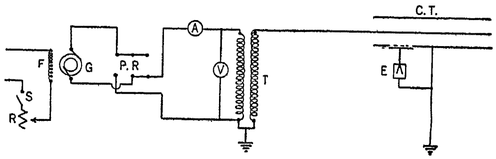


Fig. 1

low-tension winding of the transformer T through a potential regulator, $P\ R$. One terminal of the high-tension winding is connected to the low-tension winding and to ground and the other to the corona wire. $C\ T$ is the corona tube and E, an electroscope, both described in detail in a subsequent paragraph.

The motor-generator set consisted of two 5-kw., 1200-rev. per min., 120-volt alternating-current generators, one 60 and the other 20 cycles driven by a 7.5-h.p., 120-volt direct-current, shunt-wound motor. The use of a storage battery of large capacity as a source of energy for driving the motor and for exciting the two alternators made it possible to obtain a very constant voltage. Throughout the experiments only the 60-cycle unit was used. The armature was surface wound and the wave smooth.

The transformer used was rated at 3000 watts, 60 cycles, 100-25,000 volts. The low-tension winding was in two 50-volt sections, the high-tension in four 6250-volt sections. The ratio of turns of the transformer as furnished by the manufacturer was 1-250.18 and this ratio was used to determine the secondary voltage. The primary voltage was read on a standardized Weston dynamometer voltmeter *V* (Fig. 1) and was controlled by the potential regulator *P R* and a variable resistance *R* in the generator field *F*. The influence of the varying *I R* drop in the primary winding was practically negligible in its effect on the ratio of transformation owing to the small values of the magnetizing and load currents.

The corona apparatus consisted of a wire stretched co-axially in a metal tube. The wire was clamped at each end in a device which permitted stretching and centering. The tube was mounted on an insulated stand which also supported a very sensitive electroscope. Small holes were drilled in a section of the tube and a disk of the same curvature as the tube was connected to one terminal of the electroscope and placed as close to the tube as possible without actual contact. The disk, the connections to the electroscope and the electroscope proper were protected by grounded metal shields. The electroscope was charged to a potential of 440 volts from the lighting mains through lamps and condensers by parallel-series connection. The insulated stand was grounded for the alternating-current observations thus grounding the other terminal of the electroscope. The ionization accompanying the very first appearance of corona causes the electroscope to discharge rapidly, which thus served as detector of corona. Fig. 4 is an illustration of the tube as used in some later experiments.

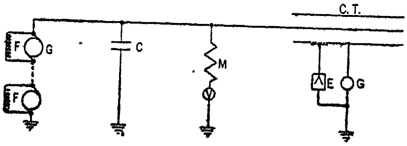


FIG. 2—CONNECTIONS FOR EXPERIMENTS WITH 15,000-VOLT DIRECT-CURRENT GENERATOR

For Continuous Voltages. Two sources of high continuous voltage were available, a high-voltage generator and a kenotron or hot-cathode rectifier. The diagram of connections in the use of the former is shown in Fig. 2. *G* the generator, supplies the high potential through parallel condensers *C* to the corona wire. *V M* and *G'* are a voltmeter, multiplier and galvanometer respectively.

The generator of special design consisted of eight units mounted on an insulating bed. Each armature is provided with two windings and commutators and rated as follows: Amperes 0.1; speed, 2700 rev. per min.; volts per commutator, 937.5; shunt wound. The available potential was, therefore, 15,000 volts, when the machine was run at normal speed with all the units in series.

The connection of a large number of low-voltage generators in series as a method of obtaining high values of continuous potential has been often used. In the present case to avoid insulation troubles, self-excited, shunt-wound generators were used with individual rheostats mounted on each frame and special precautions were taken in the design of the insulation. The generators were driven by a 5-h.p., 220-volt direct-current, shunt-wound motor through an insulating belt and insulated couplings. Voltage control was obtained in two ways. Adjustable carbon rod resistances connected in series with the generator fields and carefully insulated from the frame were mounted on the top of each machine and a movable insulated gearing provided uniform variation of all the rheostats. For fine adjustment a rheostat was inserted in the driving motor field which gave voltage control by varying the motor speed. An illustration of the unit is given in Fig. 5.

The Moscicki condensers, whose function will be explained later, were rated at 20,000 volts, effective value. They are of the Leyden jar type, the dielectric being of glass; and to prevent corona from forming on the edge of the plates, the condensers were filled with oil. Two condensers were used, consisting of eight units of 0.002 microfarad each.

The voltmeter used was a Weston intrument of the permanent magnet type, with two scales (0-150 and 0-750 volts). It was carefully compared throughout its entire range with a Weston laboratory standard instrument and found correct within the limit of observation error, *i.e.*, 0.1 per cent. The resistance of the high scale winding was 82,790 ohms. As it was desired to measure potentials up to 35,000 volts, a multiplier of 4.5 megohms was necessary. Shunting the instrument by an approximately equal resistance reduced this amount by one half although it introduced the disadvantage of doubling the amount of power required. For example, at 15,000 volts this demand is 0.0065 ampere and is great enough to impose a serious limitation on some of the later experiments, using the kenotron. A number of multipliers of the regular type supplied by the voltmeter manufacturers for increasing the range of their laboratory voltmeters were available. Their total resistance, however, was only 707,630 ohms. To obtain the remainder, six units were made up in this laboratory according to specifications of the Bureau of Standards. Each unit consisted of forty mica cards wound with manganin wire. The resistance of each card was approximately 6000 ohms. They were mounted on horizontal glass rods, which in turn were supported by hard rubber uprights, and the whole mounted on a hardwood base. The current capacity was 0.022 amperes. The resistance of each unit was measured with a standard bridge and also by placing it in series with a standard Weston voltmeter of known resistance across mains of known potential.

The diagram of connections for the second method using a kenotron is shown in Fig. 3. G the high-frequency generator

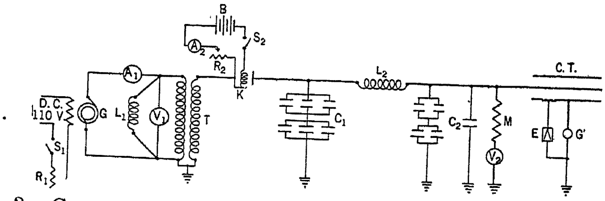


FIG. 3—CONNECTIONS FOR EXPERIMENTS WITH THE KENOTRON

supplies, through the grounded transformer T the kenotron K with high voltage. The condensers C_1 and C_2 and the inductance L_2 are introduced to smooth out the voltage which is rectified in pulsating form by the kenotron. M and V_2 are the multiplier and voltmeter, as used in the first method and $C\,T$ is the corona tube, with the electroscope E and the galvanometer G' as before. L_1 is an inductance, V_1 a hot wire voltmeter, and A_1 a hot band ammeter. Voltage control was obtained by varying the field circuit of the generator by means of the resistance R_1.

The high-frequency generator was a direct-driven unit consisting of a generator with a double field structure, one of 48 poles, the other of 240, and a 3-h.p., 240-volt direct-current, shunt-wound motor. The armature of the alternator was stationary and provided with a distributed winding. The rated full speed was 1500 rev. per min. at which one generator gave 600 cycles and the other 3000. The generators were rated at 4.4 amperes and 110 volts.

Speed or frequency control was obtained by armature and field resistance in the motor circuit, the use of the former being practically limited to starting. The driving motor and the generator field were both supplied from a storage battery which made conditions very favorable for steadiness in readings.

The kenotron used in the present investigation was designed for a maximum voltage of 40,000. It consisted of a highly evacuated glass bulb containing a cylindrical anode along whose axis was stretched a 10-mil tungsten filament, which acted as the hot cathode. The cylindrical anode was of small diameter in order to bring the anode and cathode very close together to reduce the space charge effect and thereby increase the current carrying capacity of the rectifier. One terminal was of the ordinary incandescent lamp type and served to supply energy to the cathode, acting at the same time as one terminal for the high voltage. The other end of the tube was sealed off with a metal cap and ring, which served as the other high-voltage-terminal. The theory and performance of the kenotron have been completely described by Dushman[6] and others.

For heating the tungsten filament, a 12-volt storage battery B Fig. 3 was used. A resistance R_2, a 15-ampere ammeter A_2 and a knife switch S_2 were connected in series with the battery to provide control of the filament current and indirectly the electron emission. Table I gives the electron emission and the voltage drop in the kenotron for various filament currents. A filament current of 6.1 amperes was used throughout this work.

TABLE I—CHARACTERISTICS OF THE KENOTRON

Filament current	Electron emission	Minimum voltage drop
5.0	9.5	...
5.2	16.0	86
5.4	27.0	122
5.6	40.0	158
5.8	57.0	200
6.0	82.0	250
6.1	100.0	280

The capacity C_1 (Fig. 3), was made up of six units connected two in series, three in parallel. Each unit was insulated for 10,000 volts and had a capacity of 0.01 microfarad. The condenser plates were immersed in oil. Spark gaps set for 10,000 volts were placed across the terminals of each unit to prevent breakdown of the insulation resulting from possible unequal distribution of potential. The capacity C_2 was made up of four similar units connected two in series, two in parallel together with the Moscicki condensers used in connection with the first method. The inductance L_1, consisted of two air coils of 360 turns of No. 10 B. & S. wire. The coils were in parallel but magnetically separated from each other. At 600 cycles they had an impedance of 9 ohms each. The high-tension winding of a 5-kw., 6600/110-volt, 60-cycle power transformer served for L_2. The function of these inductances is explained in connection with the experiments.

III. Preliminary Experiments

The approximate sizes of wire on which corona would appear for both alternating and continuous voltages with tubes of a given diameter were calculated from the results of foregoing researches. Two tubes were used, with inside diameters of 6.109 and 4.855 cm., respectively. Six sizes of wire were used, with diameters of 0.074, 0.090, 0.125, 0.166 and 0.231 cm. All were of brass except the largest, which was of steel. The wires were always carefully polished with crocus cloth and chamois before introducing them into the tube.

The first tests made were with the high-voltage generator as a source of potential. A number of runs were made on the same tube and wire and the voltage at which corona appeared, as determined by visual methods and temperature and atmospheric pressure, were recorded. An interesting source of error was developed at this point. It was found that successive runs under the same conditions gave values of corona voltage differing by several per cent. The trouble was traced by means of oscillograms to periodic fluctuations in the continuous voltage (see Fig. 6) of high frequencies which could not be associated with any feature of the rotating element.

In an effort to smooth out the inequalities of voltage, two Moscicki condensers (0.004 microfarad) were connected in parallel with the generator. The oscillograms shown in Fig. 6 show that the voltage fluctuation was increased under these conditions. Sixteen Moscicki units (0.032 microfarad) were then used and the oscillogram, Fig. 7, shows that the voltage was then practically constant. The frequency of the oscilla-

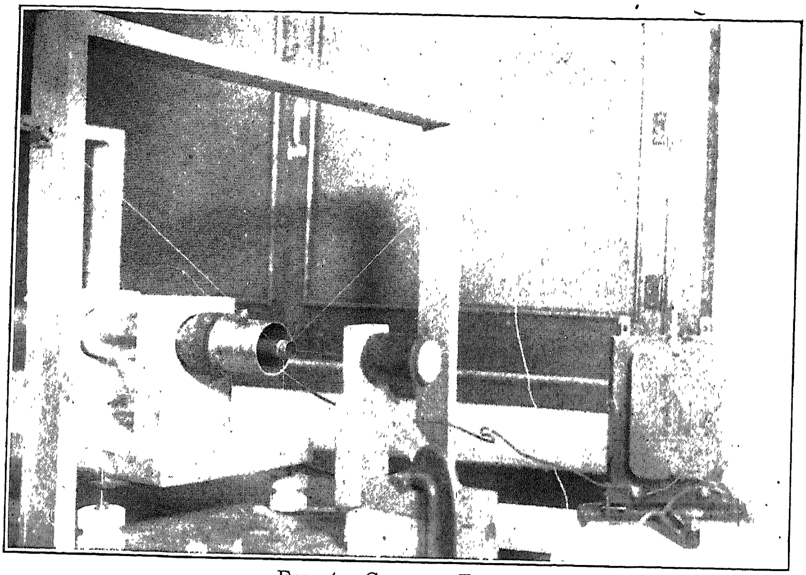
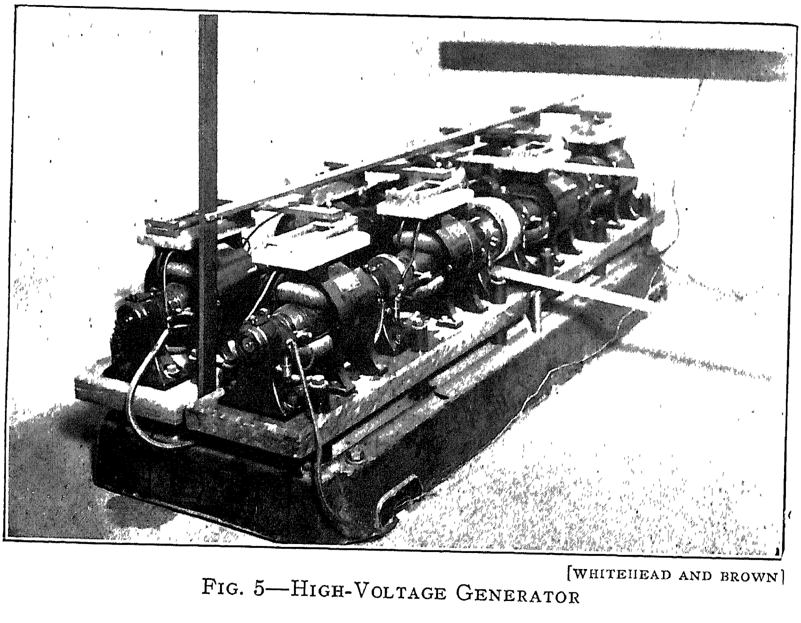


FIG. 4—CORONA TUBE



[WHITEHEAD AND BROWN]

FIG. 5—HIGH-VOLTAGE GENERATOR

PLATE IX.
A. I. E. E.
VOL. XXXVI, 1917

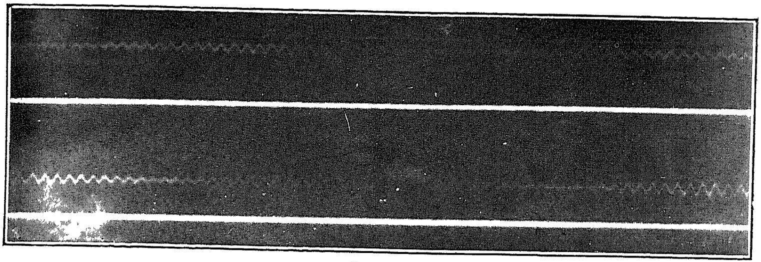


FIG. 6

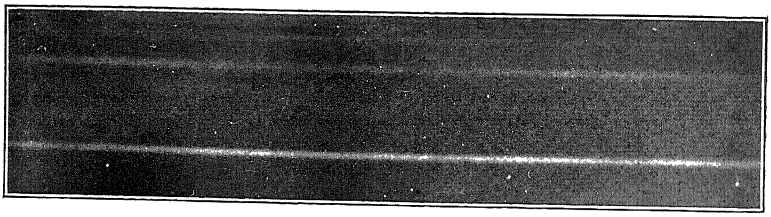


FIG. 7

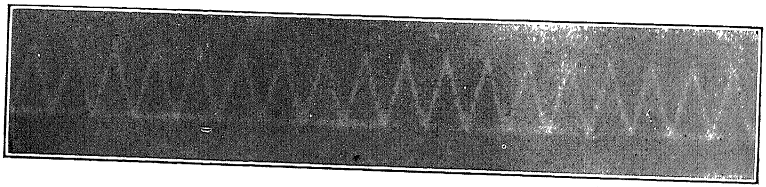


FIG. 9

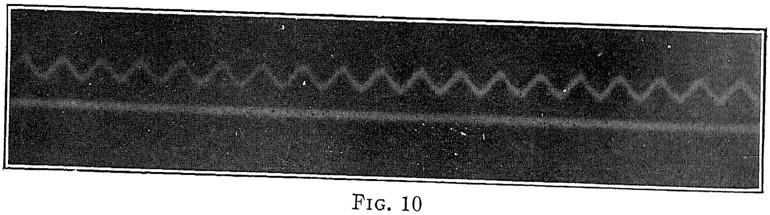


FIG. 10

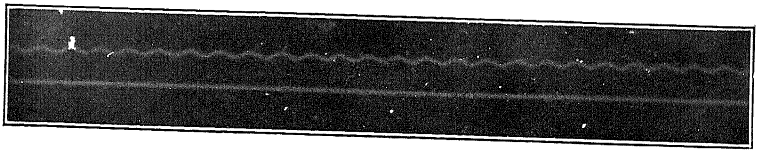


FIG. 11

[WHITEHEAD AND BROWN]

tions, when the two units were used, was obtained by superimposing a wave of known frequency, not shown in Figs. 6 and 7. The frequencies as measured in this way are for the fundamental, 359.5 cycles per second, and for the superposed, 12.3 cycles per second. An effort was made to connect these frequencies with periodic recurrencies due to the speed of the motor, the generators, the belt, the number of commutator segments, etc., but without success. The fluctuation disappeared with increased capacity in parallel and is probably due to a resonance condition between the inductance of the armatures and the capacity of the circuit. The slow superimposed fluctuation could not be definitely traced. Further investigation of this interesting phenomenon had to be postponed.

A determination of *E* as a function of the capacity in parallel with the generator showed that above 0.016 microfarad the value of *E* was constant. The results are given in Table II and are plotted in the form of a curve in Fig. 8. A capacity of 0.032 microfarad was used, twice as much as actually required.

TABLE II—APPARENT LOWERING OF THE CRITICAL SURFACE INTENSITY DUE TO RESONANCE IN THE GENERATOR CIRCUIT

No. of units	Capacity m.f.	Voltmeter reading	Critical surface intensity *E*
1	0.002	620.0	68.6
2	0.004	555.0	61.4
3	0.006	622.5	68.9
4	0.006	647.5	71.7
5	0.010	652.5	72.2
6	0.012	662.5	73.3
7	0.014	666.0	73.7
8	0.016	665.0	73.7
9	0.018	665.0	73.7
10	0.020	665.0	73.7
11	0.022	665.0	73.7
12	0.024	665.0	73.7
13	0 026	665.0	73.7
14	0.028	665.0	73.7
15	0.030	665.0	73.7
16	0.032	665.0	73.7

It must be noted that the percentage variation in voltage as shown by the oscillogram (Fig. 9) is much greater than that actually obtaining under test conditions, as the oscillograph requires a current nearly 25 times as great as that taken by the voltmeter and corona tube, and the fluctuation varies directly with the current taken from the condensers. Under test conditions therefore—voltmeter and tube connected—the voltage variation is only a very small fraction of one per cent.

The maximum voltage obtainable from the high-voltage generator was 15,000 volts so the kenotron was used for higher values. It was soon found that the transformer used imposed a serious limitation on the value of the continuous voltage obtainable by this means. At 600 cycles the high-tension windings of the transformer took a primary charging current of 20 to 25 amperes and as the full load current of the 600-cycle generator was only 4.4 amperes, the speed and consequently the frequency rapidly dropped on connecting the transformer. Although the internal reactance of the generator was considerable, the capacity of the high-tension windings of the transformer was more than sufficient to overbalance it. Additional reactance connected across the primary of the transformer was the obvious way of reducing this current. A number of air coils of low resistance were tried and after a number of trials the two coils as described in an earlier paragraph were selected, as they reduced the charging current to about five amperes and gave a maximum value of continuous voltage of about 26,500 volts.

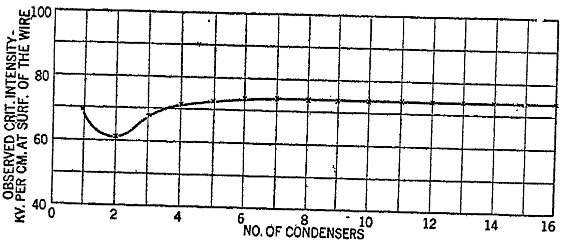


FIG. 8—APPARENT LOWERING OF CRITICAL SURFACE INTENSITY DUE TO RESONANCE IN THE GENERATOR CIRCUIT

The next step in the development of the kenotron method was to introduce parallel capacity in the direct-current side of the kenotron to smooth out the voltage by supplying energy to the load during the part of each cycle when the kenotron delivers no current. When operated alone *i.e.*, without condensers the rectifiers gave a continuous voltage which fluctuated between zero and a maximum with the frequency of the impressed e.m.f., 600 cycles. Four 10,000-volt condensers in series (0.0025 microfarad, were connected between the direct-current side of the kenotron and ground. The oscillogram, Fig. 10, shows the character of the continuous current voltage with the above capacity. Nine condensers, connected three in parallel, three in series, (0.01 microfarad) were next tried. The oscillogram, Fig. 11, shows the decided improvement. The attempt to produce further improvement in the voltage by this means was abandoned as it was evident that a far larger amount of capacity than was available would be required to obtain a voltage varying less than one per cent.

As the impedance of the load (the voltmeter and corona tube) was high, the current which flowed was very small. Under these conditions a moderate amount of series inductance (50 or 100 henrys) in conjunction with capacity offered a probable means of further improvement if connected in the manner shown in Fig. 3, where the capacity is divided into two portions. The voltage fluctuation at the terminals of C_1 is an irregular shaped wave which consists of a fundamental with a frequency of 600 cycles per second and a number of higher harmonics. The inductance L_2 and the capacity C_2 offer a definite value of impedance to each of these higher frequencies. The higher the frequency the greater the impedance offered by the inductance L_2 or in other words the voltage wave is smoothed out by removing the higher harmonics. The current drawn by the condenser C_2 over the inductance L_2 stores up electromagnetic energy in the latter. When the voltage of the condenser starts to fall, due to the current drawn from it by the load, the energy stored up in the inductance is delivered to the circuit and tends to maintain the voltage at the terminals of C_2 constant.

A number of inductances were tried in the position L_2 and the sound in a telephone used as a measure of the degree of voltage fluctuation. The intensity of sound in the telephone is proportional to the amplitude of the voltage fluctuation. The minimum sound in the telephone was obtained from one half of the high-tension winding of a 5-kw., 6600/110-volt power transformer. This final arrangement was used throughout the tests.

It is possible to calculate the voltage fluctuation under test conditions, if it is assumed that is directly proportional to the current. Referring to Fig. 3, the values were as follows, when the oscillograph was in circuit:

$$C_1 = \frac{3\,(0.01)}{2} = 0.015 \text{ microfarad}$$

$$C_2 = \frac{2\,(0.01)}{2} + 0.032 = 0.042 \text{ microfarad}$$

$$C = C_1 + C_2 = 0.015 + 0.042 = 0.057 \text{ microfarad}$$

$$= 5.7 \times 10^{-8} \text{ farads}$$

V = direct-current voltage = 4700

δV = voltage fluctuation = 13.8 per cent = 650 volts. (By measurement on the oscillogram)

i_1 = oscillograph current = 0.024 ampere (By measurement)

i_2 = voltmeter current = 0.001 ampere (By calculation)

i = total current flowing when oscillograph is connected.

$= i_1 + i_2 = 0.025$ ampere

$\omega = 2\,\pi \times \text{frequency} = 2\,\pi \times 600 = 1200\,\pi$

Hull[7] has shown that the voltage fluctuation (δV) at the terminals of C_2 may be expressed by the simple relation

$$V = \frac{8\,i\,\pi}{L_2\,\omega^3}\left(C + \frac{1}{L_2\,\omega^2}\right)^2 \qquad (2)$$

where i, ω, and C are as given above and L_2 is the magnitude of the series inductance in henrys. If we assume L_2 constant, (*i.e.*, independent of the amount of current due to the low flux density in the iron of the transformer which was used as L_2), we can write

$$\delta V = K\,i \qquad (3)$$

Let δV_a = voltage fluctuation when current i is flowing.
δV_b = voltage fluctuation when current i_2 is flowing
Then, $\delta V_a = K\,i$; and, $\delta V_b = K i_2$. **(4)** and **(5)**

Dividing (4) by (5) and transposing,

$$\delta V_b = \frac{i_2}{i} \times V_a \tag{6}$$

$$= \left(\frac{0.001}{0.025} \times 13.8\right) \text{per cent} = 0.5 \text{ per cent}$$

= percentage voltage fluctuation under test conditions, (*i.e.*, voltmeter and corona tube.)

A comparison of the different methods of detecting corona was then made. In the case of the alternating-current corona the visual and electroscope methods were compared. The curves (Fig. 13) show how closely the two methods agree, when the bend in the electroscope curve is taken as the point at which corona begins. In the case of the direct-current corona an additional method was available, *i.e.*, a galvanometer connected between the tube and ground as used by Farwell and Mackenzie[8]. A ballistic galvanometer with a sensitivity of 10^{-6} amperes per mm. scale division when the scale was placed one meter from the mirror, was used. Below corona voltage if the wire was clean, the galvanometer stood practically on zero. With the appearance of corona, the galvanometer took a large deflection (5 to 10 cm.) and any further increase in voltage threw the light spot off the scale.

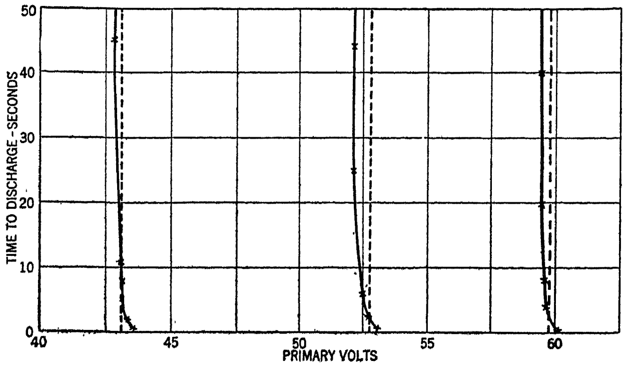


FIG. 13—VISUAL CORONA AND ELECTROSCOPE DISCHARGE
--- Visual corona
—— Electroscope discharge
Diameter tube = 6.109 cm.
Diameter wires = 0.107—0.166—0.231 cm.

The condition of the surface of the wire when fairly clean had but little effect on the value of the critical surface intensity in the case of the wire at positive or alternating-current potentials, but with the wire negative the slightest surface imperfection or particle of dust caused a lowering of the value of E. The observations on negative corona were extremely difficult on this account. On raising the voltage the slightest amount above that necessary to form corona, the blue glow changed to a purple point discharge and on lowering the voltage, corona held on to a value of voltage far below that required to start corona. The wire had to be removed and carefully polished again before another observation could be made. With precautions against the above increase of voltage beyond corona value it was possible to repeat the observations on the negative corona as often as desired.

Final Methods of Taking Observations. The wire was carefully polished, placed in the tube, accurately centered and connected to the voltage supply. In the alternating-current case the voltage was slowly raised by increasing the generator field current until the electroscope began to discharge. Readings were taken of the rate of leak of the electroscope corresponding to each increment of the voltage, until the rate of discharge became practically infinite. The field current was then decreased until the leak of the electroscope ceased and then increased again until visual corona appeared. The voltage was read at this instant on the primary voltmeter and recorded as primary volts. The temperature and pressure were also recorded. The peak factor of alternating voltage waves was determined by means of oscillograms. A large number were taken and from these the values of 1.46 at 40 volts, 1.46 at 50 volts and 1.46 at 60 volts were determined by measurement. In order to give an idea of the conditions of accuracy in making these measurements, the figures in Table III are given for one half of the wave of the oscillogram corresponding to 60 volts. The curve was carefully traced on co-ordinate paper and 26 ordinates, 1/20 of an inch (1.27 mm.) apart were read off; the height of each ordinate could be read to 1/50 of an inch (0.508 mm.) The maximum ordinate was 1.43 inches (37.08 mm.) high and the r.m.s of all the ordinates was 0.977, which gives a ratio of maximum to effective of 1.464. Similar measurements on the other oscillograms gave the values as stated above. The average of these three (1.46) was taken as the ratio of maximum to effective value over the range 40-60 primary volts.

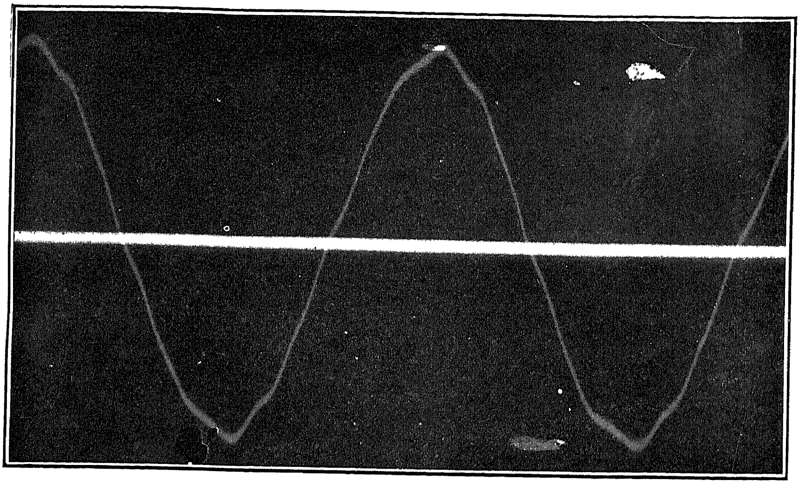
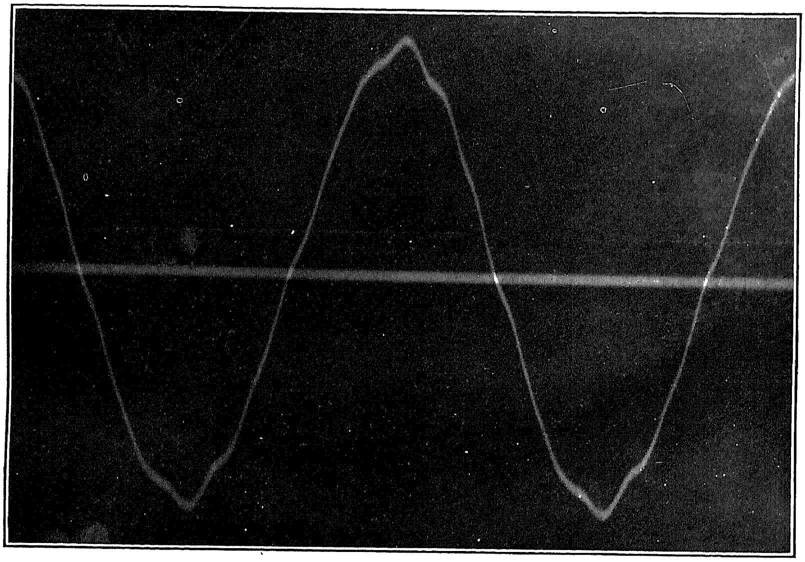


FIG. 14—AT 40 VOLTS



[WHITEHEAD AND BROWN]

FIG. 15—AT 60 VOLTS

TABLE III—OSCILLOGRAM OF THE VOLTAGE ACROSS THE LOW-TENSION TERMINALS OF THE TRANSFORMERS AT 60 VOLTS

Ordinate No.	Length	(Length)
1	0	0
2	2.5	6.25
3	5.0	25.00
4	7.0	49.00
5	11.5	132.25
6	15.3	234.09
7	17.8	316.84
8	20.3	412.09
9	22.3	497.29
10	24.2	585.64
11	25.0	625.00
12	26.0	676.00
13	27.0	729.00
14	28.0	784.00
15	28.6	817.96
16	27.0	729.00
17	25.2	635.04
18	23.5	552.25
19	22.7	515.29
20	21.0	441.00
21	17.8	316.84
22	15.0	225.00
23	12.0	144.00
24	8.8	77.44
25	5.2	27.04
26	0.0	0.00

The oscillograms taken at 40 and 60 volts respectively are given in Fig. 14 and 15.

In the direct-current experiments the procedure was much the same, the high-tension voltage however being read directly on the voltmeter in series with the multiplier, no peak factor determination being necessary. In the case of the negative corona the electroscope curve could not be obtained due to the disadvantage of raising the voltage above corona as noted above and finally, the galvanometer method of detecting the corona was used.

Accuracy of Observations. The accuracy of observation in the case of the alternating-current corona was limited only by the accuracy of reading the voltmeter, as the observations could be repeated again and again from day to day when correction was made for temperature and pressure, and the other factors entering into the determination of the value of E could be determined to a small fraction of 1 per cent. The voltmeter could be read to 1/5 of a division at 60 volts or about 1/3 per cent. It is recognized, however, that the accuracy also depended upon the measurements of peak factor on the oscillograms. The method of measuring these has already been outlined and as the value of the peak factor varied only 0.07 per cent over the range of voltage used, it is felt that the measurement of voltage is the limiting factor, as stated above. The accuracy in the continuous current case was also limited by the voltmeter reading and was about 1/3 per cent also.

IV. FINAL EXPERIMENTS

Relation Between Critical Surface Intensity and Diameter of the Wire for Alternating Voltage. In Table IV are given the results of observations on six sizes of wire in a tube 6.109 cm. in diameter. The first column gives the wire diameter in centimeters, the second column the "density factor" δ, as obtained from Peek's relation

$$\delta = \frac{3.92\,p}{273 + t} \tag{7}$$

and the third column the primary volts. V the critical voltage (maximum) is obtained by multiplying the primary voltage by the ratio of transformation and the peak factor. The relation

$$E = \frac{V}{a \log_e b/a} \tag{8}$$

was used in calculating the critical surface intensity E.

Relation Between Critical Surface Intensity and Diameter of the Wire for Positive and Negative Continuous Voltage. In Table V are given the results of observations on the same wires for positive corona and in Table VI for negative corona. The first column is again the wire diameter, the second column the density factor calculated from the observed temperature and pressure and the third column the voltmeter reading. The critical voltage V is obtained by multiplying the voltmeter reading by the multiplying factor. The last column is obtained as before.

The small number of observations on the negative corona is due to the great difficulty experienced in obtaining them as mentioned before. As many as ten trials with their attendant polishing, centering, etc., were often required to obtain a stable corona. The galvanometer in the grounded circuit proved a great help as the galvanometer only remained on zero up to corona forming voltage when the wire was absolutely clean and if at no time during the run the voltage had been raised above that required to form corona on the given wire.

Determination of the Constants A and B of Formula (**1**). As stated in the introduction the formula connecting E (the critical surface intensity) with the diameter of the wire is of the form

$$E = A\,\delta + B\sqrt{\frac{\delta}{d}} \qquad (\mathbf{1})$$

TABLE IV—ALTERNATING CURRENT

Diam. wire cm.	Density factor δ	Primary volts.	Crit. voltage (maximum) V	Crit. surface intensity E (kv.)
0.074	1.005	35.8	13,120	80.6
	1.005	35.8	13,120	80.6
	1.005	35.8	13,120	80.6
0.090	1.005	39.65	14,530	77.1
	1.005	39.65	14,530	77.1
	1.005	39.65	14,530	77.1
0.107	1.005	42.85	15,700	72.6
	1.005	43.05	15,780	73.0
	1.005	43.20	15,830	73.2
0.125	1.005	45.6	16,700	68.8
	1.005	45.65	16,720	68.9
	1.005	45.7	16,750	69.0
0.166	1.005	53.5	19,610	65.6
	1.005	53.5	19,610	65.6
	1.005	53.5	19,610	65.6
0.231	1.005	61.4	22,500	59.6
	1.005	61.4	22,500	59.6
	1.005	61.4	22,500	59.6
0.231	1.000	64.2	23,520	59.7
	1.000	64.2	23,520	59.7
	1.000	64.2	23,520	59.7

where, E = critical surface intensity in kilovolts per centimeter.

δ = Density factor.
A and B = Constants.

To determine these constants from the observed results the method of least squares was used. The calculation as applied to the positive corona is given in full in Table VII. Attention is

TABLE V—POSITIVE CORONA

Diam. wire cm.	Density factor δ	D-C. volts	Mul. fac.	Critical voltage (V)	Crit. surf. intensity (E)
0.074	1.02	228.0	52.16	11,897	77.0
	1.02	227.5	52.18	11,871	76.9
	1.02	228.0	52.18	11,897	77.0
	1.00	558.0	19.83	11,660	75.5
	1.00	587.5	19.83	11,650	75.4
	1.00	587.5	19.83	11,650	75.4
	1.022	227.5	52.18	11,871	77.1
	1.022	227.5	52.18	11,871	77.1
	1.022	227.5	52.18	11,871	77.1
	1.035	442.5	28.36	12,549	77.1
	1.035	443.0	28.36	12,560	77.2
	1.035	443.0	28.36	12,560	77.2
0.090	1.01	657.5	19.83	13,038	72.8
	1.01	657.5	19.83	13,038	72.8
	1.01	657.5	19.83	13,038	72.8
	1.02	665.0	19.83	13,187	73.6
	1.02	665.0	19.83	13,187	73.6
	1.02	665.0	19.83	13,187	73.6
	1.007	637.5	19.83	13,038	72.8
	1.007	367.5	19.83	13,038	72.8
	1.007	637.5	19.83	13,038	72.8
	1.02	665.0	19.83	13,187	73.6
	1.02	665.0	19.83	13,187	73.6
	1.02	665.0	19.83	13,187	73.6
	1.005	657.5	19.83	13,038	72.8
	1.005	657.5	19.83	13,038	72.8
	1.005	657.5	19.83	13,038	72.8
	0.998	654.0	19.83	12,969	71.4
	0.998	654.0	19.83	12,969	71.4
	0.998	654.0	19.83	12,969	71.4
	1.025	253.0	52.18	13,200	73.7
	1.025	253.0	52.18	13,200	73.7
	1.025	253.0	52.18	13,200	73.7
0.107	1.022	275.0	52.18	14,350	69.2
	1.022	275.0	52.18	14,350	69.2
	1.022	275.0	52.18	14,350	69.2
	1.035	533.0	28.36	15,116	69.9
	1.053	532.5	28.36	15,102	69.8
	1.035	533.0	28.36	15,116	69.9
0.125	1.008	292.5	52.18	15,263	66.8
	1.008	292.5	52.18	15,263	66.8
	1.008	292.5	52.18	15,263	66.8
0.166	1.003	357.5	52.18	18,654	62.4
	1.003	357.5	52.18	18,654	62.4
	1.003	357.5	52.18	18,654	62.4
0.231	1.02	422.5	52.18	22,046	58.4
	1.02	422.5	52.18	22,046	58.4
	1.02	422.5	52.18	22,046	58.4

called to the excellent agreement in the observed results and those calculated by the formula. The observed values are given in the ninth column of Table VII, the calculated values in next to the last column and the percentage error referred to the observed values in the last column. This error is less than 1 per cent with the exception of the wire of radius 0.0535 cms.

Similar calculations were made for the alternating voltage and the negative continuous voltage and the following constants were found.

TABLE VI—NEGATIVE CORONA

Diam. wire cm.	Density factor δ	D-C. volts	Mul. fac.	Critical voltage (V)	Crit. surf. intensity (E)
0.074	1.003	462.5	28.36	13,117	80.6
	1.003	462.5	28.36	13,117	80 6
	1.003	462.5	28.36	13,117	80.6
0.090	1.03	517.0	28.36	14,662	77.8
	1.03	517.5	28.36	14,676	77.9
	1.03	517.0	28.36	14,662	77.8
	0.9907	503.0	28.36	14,265	75.6
	0.9907	502.5	28.36	14,230	75.6
	0.9907	503.0	28.36	14,265	75.6
0.107	1.003	560.0	28.36	15,880	73.5
	1.003	560.0	28.36	15,880	73.5
	1.003	560.0	28.36	15,880	73.5
1.125	0.992	587.0	28.36	16,657	68.6
	0.992	587.0	28.36	16,657	68.6
	0.992	587.0	28.36	16,657	68.6
0.166	1.003	667.5	28.36	19,214	64.3
	1.003	667.5	28.36	19,214	64.3
	1.003	667.5	28.36	19,214	64.3
0.231	1.001	431.0	52.18	22,440	59.3
	1.001	431.0	52.18	22,440	59.3
	1.001	431.0	52.18	22,440	59.3

Positive Corona

$A = 33.7$ $B = 11.5$

Negative Corona

$A = 31.02$ $B = 13.5$

Alternating-Current Corona

$A = 33.7$ $B = 12.6$

TABLE VII—CALCULATION OF CONSTANTS A AND B USING METHOD OF LEAST SQUARES. POSITIVE CORONA

r	δ	δ^2	δ^3	$\frac{\delta^3}{r}$	$\sqrt{\frac{\delta^3}{r}}$	$\frac{\delta}{r}$
0.037	1.02	1.0404	1.061	28.68	5.375	27.77
0.045	1.025	1.0506	1.076	23.9	4.885	22.7
0.0535	1.022	1.0445	1.068	19.97	4.47	19.14
0.0625	1.008	1.0161	1.025	16.4	4.05	16.12
0.083	1.003	1.0060	1.01	12.16	8.85	12.07
0.1155	1.02	1.0404	1.061	9.19	3.03	8.82
Σ		6.198			25.235	106.62

r	E obs.	δE	$\sqrt{\frac{\delta}{r}}$	$\sqrt{\frac{\delta}{r}}E$	E Calc.	Error % per cent
0.037	77.0	78.54	5.27	405.79	77.1	− 0.13
0.045	73.7	75.54	4.765	351.18	73.4	+ 0.41
0.0535	69.2	70.72	4.375	302.75	70.0	− 1.1
0.0625	66.8	67.33	4.015	268.2	66.7	+ 0.14
0.083	62.4	62.59	3.475	216.84	62.1	+ 0.48
0.1155	58.4	59.56	2.97	173.45	58.5	− 0.17
Σ		414.28	24.87	1718.21		

Resultant values $A = 437.7$, $B = 11.5$.

The values of E for the six sizes of wire calculated from the formula,

$$E = A + \frac{B}{\sqrt{d}}$$

using the above constants are given in Tables VIII and IX and plotted in Fig. 16.

TABLE VIII—ALTERNATING CURRENT

RELATION BETWEEN DIAMETER AND CRITICAL SURFACE INTENSITY

Diam. wire cm.	Density factor δ	Primary volts	Critical voltage (max.) (V)	Crit. surf. intensity E (kv.)/cm.
0.074	1.005	35.8	13,120	80.6
0.090	1.005	39.65	14,530	77.1
0.107	1.005	43.0	15,760	72.9
0.125	1.005	45.7	16,750	69.0
0.166	1.005	53.5	19,610	65.6
0.231	1.005	61.4	22,500	59.6

TABLE IX—CONTINUOUS CURRENT

(POSITIVE)

RELATION BETWEEN DIAMETER AND CRITICAL SURFACE INTENSITY

Diam. wire cm.	Density factor δ	Voltmeter reading	Critical voltage (V)	Crit. surf. intensity. E (kv.)/cm.
0.074	1.02	228.0	11,897	77.0
0.090	1.025	253.0	13,200	73.7
0.107	1.022	275.0	14,350	69.2
0.125	1.008	292.5	15,263	66.8
0.166	1.003	357.5	18,654	62.4
0.231	1.02	422.5	22,044	58.4

(NEGATIVE)

Diam. wire cm.	Density factor δ	Voltmeter reading	Critical voltage (V)	Crit. surf. intensity E (kv.)/cm.
0.074	1.003	462.5	13,117	80.6
0.090	1.03	517.0	14,662	77.8
0.107	1.003	560.0	15,880	73.5
0.125	0.992	587.0	16,657	68.6
0.166	1.003	677.5	19,214	64.3
0.231	1.001	431.0	22,490	59.3

In Table X is given a comparison of the results of observations on positive, negative and alternating-current corona with the results of the more important earlier researches. The formula,

$$E = A\,\delta + B\sqrt{\frac{\delta}{d'}}$$

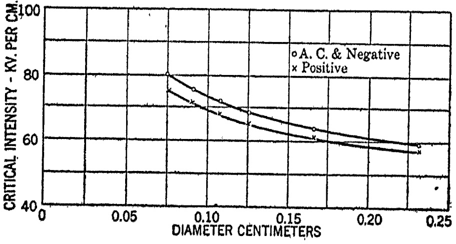


FIG. 16—CORONA VOLTAGES FOR ROUND CONDUCTORS

was used in all cases and the values of the constants A and B (as taken from the accounts of the different investigations and given in Table XI), substituted in order to obtain the values of E given in Table X.

The values of Table X for the alternating-current case are plotted in Fig. 17, those for the continuous current case (positive and negative) in Fig. 18.

TABLE X—COMPARISON OF RESULTS

Critical Surface Intensity: Kilovolts Per Centimeter

Alternating current				Continuous current			
Diam. wire cm.	White-head	Peek	White-head & Brown	White-head & Brown +	Farwell +	White-head & Brown −	Farwell −
0.074	81.2	80.7	80.3	76.0	75.7	80.8	76.9
0.090	76.6	76.0	76.0	71.9	71.6	76.1	72.0
0.107	72.9	72.3	72.5	68.9	68.3	72.4	69.8
0.125	69.9	69.2	69.6	66.2	65.5	69.3	67.2
0.166	64.8	64.1	64.8	61.9	61.1	64.2	62.9
0.231	59.9	59.1	60.1	57.6	56.6	59.1	58.7

Discussion

The fact that positive corona appears at a lower voltage than negative for the same size wire is in accord with the theory of secondary ionization if we assume that the negative ion is the principal ionizing agent. This is practically certain owing to its energy of motion being greater than that of the positive ion under a given voltage gradient. If a difference of potential be applied between a wire and concentric cylinder the negative ions or electrons present in the air will move under the influence of the electric field. In so doing they will collide with the molecules of the air and they may or may not produce other ions, depend-

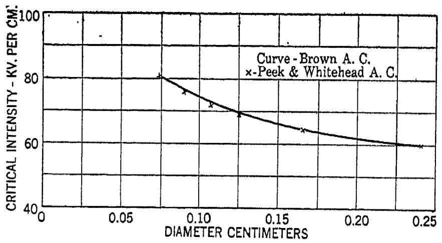


Fig. 17—Corona Voltages for Round Conductors

ing upon their velocity. Now suppose the wire is positive. In this case the electrons will move inwards (toward the wire) and in so doing will move in the direction in which the intensity of the field is increasing. Thus new ions caused by collisions are formed in stronger and stronger fields and there is a consequent tendency to multiply to the concentration corresponding to corona formation. On the other hand if the wire is negative, the electrons will move outwards in the direction of decreasing field intensity. Only those which start from near the wire pass through the field of strong electric force so that the number of ions produced by collisions is much smaller than in the first case or a higher difference of potential will be required to produce enough ions by collision to start corona.

TABLE XI

Type of voltage	Name of investigator	Constants	
		A	*B*
a-c.	Whitehead	32	13.4
a-c.	Peek	31	13.5
d-c.	Farwell	31.6	11.9
d-c.	Farwell	35.0	11.4

There is nothing new in the convergence of the curves for positive and negative corona. Schaffers, Watson, Mackenzie and Farwell all found evidences of this. As the meeting point is a function of the pressure and the diameter of the wire it is hard to compare the results of the different investigations on this basis. If the curves for positive and negative corona be extended they will meet at a point corresponding to a wire of diameter 0.285 cms.

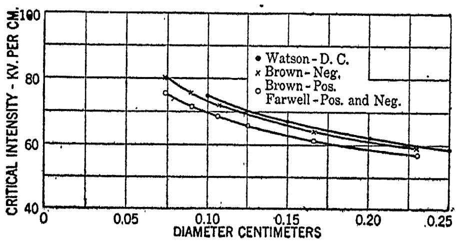


FIG. 18—CORONA VOLTAGES FOR ROUND CONDUCTORS

Farwell states that the appearance of negative corona changes with increase of wire diameter and voltage, that the negative corona started with a bright spot or two on small wires followed by a continuous brush discharge as the voltage was increased and that for the larger wires the brush discharge appeared immediately upon reaching the critical voltage. In the present experiments by careful polishing and by raising the voltage in very small steps it was found possible to make the negative corona appear sharply as a continuous bluish glow on all the wires used. If the voltage was raised the smallest amount above that necessary to produce corona, the bluish glow collapsed to a purplish, point discharge and by lowering the voltage until the discharge just disappeared, a value of E was obtained in close agreement with Farwell. It is believed that this explains to some extent the low values obtained by Farwell in this case.

V. Experiments Testing the Theory of Corona Formation

Theory of the Start of Corona. According to the theory of secondary ionization, if a difference of potential be applied to a tube and concentric wire, the free electrons present in the air between will acquire a velocity under the electric field. The velocity will depend on the applied difference of potential and the "mean free path" of the electron. In order that corona form about the wire it is necessary that the electron acquire a velocity sufficient to produce other electrons by collision and that this process multiply to some state of concentration constituting corona. If the possible free path of the electron be limited in any way, a higher difference of potential will be required to give the critical velocity to the electron or to produce corona.

To test this theory concentric metal tubes with thin walls and insulating supports were introduced between the wire and the corona tube proper, in order to reduce the possible free path of the negative ion under the influence of the electric field and to observe the effect of this reduction on the value of V, the maximum value of the voltage required to produce corona. The great difficulty in such experiments is to determine when corona starts when the secondary tube is in place. On applying continuous voltage to the wire it was found that the voltage could be carried far above that required to start corona when the auxiliary tube was not in place without the slightest deflection of a galvanometer connected between the tube and ground and without any indication of glow about the wire. It was at first thought that this could be explained on the hypothesis that the intermediate tube cut down the number of ionizing electrons already in motion in the outer portion of the field in accordance with the indications of the theory. An electrostatic voltmeter was then connected between the inner tube and the ground and the voltage again applied. On removing the voltage from the central wire the intermediate tube was found to retain a potential, as shown by the electrostatic voltmeter. An electroscope carrying a charge of known sign showed that the intermediate tube was charged to a potential of the same sign as the wire.

The failure of the galvanometer to deflect or of corona to appear was now easily explained; the presence of the high potential of the same sign on the intermediate tube lowered the potential gradient between the wire and the outer tube and therefore prevented the appearance of corona. That this is the correct explanation was shown by the absence of charge on the inner tube for voltages below corona. The conclusion therefore is that the corona may form at normal voltage, generating positive and negative ions. The ions of the opposite sign are attracted to the wire, those of like sign are driven to the intermediate tube where they raise its potential. This elevation of potential continues until it is sufficient to lower the intensity at the surface of the corona wire below corona forming value. The whole process is probably instantaneous.

Alternating current was then applied to the wire as it was thought that the inner tube would not acquire the charge noted above. The kenotron was used to rectify the current which flowed from the outer tube to the ground through the galvanometer, which was retained in these alternating-current experiments by reason of its value as a detector. Here again trouble was encountered due to a charge accumulating on the outer tube which disturbed the potential gradient and made it impossible to obtain corona even on raising the voltage far above that required to start corona when the outer tube alone was used and connected directly to ground.

The kenotron is a perfect rectifier and allows current to flow in one direction only. During the half cycle when the wire is positive a charge of the opposite sign is acquired by the tube through the kenotron. During the next half cycle the wire becomes negative, but a positive charge does not accumulate on the outer tube as the kenotron is conducting in but one direction.

The negative charge already there cannot return through the kenotron for the same reason and with each successive half-cycle more and more charge accumulates until the outer tube is considerably above ground potential, which lowers the effective potential gradient and prevents the formation of corona.

TABLE XII

INFLUENCE OF CONCENTRIC SCREENING TUBES ON CORONA VOLTAGE

(WITH INNER TUBE)

A-C. volts	Shunt in ohms	$\frac{r_1 + r_2}{r_1}$
40.0	11.0	3.780
45 0	10.0	4.060
50.0	9.0	4.400
55.0	8.5	4.605
60.0	7.5	5.080
65.0	7.0	5.340
67.5	6.0	6.100
70.0	5.0	7.120
72.5	3.0	11.200

(WITHOUT INNER TUBE)

A-C. volts	Shunt in ohms	$\frac{r_1 + r_2}{r_1}$
40.0	12.0	3.56
45.0	11.0	3.78
50.0	10.0	4.06
55.0	9.3	4.30
60.0	8.7	4.52
62.5	8.3	4.69
65.0	7.6	5.03
66.0	6.5	5.71

Galvanometer Deflection Constant = 1 cm.
Visual Corona With Tube = 69.1
Visual Corona Without Tube = 64.2

A vibration galvanometer in the ground connection was next used and after many trials the results shown in Table XII and plotted in Fig. 19 were obtained. The instrument used was of the soft iron magnet type, in which a piece of soft iron is suspended by a silk fibre between the poles of a horseshoe magnet. A coil carrying the alternating current to be measured produces a field perpendicular to that of the magnet. The temporarily magnetized piece of iron vibrates in synchronism with the alternating current and with an amplitude proportional to the strength of the current. The period of the moving system is adjusted to resonance by a magnetic shunt on the limbs of the magnet.

The first column gives the primary volts. The deflection of the galvanometer was held constant at one centimeter, by varying a non-inductive shunt across the instrument. The values of the shunt used are given in the second column. The total current flowing from the tube at any given voltage on the wire, is given by

$$i = \frac{r_1 + r_2}{r_1} + i_1 \tag{9}$$

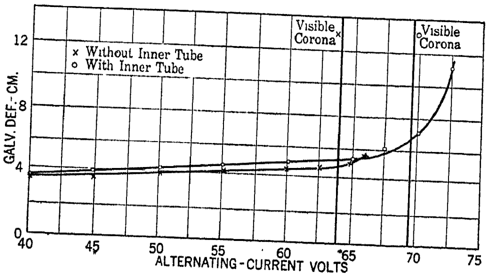


FIG. 19—EFFECT OF CONCENTRIC TUBES

Diameter outer cube = 6.98 cm.
Diameter inner tube = 2.448 cm.
Diameter wire = 0.231 cm.

where, r_1 = resistance of the galvanometer.
r_2 = resistance of the shunt.
i_1 = current through the galvanometer.

As the galvanometer current was held constant, the above equation reduces to

$$i = k\frac{r_1 + r_2}{r_1} \tag{10}$$

The values of $\frac{r_1 + r_2}{r_1}$ are entered in the last column and the curve (Fig. 19) is plotted between $\frac{r_1+r_2}{r_1}$, which is proportional to the total current flowing, and the primary volts on the transformer. The values at which visual corona appeared with and without this tube, which was 2.448 cms. in diameter, are given in Table XIII and also marked on the curve. Three other tubes were inserted and observations made on the visual corona with and without them. These values are also entered in the table.

TABLE XIII

EFFECT OF THIN CONCENTRIC INSULATED TUBES ON CORONA VOLTAGE.

Diameter inner tube cm.	Primary volts
2.448	68 7
1.829	69.1
1.209	62.0 (Spark over)
0.605	.... (" ")
No tube	64.2
Diameter outer tube 6.98 cm.	Diameter wire 0.231 cm.

Discussion. The curve (Fig. 18) shows that the charging current is higher when the intermediate tube is in place as it should be owing to the increased capacity. The curves follow straight lines up to a value of voltage required to produce corona with no intermediate tube. An indication is given that the corona current rises more rapidly when the tube is not in place which is in accord with the theory. Corona appears at a higher value of voltage when either of the larger tubes is inserted which is in accord with the theory, as they cut down the number of ionizing electrons already in motion due to the outer portion of the field. If we omit the value obtained for the 1.209 cm. tube which is uncertain owing to spark over, we see that the smaller the diameter of the inner tube the higher the voltage required which is again in accord with the theory as the smaller the inner tube the more the accelerating path is cut down and the higher the voltage required to start corona. Because of the difficulty of maintaining the galvanometer deflection constant (it is affected by the slightest variation in the frequency) further modifications in the direction of constancy of frequency are necessary, and the experiments were terminated at this point. It is believed however that the above results are a reliable qualitative indication that corona voltage is elevated by an intermediate thin screen which serves as a barrier to the free passage of the ionizing electrons.

VI. CONCLUSIONS

1. The critical corona forming intensity at atmospheric presure has been determined for six sizes of wire ranging from 0.074

to 0.231 cm. diameter for alternating and continuous potentials, in the same apparatus and under the same conditions.

2. The relation between critical surface intensity, that is, the intensity at which corona starts, and the diameter of a clean round conductor as found are expressed by the following laws:

Alternating current* $E = 33.7\,\delta + 12.6\sqrt{\frac{\delta}{d}}$

Continuous current† Positive $E = 33.7\,\delta + 11.5\sqrt{\frac{\delta}{d}}$

Continuous current‡ Negative $E = 31.02\,\delta + 13.5\sqrt{\frac{\delta}{d}}$

3. Within the range of wires used the corona appears at a lower voltage when the wire is positive than when it is negative. The maximum excess of negative over positive observed was 6.3 per cent.

4. The values with alternating voltage coincide closely with those of negative continuous voltage.

5. The curves representing the positive and negative values converge as the diameter of the wire increases.

6. The observations on the positive and alternating-current corona are in substantial agreement with those of Farwell, Peek and Whitehead, although new values of the constants of formula (**1**) are indicated.

7. The observations on the negative corona give values higher than any heretofore obtained.

8. Experiments with concentric insulated screening tubes indicate that the critical corona forming intensity is raised by their introduction. (See Table XIII and Fig. 18.)

9. The results of the present investigation are in accord with the theory of secondary ionization as proposed by Townsend.

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*(See Table VIII and Fig. 15.)

†(See Table IX and Fig. 15.)

‡(See Table IX and Fig. 15.)

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Saul Dushman: The problem of obtaining high-voltage direct current for use in such experiments as described in the above papers is of great importance. One method is to use the series of d-c. generators as described. Another method that is very convenient consists in the use of a transformer and some type of rectifier, such as that mentioned in the paper by Messrs. Whitehead and Brown. The pulsating d-c. voltage produced by the use of one half-wave or two half-wave rectifiers may then be smoothed out by means of capacity. Dr. Hull has recently shown that by dividing the capacity into two parts and inserting inductance between them, it is possible to obtain almost non-pulsating direct-current with very much less capacity than would be required if no inductance at all were used. Fig. 1 shows his arrangement for obtaining 50 milliamperes at 100,000 volts from a 2000-cycle alternator and step-up transformer, and Fig. 2 represents an oscillogram of the direct-current actually obtained.

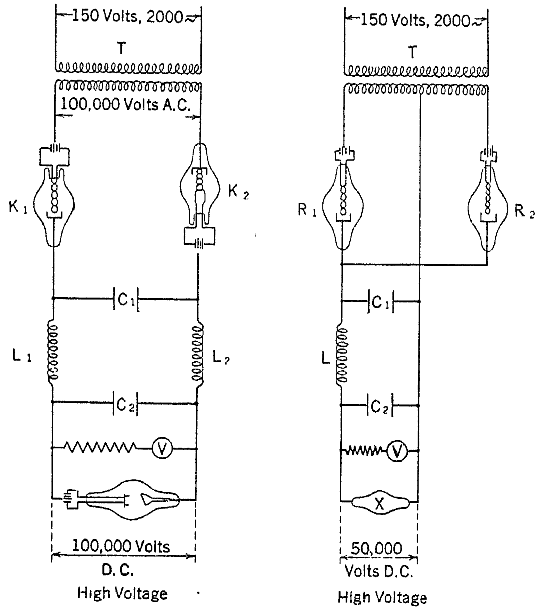


Fig. 1—Diagram of Connections for the Production of High-Voltage Direct Current

In the case of the experiments described by Messrs. Davis and Breese, the maximum current appears to have been about 4 milliamperes. This corresponds to a resistance of 5 megohms at 20,000 volts. Assuming 60-cycle alternating current and only half-wave rectification, it can be shown that in order to obtain the above current with less than one per cent fluctuation in voltage, it would require only about 0.25 microfarad, if capacity alone were used.

By using the method suggested by Dr. Hull, this could be reduced to about 0.1 microfarad by the use of an inductance of about 900 henrys. Dr. Hull's method becomes more and more advantageous as the frequency is increased, and at 500 cycles, the capacity required with the same inductance would be only about one-twenty-fifth of that required at 60 cycles.

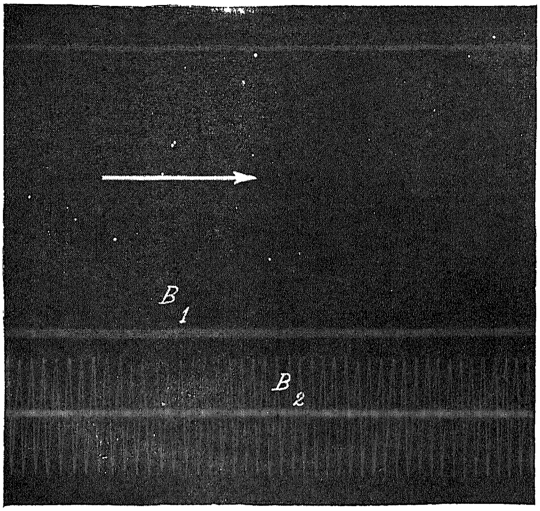
J. B. Taylor: I do not understand why, if negative corona is a different sort of action entirely from positive, which it obviously is, very decidedly in the case of hydrogen, and to a less extent in the case of air, why any equation for determining alternating-current corona point should not be the same as the minimum corona point with either alternating or direct current. Since the alternating-current wave must have a negative portion in it, why does not corona begin to form at just as low a value in alternating as in direct current, unless there is a certain lag and in that case a logical law would seem to call for the introduction of a frequency term.

L. W. Chubb: Messrs. Davis and Breese, in their paper show the difference between corona formation in hydrogen and air. In hydrogen negative corona forms at the lower voltage, while in air the reverse is true. The comparison that is shown in this paper is limited to wires of about the same size, and of a size which possibly exaggerates the difference. The Farwell paper showed the formation of negative corona first, in air, when the wire was below a certain diameter. It is possible that there might be an interchange of the phenomena also in hydrogen if wires of much smaller or much larger diameter are used, and that this apparent difference is only because of the relatively different parts of the curve.

I want to call attention to some facts regarding rectification. Apparently, the scheme given looks like a practical rectifier, but if you consider the losses that would occur in such rectification, and also consider the fact that it can work against a counter e.m.f. equal to only one-half of the difference between the positive and the negative corona-forming voltages, it does not make a very attractive scheme.

The increase of the pressure in the tube when ionization starts has come up before in the Farwell paper, and the change was denied in some discussion last year. It seems that there is a possibility that this pressure change is local, and will depend upon where the pressure is taken, whether near the corona wire or near the outside cylinder.

Dr. Whitehead's method of inserting tubes to prevent the multiplication of ionization is rather interesting, but I think is



[DUSHMAN]

open to some objection in that the tubes may become charged with electrons, and have effects on the field. They may produce a space charge which will change the natural gradient, and may also act as condenser plates, because they have low conductance and may distribute the gradient like the plates of the concentric series of foil of condenser bushing.

Dr. Whitehead reviews the common theory of ionization by bombardment by the electrons near the wire when it is positive, and explains by this that it is natural that the positive corona should form at a lower voltage. That is the way I have always looked at it myself, but the reverse effect in hydrogen gives us something more to think about.

F. W. Peek, Jr.: The tests that Messrs. Davis and Breese have made indicate that corona formation in hydrogen follows the same laws as in air. The constants which they have obtained for the hydrogen corona formula are, however, lower. Dr. Whitehead found that CO_2, a much heavier or denser gas than air, had the same dielectric strength at the same "relative" density—at the same temperature and pressure. The strength of air varies with the "relative" density, as shown by the standard formula. The dielectric strength of different gases should be measured at various densities, to determine if the strength is affected by the "absolute" density or only by the "relative" density of the gas. The effect of molecular spacing would thus be determined.

Some years ago, I was able to rectify by corona, and obtained about 100 kv., direct current, with power of a few kw. A description of this investigation will be found in an A. I. E. E. discussion.*

In my experiments with d-c. corona, I found it extremely difficult to determine the exact starting voltage when the wire was negative. The positive corona was quite definite, and apparently about the same for direct-current or for alternating-current.† The slightest irregularity on the negative wire made the corona starting point very irregular. What is recorded as the starting point of negative visual corona will depend to a great extent upon the observer and the condition of the wire. Dr. Whitehead gives constants for the corona formula for a-c., and negative and positive d-c. corona. When these constants are inserted in the formula, it is found that for very large wires (radii $= \infty$) the a-c. and the positive d-c. corona voltages are equal, while the negative d-c. voltage is much lower. For wires about 0.5 cm. in diameter the a-c. and positive and negative d-c. are approximately equal; for small wires the negative corona voltage is higher than the positive. For the small wires actually used in Dr. Whitehead's investigation, he found that the negative corona starting point was approximately equal to the a-c. while the positive corona starting point was lower. In this connection, Mr. Taylor's question is a good one. If there is a difference between positive and negative starting voltages, it would be expected that the a-c. would correspond to the lower voltage, or to the positive. This was not the case over the range investigated by Dr. Whitehead. The time necessary for corona to start should not affect this, as this time is relatively very small. I believe the g_0 constant of 33.7 is too high since spark-over in an approximately uniform field, where the radius is infinity, apparently takes place at a gradient of about 31 kilo-volts per centimeter.

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*TRANS. A. I. E. E., p.1062 Vol. 34., 1915.

†F. W. Peek, Jr., "The Effect of High Continuous Voltages on Air, Oil, and Solid Insulation." TRANS. A. I. E. E., 1916, Vol. XXXV, Part II, p. 783.

Some time ago, I investigated corona formation and spark-over with impulse voltages. The duration of these voltages was sometimes less than a microsecond. It is interesting that the same general laws hold for these transients. The time lag for the starting of visual corona on wires is very small, and measured in microseconds. For spark-over, the time lag has very little effect on sphere-gap spark-over voltages, or for spark-over voltages, where the fields are uniform. For needle-gap spark-over, or the spark-over of other gaps, where the field is non-uniform, the time lag has a decided effect. For such gaps the impulse spark-over voltage may be many times the continuously applied spark-over voltage. This investigation, which also includes the effect of transient or impulse voltages on liquid and solid insulations, surface arc-over, etc., is given in the A. I. E. E. TRANSACTIONS.*

J. B. Whitehead: The conclusion of Messrs. Davis and Breese, that hydrogen does not obey the law of corona that is followed by air, is not warranted by the results described in the paper. The observations are, in large part, qualitative and in order to test the conclusions, the actual observations and methods of reaching the conclusions should be given. No readings of pressure are given, and above all no readings of temperature. While it is probable that the conclusion that hydrogen does not obey the law is correct, it should be noted that temperature has a large influence on corona voltage. I have often observed that it is possible to reduce corona forming voltage below that at which corona starts, but have invariably been able to attribute it to a rise in temperature.

The authors are working with extremely small wires and small tubes, the latter about four centimeters in diameter. The corona is run far above the value at which it starts, and it is certain that the temperature in the tube is considerably elevated and that means a lowering of corona voltage. I am not prepared to say that the explanation of the authors, that space charge is the reason for the lowering of the voltage, is incorrect, but the paper does not show this, nor does there appear to be any reason why the same effect should not occur in air. The authors are working

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*F. W. Peek, Jr., "Effect of Transient Voltages on Dielectrics." TRANS. A. I. E. E., Sept. 1915, Vol. 34, p. 1857. F. W. Peek, Jr., "Lightning", *G.E. Review*, July 1916.

in a most important and interesting field, but in order that certain results may be drawn, why should not the conditions of accuracy, the methods of observation, and the record of observations themselves be given?

There is one indication in Fig. 8 which the authors have not mentioned, and which nevertheless may have some bearing upon the fact that corona voltage may be reduced below the point at which it starts. The curve for voltage in Fig. 8 is that having a peak in the upper half. The upper half of these waves is considerably shorter in time than the lower half, showing that the wave has been distorted not only on the upper or positive side, but also on the lower side. That, it seems to me, is an indication that there may be some distortion of the field in the tube, that is to say, some form of influence by foregoing ionization.

The results in hydrogen are, of course, of great interest, and the authors are to be congratulated for bringing them forward. Those shown in Fig. 7, in which the relation of positive and negative potential for corona is reversed as regards air, and the very marked change in the form of the curve connecting the voltage and the current are especially striking. However, it is only fair to note the extremely small wires with which the authors are working, and the difficulty of making the surface of such wires absolutely uniform. Surface irregularities on small wires cause wide variations in the value of corona forming voltage. You will notice, that the lower curve for hydrogen is the negative one, and if the wire did, by chance, happen to be irregular, it would tend to give the results as shown.

The authors state that: "The voltage at which negative corona starts, as shown in this oscillogram, is approximately twice the voltage across the tube at the instant that the discharge ceases."

They state that this difference between the initial and final corona voltage has not been observed before. They are correct in stating that it has not been observed for continuous potentials, but the first paper of our own series shows a wave form on which is marked the values of voltage at which corona started and those at which it ended, and the last were invariably lower. In that paper we attributed the difference to the slight elevation of temperature due to the corona, and the consequent hanging on of corona to a somewhat lower voltage.

The spots on the negative corona wires are more or less natural, by reason of the electro-magnetic reaction between successive discharge points, considered as electric currents. If the surface of a wire is irregular, either at the beginning, or becomes so as a result of discharge, it is natural that successive discharge points should space themselves more or less uniformly along the wire, as representing an equilibrium condition of a number of similar forces.

Coming to the comments that have been made on the paper of Mr. Brown and myself, Mr. Taylor's comment as to the absence of units in the formulas is quite pertinent. The units for these constants as they are given in the paper are in kilovolts per centimeter throughout. The density factor δ is a ratio; somewhat later in the paper the exact value of δ and the methods by which it is derived, are given. Its value is unity at a pressure of 76 cm. and a temperature of 75 deg. cent. The radius and diameter of the wires are to be expressed in centimeters.

Mr. Taylor also asks, why, if with continuous potential, the positive corona starts earlier, should not the a-c. corona at a corresponding value, inasmuch as it has a positive half wave. I cannot answer that question, and I shall be very glad indeed to have anybody else do so. It undoubtedly has to do with a time element involved in the rate of recombination of ions. It is quite possible to imagine conditions under which it will be necessary to reach a potential in which both halves of the wave are active before corona starts. In that event, we would expect coincidence between the alternating and the negative continuous potential, as regards corona voltage.

Mr. Chubb has commented on the change of pressure with the appearance of corona. That question recalls the paper of Farwell, which showed that there is an increased pressure due to the appearance of corona. A number of comments have been made upon that fact, and nobody questions that it is so. When a gas is ionized, there are a greater number of free particles, and these free particles obey the kinetic theory of gases, which indicate that there is an elevation of pressure. The question has no bearing, in my opinion upon the conditions which regulate the beginning of corona, but it may well regulate the value of the current above the corona voltages.

Mr. Chubb's second point is a comment on the use of the intermediate barrier tubes, and their effect upon corona voltage. He points out two possible sources of error, one the variation in the capacity of the system and the possibility of appearance of charges on the surface of the tube. The observations with these tubes in place show a raising of corona voltage which is far greater than that which can be accounted for by any change in the capacity of the system. With reference to the appearance of charges on the tubes, the paper shows that these charges are a distinct source of error, so long as we try to work with a galvanometer or kenotron as a detecting instrument, and also if we try to work with continuous potentials. Working with alternating potentials we had conditions under which these charges were not present. It is possible to show the presence of these charges by connecting the electroscope to the tube immediately after the voltage is thrown off. I think that both these errors suggested by Mr. Chubb are shown to have been eliminated by the subject matter in Section 5 of the paper.

There is undoubtedly a difference between the corona voltage for positive and negative continuous potentials. The curves approach each other, but I believe they will never be found to cross. I think they will converge into one with the increasing diameter of the wire. I think if the wire gets to be of sufficient size, the field between two parallel wires, will be sufficiently similar in the case of the passage of both positive and negative ions to give the same potential. If that is true, the constants of the formula as used in the paper would have to suffer some modification, but I do not believe we will ever find in air that the negative wire will show corona first.

Presented at the Fifth Midwinter Convention of the American Institute of Electrical Engineers, New York, February 15, 1917.

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OSCILLATING-CURRENT CIRCUITS BY THE METHOD OF GENERALIZED ANGULAR VELOCITIES

BY V. BUSH

ABSTRACT OF PAPER

The oscillating-current circuit differs from the alternating-current circuit in that the sinusoids involved are damped.

In the same way that alternating-current theorems are obtained by generalizing direct-current theorems, it is possible to obtain oscillating-current theorems by still further generalization. This involves a generalized angular velocity and a generalized impedance which are complex quantities. Kirchoff's law may be thus generalized; and yields a simple method of determining the generalized angular velocities, and thus the frequencies and decrements, of free oscillation.

Since, in an oscillating-current circuit, there is no impressed voltage, the initial voltage present in the circuit must be used in determining the amplitudes of oscillation. This results in the introduction of a "threshold impedance" for use in determining the amplitudes, which is obtained by a single differentiation from the generalized impedance. Further direct-current rules may also be readily generalized for convenient use in the solution of oscillating-current circuits.

A simple example is given to show the method of procedure. The discharge of a leaky condenser through a reactor furnishes a second example which better illustrates the convenience of the method for numerical computations. As a third example the oscillation of a three-section artificial cable under particular terminal conditions is chosen. The algebraical portion of this example is given in an appendix.

A list of symbols used will be found at the end of the paper.

INTRODUCTORY

IN SOLVING alternating-current networks it is convenient to make use of the pure imaginary, j. A sinusoidally varying quantity is represented by a revolving plane vector, and this in turn by the expression:

$$A\,\epsilon^{j\omega t}$$

A is the amplitude, and ω the "angular velocity" of the harmonic quantity or of the vector which represents it. When such expressions are combined according to the laws of algebra, true physical results are obtained because of the fact that harmonically varying quantities combine in accordance with the same laws as does this easily handled mathematical expression.

The use of this symbolism much simplifies the solution of such circuits; and much of this simplification results from the translation of direct-current rules into alternating-current rules by a proper exchange of terms. Thus Ohm's law is retained in its simplicity for the alternating-current case by the substitution of impedance for resistance; and the impedance of a circuit is defined in such a manner that the law will hold.

In oscillating-current circuits we consider the subsidence of a system from one state to another in the absence of impressed forces. The quantities involved vary in accordance with damped sinusoids. They may be represented by revolving plane vectors, which shorten exponentially as they revolve. Such a vector may be represented by the expression:

$$A \epsilon^{n t}$$

where n is a complex quantity of the form:

$$n \equiv - \alpha + j \omega$$

If we rewrite the expression in the form:

$$A \epsilon^{-a t} \epsilon^{j w t}$$

it is seen to consist of the term of the alternating-current case multiplied by a damping factor. In such an expression the complex quantity n may be conveniently called a generalized angular velocity; and we may form from it generalized impedances, generalized admittances etc. in a manner exactly analagous to the alternating-current case.

THE FREQUENCIES AND DECREMENTS OF FREE OSCILLATION

Kirchoff's law, that the sum of the voltages around a closed path in a direct-current circuit is zero, is translated for use in alternating-current circuits by substituting the term impedance for resistance.

In similar manner it may be extended for use in oscillating-current circuits by putting in instead the term, generalized impedance, Since there are in this case no impressed voltages to be considered, the law may be more conveniently expressed by stating that the generalized impedance of a closed circuit to free oscillations is zero.

In the direct-current case the law furnishes an equation for determining an unknown constant. In the alternating-current case it may be used also to determine the frequency if everything else is known. In the oscillating-current case it may be used to determine the unknown generalized angular velocities of oscillation. This proposition was used by Rayleigh and Heaviside; and has been presented in more explicit form for convenient use by Eccles, Campbell, and Kennelly.†

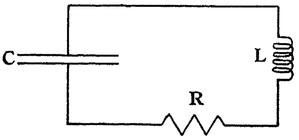

FIG. 1—SIMPLE SERIES CIRCUIT

One example will be sufficient to show the convenience of the law for practical use. Consider the series circuit of Fig. 1; a condenser of capacitance C, discharging through a reactor of inductance L and resistance R.

If we let n be the unknown generalized angular velocity, and apply the law to this circuit, we obtain

$$R + L n + \frac{1}{C n} = 0$$

From which, solving for n, there results:

$$n = -\frac{R}{2L} \pm j\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}.$$

Assuming for convenience the quantity under the radical positive, the angular velocity of free oscillation of this circuit is thus:

$$\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

or the frequency of oscillation is

$$\frac{1}{2\pi}\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}.$$

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†Rayleigh, Theory of Sound.
Heaviside, Electromagnetic Theory, Vol. 2.
Eccles, *Electrician*, 1915. Phys. Soc. Proc. 24, 1912.
Campbell, TRANS. A. I. E. E., 1911. Vol. XXX, Part II, p. 873.
Kennelly, Proc. Institute Radio Engineers, 1915.

and the circuit has a decrement expressed by

$$\frac{R}{2L}$$

This familiar result, in which there may be three cases according to the sign of the quantity under the radical, is thus arrived at simply by the solution of an algebraic equation, without the necessity of employing the calculus.

THE AMPLITUDES OF FREE OSCILLATION

If we attempt to translate Ohm's law in analogous manner, in order to arrive at the amplitudes of oscillation, a difficulty arises because of the fact that there is no impressed voltage present in an oscillating-current circuit. The impressed voltage, and the generalized impedance, are both zero; and their quotient is indeterminate.

There is however always an initial voltage present in such a circuit due either to an initial charge on a condenser, or to an initial current through a resistor. We seek then a function of the constants of the circuit, the generalized impedance Z, and the free generalized angular velocities n which shall give the amplitudes of oscillation when divided into the initial voltage. Such a function will be found in the expression.

$$n \frac{dZ}{dn}$$

It may well be called on account of its peculiar properties the "threshold impedance" of the circuit.

If a circuit, initially containing a voltage E, be allowed to oscillate freely, the initial amplitude of current oscillation will be.

$$\frac{E}{n \frac{dZ}{dn}}$$

and the current expression accordingly, the sum of terms of the form

$$\frac{E}{n \frac{dZ}{dn}} \epsilon^{nt}$$

where each term is formed from one of the values of n resulting from the solution of the equation

$$Z = 0$$

Z is formed for the entire circuit, considering the element initially charged as the main branch.

The proof of this second law is neccesarily long; and may well be omitted in a paper of this character. The "threshold impedance" appears in papers by Heaviside.‡

When there is only a single frequency of oscillation present, as in the series circuit treated above, this second law is unnecessary; for the amplitude may be written down by inspection. In more involved networks, where several frequencies of oscillation exist simultaneously, the law furnishes a very convenient means of determining the amplitudes of the several terms in the solution.

Solution of Circuits

To completely solve an oscillating-current network, the method of generalized angular velocities thus requires the following steps:

1. Form the generalized impedance of the network, considering as the main branch the element initially charged.
2. Equate to zero, and solve for the generalized angular velocities of free oscillation: $n_1\ n_2\ n_3 \ldots\ldots$
3. Form the threshold impedance,

$$T = n \frac{d\,Z}{d\,n}$$

and insert the roots for n found above to form a series of values of the threshold impedance: $T_1\ T_2\ T_3 \ldots\ldots$

4. Divide the initial voltage E of the circuit by each of these values to form a corresponding set of amplitudes.
5. Write the oscillating-current expression in the form

$$i = \frac{E}{T_1}\,\epsilon^{n_1 t} + \frac{E}{T_2}\,\epsilon^{n_2 t} + \frac{E}{T_3}\,\epsilon^{n_3 t} + \ .\ \ .\ \ .$$

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‡Heaviside, Electrical Papers, Vol. 2, p. 373., Electromagnetic Theory. Vol. 2, p. 135.

Since this paper was written, an excellent proof of the second law by Wagner has appeared in *Archiv für Elektrotechnik*, 1916, Band IV, 5/6 Heft.

or more concisely,

$$i = \sum \frac{E}{T} \epsilon^{n\,t}$$

where the summation extends over the values of n found from the equation,

$$Z = 0$$

It will be noted that in the above the generalized angular velocities, generalized impedances, threshold impedances and generalized amplitudes are all complex quantities. Upon reducing the final expression to the usual trigonometric form by the use of the identity,

$$\epsilon^{j\,\omega\,t} \equiv \cos\,\omega\,t + j\,\sin\,\omega\,t$$

the imaginary portions of the expression will of necessity cancel out, and leave a real trigonometric expression for the current.

Further rules for the examination of oscillating-current circuits may be readily obtained by generalization.

The current or voltage in a distant portion of the network may be found by combining the generalized impedances of the elements in the manner of simple resistances. Here as before the several terms of the current or voltage expressions are to be treated separately. An element has a generalized impedance corresponding to each free generalized angular velocity, to be used with the term of the current or voltage expression containing the same value of n.

When several stores of energy are simultaneously discharged, they may be treated separately and the results added.

Cases of oscillation under a suddenly applied steady electromotive force may be treated as the inverse of the discharge from the final state attained.

Oscillations under suddenly applied alternating voltages may be similarly treated; but the solution here rapidly becomes involved algebraically.

Examples of the Method. Three examples of the practical use of the method will be given. The first is very simple, to outline the procedure only. The second is of somewhat more complexity and is solved numerically. The third is intended to exemplify the method when several frequencies are present simultaneously. The inductively coupled circuit, which furnishes an excellent example for this purpose, is reserved because of its interest for separate treatment. The method as used has been checked in numerous cases by the use of parallel solutions of the differential equation. Circuits such as that of example III have also been checked by the oscillograph and the artificial line at Harvard University.

I. Consider the circuit of Fig. 2: a condenser of capacitance C, initially charged to potential E, and discharging through simple resistance R.

The generalized impedance is,

$$Z = R + \frac{1}{C\,n} \qquad \text{ohms} \angle$$

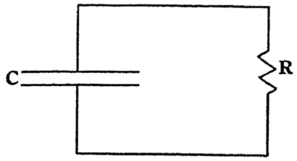


FIG. 2—CIRCUIT OF EXAMPLE I

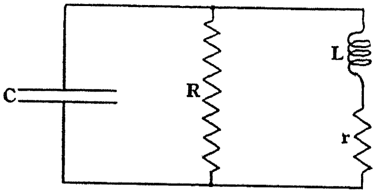


FIG. 3—CIRCUIT OF EXAMPLE II

Equate to zero, solve for n, and there results,

$$n = -\frac{1}{R\;C} \qquad \text{hyp. rad./sec.}$$

The threshold impedance is obtained from Z by differentiation;

$$T = n\frac{d\,Z}{d\,n} = -\frac{1}{C\,n} \qquad \text{ohms} \angle$$

For the free value of n found, this has the value,

$$T = -\frac{1}{C\left(-\frac{1}{R\,C}\right)} = R \qquad \text{ohms}$$

The current in the circuit is thus,

$$i = \frac{E}{T}\,\epsilon^{n\,t} = \frac{E}{R}\,\epsilon^{-\frac{t}{R\,C}} \qquad \text{amperes}$$

which may be checked by inspection and is the well known result.

II. As a better example, consider the network of Fig. 3; a condenser of capacitance C, initially charged to potential E, shunted by resistance R, and discharging through a coil of inductance L and resistance r.

Consider the condenser as the main branch, and form the generalized impedance:

$$Z_C = \frac{1}{C\,n} + \frac{1}{\dfrac{1}{R} + \dfrac{1}{r + L\,n}}$$

$$= \frac{R + r + L\,n + R\,r\,C\,n + R\,L\,C\,n^2}{C\,n\,(R + r + L\,n)} \quad \text{ohms} \angle$$

Equate to zero, and solve for n, and there results,

$$n = -\left(\frac{r}{2\,L} + \frac{1}{2RC}\right) \pm j\sqrt{\frac{1}{L\,C} - \left(\frac{r}{2\,L} + \frac{1}{2RC}\right)^2} \quad \text{hyp. rad./sec.}$$

It may be noted in passing that if we had formed Z with another branch, say R, as the main branch;

$$Z_R = R + \frac{1}{C\,n + \dfrac{1}{r + L\,n}}$$

$$= \frac{R + r + L\,n + R\,r\,C\,n + R\,L\,C\,n^2}{1 + r\,C\,n + L\,C\,n^2} \quad \text{ohms} \angle$$

the same roots for n would have resulted upon equating to zero. Owing to the difference in the denominator, however, Z_C should be used in forming the threshold impedance.

Form the threshold impedance:

$$T = n\,\frac{d\,Z_C}{d\,n}$$

$$= \frac{(R^2\,L\,C^2 - L^2\,C)\,n^2 - 2\,L\,C\,(R + r)\,n - C\,(R + r)^2}{C^2\,n\,(L\,n + R + r)^2} \quad \text{ohms} \angle$$

For the sake of brevity let us now introduce the numerical values of the constants:

$$\begin{aligned} C &= 10^{-5} \text{ farad.} \\ L &= 0.1 \text{ henry.} \\ r &= 10 \text{ ohms.} \\ R &= 100 \text{ ohms.} \\ E &= 100 \text{ volts.} \end{aligned}$$

Substitute these values in the expressions above, and we obtain

$$n = -\,550.0 \pm j\,835.2$$

$$T = +\,101.2 \pm j\,153.8$$

We may now write the expression for the oscillatory current in the condenser,

$$i_C = \frac{100}{101.2 - j\,153.8}\,\epsilon^{(-550.0\,+j\,835.2)\,t} + \frac{100}{101.2 + j\,153.8}\,\epsilon^{(-550.0 - j\,835.2)\,t} \quad \text{amperes } \angle$$

Note that in this expression each threshold impedance is associated with the term containing the generalized angular velocity from which it was derived.

Reduce the expression to trigonometric form, and combine terms, and it becomes,

$$i_C = 1.089\,\epsilon^{-550.0\,t} \sin(835.2\,t + 0.9882) \qquad \text{amperes.}$$

Suppose we now wish the current in the coil. The condenser current divides between the other two branches in the ratio of their generalized admittances.

This ratio is,

$$\frac{R}{R + (r + L\,n)} \qquad \text{numeric } \angle$$

or numerically,

$$0.550 \pm j\,0.8352 \qquad \text{numeric } \angle$$

Multiply the exponential expression for i_C by this ratio; paying due attention to signs, and the coil current results:

$$i_L = (-\,j\,0.5443)\,\epsilon^{(-550.0\,+j\,835.2)t} + (j\,0.5443)\,\epsilon^{(-550.0 - j\,835.2)\,t} \quad \text{amperes } \angle$$

and this may be reduced to,

$$i_L = 1.0886\ \epsilon^{-550.0t}\ \sin 835.2\, t \qquad \text{amperes.}$$

If we desire the voltage across the inductance L, multiply i_L by $L\, n$, or by,

$$(55.0 \pm j\, 83.52) \qquad \text{ohms } \angle$$

and obtain on reducing,

$$e_L = 108.9\ \epsilon^{-550.0t} \sin (835.2\, t - 0.9882)$$

and so on.

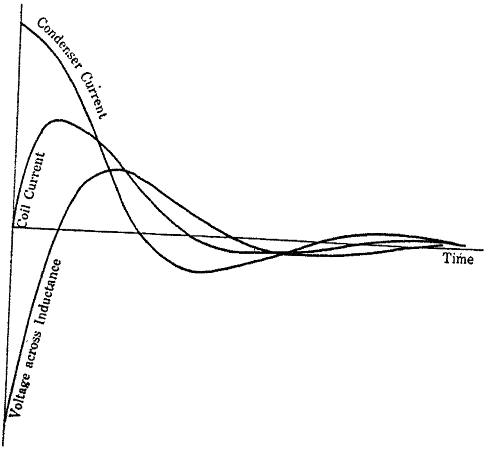


FIG. 4—DISCHARGE OF LEAKY CONDENSER—EXAMPLE II

These values are plotted in Fig. 4.

III. In the above example there are two conjugate roots for n, and hence a single frequency of free oscillation. The third example shows a case where more than one frequency is present. It is chosen from a series of solutions of the oscillations of the lumped artificial line under suddenly applied voltage. The three section artificial cable, grounded at one end, and suddenly connected at the other to a source of potential E, is chosen; as it illustrates the method with the least complexity of expression. The method proved adequate for the solution of the more general case with inductance present; and a series of solutions for various numbers of sections was used to derive a general expression for the lumped line and this type of transient. This expression, compares with that of a smooth line under similar conditions, furnished a criterion of the value of experimental results obtained on the lumped line for this type of transient. A lower limit for the number of sections could be set if such results were to apply to the smooth line case with allowable error.

If the total cable capacitance is C, and resistance R, the three section artificial cable is as shown in Fig. 5.

Putting for brevity,

$$c = \frac{C}{3}$$

$$r = \frac{R}{3}$$

this circuit is equivalent to that of Fig. 6.

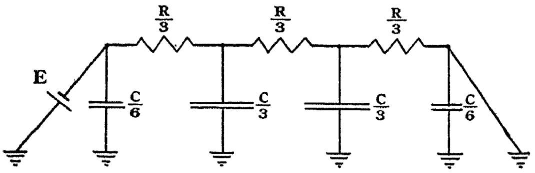


FIG. 5—THREE SECTION ARTIFICIAL CABLE

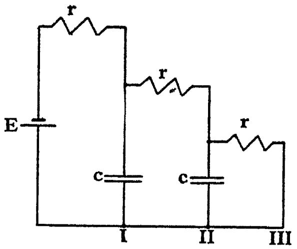


FIG. 6—CIRCUIT OF EXAMPLE III

This is a case of charge, to be treated as the inverse of discharge from the final state attained. We are interested merely in the current in branch III.

In the steady state there is a charge in condenser I with terminal voltage: $\frac{2E}{3}$, and in condenser II: $\frac{E}{3}$.

If we compute the discharge current of each of these condensers, by the method of the previous example, and find the proportional part of each which passes through branch III, we obtain respectively,

$$i_{\text{III}}' = \frac{E}{3r}\,\epsilon^{-\frac{T}{rc}} - \frac{E}{3r}\,\epsilon^{-\frac{3T}{rc}} \qquad \text{amperes}$$

$$i_{\text{III}}'' = \frac{E}{6r}\,\epsilon^{-\frac{T}{rc}} + \frac{E}{6r}\,\epsilon^{-\frac{3T}{rc}} \qquad \text{amperes}$$

In the steady state there is a current in branch III of value,

$$\frac{E}{3r} \qquad \text{amperes}$$

Adding to this the negatives of the two discharge currents above gives the current in branch III due to the sudden application of voltage,

$$i_{\text{III}} = \frac{E}{3r} - \frac{E}{2r}\,\epsilon^{-\frac{t}{rc}} + \frac{E}{6r}\,\epsilon^{-\frac{3t}{rc}} \qquad \text{amperes}$$

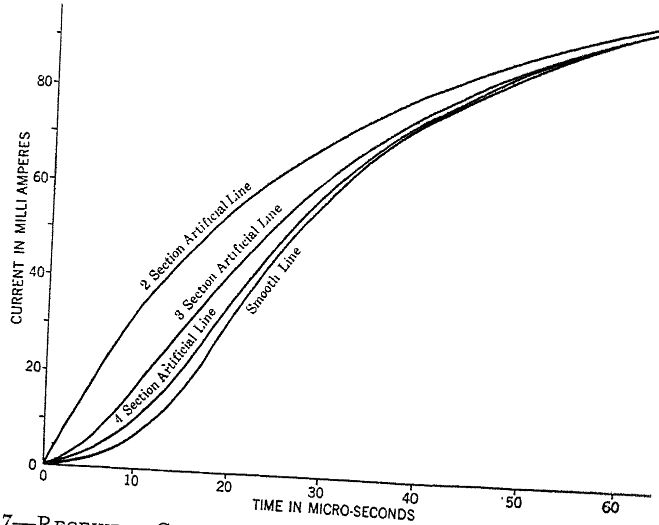


FIG. 7—RECEIVED CURRENT ON GROUNDED TRANSMISSION LINE— SMOOTH AND LUMPED LINES

Suddenly applied steady voltage at home end—20 volts
Line resistance 200 ohms—line capacity 10^{-6} farads

or in terms of total constants,

$$i_{\text{III}} = \frac{E}{R} - \frac{3E}{2R}\,\epsilon^{-\frac{9t}{Rc}} + \frac{E}{2R}\,\epsilon^{-\frac{27t}{Rc}} \qquad \text{amperes}$$

A part of this example is treated more fully in the appendix. A series of curves of this type, together with the smooth line curve from Kelvin's formula, are plotted in Fig. 7 for typical constants.

APPENDIX

ALGEBRA OF EXAMPLE III

In the steady state condenser I (Fig. 5) is charged to potential $\frac{2E}{3}$. The discharge of this condenser is computed as follows:

Consider branch I the main branch, and form the generalized impedance of the network by combining the generalized impedance of the elements in the manner of simple resistances.

$$Z_{\text{I}} = \frac{1}{c\,n} + \cfrac{1}{\cfrac{1}{r} + \cfrac{1}{r + \cfrac{1}{c\,n + \cfrac{1}{r}}}} \qquad \text{ohms}$$

or reducing,

$$Z_{\text{I}} = \frac{r^2\,c^2\,n^2 + 4\,r\,c\,n + 3}{c\,n\,(2\,r\,c\,n + 3)} \qquad \text{ohms}$$

Equate to zero, solve for n, and we obtain as free generalized angular velocities,

$$n_1 = -\frac{1}{r\,c} \qquad \text{hyp. rad./sec.}$$

$$n_1 = -\frac{3}{r\,c}$$

Differentiate Z_{I} with respect to n, multiply by n, and we obtain the threshold impedance. It is well to use for this purpose the form of Z_{I} which is most easily differentiated.

$$T_{\text{I}} = n\frac{d\,Z_{\text{I}}}{d\,n} = -\frac{1}{c\,n} - \frac{r^2\,c\,n}{(2\,r\,c\,n + 3)^2} \qquad \text{ohms.}$$

This threshold impedance has two values corresponding to the two free generalized angular velocities obtained above. Substituting them in and reducing, there results,

$$T_1 = 2\,r \qquad \text{ohms}$$

$$T_2 = \frac{2\,r}{3}$$

Divide the initial voltage, $\frac{2E}{3}$, by each of these values of the threshold impedance to give the amplitudes of oscillation of the discharge current:

$$A_1 = \frac{E}{3\,r} \qquad \text{amperes}$$

$$A_2 = \frac{E}{r}$$

Associate these amplitudes with the corresponding generalized angular velocities, and we have the discharge current of condenser I:

$$i_{\mathrm{I}} = \frac{E}{3\,r}\,\epsilon^{-\frac{t}{r\,c}} + \frac{E}{r}\,\epsilon^{-\frac{3\,t}{r\,c}} \qquad \text{amperes.}$$

This current divides among the several branches in proportion to their generalized admittances. The proportional part which passes through branches II and III is thus,

$$\frac{r}{2\,r + \cfrac{1}{c\,n + \cfrac{1}{r}}} \qquad \text{numeric}$$

and of this the part

$$\frac{\cfrac{1}{c\,n}}{r + \cfrac{1}{c\,n}} \qquad \text{numeric}$$

goes through branch III. The proportional part in branch III is thus,

$$\frac{r}{2\,r + \cfrac{1}{c\,n + \cfrac{1}{r}}} \times \frac{\cfrac{1}{c\,n}}{r + \cfrac{1}{c\,n}} \qquad \text{numeric}$$

or

$$\frac{1}{2\,r\,c\,n + 3} \qquad \text{numeric.}$$

This ratio again has two values corresponding to the two free values of n. Substituting these we obtain for the ratio the two values,

$$1 \text{ and } -\frac{1}{3}$$

To multiply i_{I} by the ratio, we multiply each term by the value of the ratio which corresponds to the value of n already involved in the term. In this manner we obtain the portion of the discharge current of condenser I which passes through branch III:

$$i_{\text{III}}' = \frac{E}{3\,r}\,\epsilon^{-\frac{t}{r\,c}} - \frac{E}{3\,r}\,\epsilon^{-\frac{3\,t}{r\,c}} \qquad \text{amperes.}$$

The discharge of condenser II is treated in like manner.

In this example the generalized angular velocities contain no imaginary part. In the more general case with inductance present, the generalized angular velocities are complex. The method of procedure is identical in the two cases; and the only difficulty results from the length of the expressions involved.

List of Symbols Employed

A = Amplitude of sinusoidally varying quantity.
j = The pure imaginary $\sqrt{-1}$.
ϵ = Naperian base. 2.718 . . .
ω = Angular velocity. Radians per second.
t = Time in seconds.
n = Generalized angular velocity. Hyperbolic radians per second. $\angle$
α = Logarithmic decrement. Hyperbolic radians per second.
C,c = Capacitance. Farads.
L = Inductance. Henries.
R,r = Resistance. Ohms.
Z = Generalized impedance. Ohms. $\angle$
T = Threshold impedance. Ohms. $\angle$
E = Initial potential. Volts.
e = Instantaneous potential. Volts.
i = Instantaneous current. Amperes
$\angle$ = Sign used to denote a complex quantity.

DISCUSSION ON "OSCILLATING-CURRENT CIRCUITS BY THE METHODS OF GENERALIZED ANGULAR VELOCITIES" (BUSH), NEW YORK, FEBRUARY 15, 1917.

A. E. Kennelly: Mr. Bush's paper points out a simple method of making the oscillating-current circuit as easily managed by engineers as the alternating-current circuit. The only essential difference is that the coefficient with which you multiply the inductance, or multiply the capacitance of a circuit, instead of being a pure j quantity, or a pure imaginary, is a complex quantity, which contains a real component.

The important and new material which Dr. Bush presents here is the *threshold impedance.* By the use of this conception, you are enabled to obtain the initial strength of the oscillating current in the circuit at a time when the impedance has to be made zero. If you make the impedance of a circuit zero you are unable, ordinarily, to determine what the initial current strength of discharge will be; but by taking this threshold value of the impedance just at the time when it is about to become zero (the disappearing value, so to speak), you are able in the manner in which he shows here to determine the starting-off current, and then the natural decay, using the real part of the complex angular velocity, to determine the strength of the current at all subsequent times. I think as soon as this method is clearly understood by engineers, they will be able to use it with great effectiveness, because it destroys, as the author has said, the necessity of introducing differential equations every time we have an oscillating-current system to deal with. In the early days of the a-c. circuit, we all had to start off with differential equations, and get the solutions of an alternating current in that manner. Since then, we have found it unnecessary to go to differential equations, we have reserved that for final work in physics, and we can work with a generalized Ohm's law. Now, here is a second generalization and extension of Ohm's law, whereby, without any reference to differential equations, we get right down to Ohm's-law equations of an oscillating-current circuit and obtain solutions by the ordinary method of vector arithmetic.

C. Fortescue: It appears to me that there is not much advantage in this method of considering the free oscillations of a network, if the order of the differential equations for the problem under consideration is beyond the third order. In fact, the method is really based on the use of differential equations. The generalized angular velocity is simply an equivalent for differential operator D, and I think sometimes that we lose something in adopting a narrower method of dealing with these problems than that of differential equations. It seems to me that on a new problem, if one were not able to form the differential equations of the system some difficulty would be found in using this method.

H. Fletcher: I would like to inquire whether the method can be used in cases where the e.m.f. was different from the steady e.m.f.

Thornton C. Fry: I want to call attention in the first place to the fact that the chief element of interest in this method lies in the fact that it brings in the natural periods of the circuit on the application of the e.m.f. I don't think there are many people who are interested in a-c. problems who have not known that the use of the generalized impedance formula, with $j\omega$, was also possible where the alternating e.m.f. was damped, and consequently *dt* must be replaced by a complex, as distinguished from a pure imaginary quantity.

The second point that I would like to make is the fact that while Mr. Bush has given a proof of Heaviside's formula from the point of his Fourier integral on, he did something just at the time of introducing his Fourier integral which is not justified mathematically. He introduced an operator in the integral sine. That is equivalent to a differentiation in the sine of integration, or an integration in the sine of integration, or more generally a combination of the two; and at the same time, in having thrown mathematical rigor to the winds at that point, he would have been justified in going ahead and throwing mathematical rigor to the winds further, and carrying out his solution by purely operational methods. It is much simpler, and undoubtedly Mr. Bush has done that for himself. The method simply consists in noting that the rate is itself a fraction introducing *D* only to integral powers, and can consequently be expanded into a series of partial fractions. Everybody knows how to do that, but everybody does not know Lorentz' series; and then by the theory of Lamique, with which the author seems to be familiar, he can perfectly well integrate each one of those partial fractions and come to the result that he has. This is a thing which is more useful for the practical man, because he can expand a long series of partial fractions, but he cannot know Lorentz' series. Of course, Mr. Bush can do that by introducing his arbitrary e.m.f. as distinguished from his absolutely constant e.m.f., but in each case he would have to carry out his line of reasoning to evaluate the residue, this function at the various variants in order to be able to write down his Lorentz series, which is entirely unnecessary by the other method.

V. Bush: On the first point, made by Mr. Fortescue, perhaps I can give an idea of the amount of simplification which results from the use of this method by quoting one problem. Take the case of an inductively-coupled circuit, which I left out of the paper, because I believe it is of sufficient importance to warrant special treatment. In that case we encounter the same fourth degree equation, whether we solve by differential equations or by threshold impedance. But in the determination of the constants of integration, in the differential equation method, we encounter fourth order determinants with complex members; whereas in the case of using the threshold impedance it is merely a question of substituting our complex values of the generalized angular velocity into a simple algebraic expression. The work for a typical circuit I believe will be much more than cut in half for a complete solution.

Now, in regard to whether this method can be used for the study of the application of other voltages than the steady voltage: Heaviside, I believe, does this in a few special cases. I have used the method and obtained correct results in the study of the sudden application of an alternating e.m.f. to certain simple networks, but the method there rapidly becomes involved, so that there is no great advantage.

Concerning the rigor of Wagner's proof of the Heaviside formula: I shall be glad to study this proof again. My purpose in presenting it here was rather to show the limitations of the method than to rigorously establish it.

In regard to the expansion of the operator into partial fractions: I think you will find that for circuits with lumped constants the threshold impedance method will be shorter. For circuits with distributed constants, to which the method can also be applied, an expansion in partial fractions I have found to be valuable; in fact, I have found it to be superior to Heaviside's method of expansion in power series; but for the ordinary network with lumped constants which we meet in practise, I think you will find the threshold impedance method superior, the difficulties being simply algebraic ones, with the greater part of the difficult mathematical work left out.

A. Press (Communicated after adjournment): Mr. Bush has done the electrical profession a very great service in drawing attention to the Heaviside Symbolic Method of dealing with Transient, *i.e.*, Impact Phenomena. The paper on "Oscillating Currents by the Method of Generalized Angular Velocities" is particularly well timed, especially since the "Shock Problems" are forcing consideration with their Transient Oscillating Currents, whether in interconnected power circuits or in heavy electrical machinery. However, it is highly regrettable that Mr. Bush did not give a more complete theoretical development of the Heaviside operator methods used, but rather assumed that his readers were, or could easily become, acquainted with them.

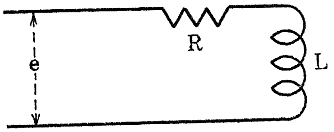
A more detailed exposition would have brought out for example that Kirchoff's laws are being interpreted in two totally different ways, which can lead to error if care is not taken. In the case of networks it is very important to state that Kirchoff's laws really apply to *instantaneous values* and it is only by means of peculiar extensions that a so-called "Ohms Law" very much similar to the direct-current law can be employed in practical problems.

The basis of all such symbolic work is really the set of simultaneous differential equations linking up the current in the network, and from which the resolved expressions of current are to be derived algebraically to obtain the *threshold impedance factors*. Another matter dealt with is the proper expression for what Mr. Bush calls the "Generalized Velocities."

Undoubtedly Mr. Bush's method will give the independent frequency terms but to obtain the amplitude factors it is still necessary to fall back on the fundamental differential equations of the net-work. The paper is replete with suggestions, yet, because the import of Heaviside methods are so little understood the writer takes the liberty of dilating on Mr. Bush's work and incidentally gives a simple and original proof of Heaviside's famous "Expansion Theorem".

The Basis for Vectorial Representation. For a simple circuit with resistance and self induction the differential equation is

$$e = i\,R + L\,\frac{d\,i}{d\,t} \qquad (1)$$



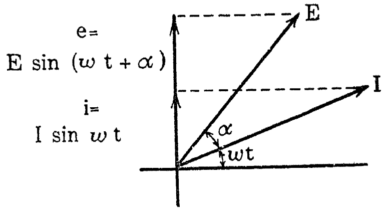
if then

$$i = I \sin \omega\, t, \text{ then substituting,}$$

$$e = I\,R \sin \omega\, t + L\,\omega\, I \cos \omega\, t \qquad (2)$$

These are all instantaneous values and therefore equation (2) suggests a method of representing the relation of the two variables e and i.

If the amplitudes I and E are to be taken, then geometrically one can represent the state of affairs indicated by equation (1) as follows: Where α is the angle of lag between E and I and is a constant of the circuit depending in fact upon the constants occurring in the differential equation (1). The geometric representation therefore is a complete solution of the steady case.



Projecting the lines E and I upon the axis at right angles to the one from which $\omega\, t$ is reckoned one has

$$e = E \sin (\omega\, t + \alpha)$$

$$i = I \sin \omega\, t$$

and therefore it is required to show that

$$e = E \sin (\omega\, t + \alpha) = I\,R \sin \omega\, t + L\,\omega\, I \cos \omega\, t \qquad (3)$$

by equation (2). Developing sin $(w\, t + \alpha)$ one has by ordinary trigonometry

$$e = E \sin \omega\, t \cos \alpha + E \cos \omega\, t \sin \alpha = I\,R \sin \omega\, t + L\,a\,I \cos \omega t \qquad (4)$$

If then (**4**) is true it will be true for $\omega t = 0$ and for $\omega t = \pi/2$. Thus for $\omega t = \pi/2$ one has

$$E \cos \alpha = I R \tag{5}$$

Again for $\omega t = 0$ one has.

$$E \sin \alpha = L I \omega$$

and dividing (**6**) by (**5**) one obtains

$$\sin \alpha \cos \alpha = \tan \alpha = L \omega / R.$$

Basis for Symbolic and Exponential Representations. However, because we are dealing with sinusoids we have the following:

$$\frac{d}{d t} \cdot \sin \omega t = \omega . \cos \omega t$$

$$\left(\frac{d}{d t}\right)^2 \cdot \sin \omega t = - \omega^2 . \sin \omega t$$

$$\left(\frac{d}{d t}\right)^3 \cdot \sin \omega t = - \omega^3 . \cos \omega t$$

$$\left(\frac{d}{d t}\right)^4 \cdot \sin \omega t = \omega^4 . \sin \omega t, \text{ etc.}$$

so that for $d/d t$ one can actually replace $w j$, and where for any even power of j one is entitled to write $j^2 = (\sqrt{-1})^2 = -1$, but for odd powers one must fall back upon differentiation when necessary.

Thus $j^3 = j^2 \cdot j = -1 j = -\dfrac{1}{\omega} \cdot \dfrac{d}{d t}$, because equivalently

$$\frac{d}{d t} = \omega j$$

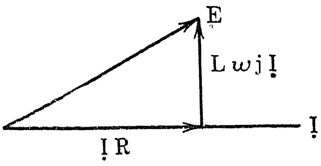
Equation (**1**) above can therefore be written *algebraically*

$$e = (R + L \omega j) i \tag{7}$$

and from the geometric construction one can write (**7**) vectorially by capitalization and dotting as

$$\underset{\bullet}{E} = (R + L \omega j) \underset{\bullet}{I}$$

so that $\underset{\bullet}{E}$ and $\underset{\bullet}{I}$ are now vectors in the amplitude time-phase plane. The multipliers j here must mean that a line $L \omega$ is to be drawn perpendicular to R which latter is to be drawn in the direction of $\underset{\cdot}{I}$ and thereby one obtains the angle α and the direction of $\underset{\cdot}{E}$.



The quantity j therefore is an indicator how to draw the quantity $L\omega\, \underset{\cdot}{I}$, for $L\,\omega\, j\, \underset{\cdot}{I}$ is perpendicular to the quantity $\underset{\cdot}{I}$.

There is however, an exponential method of representation, for since one has the equation (**7**)

$$e = (R + L\,\omega\, j)\, i,$$

and by ordinary algebra

$$e^{\omega t} = 1 + \frac{\omega\, t}{\underline{/1}} + \frac{\omega^2\, t^2}{\underline{/2}} + \frac{\omega^3\, t^3}{\underline{/3}} +$$

$$\cos\,\omega\, t = 1 - \frac{\omega^2\, t^2}{\underline{/2}} + \frac{\omega^4\, t^4}{\underline{/4}} - \frac{\omega^6\, t^6}{\underline{/6}} +$$

$$\sin\,\omega\, t = \omega\, t - \frac{\omega^3\, t^3}{\underline{/3}} + \frac{\omega^5\, t^5}{\underline{/5}} -$$

and it follows that

$$A\, e^{j\omega t} = A\,(\cos\,\omega\, t + j\sin\,\omega\, t).$$

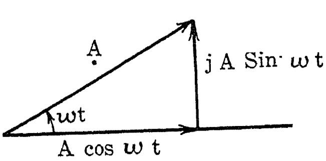
The utility of this mode of representation is important.

Any vector such as $\underset{\cdot}{E}$ or $\underset{\cdot}{I}$ in a time-phase diagram can always be considered as made up of two components at right angles to each other. Thus choosing any datum line for $\omega\, t$ one can write

$$\underset{\cdot}{I} = I\,(\cos\,\omega\, t + j\sin\,\omega\, t).$$

Thus when one writes $A\, e^{j\omega t}$ it really means

$$A\,(\cos\,\omega\, t + j\sin\,\omega\, t) = \underset{\cdot}{A} = A\, e^{j\omega t}$$



Then ω is the "angular velocity" and really is given by $\frac{2\,\pi}{T} = 2\,\pi f$.

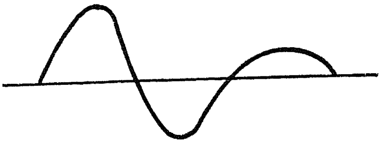
Of course here the amplitude A does not vary and the vector $\underset{\cdot}{A}$ keeps the same magnitude but only varies its position in the plane.

The current vector $\underset{\cdot}{I}$ before mentioned could be written therefore

$$\underset{\cdot}{I} = I\, e^{j\omega t} \tag{9}$$

Such a wave of current would be represented by a true sine wave.

Bush's Generalized Exponential Representation. Supposing, however, that we wish to represent a decremental wave component of the following form:



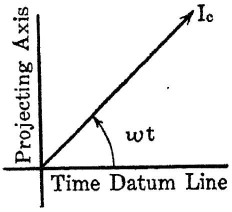
such as one has in wireless or as one has in some direct-current circuits when the switch is suddenly closed or opened and there is considerable capacity and self-induction. A wave of this type is represented by

$$i = I\, e^{-\alpha t} \sin \omega\, t, \text{ or}$$

$$i = I\, e^{-\alpha t} \cos \omega\, t$$

Here counting $\omega\, t$ as before the projection of the constant amplitude vector $\underset{\cdot}{I}$ upon the projection axis would not give the true instantaneous value because as t continually grows in $\omega\, t$ it is necessary also to always consider the amplitude multiplying factor $e^{-\alpha t}$. This can be corrected for by applying the factor $e^{-\alpha t}$ to each position of the original amplitude vector $\underset{\cdot}{I_c}$ as in fact this original amplitude vector travels around.

As a true vectorial representation one can therefore write



$$\underset{\cdot}{I} = e^{-\alpha t}\, I_c\, (\cos \omega\, t + j \sin \omega\, t) = I_c\, e^{-\alpha t} e^{j t \omega}$$
$$= I_c\, e^{(-\alpha + j\omega)\, t} = I_c\, e^{n\, t},$$

where n is a complex and is of the form

$$n = -\alpha + j\, \omega.$$

In the diagram just given $\omega\, t$ was drawn as a real angle. Supposing however, that it is assumed that in some imaginary geometric system complex angles of the form

$$\frac{n}{j} = -\frac{\alpha}{j} + \omega = j\, \alpha + \omega$$

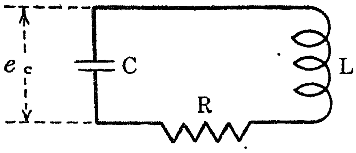
could be drawn then as the vector $\underset{\cdot}{I_c}$ travelled around according to the angular velocity $j\, \alpha + \omega$, projecting on to the projecting axis would actually give the time instantaneous values of i.

The complex angular velocity should strictly speaking be given by dividing the complex exponent in $e^{j\omega t}$ by $j\, t$ and the same thing should hold for e^{nt}. This view point as to complex angles is not Heavisidean but is really due to Mr. Bush in his paper except for the fact that a division by $j\,t$ is not resorted to. Thus if $\alpha = 0$, the amplitude factor is $e^{-\alpha t} = 1$ then,

$$n = j\,\omega$$

and it was seen that ω was called the angular velocity and not $j\,\omega$.

Application of Generalized Velocity Components to Circuits. Now take the equation of a circuit in which there is capacity, inductance and resistance, if i is the current then we have as the voltage across the resistance, $i\,R$: for that across the inductance $L\dfrac{d\,i}{d\,t} = n\,L\,i$ writing n, for $d/d\,t$; but for the capacity the voltage is not so straight forward, if we wish to express the voltage in terms of i and C. We have



$$q = C\,e_c$$

where q is the charge and C is the capacity in farads for an impressed e.m.f. e_c, so that

$$\frac{d\,q}{d\,t} = i = \frac{d\,e_c}{d\,t} = n\,C\,e_c, \text{ since } n = \frac{d}{d\,t}, \text{ whence}$$

$$e_c = \frac{i}{C\,n}$$

The equation of a circuit in which the impressed voltage is zero and forms the right hand term is therefore

$$\left(R + L\,n + \frac{1}{C\,n}\right) i = 0 \tag{10}$$

Heaviside's Treatment of Zero Operands. An equation of this type is to be solved in the following manner by Heaviside. If both sides of the equation are operated upon with what really is a differential operator function, $\left(R + n\,L + \dfrac{1}{C\,n}\right)^{-1}$ whatever this may mean, will give

$$\left(R + n\,L + \frac{1}{C\,n}\right)^{-1} \cdot \left(R + n\,L + \frac{1}{C\,n}\right) \cdot i$$

$$= \left(R + n\,L + \frac{1}{C\,n}\right)^{-1} \cdot 0$$

The left-hand side gives the i because any quantity (or operation) in ordinary algebra when multiplied by its reciprocal is assumed to give unity and therefore one has

$$i = \frac{0}{R + nL + \dfrac{1}{Cn}} \S \qquad (\mathbf{11})$$

where it must now be understood that zero is the operand and corresponds to the impressed disrupted voltage.*

"Determinantal Equation". Resolution into Partial Fractions. Supposing, however, the right-hand side is developed in partial fractions then algebraically it will be necessary to put

$$R + nL + \frac{1}{Cn} = 0$$

This equation of condition is called by Heaviside the "Determinnantal Equation."†

As an equation in n we will have

$$n = -\frac{R}{2L} \pm j\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2} = -\alpha \pm j\,\omega$$

so that by the rule for partial fractions one can write (**11**) as

$$i = i_1 + i_2 = \frac{0}{n + (\alpha - j\,\omega)} + \frac{0}{n + (\alpha + j\,\omega)} \qquad (\mathbf{12})$$

But then how interpret the individual fractions?‡

Let us take a single term given by

$$i_1 = \frac{0}{n + (\alpha - j\,\omega)} \text{ or, } \{n + (\alpha - j\,\omega)\}\, i_1 = 0$$

$$\frac{d\,i_1}{d\,t} + (\alpha - j\,\omega)\, i_1 = 0.$$

The solution of this last is very well known and gives

$$i_1 = A\, e^{-(\alpha - j\omega)\,t}$$

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*For fractions of this type compare Perry's Calculus, page 238.

†See also Bush's paper page 209.

‡See Perry's Calculus, page 238.

§One must be very careful not to treat the numerator zero, in such an operator function as still zero when multiplied by any function. To do so will result in wrong amplitude factors or determinantel equations.

o that the complete solution of the above is really

$$i = \left\{ e^{-(\alpha - j\omega)\,t} + e^{-(\alpha + j\omega)\,t} \right\} A = A\, e^{-\alpha t} \left(e^{j\omega t} + e^{-j\omega t} \right) §$$

$$= A\, e^{-\alpha t} \cos \omega\, t$$

Turning now to what has been called the "Generalized Angular Velocity" it is in fact nothing other than what constitutes the roots of the Determinantal Equation in *n* considered as an ordinary algebraic equation and divided by *j*. This is the complex "Generalized Velocity" of Bush. However, *n* itself is not a generalized velocity but rather a Heaviside-Operator function. The actual velocity assuming the resistance to be sufficiently small is given by,

$$\frac{j\,\omega}{j} = \sqrt{\frac{1}{L\;C} - \left(\frac{R}{2\,L}\right)^2} = \omega$$

$$f = \frac{1}{2\,\pi} \sqrt{\frac{1}{L\,C} - \left(\frac{R}{2\,L}\right)^2}$$

where *f* is the frequency of the oscillations.

Heaviside's Expansion Theorem. Engineers encounter linear differential equations when dealing with the "Forced Vibrations" of a steadily applied alternating force and "Natural Vibrations" set up by the shock application of some force. The latter call for evanescent terms which mathematically correspond to the "complementary function" whereas for that part corresponding to the steady state *particular* solutions are found necessary.*

Heaviside's Expansion Theorem is directed toward determining the evanescent terms in a mechanical or electrical dynamic system, without having to laboriously evaluate constants as required by the usual methods of solution† Moreover it allows of semi-graphic methods of obtaining the amplitude coefficients in complicated and transcendental expressions. It is especially important in net works‡ where make and break contacts are made. Yet on closer examination the whole complicated phenomenon can be looked upon as being due to a series of transient terms 'X' having their origin in the steepness of the voltage curve at the make and break (really the complementary solution in mathematics) plus a term due to the steady variable (or non-variable) state corresponding to the forced vibration. The latter is the so-called *particular* solution.

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§See Perry, page 520.

*See Differential Equations by Abraham Cohen, page 91.

†See Heaviside, E. M. T. Vol. II, pages 127-129.

‡Alternate-current networks should be solved by means of Kirchoff's Laws vide Fleming. Alternate Current Transformer, Vol. I, page 234.

For the general case with the suddenly applied voltage E_0 impressed applied to a circuit of which the differential equation of current is

$$F(\theta).\, i = E_0 \dagger \tag{13}$$

the steady state will be given by placing $\theta = 0$ in $F(\theta)$ so that

$$i_{steady} = \frac{E_0}{Z_0} \tag{14}$$

where Z_0 means $F(\theta)$ in which θ is put equal to zero. To find the transient component X due to shock one has by (**13**) and (**14**)

$$X = \frac{E_0}{Z} - \frac{E_0}{Z_0} = \frac{Z_0 - Z}{Z_0 Z} \cdot E_0 \tag{15}$$

In general all the terms in $Z_0 - Z$ will contain θ whence dividing by θ one can write

$$X = \frac{(Z_0 - Z)/(\theta Z_0)}{Z} .\, \theta E_0 \tag{16}$$

Now splitting up $\dfrac{Z_0 - Z}{\theta Z_0 Z}$* into partial fractions of the form $\sum \dfrac{A_n}{\theta + a_n}$, let $\varphi(\theta) = \dfrac{Z_0 - Z}{\theta Z_0}$; $F(\theta) = Z$ then

$$A_n = \frac{\varphi(\theta)}{\left\{ \dfrac{d}{d\theta} \cdot F(\theta) \right\}_{\theta = \theta_n}} \tag{17}$$

where θ_n is the nth root of the determinantal equation $F(\theta) = 0$ regarded as an ordinary algebraic equation in θ. Since

$$\varphi(\theta) = \frac{Z_0 - F(\theta)}{\theta Z_0} \tag{18}$$

then replacing θ by one of the roots it follows that $F(\theta) = 0$ and

therefore $\varphi(\theta)_n = \dfrac{1}{\theta_n}$, $A_n = \dfrac{1}{\left\{ \theta \dfrac{dZ}{d\theta} \right\}_{\theta = \theta_n}}$

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*Todhunter's Integral Calculus, 7th edition, page 26.

†The symbol θ is used for d/dt.

$$X = \sum \frac{1}{\left(\theta \frac{d Z}{d \theta}\right)_{\theta=\theta_n}} \cdot \frac{\theta E_0}{\theta - \theta_n} \tag{19}$$

As already pointed out

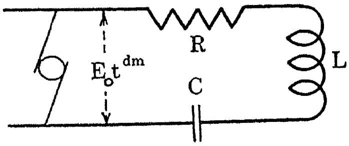
$$\theta E_0/(\theta - \theta_n) = E_0 . \frac{0}{\theta - \theta_n} = E_0 \, e^{\theta_n t} \tag{20}$$

and therefore finally, for the transient component with suddenly applied voltage

$$X = E_0 \sum \left\{ \frac{e^{\theta t}}{\theta \frac{d Z}{d \theta}} \right\}_{\theta=\theta_n}$$

which expresses Heaviside's Expansion Theorem.

Application. A direct-current voltage is suddenly impressed upon a series circuit containing inductance resistance and capacity. The differential equation is



$$L \frac{d i}{d t} + R i + \frac{1}{C} \int i \, d t = E_0$$

or

$$i = \frac{E_0}{L \theta + R + \frac{1}{C \theta}} = \frac{\text{Voltage}}{\text{Impedance Operator}}$$

Thus for the impedance operator one has

$$Z = L \theta + R + \frac{1}{C \theta}$$

and regarding momentarily $Z = 0$ as an algebraic equation in θ the roots are

$$\theta_1 = - \frac{R}{2 L} + \sqrt{\frac{R^2}{4 L^2} - \frac{1}{L \; C}}$$

$$\theta_2 = - \frac{R}{2 L} - \sqrt{\frac{R^2}{4 L^2} - \frac{1}{L C}}$$

Thus $\theta \frac{dZ}{d\theta} = L\theta - \frac{1}{C\theta}$ and therefore for the transient current

$$i_x = \frac{E_0 . e^{\theta_1 t}}{L\theta_1 - \frac{1}{C\theta_1}} + \frac{E_0 e^{\theta_2 t}}{L\theta_2 - \frac{1}{C\theta_2}}$$

This can be put in another form, since in developing the same, use was made of the equation $Z = 0$ in determining the roots. Thus

$$L\theta_1 + R = -\frac{1}{C\theta_1};\quad L\theta_2 + R = -\frac{1}{C\theta_2}\quad \text{whence}$$

$$i_x = \frac{E_0}{2L\theta_1 + R} e^{\theta_1 t} + \frac{E_0}{2L\theta_2 + R} e^{\theta_2 t}.$$

A lecture delivered at the Fifth Midwinter Convention of the American Institute of Electrical Engineers, New York, February 15, 1917.

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MODERN PHYSICS

BY R. A. MILLIKAN

MY POSITION tonight is somewhat reminiscent of a story that my father used to be fond of telling of a Scotch preacher who thought that all of the italicised words in the Holy Writ were meant to be emphasized, and so he read: "And the Prophet said unto his servants, saddle me an ass, and they saddled *him*." When one of his parishioners expostulated with him and told him that he didn't think that was really what was meant, he, being a Scotchman, kept his own opinion, but being more or less politic, he said he would change it, and so the next time he read: "And the Prophet said unto his servant, saddle *me* an ass, and they *saddled* him." (Laughter.)

The point of the story is that it doesn't make any difference where the emphasis is placed, the situation remains entirely unchanged. I am, however, really glad to be saddled tonight, because I should like to do a little bit, if I can, towards bridging the chasm which we have foolishly—I had almost said idiotically—allowed to grow up between the physicist and the applied physicist, who commonly is called an engineer. The chemists have been very much more sensible, they have not split up into two groups, called chemists and applied chemists, and there is absolutely no more reason why we should have done so, because obviously the physicist is merely the vanguard in the army of engineers, the scout, the explorer, who is given the task of trying to open up new paths for human progress, of prospecting for new leads to nature's gold, and it is just as important that the engineer know where he is and what he is doing as it is that he know where the army is which is behind and which supports him.

If you have any respect for my subject or any respect for me, you will not expect me to outline in the space of one brief hour the work of modern physics. It is utterly impossible to do, and I can say that without affecting an inordinate egotism. I had the good fortune to listen a little while ago to a series of lectures by our honored ex-President, Mr. Taft, on "The Executive Power," and he said in those lectures that his friend, Mr. Roosevelt, had somewhere classified the Presidents of the United States into two groups, the first, the group that had interpreted the executive power broadly and exercised it largely, and the second, the group that had interpreted it narrowly and exercised it sparingly, and he said that he began the first group with Lincoln and closed it with himself, and that he began the second group with Buchanan and closed it with myself—that is, with Mr. Taft. Then, with his inimitable chuckle, Taft said, "That reminds me of a story about a little boy who came home from school and said, 'Papa, did you know I was the brightest student in class?' His father replied, 'No, I didn't know it. When did the teacher tell you so?'

"The boy answered, 'The teacher didn't tell me so, I just noticed it myself.' " (Laughter.)

If I appear at all in what I say to have just noticed it myself, I beg of you at any rate to believe me that the appreciation is an appreciation of the subject, of the method that it uses, of the spirit that underlies it, and of the results that have actually flowed from it, and not an appreciation of any individual or group of individuals.

The spirit of modern science is something relatively new in the world's history, and I want, as an introduction to the main address, to give an analysis of what it is. I want to take you up in an aeroplane which flies in time rather than in space, and look down with you upon the high peaks that distinguish the centuries, and let you and me see together what is the distinguishing characteristic of this century in which we live. I think there will be no question at all, if you get far enough out of it so that you can see the woods, without having your vision clouded by the proximity of the trees, that the thing which is characteristic of our modern civilization is the spirit of scientific research—a spirit which first grew up in the subject of physics, and has spread from that to all the other subjects of modern scientific inquiry.

That spirit has three elements. The first is a philosophy, the second is a method, and the third is a faith. Look first at the philosophy. I say that is new for the reason that all primitive peoples, and many that are not primitive, have held a philosophy that is both animastic and fatalistic. Every phenomenon which is at all unusual or for any reason not immediately intelligible used to be attributed to the direct action of some invisible personal being. Witness the peopling of the woods and streams with spirits by the Greeks; the miracles and possession by demons of the Jews; the witchcraft manias of our own Puritan forefathers, only two or three hundred years ago.

Now, that a supine fatalism results from such a philosophy is to be expected, for according to it everything that happens is the will of the gods, or the will of some more powerful beings than ourselves. And so, in all the ancient world, and in much of the modern also, three blind fates sit down in dark and deep inferno and weave out the fates of men. Man himself is not a vital agent in the march of things, he is only a speck, an atom which is hurled hither and thither in the play of mysterious, titanic uncontrollable forces.

Now, the philosophy of physics, a philosophy which was held at first timidly, always tentatively, always as a mere working hypothesis, but yet held with ever increasing conviction from the time of Gallileo, when the experimental method may be said to have had its beginnings, clear up to the present time, is the exact antithesis of the above. Stated in its most sweeping form it holds that the universe is ultimately rationally intelligible, no matter how far from a complete comprehension of it we may now be or indeed may ever come to be. It believes in the absolute uniformity of nature. It views the world as a mechanism, every part and every movement of which fits in some definite, invariable way, into the other parts and the other movements; and it sets itself the inspiring task of studying every phenomenon in the confident hope that the connections between it and other phenomena can ultimately be found. It will have naught of caprice in nature. It looks askance at mysticism in all its forms whether put forth by Dionysus in Greece in 300 B. C. or by the devotees of Bergson in Paris in 1915. That is the spirit, the attitude, the working hypothesis of all modern science, and let me say that this philosophy is in no sense materialistic, because good, and mind, and soul, and moral values, which is only another word for God, these things are all here just as truly as are any physical objects, and with that kind of a creed they must simply be inside and not outside of this matchless mechanism.

Second, as to the method of science, it is a method practically unknown to the ancient world; for that world was essentially subjective in all its thinking and built up its views of things largely by introspection. The scientific method on the other hand is a method which is completely objective. It is the method of the working hypothesis which is ready for the discard the very minute it fails to work. It is the method which believes in a minute, careful, wholly dispassionate analysis of a situation; and any physicist or engineer who allows the least trace of prejudice or preconception to enter into his study of a given problem violates the most sacred duty of his profession. This present cataclysm which has set the world back a thousand years in so many ways, has shown us the pitiful spectacle of scientists who have forgotten completely the scientific method, and have been controlled simply by prejudice and by preconception. This is no reflection on the scientific method, it merely means that these men have not been able to carry over the methods they use in their science into all the departments of their thinking. The world has been controlled by prejudice and emotionalism so long that reversions still occur, but the fact that these reversions occur after all does not discredit the scientist, or make him disbelieve in his method. Why? Simply because that method has worked, it is working today, and its promise of working tomorrow is larger than it has ever been before in the world's history. Do you realize that within the life time of men now living, within a hundred years, or one hundred and thirty years at most, all the external conditions under which man lives his life on this earth have been more completely revolutionized than during all the ages of recorded history which proceeded? My great grandfather lived essentially the same kind of a life, so far as external conditions were concerned, as did his Assyrian prototype six thousand years ago. He went as far as his own legs or the legs of his horse could carry him. He dug his ditch, he mowed his hay, he did all the operations of his industrial life, with the power of his own two arms, or the power of his wife's two arms, with an occasional lift from his horse or his ox. He carried a dried potato in his pocket to keep off rheumatism, and he worshiped his God in almost the same superstitious way. It was only in the beginning of the nineteenth century that the great discovery of the ages began to be borne in upon the consciousness of mankind through the work of a few patient, indefatigable men who had caught the spirit which Gallileo perhaps first notably embodied, and passed on to Newton, to Franklin, to Faraday, to Maxwell, and to the other great architects of the modern scientific world in which we live,—the discovery that man is not a pawn in a game played by higher powers, that his external as well as his internal destiny is in his own hands. You may prefer to have me call that not a discovery but a faith. Very well! It is the faith of the scientist, and it is a faith which he will tell you has been justified by works. Take just this one illustration, suggested by the opening remarks of your President. In the mystical, fatalistic ages which preceded, electricity was simply the agent of inscrutable Providence; it was Elijah's fire from Heaven sent down to consume the enemies of Jehovah; or it was Jove's thunderbolt hurled by an angry God; and it was just as impious to study so direct a manifestation of God's power in the world as it would be for a child to study the strap with which he is being punished, or the mental attributes of the father who is behind the strap. It was only one hundred and fifty years ago that Franklin sent up his famous kite, and showed that these thunder bolts were identical with the sparks which he could draw on a winters night from his cat's back. Then, thirty years after that Volta found that he could manufacture these same thunderbolts artificially by dipping dissimilar metals into an acid. And, thirty years farther along Oersted found that those same thunderbolts when tamed and running noiselessly along a wire would deflect a magnet, and with that discovery the electric battery was born, and the erstwhile blustering thunderbolts were set the inglorious task of ringing house bells, primarily for the convenience of womankind. Then ten years later Faraday found that all he had to do to obtain a current was to move a wire across the pole of a magnet, and in that discovery the dynamo was born, and our modern electrical age, with its electric transmission of power, its electric lighting, its electric telephoning, electric toasting, electric foot warming electric milking,—all that is an immediate and inevitable consequence of that discovery—a discovery which grew out of the faith of a few physicists that the most mysterious, most capricious and the most terrible of natural phenomena is capable of a rational explanation and ultimately amenable to human control.

In that statement, I have revealed the top root of the civilization of the nineteenth century. Add to it a bit to cover the harnessing of steam, and the development of the principle of the conservation of energy, and you have an epitome of the progress of the nineteenth century. It all grew out of the application of a few, a relatively few, discoveries as to the way in which nature works.

And at the end of the nineteenth century there were many of us physicists and engineers who thought that all the great discoveries had been made. It was a common statement that this was so. I heard it publicly made in 1894, and yet within a year of that time I happened to be present in Berlin at the meeting of the Physical Society at which Roentgen showed his first photographs, and since that time we have had a whole new world, the very existence of which was undreamed of before, opened up to our astonished eyes. We have found a world of electrons which underlies the world of atoms and molecules with which we had been familiar, and the discoveries in that world have poured in so rapidly within the last twenty years that there are no two decades in human history that compare at all with them in the rapidity of the advance. And these discoveries have been made too for the most part by groups of men interested merely in finding out how nature works. They have been made almost exclusively by college professors; and for ten years they remained the exclusive property of these professors. What has happened in the last ten years? The industrial world has fallen over itself in the endeavor to get hold of these advances, and by their aid it has increased ten-fold the power of the telephone, it has obtained four or five times as much light as we got a few years ago out of a given amount of electrical power, it has developed new kinds of transformers the existence of which was never dreamed of before—all these things are coming *now*, it is not in the distant future, that we are going to find the applications; we have found in the last five years a great quantity of them, and how many more are going to come, no man can tell.

And yet we must not focus our attention too intently upon the utility of a discovery. Did you ever hear the story of what happened when Faraday was making before the Royal Society, in 1831 that experiment to which your Chairman referred. He performed his experiment, and then explained it. It was simple, it did not look particularly interesting. One saw only a deflection of a needle. And some woman in the audience said, "But Prof. Faraday, of what *use* is it?" His reply was, "Madam will you tell me of what use is a new born babe?"—and what a reply it was? Infinite possibilities—possibilities which may indeed not be realized, but at any rate something *altogether new*. Faraday did not care of what *immediate* use that new thing was, for he was one of the great souls who had caught the spirit of Gallileo. He knew that human progress depends primarily upon *the growth of the human mind*, the ability of man to get hold of nature. The utilities might come, they always do come, but they generally crop out as by-products, and the man who has got his mind fixed merely on utilities is simply the man who kills the hen to get the golden egg. I have just as much respect for utilities as you or anybody has, I believe that nothing is worth while except as it contributes in the end to human progress, but the difficulty is you cannot tell, nor can I or anybody else tell what is going to contribute to human progress. The thing that is important is that the human mind should grow. That is the sine qua non of progress. At the Capitol in Harrisburgh there is a picture by Sir Edwin Abbey, which is entitled, "Wisdom, or the Spirit of Science." It consists of a female veiled figure with the forked lightnings in one hand, and in the other the owl and the serpent, the symbols of mystery; and beneath is the inscription,—

"I am what is, what hath been and what shall be.
My veil has been disclosed by none
What I have brought forth is this: The sun is born."

It is to lighten man's understanding, to illuminate his path through life, and not merely to make it easy, that science exists. Hence, if you ask me what are the utilities of the particular category of discoveries which I am going to run over here very rapidly, I may be able to tell you of a good many of them, but I shall not try to catalogue them all, because that is not where our immediate interest lies, It is "where there is no *vision*" That "the people perish."

Finally, before launching upon the sea of recent discovery, I wish to make one more remark about the method of science, namely this: The progress of science is almost never by the process of revolution. You see a great deal in your newspaper headings about revolutionary discoveries. They almost never happen. Thus when the atom was found not to be an ultimate but a divisible thing, there was no revolution, there was not a single law that had to be given up. We had simply opened up a new field, tapped a new lead, found an unexplored region, a sub-atomic region, and all that was above it remained just exactly as it had been, and no chemist had any occasion to be disturbed, for the chemists' laws were just as precise as they had been before. Sometimes we do indeed find that we have generalized too far, and that some law which we had supposed to be of universal application is limited in its scope, but this does not alter the fact that the growth of science is in general by a process of accretion, almost never by that of revolution. Once in a while we have something revolutionary, but not often.

Let me now run over a list of ten discoveries which I will call the ten most important advances of the last twenty years. I will not keep you long upon them, I will just touch upon them, because I could spend the whole evening on any one of them.

We may aptly characterize the physics of the last twenty years as the physics of atomism, and the first discovery on my list of ten advances is the recent verification of the adumbrations of the Greeks regarding the atomic and the kinetic theories—the proof that, as Democritus had imagined 500 B. C., this world does indeed consist, in every part of it, of matter which is in violent motion.

Up to within six years there were not a few distinguished scientists who withheld their allegiance even from these atomic and kinetic theories of matter. The most illustrious of them was Professor Wilhelm Ostwald, but in the preface to a new edition of his *Outlines of Chemistry* he now says frankly:

"I am convinced that we have recently become possessed of experimental evidence of the discrete or grained nature of matter for which the atomic hypothesis sought in vain *for hundreds and thousands of years.* The isolation and counting of gaseous ions on the one hand....and on the other the agreement of the Brownian movements with the kinetic hypothesis....justify the most cautious scientist in now speaking of the experimental proof of the atomic theory of matter. The atomic hypothesis is thus raised to the position of scientifically well-founded theory."

I think you all know what the Brownian Movements are but I wish especially to call attention to the fact that this advance was made not by a practical man, but by a man who never did any experimental work in his life, Einstein, a mathematician, a man who was capable of analyzing a theory and predicting results, and the experimentalists have checked those results. The results consist in predicting how far a given particle that you can see in an ultra microscope will drift in a given time, and our own experiments have checked this prediction to within one-half per cent. It is that sort of evidence that has convinced Ostwald of the correctness of the kinetic and the atomic theories.

The second advance is the proof of the divisibility of the atom, a proof which grew out of the discovery of X-rays. Let me tell you how. If you have here two plates with an electric field between them, and nothing else but a monatomic gas like helium, then it is found that when the field is thrown on the helium is perfectly stagnant, but when a beam of X-rays is shot between the plates some of the molecules become electrically charged and begin to jump, some of them toward the upper plate and some toward the lower plate, where their presence can be detected by an electrical measuring instrument. What does that show? It shows that the thing which we call an atom has electrical charges as its constitutents; and the history of the last twenty years in physics has consisted pretty largely in determining what are the properties of these electrical constituents.

The third is the discovery of radio-activity, which occurred just a little after the discovery of X-rays. And here again we found matter doing things we had never dreamed it was doing viz: shooting off from itself both negatively and positively charged particles, the negatives with a speed which may approach close to the velocity of light, 186,000 miles per second, and positives with a speed of one-tenth of that, or 18,000 miles. The fact that such speeds could be imparted to projectiles of any kind was undreamed of twenty years ago.

The fourth discovery that I wish to mention is the discovery of the atomicity of electricity, the proof that the thing we call electricity is built up out of a definite number of specks of electricity, all exactly alike, and that what we call an electrical current consists simply in the journey along the conductor of these electrical specks, which we may call with perfect justice definite *material* bodies. Now, I can give you in just a word the proof of that statement. There are half a dozen ways in which it could be approached. I will mention the one with which I am most familiar, because it is the particular proof which we worked out at our laboratory.

We took these plates with a field of 10,000 volts between them, with a little hole in the top plate, and we blew an oil spray above the top plate so as to get an electrically charged body just as small as we could, for we expected that the frictional process involved in blowing the spray would charge the drops, which it was found to do. We let one of those drops come into the space between the plates and then moved it up and down by an electrical field, throwing on the field as it came close to the bottom plate, and throwing it off as it approached the upper one, and so we kept that oil drop going up and down between the plates, in the hope that it would capture some of the ions which we knew existed in the air, put there by radium or other agencies. The drop met our fullest expectations as a police officer capturing ions frequently and signalling the fact of each capture to the observer by the change in its speed in the field. For the oil drop is an electrically charged body, and in a given field it moves with a definite speed. If however it captures an ion, its charge increases or decreases, and hence its speed increases or decreases. If the charges on ions are all alike, then we can only get one particular change in speed. If the charge that is already upon it, put there by the frictional process, is built up out of these same units, then the total speed which the field will impart must be an exact multiple of the change in speed which the capture of an ion produces. In other words, if electricity is atomic in structure, you cannot get in a given field anything except a definite number of speeds, which will make an arithmetical series, that is, will come up by steps, one, two, three, etc. That is exactly what we found. We have experimented with thousands of drops and scores of different substances, and they always work exactly that way. Both positively and negatively charged drops are found to act in quite the same way, showing that both positive and negative electrical charges are built up of specks of electricity. Further we can count the number of those specks, which we will call electrons, in a given drop, with the same certainty with which you can count the number of fingers that are before you now. And again since Rowland showed that an electrical current is nothing but a charge in motion, you have here the proof that the electrical current that goes through these lamps, for example, is nothing except the motion of a certain number of electrical specks through or over the filament of the lamp. Add to that J. J. Thomson's discovery made in 1881, that an electrical charge possesses inertia, the only distinguishing property of matter, and you have made it perfectly legitimate to say that an electrical current in a wire is a definite, material, granular something which is moving along that wire.

The fifth great discovery of modern physics is the bringing forward of evidence for the electrical origin of mass. I have just said that electricity is material. Can we turn it around, and say that all matter is electrical in origin? The last is not exactly the same as the first, and it needs evidence. When we have proved that an electrical charge possesses inertia or mass we have not shown that there is no inertia in matter which is not electrical in its origin? Now we have a certain amount of evidence upon this point and I wish to state what that evidence is. We can measure the inertia of the negative electron and it is found to be $\frac{1}{1845}$ of the inertia of a hydrogen atom, but the positive electron is never found with an inertia less than the inertia of a hydrogen atom. Let us consider the inertia of the negative. So long as it is moving slowly compared with the speed of light its inertia remains constant because the shape of its electromagnetic field is not appreciably distorted by its motion. But as soon as you imagine it to be moving with a speed which is close to the speed of light, that is with a speed which is nearly as great as the speed with which its own electromagnetic field can travel forward, then further change in speed will distort the field and hence change the inertia. In other words, the inertia of a charge ought to be a function of speed only when the speed approaches the speed of light. As a matter of fact, when it is from 0.1 up to 0.9 of the speed of light, you can compute just how it ought to vary. Now, by some happy chance the physicist has found negative electrons, namely those shot off by radium, which are going with these speeds, and hence it is possible to test our theory for these particles and see whether the rate of change of their inertia with the speed checks with the theoretical value. It is found that there is such a check. This means that there isn't any inertia in those particles which does not obey the electromagnetic laws. Therefore, we *have good reason for assuming that the negative electron is nothing but a disembodied electrical charge, and that its inertia is wholly of electrical origin.*

Now, with respect to the positive electron, we haven't that evidence as yet, but it is obviously in the interest of simplicity to assume one kind of inertia rather than two. Further, we have a little bit of evidence of this kind, and I wish to mention what it is, because that will furnish an introduction to my sixth important modern discovery. We have good reason to think, at any rate, that there is only one positive electron in the hydrogen atom, but that the mass, or inertia of that positive electron is almost the mass of the hydrogen atom—at any rate we never find it any less. Now if this inertia is all electrical then we know from theory that the charge must be more condensed in the positive than in the negative, consequently, if we are going to make the observed inertia of the hydrogen nucleus all electrical, it must be even a more dense charge, that is a smaller body, than is the negative charge. So, we first get the picture by that kind of a theory of an atom which has an extraordinarily minute single positive nucleus, and negative electrons around the outside. We first got this picture by that kind of a theory, but we don't depend upon that theory now, because we know that the conclusion is correct. We know that the atom does consist of a body with a positive nucleus which is extraordinarily minute, and we can tell just how large it is, if we define the nucleus as the part of the atom that is impenetrable to the alpha rays of radium.

This brings me to the sixth of our discoveries namely *the discovery of the nucleus atom.* Let me give you just a brief statement of how we know that the atom is somewhat like a miniature solar system, with an extraordinarily minute nucleus, the size of the nucleus never being more than 1/100,000 part of the diameter of the atom, with a certain number of subsidiary bodies—negative electrons—which we should liken to the planets, somewhere around the outside. How do we know that is the case? We have this direct evidence. Nature takes a helium atom which is going with a speed of 18,000 miles per second, and nature shoots that atom right through a glass wall without leaving any hole behind, and without in any way interferring with the structure of the molecules of the glass. I can show you photographs that make the thing so clear that the wayfaring man can see it, you don't need to be a physicist. I will do so at the end of the hour, if there is time. This obviously means that the positive nucleus itself must be extraordinarily minute. Indeed the fact that the negative electron actually shoots through those hundreds of thousands of atoms without ever going near enough to any constituent of those atoms to knock any one of them out, and the fact that the positive nucleus of helium, viz; the alpha particle, shoots through even more molecules without being deflected at all from its course, causes one to wonder whether there is anything at all that is impenetrable to the atom. Why do we say there is a nucleus there? Because direct experiment says there is. There is a certain portion of the atom which the alpha particle itself cannot penetrate. If the impact is head on, the alpha particle goes right up to the atom and then it backs straight back again, or if it comes up to the atom at an angle like this it goes off that way. (Illustrating.) It is only rarely that that happens, but Rutherford and Geiger and Marsden counted the percentage of alpha particles which goes straight on, and the percentage which goes off here, and in that manner, by perfectly simple algebraic analysis that any one of you can understand, without any assumption at all except the law of inverse squares, which can hardly be called an assumption, at least so far as the attraction between the positive nucleus and the negative electron is concerned we find how big that nucleus is. By the size of the nucleus I mean the size of that portion of the atom which is impenetrable to the alpha particles. It comes out something like 10^{-13} centimeters. The diameter of the atom is 10^{-8}. Furthermore, by counting how the deflections of the alpha particles are distributed around this sphere, which we can do directly with the aid of zinc sulphide spread over the inside of the sphere we can obtain the number of alpha particles deflected through any given angle, and then with a little analysis of unquestionable correctness, we find how many unit charges, positive electrons, there are in this exceedingly small nucleus, and it comes out approximately one-half of the atomic weight.

Now, I come to another extraordinary discovery which did not merely tell us approximately how many electrons there are in the nucleus but it told us *exactly* how many there are, and the result checked too with the number obtained by the foregoing approximate method. This brings me to the recent discoveries in the field of X-rays, and I will call the seventh of the modern advances the discovery of the nature of X-rays, which was virtually made by Barkla in 1904. For Barkla and others had proved that there are two types of X-rays, first, X-rays which consist in simple ether pulses pushed off from an electron when it changes its speed; and second so-called characteristic X-rays which are formed thus. When the electrons bump into a target they set something in the target into vibration, and this something sends off perfectly definite characteristic X-rays, which are like monochromatic light. So, we have two types of X-rays, pulse X-rays, like white light, and monochromatic X-rays, like monochromatic light, such as mercury gives rise to. That is the seventh of our great modern discoveries and it must be credited chiefly to Barkla.

The eighth I will call the discovery of crystal structure by the study of X-rays, which is due to Laue in Munich, and Bragg, in England. The method is simply this. You know that we analyze light by a grating which consists of a series of equally spaced lines on a reflecting or transmitting surface. With such a device we can split light up into a spectrum but we cannot thus split it up unless the width of the grating space is comparable with the wave length of the light. In the case of X-rays, we had no knowledge of gratings whose grating spaces were anything like as small as the wave length of X-rays, in fact such gratings were unknown until Laue had the bright idea of using the regular arrangement of the atoms in a crystal for a grating to see whether that would not do the work, and it did the work marvelously well. It was found that we could compute the grating space of certain crystals from the density and the atomic weight and then from the observed spectrum find the wave length of X-rays. And now knowing the wave length we can work backward and find the grating-space for other crystals. We are now using this method for finding the positions and the arrangements of the atoms in crystalline bodies. Prof. Bragg in his recent book on X-rays and crystal structure has described this work very beautifully. Thus a whole new field of experimentation has been opened up and is being pursued in a great many laboratories, and with particular success by A. W. Hull at the laboratory of the General Electric Company. There are almost unlimited possibilities for the chemist in the discovery of the exact position of the atoms in any kind of crystal by this method.

But the results of this discovery as of most of the others which I have mentioned are rather insignificant when compared with those of the ninth which I am going to mention, namely the discovery of the relations between the elements, and the extension of our knowledge of the radiations emitted by different substances. This discovery was made by a young Englishman only twenty-six years old, Moseley, who has already, unfortunately, fallen a victim to this juggernaut which is at the present time crushing out the finest scientific brains in the world. Moseley was killed at the age of twenty seven a year after he made his epoch-making discovery, and all the lives and all the interests of the eternally infamous men who made this war are not to be compared in value to the world with a hair of Moseley's head. Yet he had to be sacrificed to save a threatened civilization. A double honor to Moseley.

His discovery was this: He was analyzing the characteristic X-rays which are given off when any kind of a substance is bombarded with cathode rays. The experiment was in my judgment as brilliantly conceived, as carefully and skillfully carried out, and as illuminating in its results, as any which has been done in the last fifty years. What he found was this, that the atoms of all the different substances emit radiations or groups of radiations which are extraordinarily similar, but that these radiations differ as we go from substance to substance in their wave lengths. The whole discovery can be stated in this fashion: If you take the highest frequency emitted by a given atom, and if you lay down on a table a length which is equal to the square root of this frequency, and if on top of that you lay down the square root of the frequency of the atom which has the next lower frequency, and so if you continue to lay down, with one group of ends together, the measured square root frequencies of all the elements that you can study, then what have you got? You find that you have a flight of stairs, with perfectly definite equal treads; that is, the frequencies change by definite steps as you go from element to element. And there are only four vacant treads between the lightest element which Moseley could study, namely aluminum and the heaviest one, namely, lead, thus indicating that there are only four elements in this range which we have not already found. An extremely interesting question is, what is the greatest common divisor of this series of steps, that is, what is the top step? There are two ways to get at it. One is by filling all the spaces up to aluminum with known elements in the order of their weights—we cannot investigate the lighter ones by the X-ray method because their frequencies are too low; at least we have not yet found how to investigate them. Now there are just twelve elements below aluminum, so we may put them all in, starting with hydrogen. That would make hydrogen correspond to the top step. The second way is to find arithmetically the greatest common divisor of the square root frequencies. This gives us a frequency which is within a few per cent of the highest frequency which hydrogen can produce, according to Lyman's measurements in the ultraviolet region of optical radiations. This indicates again that hydrogen is the element corresponding to the first step. All of this seems to mean three things, it means first that the X-rays of hydrogen are just its ordinary visible radiations; second it means that Moseley opened up a whole new field of radiation, beginning with the radiations of hydrogen, and extending up to a frequency $(92)^2$ or 8464 times as high as that given by hydrogen. I have squared 92 because 92 is the number of the step corresponding to uranium, the heaviest known element, and the one having the highest frequency in its characteristic X-rays. Moseley's discovery means in the third place, almost certainly, that the elements are built up one from another by successively adding the nucleus of the hydrogen atom. The probable reason for the change in frequency as the nucleus takes on a stronger and stronger charge is that the electron sending off say the highest characteristic frequency is in a stronger electrical field in the helium atom, for example, than in the hydrogen atom, and so as the charge on the nucleus goes up by successive steps in going from element to element frequencies go up by corresponding steps.

We may then picture with considerable confidence this whole physical world as built up out of one positive and one negative electron. The positive electron is the nucleus of the hydrogen atom. It is very minute in comparison with the negative, but much more massive. When two free positive electrons are tied together we have the helium atom. We don't know why these positives cling together. We can assume, as an hypothesis, that there are four positives in helium which are held together by two negatives, thus leaving but two free positions as experiment indicates is the case. The assumption here is that in the nucleus one negative is capable of holding two positives. This assumption would make the nucleus of any atom contain a number of negatives equal to the atomic number and a number of positions equal to twice the atomic number. So much for a very brief and incomplete sketch of Moseley's contribution to modern physics.

My last of the great discoveries of modern physics is one that I will just touch upon. It is the discovery of quantum relations in photo-electricity, in X-rays, and in optical spectra; but here I am coming to a field which we do not know very much about, which we do not yet understand, and my main motive in introducing it is to convince you that the physicist, in spite of all he knows, or thinks he knows, is a fairly modest fellow, because there are some things he knows he doesn't know, and one at present is the nature of radiation. However, we know some things about it that are new. For example, it is an extraordinarily interesting fact that when light of the X-ray type, or indeed, light of any frequency falls upon say a lithium or sodium surface, or upon almost any surface, it has the property in some way of taking hold of a negative electron in the atoms of that surface and of hurling that electron out with a perfectly definite speed, which we can measure, and which we find to be exactly proportional to the frequency of the light. That is an extraordinary phenomenon, and it is one that we explain on a kind of quantum theory which I will not attempt to go into here, because of the fact that we have not yet worked it out fully, so that I cannot give you anything very definite about it; but at any rate, the quantum constant comes out of the photoelectric effect, as shown in my own work, out of X-ray work as discovered by Duane and Hunt at Harvard, and out of spectroscope work, as shown by Bohr in the beautiful theory of the atom which he has developed within the last three or four years.

I think I have brought you in this brief survey to the very outmost boundaries of our present knowledge. Bring me back ten years from now, and we will know more about these quantum theories; but for the present I will stop, and close this hasty survey of the problems and successes of modern physics with a few reflections which are based upon historical studies.

At the University of Chicago I have a friend by the name of Braested, who is an Egyptologist. Braested tells me that he and his fellow Egyptologists have proof that less than 100 years elapsed from the time when, about five thousand years ago, the Egyptians knew so little about building that the best they could do was to pile crude rows of uncut stone around their dead, to the time when some of the great pyramids were built, structures which represent in some ways the height of the builder's art, structures on which the surfacing is so perfect that huge granite blocks eighteen feet on a side are joined together without cement and with not as much 1/100 of an inch of space anywhere between them. That kind of engineering we do not do now, luckily we do not have to do it, but it is doubtful if we could do it if we would. I am mentioning this to bring out the fact that Egypt, at that time, got the key to a certain kind of development, and pushed that development to a marvelous degree of perfection. Indeed there was in that century, so Professor Braested says, an industrial progress which has never been equalled at any time in the world's history until within the last one hundred years, when the modern industrial revolution set in.

Go now to Greek history, and we find the same sort of a situation. About 500 B. C. the Greeks got the key to a certain type of progress, and they developed a civilization which on the intellectual side, and on the artistic and aesthetic side, has never been equalled. The Greeks, like the Egyptians, got the key to a certain kind of civilization and they worked it out to marvelous perfection; but in neither case did these men or these races go on; they did not open up new fields; they did not tap new mines. Their civilization came to an apex, and then decayed, and the question has often arisen in your minds, as it has in mine, is this age in which we are living going to follow in the same way? Have we risen to a maximum? Have we had a period of marvelous development which is going to be followed by one of decay and stagnation, or are we going to ascend to higher and higher levels? No man can answer that question, but this I know and this you know, that it was wholly unnecessary that Greek civilization, or that Egyptian civilization should stop when it did. If they had developed the modern scientific spirit, the spirit of search for new phenomena and new methods, they could have found them. There were plenty of new mines for them to tap, plenty of unexplored fields to search out. But they did not do it. As for us I feel just as sure as Shakespeare did that "there are still more things in heaven and earth than are dreamed of in our philosophy," and if we stop, it will be because we have forgotten the lesson which Gallileo first tried to teach, and which we have been learning in the last one hundred years, and that is the lesson of research. It is the lesson, the philosophy, the method and the faith of modern physics. That is our hope, and if we keep that, if we don't call in our scouts because the rewards are larger in the applications, then I haven't any doubt that our civilization will go on; and if we do call in our scouts here in this country, then our civilization will give place to that of some other country which does not do so, but which learns the value for the human race of the spirit of modern scientific research.

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INDUSTRIAL CONTROLLERS—WITH PARTICULAR REFERENCE TO THE CONTROL OF DIRECT-CURRENT SHUNT MOTORS

BY H. D. JAMES

ABSTRACT OF PAPER

Stating that present day development of motors and controllers is such that former limitations as to regulation of starting current from the viewpoint of motor, controller and supply system have been largely removed, the author presents a series of tests on 15 and 20-h.p., 230-volt, d-c. shunt motors, started from rest. The motors are of two types, constant and adjustable speed.

These tests were made to determine: first, the load driven by the motor; second, the power supply; third, the motor operation.

From these tests the author concludes, in part; that it is practical with automatic acceleration to use one switch to short-circuit the armature resistor used with motors as large as 15 h. p.; that the shunt field of small adjustable speed motors can be reduced in one step under normal load conditions (this practise can be safely followed up to motors of 50 h. p.); and that adjustable speed motors can use one step resistance for dynamic braking.

THE PRESENT practise in industrial control is the result of accumulated experience extending over the past twenty-five years. The theory consists in analyzing the characteristics of the motor to be controlled, and designing a controller which will furnish the characteristics not incorporated in the motor design. An analysis of the motor, from this standpoint, has brought out a number of features which were not generally appreciated.

The application of the controller requires an analysis of the load to be driven by the motor, as well as the characteristics of the motor. The controller must cause the motor to operate through a cycle which is best adapted to the motor load, or the machinery to which the motor is connected. The engineer must never lose sight of the fact that the purpose in using a motor and controller is to do certain work, and that he should select a type of motor and controller that will perform this work under existing conditions with the least apparatus, and require the minimum of attention.

In the early days of the d-c. motor, great care had to be taken in starting, and in changing the motor speed, to prevent bad sparking or flashing. This led to the practise of using a considerable number of steps in the external armature resistor, to give a small change in amperes per step. The most common form of motor starter, even today, is a face plate, the design lending itself readily to the use of a large number of steps. Furthermore, the characteristics of this starter are such that considerable burning would result if there were a large change in current per step, so that a relatively large number of steps are necessary to protect the starter against excessive wear.

The steady improvement in the design of motors and controllers has removed many of the limitations formerly existing. At one time the permissible starting current was seriously limited by the danger of disturbance to the supply voltage. The increase in the capacity of systems supplying power now permits small and medium size motors to start with twice full-load current, or even more, without disturbing the supply voltage. Many motor applications are made in factories and mills where there is a continual change in the power demand, so that the starting of relatively small motors makes little difference in the total load.

However, there still exists a number of applications where starting current is an important feature, such as elevators and some other apparatus used in public buildings. Printing presses, paper machinery and applications of similar nature, require special controllers to give a slow start to properly manipulate the product.

Direct-current series and compound motors, as large as 20 h.p. are in successful operation, without the use of an external starting resistor. Mr. Hansen, in his paper published elsewhere in this issue, shows oscillograph records illustrating the starting of these motors. Shunt motors as large as 15 h.p. connected to drum controllers, short-circuiting the starting resistor in one step, have been in successful operation for over ten years.

In order to get a better understanding of the conditions when a d-c. shunt motor is started from rest, the writer arranged for a number of tests on standard motors for general purposes, equipped with automatic starters. These tests were made to determine the effect upon:

First, the load driven by the motor.

Second, the power supply.

Third, the motor operation.

No tests were made upon the controller itself, as commercial controllers are now available which are well able to meet all service requirements. A magnetic contactor controller, of modern design, should be able to take care of any starting conditions which can be met by the motor, without causing undue wear upon any part of the controller.

Most d-c. shunt motors, designed for general purposes, start with less than full load, so that the tests were made with different methods of loading.

The controller was adjusted to short-circuit the starting resistor, while the counter e.m.f. of the motor was quite low. The starting current was also kept small in order to exaggerate the starting conditions. The armature voltage at which the resistance was closed can readily be seen by the notch in the counter e.m.f. curve.

This investigation covered only small motors, as these are typical of most of the motors used, and cover a field where very little application engineering work can be done. It is very desirable to have a simple and strong controller for these motors, as they are often used where the operating conditions are bad and expert attention is not given them. The application of larger motors frequently requires the investigation of the load conditions by engineers, who can specify the correct motor and controller, and modify the controller to adapt it to any special requirements, if desirable. Furthermore, large motors usually receive better care.

Apparatus Used in Making Test.—An oscillograph was employed in all these tests. Figs. 2 to 14 for the 20-h.p. motors were taken on a cylinder, giving the total time of approximately two seconds. Figs. 15 to 21 for the 15-h.p. motor were taken on a long film attachment, and the time for these curves was four to six seconds.

In all cases, the motors used were standard 230-volt shunt-wound d-c. motors. The voltage at the supply circuit dropped a little below this value, and varied from 220 to 225 volts.

The load, in all but two cases, consisted of a d-c. generator, slightly larger in size than the motor, belted to the motor. The generator exerted no retarding force, other than that due to friction and inertia, except where noted. This will represent the majority of loads which a motor of this class will be called upon to start. In one case, the field of the generator was excited, and the armature short-circuited through sufficient resistance to develop full motor torque at full speed. This condition more nearly represents the starting of a heavy fan, or centrifugal pump.

The tests on all 20-h.p. motors were made with an automatic controller having a wiring diagram shown in Fig. 1. The switches used on this controller were standard type *C* contactors, of 125-ampere capacity. The first resistance switch was of the series lock-out type, and the second switch of the counter e.m.f. type. Where the resistance was cut out in one step, the first switch was blocked in the open position. With all starting resistance short-circuited the total resistance of the controller and motor from + line to − line is $\frac{25}{100}$ ohms for the constant speed motor and $\frac{34}{100}$ ohms for the adjustable speed motor.

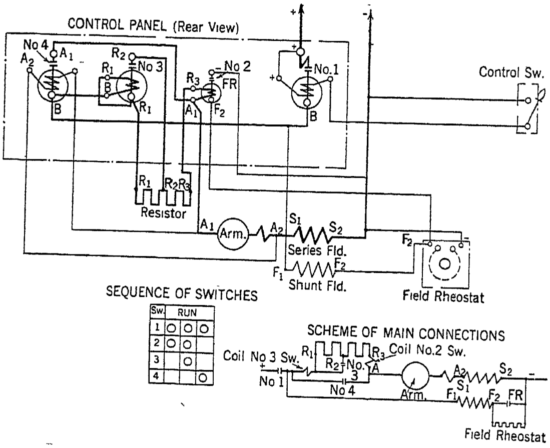


FIG. 1—DIAGRAM OF CONNECTIONS

For the adjustable speed motors, a series relay was used to short-circuit the field resistor during acceleration. This relay was open when the armature resistor was short-circuited, and allowed the field to decrease to its minimum value. When the motor was at rest, the shunt-field strength was either zero or equal to that for the maximum speed condition. The constant speed shunt motor had its field disconnected in the off position, in all of the tests. For the internal resistance of the motors and controller see the curves and diagrams.

Tests on 20-h. p., 750-rev. per min., 230-volt, d-c., Shunt Motor. See Fig. 22. Fig. 2 shows the motor started with 1.35 ohms external armature resistance, which was reduced to 0.35 of an ohm when the counter e.m.f. equalled 120 volts. The second step of resistance was short-circuited when the counter e.m.f. reached 160 volts. This is a very good record of a two-step automatic starter.

Fig. 3 shows the same motor, with one ohm external armature resistance. The entire resistance in this case was short-circuited in one step. The resistance switch was not closed until the counter e.m.f. of the motor equalled 190 volts.

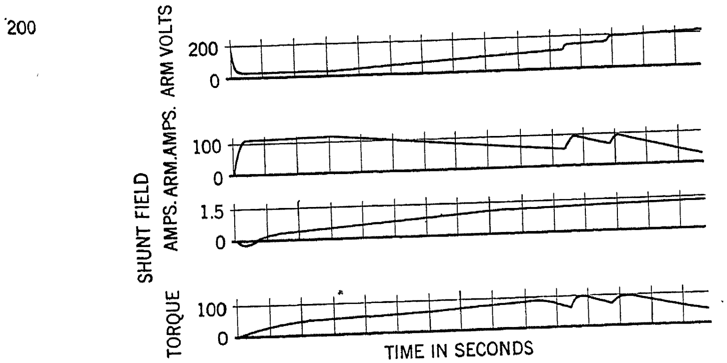


Fig. 2—20-h.p. Motor—750 rev. per min.—Two-step Starting Resistance 1.35 Ohms

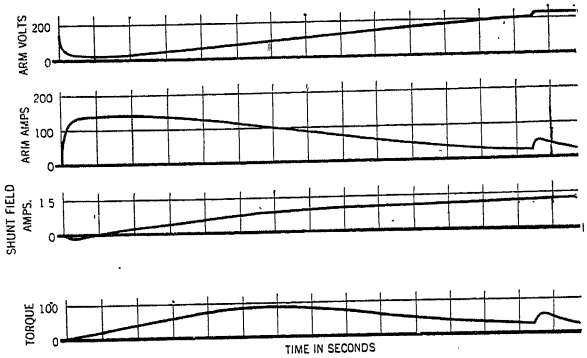


Fig. 3—20-h.p. Motor—750 rev. per min.—One-step Starting Resistance 1.00 Ohm

This represents an adjustment which is better than can usually be obtained in practise.

Fig. 4 shows the motor started with 1.35 ohms external resistance, the resistance being short-circuited in one step, the switch closing when the counter e.m.f. of the motor was 150 volts. This adjustment gives equal current peaks, and represents a practical controller.

Fig. 5 shows a start under the same conditions as Fig. 4, except that the short-circuiting switch closed at 120 volts. This shows a peak at the time of short-circuiting the armature resistance, in excess of the starting current, and represents very bad commercial practise. Figs. 3, 4 and 5 show very clearly the results obtained when the short-circuiting switch closes at different values of counter e.m.f.

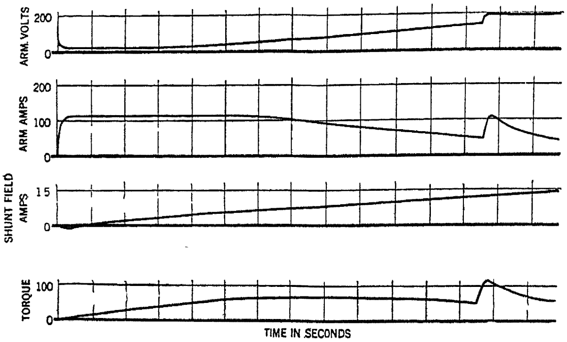


FIG. 4—20-H.P. MOTOR—750 REV. PER MIN.—ONE-STEP STARTING RESISTANCE 1.35 OHMS

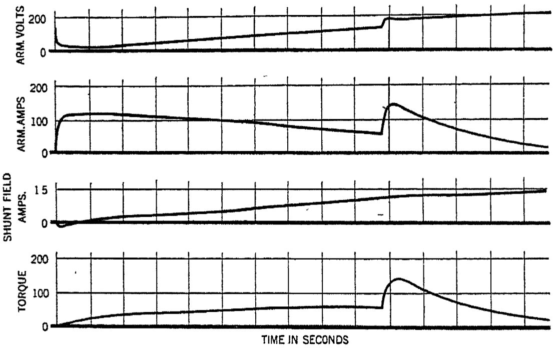


FIG. 5—20-H.P. MOTOR—750 REV. PER MIN.—ONE-STEP STARTING RESISTANCE 1.35 OHMS

Fig. 6 shows a one-step start, with one ohm external armature resistance, the short-circuiting switch closing at about 125 volts, counter e.m.f. This curve has the same adjustment for short-circuiting the armature resistance as Fig. 5, and shows that the current peaks can be made approximately the same by increasing the initial current.

Fig. 7 shows a start with 0.765 of an ohm external armature resistance, with one short-circuiting switch. The load on the motor, in this case, was a prony brake, set to give full-load torque. This represents a very difficult starting condition, as the static friction of the brake is considerably in excess of full load torque. The start, however, was successfully made with one step of resistance.

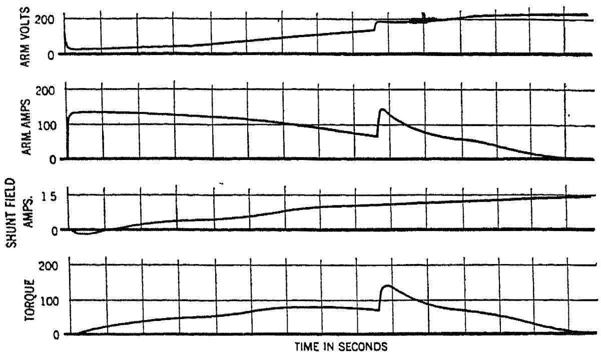


Fig. 6—20-h.p. Motor—750 rev. per min.—One-step Starting Resistance 1.00 Ohm

In the curves referred to, armature volts, armature amperes and shunt field amperes were taken from the oscillograph records. The torque curve was plotted by assuming that the field flux was proportional to the shunt field amperes, except that the negative value of the shunt-field current, at starting,

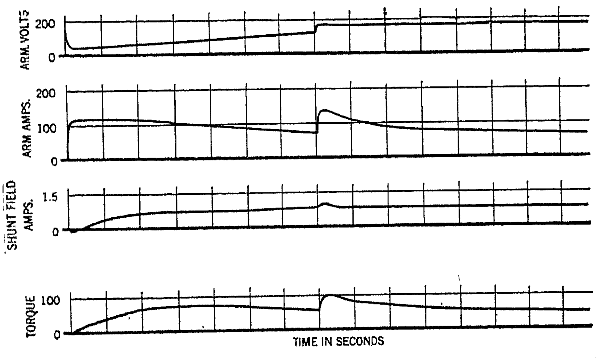


Fig. 7—20-h. p. Motor—700 rev. per min. Full Load—One-step Starting Resistance 0.765 Ohm

was neglected, and the torque curve started at zero value. While these assumptions are not strictly correct, they serve to show the characteristic shape of the torque curve. Since the motor was connected to the line with zero field current, it can be assumed that the initial torque was approximately zero, and independent of the armature current. This torque shows a gradual increase, the armature current remaining constant for a brief period, until sufficient torque is developed to start rotation. Fig. 3 shows how this torque not only increased gradually, but decreased again, up to the point of short-circuiting the armature resistance. At this instant, the torque has a momentary increase, and again tapers off. Some of the curves show a slight change in the field current at this instant, due to the transformer action between the armature current and the field windings. This momentary change in field current was neglected in plotting the torque curve, as it was assumed that the field flux would show an opposite change.

I wish to lay special emphasis upon the characteristic shape of this torque curve, at the time of starting the motor from rest. This curve shows that the motor will start its load gradually, independent of the value of armature current. A heavy armature current, several times the value of full-load current, can be used with safety, providing there is no serious effect on the voltage of the supply circuit, which might be objectionable for other reasons. An easy start of the load is very desirable, as there usually is considerable lost motion in the gearing, or other mechanical connections, between the motor and its load. The sudden application of torque would cause a hammer blow, and might injure the machinery. The sudden increase in torque, at the time the section of the armature resistance is short-circuited, is not so dangerous, as long as a positive torque is maintained, and there is no lost motion in the machinery. Figs. 2 and 4 show that very little improvement can be made by using two switches to short-circuit the starting resistor.

Tests on a 20-*h. p., Adjustable Speed,* 230-*Volt, D-C., Shunt Motor, Having Speed Adjustment of* 500 *to* 1500 *rev. per min.* (*See Figs.* 23 *and* 24) These tests are duplicates of the previous ones, except that, in some cases, the shunt-field current has an initial value equal to the value when the motor is operating at 1500 rev. per min. Also, the field resistance, at the instant of starting the motor, is short-circuited, and again open-circuited at the instant of short-circuiting all of the external resistance.

Fig. 8 shows a typical start, short-circuiting the external armature resistance in two steps, the initial resistance being one ohm, and the resistance of the last step 0.25 of an ohm.

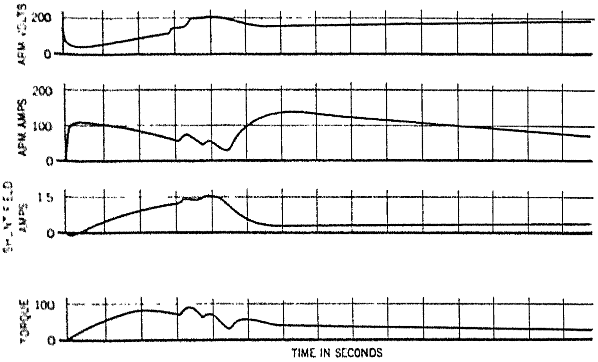


Fig. 8 20-h. p. Motor 500-1500 rev. per min. Full Load—Two-step Starting Resistance 1.00 ohm

The first switch closed at approximately 125 volts and the second switch at 175 volts, counter e.m.f. It is evident, from this curve, that the second step of starting resistance was not necessary.

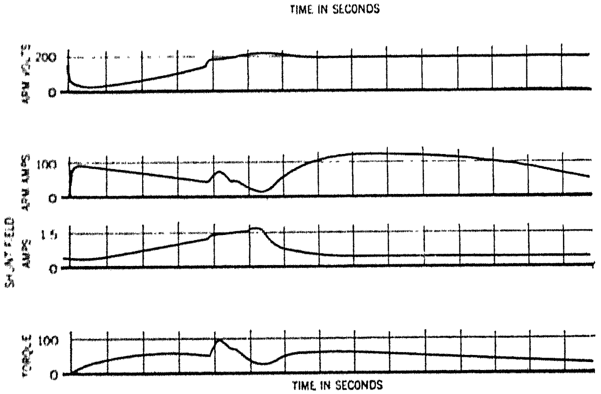


Fig. 9 20-h. p. Motor 500-1500 rev. per min.—Two-step Starting Resistance 1.50 Ohms

Fig. 9 shows a similar two-point starter, having an initial resistance of 1.5 ohms, the second step of resistance being 0.25 of an ohm.

Figs. 10 and 11 show similar starts, with 0.765 ohms external resistance. In these two tests the armature resistance is short-circuited in one step.

Fig. 12 shows a single-step start, with the same amount of

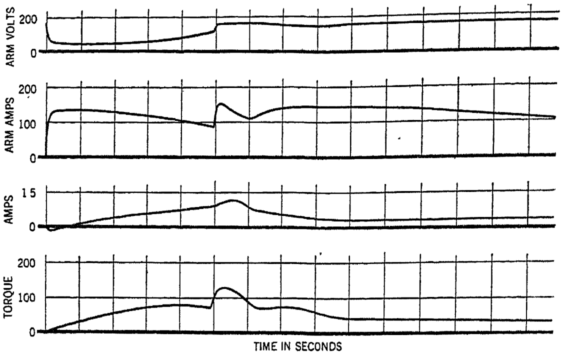


Fig. 10—20-h. p. Motor—500-1500 rev. per min.—One-step Starting Resistance 0.765 Ohms

armature resistance (0.765 ohms), but with the generator loaded, so that it developed full-load torque, at full speed.

Fig. 13 shows a start under similar conditions, except that the motor was loaded with a prony brake, giving full-load torque.

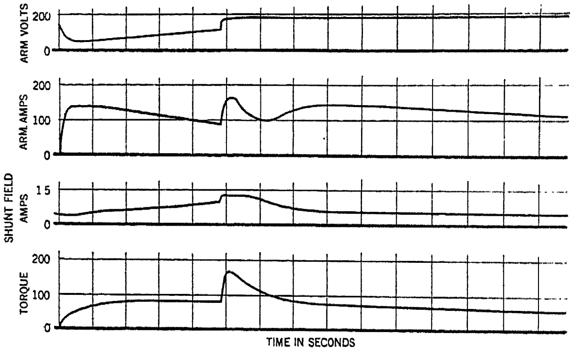


Fig. 11—20-h. p. Motor—500-1500 rev. per min.—One-step Starting—Resistance 0.765 Ohms

Fig. 14 shows a single-step start, having an armature resistance of 1.5 ohms, the short-circuiting switch closing at about 140 volts, counter e.m.f., giving equal starting and short-circuiting current peaks.

These curves differ from the preceding set in characteristic shape, after the armature resistance has been short-circuited. This is due to a gradual decrease in the shunt field strength, with a corresponding increase in the armature current. This change in field strength and current is gradual, and it does not seem necessary to use automatic means for delaying this change in field strength.

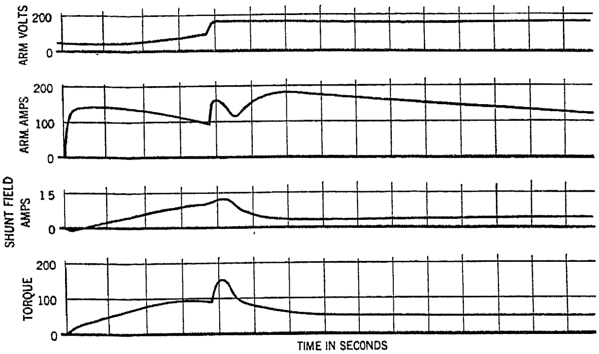


FIG. 12—20-H. P. MOTOR—500-1500 REV. PER MIN. FULL LOAD—ONE-STEP STARTING RESISTANCE 0.76 OHM

Different methods of loading the motor do not show much change in the current peak when the resistance is short-circuited. This is largely dependent upon the value of the counter e.m.f. at the time of short-circuiting the starting resistance. In none of the tests was this peak excessive.

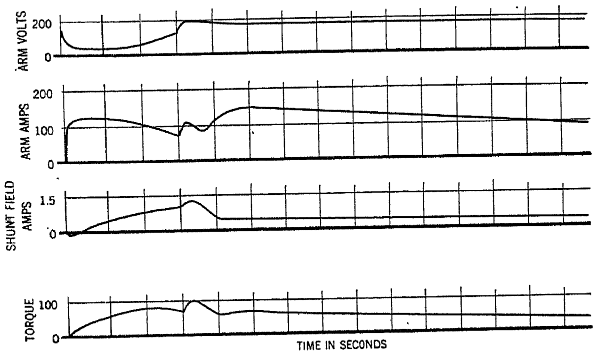


FIG. 13—20-H. P. MOTOR—500-1500 REV. PER MIN. FULL LOAD—ONE-STEP STARTING RESISTANCE 0.765 OHMS

Tests on a 15-*h.p.*, 230-*Volt*, 400 *to* 1600 *rev. per min.*, *D-C.*, *Shunt Motor.* (*See Figs.* 25 *and* 26). A number of tests were made on this motor, using a controller connected as shown in Fig. 27. In these tests the motor was belted to a generator a little larger than the motor. The only load to be overcome was that due to friction and inertia.

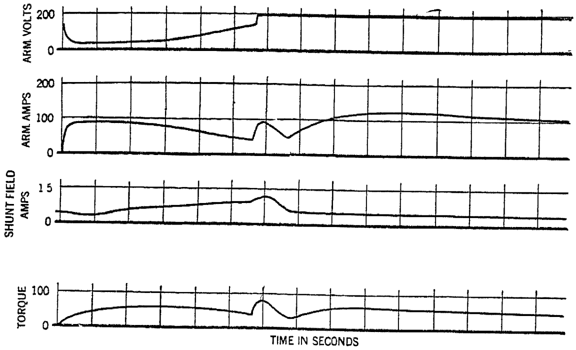


FIG. 14—20-H.P. MOTOR—500-1500 REV. PER MIN.—ONE-STEP STARTING RESISTANCE 1.5 OHMS

The controller differs from that shown in Fig. 1 in the connections to the shunt field relay. In this case a "fluttering" or "vibrating" relay was substituted for the series switch. Provision is made for keeping these relay contacts closed during acceleration, by using more ampere turns on the relay during this period. (See Fig. 27.)

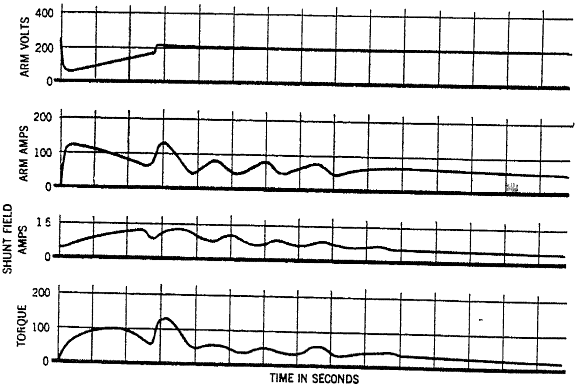


FIG. 15—15-H.P. MOTOR—400-1600 REV. PER MIN.

This fluttering relay is arranged to close its contacts when the current exceeds a fixed value, and to open the contacts when the current is reduced below this value. The difference between the closing and opening values of current is about 10 per cent.

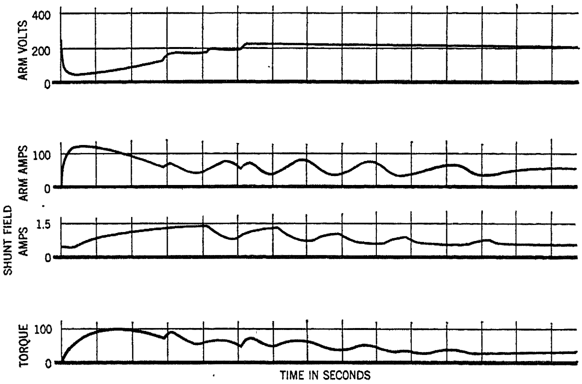


Fig. 16—15-h.p. Motor—400-1600 rev. per min.

The object of the relay is to control the weakening of the shunt field during the period of acceleration from 400 rev. per min. to a higher speed, depending upon the setting of the field rheostat. It operates in a similar manner to a voltage regulator. The relay contacts short-circuit the field rheostat, the rapid cutting in and out of the field rheostat tending to retard the weakening of the shunt field.

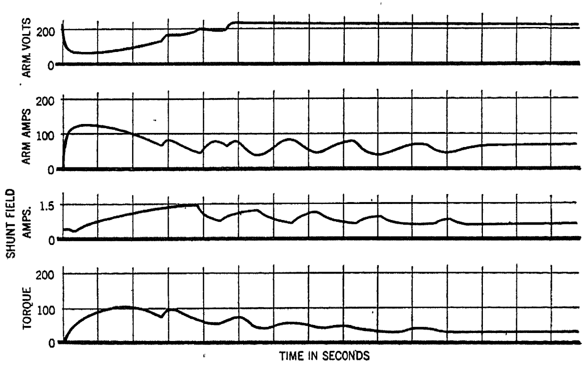


Fig. 17—15-h.p. Motor—400-1600 rev. per min.

Tests shown by Figs. 15, 16 and 17 indicate that the action of such a relay is not very smooth. When this principle is applied to the exciter of a large d-c. generator, the pulsations are damped out to a considerably greater extent. Moreover, the parts of the voltage regulator are lighter and the regulator is much quicker in its operation and much more sensitive to a change in current.

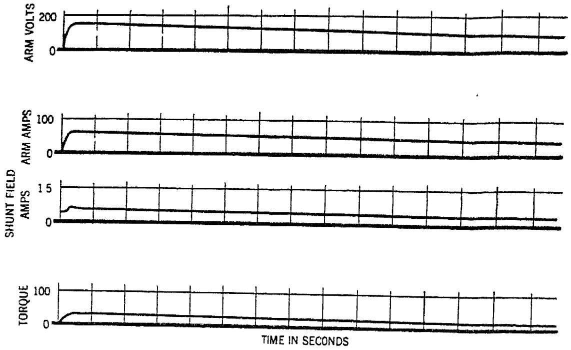


FIG. 18—15-H.P. MOTOR—400-1600 REV. PER MIN.

By comparing these curves with the preceding curves, made on a three-to-one adjustable speed motor, it will be seen that the armature current and torque are considerably more disturbed when the relay is used, than in the cases where this relay is omitted. It would, therefore, seem unwise to add this complication to the average small motor.

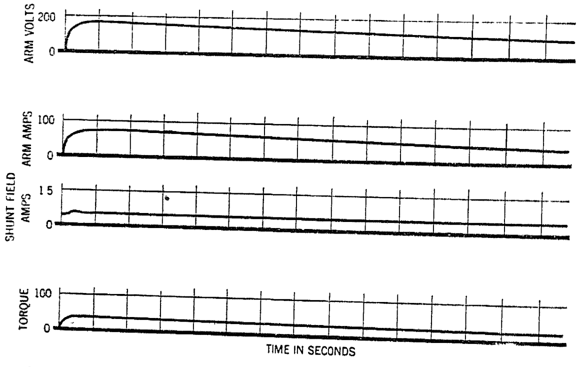


FIG. 19—15-H.P. MOTOR—400-1600 REV. PER MIN.

There is another objection to the use of this relay. If we consider the above motor operating at 1600 rev. per min., and quickly move the field rheostat to reduce the motor speed to

1200 rev. per min., the motor will regenerate and cause a heavy reverse current to flow. This excess current will lift the fluttering relay and short-circuit the field rheostat. This action still further increases the field strength of the motor and causes an increase in the armature current. The action of the relay in this case is the reverse of its operation during acceleration, and has a very undesirable effect.

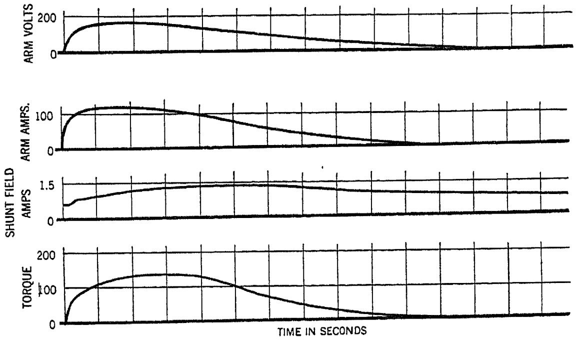


Fig. 20—15-h.p. Motor—400-1600 rev. per min.

In order to correct this difficulty, a second relay known as a "transfer" relay has been used. This relay has a compound winding so arranged that it reverses the connections to the fluttering relay when the motor regenerates. In order to properly protect the motor it is, therefore, necessary to use two relays instead of one. Controllers for paper calenders, printing presses

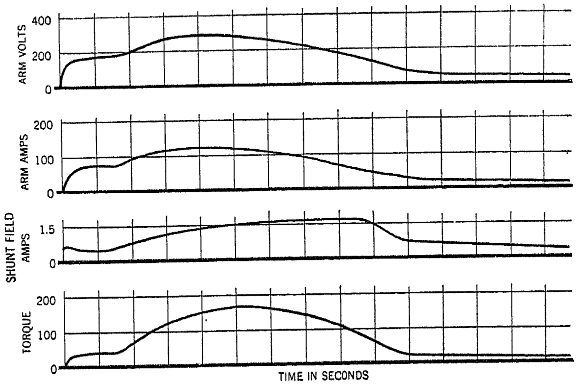


Fig. 21—15-h.p. Motor—400-1600 rev. per min.

and other applications having motor-operated field rheostats, can use the fluttering relay without a transfer relay, as the movement of the field rheostat is slow.

Dynamic Braking. This same 15-h.p. four-to-one adjustable speed motor was arranged for dynamic braking. The connections were such that the field rheostat could be short-circuited during the period of braking. Figs. 18 and 19 show the braking characteristics with the field strength remaining constant and the motor operating at 1600 rev. per min. when the dynamic brake was applied. The drift in this case was considerable,

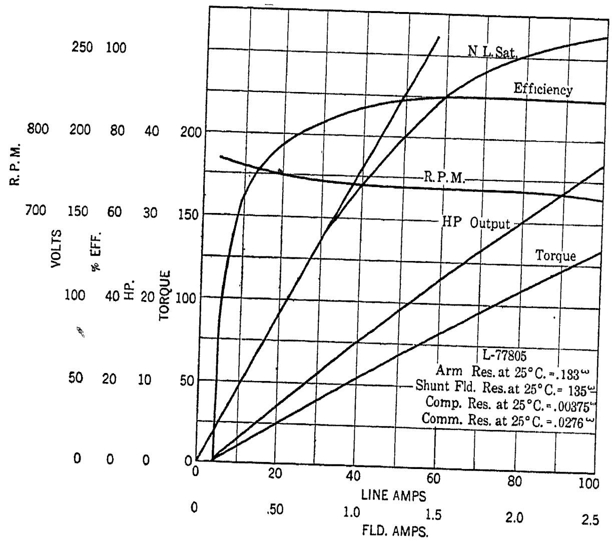


FIG. 22—CONSTANT-SPEED MOTOR—20-H.P.—230 VOLTS—750 REV. PER MIN.

as the motor was belted to a generator and there was very little friction to assist in bringing the load to rest.

Figs. 20 and 21 show dynamic braking with exactly the same conditions as the two previous curves, except the field rheostat was short-circuited during this braking period. The action of the shunt field in this case is very interesting. The increase in the field strength maintained the armature current at a fairly constant value over a considerable part of the retardation, the resultant torque curve showing very well sustained retarding effort. This indicates that the action on an adjustable speed motor gives very satisfactory results without the use of additional steps in the dynamic brake resistance. Under normal operating conditions the friction load would be sufficient to bring the motor quickly to rest from the low speed. In the absence of considerable friction a mechanical brake should be used to obtain a quick stop, as very little dynamic braking can be obtained when the motor reaches low speeds.

Summary of Tests

1. It seems unnecessary, with automatic acceleration, to use more than one switch to short-circuit the armature resistor used with small motors except where special requirements are to be met. I believe it is practicable to use one switch with motors as large as 15 h.p. for general purposes, and operate this switch by counter e.m.f., setting the switch to close at 75 per cent of normal voltage.

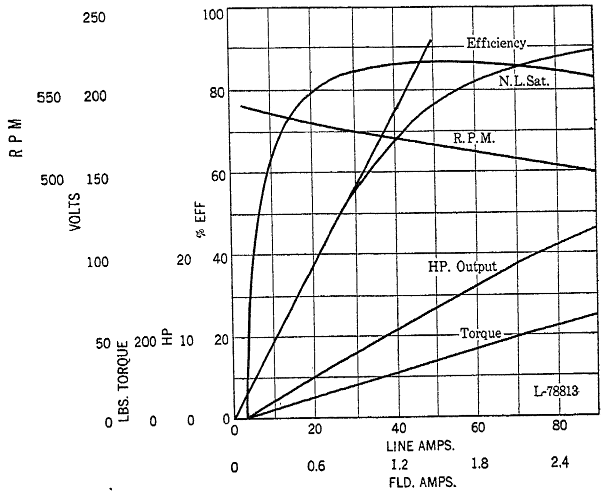


Fig. 23—Low-Speed—20-h.p. Motor—230-Volts—500—1500 rev. per min.

2. If the motor field is zero, or has a small value, when the line switch is closed, the starting torque is also zero or has only a small value and it will increase gradually so that the motor, or its load, will not be subjected to a heavy shock or jar when the lost motion in the drive is taken up.

3. The shunt field of small adjustable speed motors can be reduced in one step under normal load conditions without fear

of undue torque or current. I believe this practise can be safely followed with 50-h.p. motors and perhaps larger. This covers the usual range of sizes for this type of motor.

I believe that most machine tool motors are always started light. Under this condition the motor can be started successfully with minimum field strength and the field relay omitted. This will enable us to use the same controller for constant speed and adjustable speed motors supplying a separately mounted field rheostat for the latter.

4. Adjustable speed motors can use one step of resistance for dynamic braking as the change in field strength tends to maintain the braking current constant over a considerable range of speed.

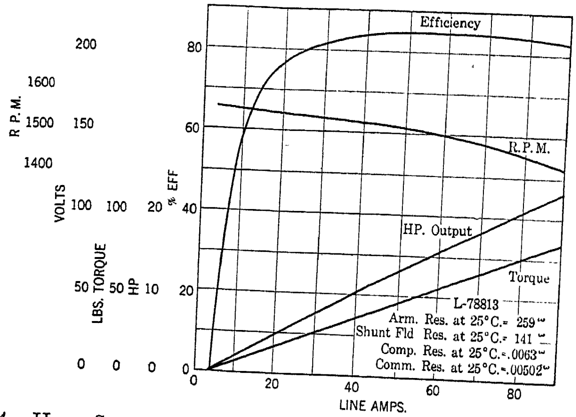


FIG. 24—HIGH-SPEED—20-H.P. MOTOR—230 VOLTS—500-1500 REV. PER MIN.

5. The use of a "vibrating" or "fluttering" relay is bad engineering for hand-operated field rheostats, unless a "transfer" relay is also used to reverse its action when the motor regenerates.

6. The time required to accelerate to 95 per cent of speed is very short. In these tests the time did not exceed three seconds.

NEW PROBLEMS

There are very few problems which present a more fertile field than the control of adjustable speed motors. The foregoing discussion covers only a few of the questions involved. The writer has observed cases where gearing has broken, due to a momentary reverse torque of the motor. It is important to see that all interlocks or relays connected to the shunt field resistance are so arranged that they cannot cause a momentary reverse torque of the motor. After a positive torque has been applied to machinery, and all of the lost motion taken up, it is seldom that any part of the apparatus is injured, even if the driving torque varies over a considerable range, provided this torque is always positive. A slight negative torque allows lost motion to occur, and is apt to break some part of the apparatus, due to the hammering blow effect, when the positive torque is resumed.

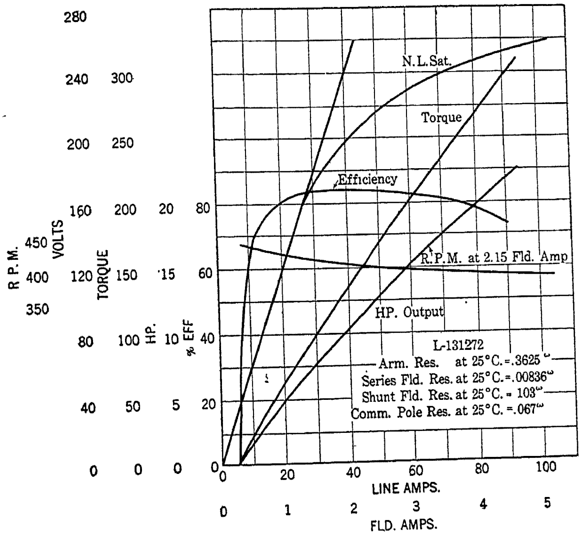


FIG. 25—LOW-SPEED—15-H.P. MOTOR—230 VOLTS—400-1600 REV. PER MIN.

This reverse torque may also be caused by a drop in the line voltage below the counter e.m.f. of the motor, or it may occur due to a rapid movement of the field rheostat.

Trouble may also result from increasing the field strength of a motor when it is disconnected from the line. For example, a 400 to 1600-rev. per min. motor is operating at its maximum speed. If the controller disconnects the motor armature from the line, and at the same time increases the field strength to its maximum value, the voltage across the brushes of the motor will rise to something over three times normal voltage, unless the armature is slowed down very quickly. It can readily be seen that a 500-volt motor, having over 1700 volts across its brushes, is apt to flash across at some part of its insulation and untimately be damaged. This rise of voltage has actually been observed, and care must be taken not to manipulate the shunt field so as to cause it.

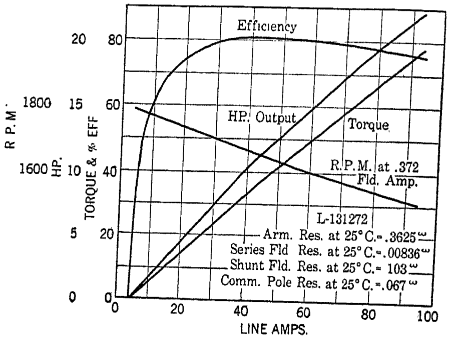
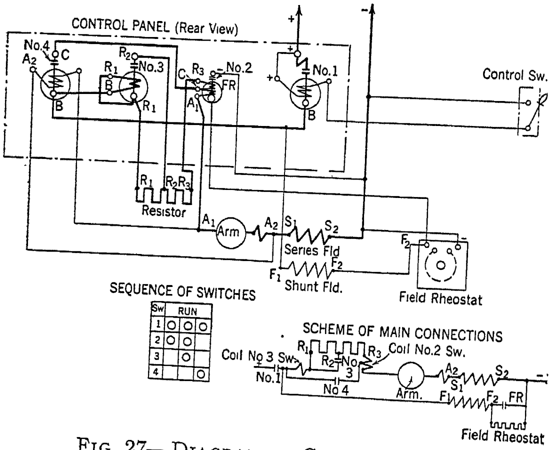


Fig. 26—High-Speed—15-h.p. Motor—230 Volts—400-1600 rev. per min.

The use of dynamic braking introduces a good many interesting problems, both mechanical and electrical. The additional heating effect on the motor is well known. The effect, however, upon the loading and lubrication of bearings, is not so well understood. The resultant change in the field flux of the motor, and the rate of change of current, are other problems.



Sw	RUN		
1	O	O	O
2	O	O	
3		O	
4			O

Fig. 27—Diagram of Connections

The whole field of controller engineering presents many opportunities for the investigator. It has many problems that have not yet been solved, and I believe that engineers should not take the existing practise in controller engineering for granted, but study each application in a thorough manner, and design as simple a controller as possible for the work.

The author wishes to acknowledge the assistance of Professor D. Rowell, of Purdue University, in analyzing some of these curves and in making suggestions for future tests.

Presented at the Fifth Midwinter Convention of the American Institute of Electrical Engineers, New York, February 16, 1917.

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ANALYSIS OF STARTING CHARACTERISTICS OF DIRECT-CURRENT MOTORS

BY K. L. HANSEN

ABSTRACT OF PAPER

In this paper are derived the mathematical expressions for the current, speed, torque and power at any time during the accelerating period when a shunt, series or compound motor is connected to a supply line.

THE determination of the torque required to accelerate a given mass to any desired speed in a given time is a problem frequently met with in engineering practise. It has been shown in text books on mechanics, that the speed of a revolving mass after a time of t seconds is

$$\text{rev. per min.} = \frac{307.5 \times T_a \times t}{W \times G^2} \tag{1}$$

where T_a = accelerating torque in pounds at one foot radius
W = weight in pounds
G = radius of gyration in feet.

Formula (1) holds only when the torque remains constant. In d-c. motors the torque is constant only when the current and flux remain constant. In many applications, especially where a smooth acceleration is required, as for example in starting trains, street cars, elevators, etc., this condition is approximately realized by gradually cutting out the starting resistance. With a sufficiently large resistance and a great number of steps, the current may be limited to a given percentage above normal (as 20 per cent or 25 per cent) and the torque may be considered constant.

There are, however, many applications in which the object is to accelerate as rapidly as the motors will stand, and the starting resistance has only a few steps or is left out altogether. In such cases it is evident that the current and torque do not remain constant, and formula (1) would not hold. When a motor is thrown directly on the line or the starting resistance is low, the starting current may reach a value many times the normal current. It is then not sufficient merely to know the time required to accelerate to a certain speed, but the power developed by the windings must be determined to insure that dangerous overloads are not reached. There are, moreover, many industrial operations where the motor is started and reversed at frequent intervals and the power required for acceleration may be a considerable portion of the total load. In such cases it is necessary to know the r.m.s. value of the current in order to choose a motor with proper capacity and avoid excessive heating.

The problem then is to obtain mathematical expressions for the current, speed, torque and power at any time during the accelerating period when a motor is connected to a supply circuit. Closely connected with this, is the problem of dynamic braking where a revolving mass is brought to a stop by dissipating part of the stored energy in a resistance through which the armature windings have been short-circuited. Another application involving both phases of the problem is the flywheel motor-generator set, in which electrical energy is converted into mechanical energy and partly stored in a flywheel. At peak loads the stored energy is again converted into electrical energy.

Approximations of the speed, torque and current curves during the acceleration may be obtained by assuming the torque constant for a small period of time. The speed and current at the end of this period may then be calculated, and the torque based on this new value of the current again assumed constant for a small period. A sufficient number of points may be obtained by this step by step calculation to draw the curves. This method has the disadvantage, however, that it does not show at a glance the relation which the quantities entering into the problem bear to one another, and the way in which a change in one or more of them will modify the curves. This paper is an attempt to arrive at a more general solution.

SHUNT MOTORS

It has been shown in numerous text books and articles on armature reaction in d-c. machines that there is no demagnetizing effect due to the main armature current when motors are operated on no-load neutral. There is, however, a distortion of the main field under load due to cross-magnetization, and as the teeth and pole tips are usually saturated to some extent, the increase in flux on one side of the pole is less than the reduction on the other. The net result is then a decrease in the total flux per pole under load. In commutating-pole motors where relatively small air gaps are used, it is customary to supply a compensating winding in order to insure stability. It will be assumed that this winding is just about sufficient to compensate the flux reduction due to distortion, and the flux may be considered constant.

When the terminals of a shunt motor are connected with the supply line, the impressed voltage must be balanced by the counter e.m.f. due to resistance and the counter e.m.f. due to rotation. If R be the resistance of all circuits in series, the first of these is $R\,i$. The counter e.m.f. of rotation is expressed by the formula

$$e = \frac{B \times T \times \phi \times K \times \text{rev. per min.}}{6 \times 10^6} \tag{2}$$

where B = number of commutator bars
T = number of turns in series per commutator bar
ϕ = flux per pole in kilolines
K = 2 in multiple wound armatures
= number of poles in two-circuit-wound armatures.

As previously stated, when the torque is constant, the acceleration is constant, and the speed is directly proportional to the accelerating torque and the time. When the torque is not constant the speed is no longer proportional to the time, but the time rate of change in speed is proportional to the accelerating torque and if S be the number of revolutions per minute;

$$\frac{d\,S}{d\,t} = \frac{307.5 \times T_a}{W \times G^2} \tag{3}$$

when starting from standstill the speed at any time t is

$$S = \frac{307.5}{W \times G^2} \int_0^t T_a\,d\,t \tag{4}$$

The accelerating torque T_a is the difference between the torque developed by the motor windings and the counter torque.

The torque developed by the windings expressed in pounds at one foot radius is

$$\text{Torque} = \frac{B \times T \times \phi \times K \times i_a}{8.52 \times 10^5} \qquad (5)$$

Where i_a is the armature current and the remaining symbols have the same meaning as in the counter e.m.f. formula.

The counter torque T_c consists of bearing friction, windage, hysteresis and the friction load. The friction load may be approximately constant or may vary with the speed, according to some law, in which case it can be expressed as a function of the speed. Assume first, the counter torque to be constant; the accelerating torque then is

$$T_a = \frac{B \times T \times \phi \times K \times i_a}{8.52 \times 10^5} - T_c \qquad (6)$$

Substitute formula (6) in speed formula (4)

$$S = \frac{307.5}{W \times G^2} \int_0^t \left(\frac{B \times T \times \phi \times K \times i_a}{8.52 \times 10^5} - T_c \right) d\,t \qquad (7)$$

Substituting formula (7) for rev. per min. in formula (2) for induced volts, we get

$$e = \frac{6 \times (B \times T \times \phi \times K)^2}{10^{11} \times W \times G^2} \int_0^t i_a \, d\,t$$

$$- \int_0^t \frac{5.12 \times B \times T \times \phi \times K}{10^5 \times W \times G^2} T_c \, d\,t \qquad (8)$$

Let $K_1 = \dfrac{6 \times (B \times T \times \phi \times K)^2}{10^{11} \times W \times G^2}$ and $K_2 = \dfrac{5.12 \times B \times \phi \times K \times T}{10^5 \times W \times G^2}$

The e.m.f. equation may then be written:—

$$E = R\,i_a + K_1 \int_0^t i_a \, d\,t - K_2 \int_0^t T_c \, d\,t \qquad (9)$$

Equation (9) is not complete because it does not take into account the counter e.m.f. of self induction and the mutual induction between the shunt and compensating coil. It will be shown later that the effect of these is of so short duration, compared with time to accelerate to full speed, that, except in special cases, they may be omitted.

Differentiating equation (9) with respect to t to free it from integral sign,

$$R \frac{d\,i_a}{d\,t} + K_1\,i_a - K_2\,T_c = 0$$

Dividing by R and transposing,

$$\frac{d\,i_a}{d\,t} + \frac{K_1}{R}\,i_a = \frac{K_2\,T_c}{R}$$

A linear differential equation of first order integrated by the equation

$$i_a = \epsilon^{-\frac{K_1}{R}t} \int \epsilon^{\frac{K_1}{R}t}\,\frac{K_2\,T_c}{R}\,d\,t + A\,\epsilon^{-\frac{K_1}{R}t}$$

where A is a constant of integration.

Integrating and reducing we have

$$i_a = \frac{K_2}{K_1}\,T_c + A\,\epsilon^{-\frac{K_1}{R}t}$$

If $i_a = I_0$ when $t = 0$ then

$$A = I_0 - \frac{K_2}{K_1}\,T_c$$

and

$$i_a = \frac{K_2}{K_1}\,T_c + \left(I_0 - \frac{K_2}{K_1}\,T_c\right)\epsilon^{-\frac{K_1}{R}t}$$

when starting from a standstill $I_0 = \frac{E}{R}$ and therefore

$$i_a = \frac{K_2}{K_1}\,T_c + \left(\frac{E}{R} - \frac{K_2}{K_1}\,T_c\right)\epsilon^{-\frac{K_1}{R}t} \qquad \textbf{(10)}$$

After a theoretically infinite (but practically very short) time the term containing the factor $\epsilon^{-\frac{K_1}{R}t}$ becomes equal to zero and the current becomes $\frac{K_2}{K_1}\,T_c$, which is the current required to develop full-load torque. From the value of the current, the speed and torque and power output can easily be calculated.

If the service is intermittent and the motor is on the line for a period of t_1 seconds and the shut down for t_2 seconds, the r.m.s. or effective value of the current is

$$I_{eff.} = \sqrt{\int_0^{t_1} \frac{i^2\,dt}{t_1 + t_2}}$$

$$= \frac{1}{\sqrt{t_1 + t_2}} \sqrt{\int_0^{t} i^2\,d\,t}$$

$$= \frac{1}{\sqrt{t_1 + t_2}} \sqrt{\int_0^{t_1} \left[\left(\frac{E}{R} - \frac{K_2}{K^1}\,T_c\right) \epsilon^{-\frac{K_1}{R}\,t} + \frac{K_2}{K_1}\,T_c\right]^2 d\,t}$$

$$= \frac{1}{\sqrt{t_1 + t_2}} \sqrt{\frac{R}{2\,K_1}\left(\frac{E}{R} - \frac{K_2}{K_1}\,T_c\right)^2\left(1 - \epsilon^{-\frac{2\,K_1}{R}\,t_1}\right) + \frac{2\,R\,K_2\,T_c}{K_1^2}\left(\frac{E}{R} - \frac{K_2}{K_1}\,T_c\right)\left(1 - \epsilon^{-\frac{K_1}{R}\,t}\right) + \frac{K_2^2\,T_c^{\;2} t_1}{K_1^2}}$$

If t is sufficiently long to let the motor come up to speed, the terms containing the factors $\epsilon^{-\frac{2\,K_1}{R}t_1}$ and $\epsilon^{-\frac{K_1}{R}t}$ become negligible, and we have

$$I_{eff.} = \frac{1}{\sqrt{t_1 + t_2}} \sqrt{\frac{R}{2\,K_1}\left(\frac{E}{R} - \frac{K_2}{K_1}\,T_c\right)^2 + \frac{2\,R\,K_2\,T_c}{K_1^2}\left(\frac{E}{R} - \frac{K_2}{K_1}\,T_c\right) + \frac{K_2^2\,T_c^2\,t_1}{K_1^2}}$$

If the counter torque T_c is neglected, formula (**10**) reduces to

$$i_a = \frac{E}{R}\,\epsilon^{-\frac{K_1}{R}t} \qquad \textbf{(11)}$$

The instantaneous value of the power input is

$$P_i = E\,i_a = \frac{E^2}{R}\,\epsilon^{-\frac{K_1}{R}t} \qquad \textbf{(12)}$$

The total watt-seconds energy to bring the motor up to speed is

$$\int_0^\infty P_i\,d\,t = \frac{E^2}{R}\int_0^\infty \epsilon^{-\frac{K_1}{R}t}\,d\,t$$

$$= \left[-\frac{E^2}{K_1}\,\epsilon^{-\frac{K_1}{R}t}\right]_0^\infty = \frac{E^2}{K_1}$$

The total energy expended in heating the windings is

$$\int_0 R\,i^2\,d\,t = \frac{E^2}{R}\int_0^\infty \epsilon^{-\frac{2K_1}{R}t}\,d\,t$$

$$= \frac{E^2}{2\,K_1}$$

The instantaneous value of the power output is

$$P_o = (E - R\,i)\,i = \frac{E^2}{R}\,\epsilon^{-\frac{K_1}{R}t} - \frac{E^2}{R}\,\epsilon^{-\frac{2K_1}{R}t}$$

and the total energy developed by the motor windings is

$$\int_0^\infty P_o\,d\,t = \frac{E^2}{R}\int_0^\infty \epsilon^{-\frac{K_1}{R}t}\,dt - \frac{E^2}{R}\int_0^\infty \epsilon^{-\frac{2K_1}{R}}\,d\,t$$

$$= \frac{E^2}{K_1} - \frac{E^2}{2\,K_1} = \frac{E^2}{2\,K_1} \qquad \textbf{(13)}$$

which is the energy stored in the revolving parts. If T_c is negligible, the counter e.m.f. at full speed becomes equal to the impressed e.m.f. Substituting the values of E (or e) and K_1 in this formula and multiplying by $\frac{550}{746}$ to convert from watt-seconds to foot-pounds, we have

$$\text{Foot-Pounds} = \frac{1}{5860} (\text{rev. per min.})^2 \times W \times G^2$$

$$= \frac{1}{2 \times 32.2} \times \left(\frac{2\,\pi \times \text{rev. per min.}}{60}\right)^2 \times G^2 \times W \qquad \textbf{(14)}$$

which is the usual expression for the energy stored in a revolving mass. Hence the total energy input is divided in two halves, one half being expended in heating the windings and the other half stored in the rotating mass.

This result can be arrived at directly from physical considerations. Suppose a motor to be running at no load and that the friction losses are negligible. The counter e.m.f. is equal to the impressed e.m.f. E, and if the motor terminals are disconnected from the line and short-circuited upon themselves, the momentary value of the short-circuit current (neglecting self-induction) is $\frac{E}{R}$. The torque developed by the motor windings now becomes the counter or decelerating torque, and if a constant flux is maintained, the current decreases to zero in the same manner as when accelerating from stand still to full speed. The copper losses in the two cases are therefore the same, and in the decelerating case the only way in which the stored energy can be dissipated is in heating the windings. When accelerating from zero to full speed, the energy stored in the moving parts is therefore equal to the energy dissipated in the resistance. While the copper losses in the two cases are the same, from the standpoint of commutation, the condition is much more severe when the windings are short-circuited and the motor brought to a stop. The induced voltage and current are in that case a maximum at the same instant and the maximum power is much greater.

As an illustration, consider a 5-h.p., 230-volt, 525-rev. per min. motor with the following characteristics: $R = 1.8$, $B = 111$, $T = 6$ and $\Phi = 840$.

Constant Torque. Assume the moment of inertia of the armature and load $= W \times G^2 = 75$

Hence,

$$K_1 = \frac{6 \times (111 \times 6 \times 840 \times 4)^2}{10^{11} \times 75} = 4$$

$$K_2 = \frac{5.12 \times 2{,}240{,}000}{10^5 \times 75} = 1.53$$

$$T_c = 50$$

Armature current at full load $\frac{K_2}{K_1} \; T_c = 19.2$ amperes

From (**10**) we get: $i_a = (128 - 19.2)\, \epsilon^{-\frac{4}{1\ 8}t} + 19.2$

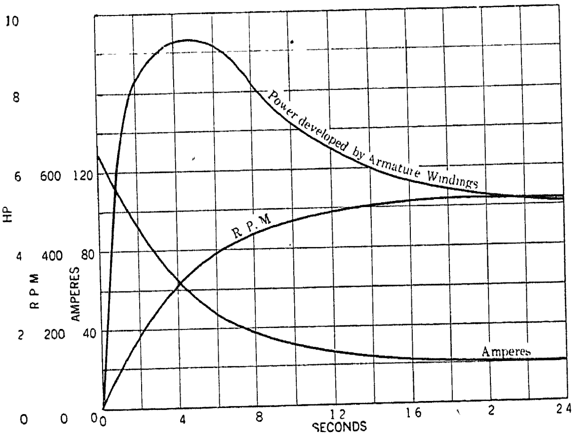

Fig. 1

The current, speed and power curves are shown in Fig. 1 plotted against time in seconds, as abscissa.

Assuming now that a resistance is inserted to limit the starting current to approximately three times the full-load current. This would require a total resistance of four ohms or an external resistance of 2.2 ohms. The curves are shown in Fig. 2. It will be noticed that the maximum power output when the resistance is cut out is almost as great as when no resistance was used. In order to limit the power output the resistance must be cut out in two or more steps.

So far in this discussion it has been assumed that the counter torque was constant. In practise it may be approximately constant or may vary with the speed in a great variety of ways, giving rise to many interesting problems. It may vary periodically becoming zero at regular intervals, and it may become negative or in the same direction as the motor torque through part of the cycle.

Varying Counter Torque. As an illustration of a case in which the torque is some function of the speed assume that the torque varies as the square of the speed. A direct-connected fan is an example of this, because the air pressure and consequently the torque varies as the square of the speed.

The counter torque may then be expressed as the product of a constant and the square of the speed thus:

$$T_c = C\,S^2$$

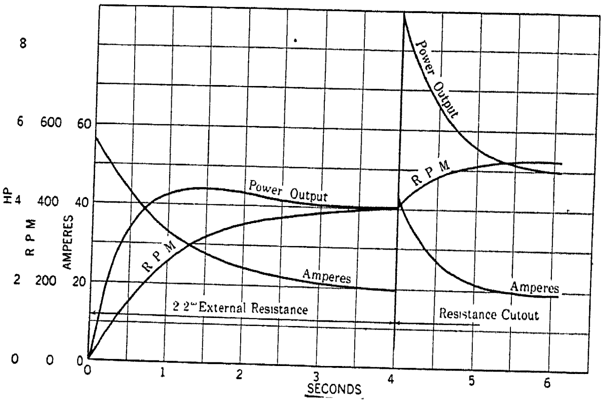

FIG. 2

and the accelerating torque becomes

$$T_a = \frac{B \times T \times \phi \times K \times i_a}{8.52 \times 10^5} - C\,S^2 \qquad \textbf{(15)}$$

Substituting then in formula (**15**) the value of S,

$$S = \frac{e \times 6 \times 10^6}{B \times T \times \phi \times K} = \frac{(E - R\,i_a) \times 6 \times 10^6}{B \times T \times \phi \times K}$$

$$T_a = \left\{ \frac{B \times T \times \phi \times K \times i_a}{8.52 \times 10^5} - C \left[\frac{(E - R\,i_a) \times 6 \times 10^6}{B \times T \times \phi \times K} \right]^2 \right\}$$

As in (**3**) the time-rate of change of speed equals accelerating torque:

$$\frac{d\,s}{d\,t} = \frac{307.5}{W \times G^2} \left\{ \frac{B \times T \times \phi \times K \times i_a}{8.52 \times 10^5} - C\left[\frac{(E - R\,i_a) \times 6 \times 10^6}{B \times T \times \phi \times K}\right]^2 \right\} \tag{16}$$

As before the sum of the induced volts and the $R\,i_a$ drop must be equal to the impressed volts thus:

$$E = R\,i_a + \frac{B \times T \times \phi \times K}{6 \times 10^6}\,S$$

Differentiating with respect to t

$$R\frac{d\,i_a}{d\,t} + \frac{B \times T \times \phi \times K}{6 \times 10^6}\,\frac{d\,s}{d\,t} = 0$$

Substituting the value of $\frac{d\,s}{d\,t}$ from (**16**)

$$R\frac{d\,i_a}{d\,t} + \frac{307.5}{W \times G^2} \times \frac{B \times T \times \phi \times K}{6 \times 10^6} \left\{ \frac{B \times T \times \phi \times K\,i_a}{8.52 \times 10^5} - C\left[\frac{(E - R\,i_a) \times 6 \times 10^6}{B \times T \times \phi \times K}\right]^2 \right\} = 0$$

Multiplying and reducing

$$R\frac{d\,i_a}{d\,t} + \frac{6 \times (B \times T \times \phi \times K)^2}{10^{11} \times W \times G^2}\,i_a - \frac{1.845 \times 10^9}{W \times G^2 \times B \times T \times \phi \times K}\,C\,(E - R\,i_a)^2 = 0$$

now let $K_1 = \frac{6 \times (B \times T \times \phi \times K)^2}{10^{11} \times W \times G^2}$

and $$K_2 = \frac{1.845 \times 10^9 \times C}{B \times T \times \phi \times K \times W \times G^2}$$

$$R \frac{d\, i_a}{d\, t} + K_1\, i_a - K_2\, (E - R\, i_a)^2 = 0$$

$$R \frac{d\, i_a}{d\, t} - K_2\, E^2 + (K_1 + 2\, K_2\, R\, E)\, i_a - K_2\, R^2\, i_a^2 = 0$$

$$\frac{R\, d\, i_a}{-K_2\, E^2 + (K_1 + 2\, K_2\, R\, E)\, i_a - K_2\, R^2\, i_a^2} = -d\, t$$

Let $K_2\, E^2 = a, K_1 + 2\, K_2\, R\, E = 2\, b,\ K_2\, R^2 = c$, and substitute,

$$\frac{R\, d\, i_a}{-a + 2\, b\, i_a - c\, i_a^2} = -\, d\, t$$

Multiplying numerator and denominator by c,

$$\frac{R\, d\, c\, i_a}{-\, a\, c + 2\, b\, c\, i_a - c^2\, i_a^2} = -\, d\, t$$

adding and subtracting b^2 in the denominator,

$$\frac{R\, d\, c\, i_a}{b^2 - a\, c - (c^2\, i_a^2 - 2\, b\, c\, i_a + b^2)} = -\, d\, t$$

$d\, (c\, i_a - b) = d\, c\, i_a\, b$ being a constant, hence

$$\frac{R\, d\, (c\, i_a - b)}{b^2 - a\, c - (c\, i_a - b)^2} = -\, d\, t$$

$$R \int \frac{d\, (c\, i_a - b)}{b^2 - a\, c - (c\, i_a - b)^2} + \text{a constant of integration} = -\, t \quad \mathbf{(17)}$$

The expression under the integral sign is of the form $\frac{d\, x}{p^2 - x}$

which is integrated by $$\int \frac{d\, x}{p^2 - x^2} = \frac{1}{2\, p} \log \left[\frac{p + x}{p - x}\right]$$

Integrating (**17**)

$$\frac{R}{2\sqrt{b^2-a\,c}}\log\left[\frac{\sqrt{b^2-a\,c}+(c\,i_a-b)}{\sqrt{b^2-a\,c}-(c\,i_a-b)}\right]+\text{a constant}=-t$$

Let constant of integration $= -\dfrac{R}{2\sqrt{b^2-a\,c}}\log A$

and we have

$$\frac{R}{2\sqrt{b^2-a\,c}}\left\{\log\left[\frac{\sqrt{b^2-a\,c}+(c\,i_a-b)}{\sqrt{b^2-a\,c}-(c\,i_a-b)}\right]-\log A\right\}=-t$$

$$\frac{\sqrt{b^2-a\,c}+(c\,i_a-b)}{\sqrt{b^2-a\,c}-(c\,i_a-b)}=A\,\epsilon^{-\frac{2\sqrt{b^2-a\,c}}{R}t} \qquad (\mathbf{18})$$

Now when $t=0 \quad i=\dfrac{E}{R}$ and

$$A=\frac{R\sqrt{b^2-a\,c}+c\,E-b\,R}{R\sqrt{b^2-a\,c}-c\,E+b\,R}$$

Solving equation (**18**) for i_a

$$i_a=\frac{A\,\epsilon^{-\frac{2\sqrt{b^2-a\,c}}{R}t}\left(\sqrt{b^2-a\,c}+b\right)-\sqrt{b^2-a\,c}+b}{c\left(1+A\,\epsilon^{-\frac{2\sqrt{b^2-a\,c}}{R}t}\right)} \qquad (\mathbf{19})$$

Using same illustration as before $C\times(525)^2=50$, $C=0.000182$ $K_1=4$, $K_2=0.00199$, $a=105$ $b=2.83$ $c=0.00642$ $\sqrt{b^2-a\,c}=2.705$, $A=0.148$ and substituting in (**19**) and reducing

$$i_a=\frac{0.817\,\epsilon^{-3t}+0.125}{0.00642\,(1+0.148\,\epsilon^{-3t})}$$

The curves are shown in Fig. 3. The current decreases and the speed rises more rapidly than with constant torque, but the difference is not very marked because the counter torque is small compared with the motor torque.

If, as before, we assume an external resistance of 2.2 ohms inserted in the armature circuit, we have

$$R = 4,\ K_1 = 4,\ K_2 = 0.00199,\ a = 105,\ b = 3.85$$
$$c = 0.0318,\ A = 0.25$$

then
$$i_a = \frac{1.82\,\epsilon^{-1.69t} + 0.47}{0.0318\,(1+0.25\,\epsilon^{-1.69t})}$$

The curves are shown in Fig. 4 and it will be noticed that the difference between these and the ones shown in Fig. 2 is more pronounced.

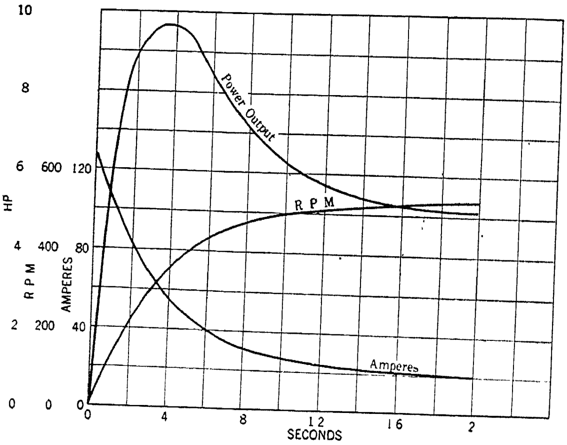

FIG. 3

In general when a motor is thrown directly on the line the difference between the curves taken with and without a friction load is hardly noticeable, the shape of the curves and time required for acceleration being determined almost entirely by the inertia of the mass to be accelerated. As the starting resistance increases, however, the counter torque becomes of increasing importance.

SERIES MOTORS

In series motors the flux is not constant, but must be expressed as a function of the current. As was first shown by Frölich, the flux can approximately be represented by a parabolic curve.

$$\Phi = \frac{\phi\, i}{1 + b\, i}$$

Where ϕ is the flux per ampere at low density. A still closer approximation can be obtained if a constant a is substituted for 1, thus

$$\Phi = \frac{\phi\, i}{a + b\, i}$$

Substituting this value of the flux in formula (5) the torque becomes,

$$\text{Torque} = \frac{B \times T \times K \times \phi \times i^2}{8.52 \times 10^5 \times (a + b\, i)}$$

and if the counter torque be neglected, the speed is

$$\text{Rev. per min.} = \frac{307.5 \times B \times T \times K \times \phi}{8.52 \times 10^5 \times W \times G^2} \int \frac{i^2\, d\, t}{a + b\, i}$$

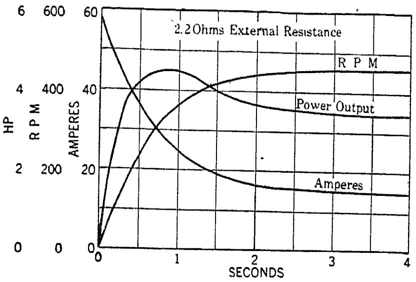

FIG. 4

Substituting these values of flux and speed in formula (2) we have for induced volts

$$e = \frac{6 \times (B \times T \times K \times \phi)^2}{10^{11} \times W \times G^2} \frac{i}{a + b\, i} \int \frac{i^2\, d\, t}{a + b\, i}$$

Let $$\frac{6 \times (B \times T \times K \times \phi)^2}{10^{11} \times W \times G^2} = K_1$$

The e.m.f. equation then becomes

$$R\, i + \frac{K_1\, i}{a + b\, i} \int \frac{i^2\, d\, t}{a + b\, i} = E$$

Dividing by $\left(\frac{i}{a + b\, i}\right)$

$$R\, a + b\, R\, i + K_1 \int \frac{i^2\, d\, t}{a + b\, i} = \frac{a\, E}{i} + b\, E$$

Differentiating with respect to t:

$$b\,R\frac{d\,i}{d\,t} + K_1\frac{i^2}{a + b\,i} = -\frac{a\,E}{i^2}\frac{d\,i}{d\,t}$$

Dividing by $\dfrac{i^2}{a + b\,i}$.

$$\left(\frac{b\;a\;R}{i^2} + \frac{b^2\,R}{i}\right)\frac{d\,i}{d\,t} + K_1 = -\left(\frac{a^2\;E}{i^4} + \frac{b\;a\;E}{i^3}\right)\frac{d\,i}{d\,t}$$

Transposing:

$$\left(\frac{b^2\;R}{i} + \frac{b\;a\;R}{i^2} + \frac{b\;a\;E}{i^3} + \frac{a^2\;E}{i^4}\right)\frac{d\,i}{d\,t} + K_1 = 0$$

$$b^2\,R\int\frac{d\,i}{i} + b\,a\,R\int\frac{d\,i}{i^2} + b\,a\,E\int\frac{d\,i}{i^3} + a^2\,E\int\frac{d\,i}{i^4} + K_1\int d\,t = A$$

$$b^2\,R\log i - \frac{b_a\,R}{i} - \frac{b_a\,E}{2\,i^2} - \frac{a^2\;E}{3\,i^3} + K_1\,t = A$$

$$t = \frac{A + \dfrac{b\;a\;R}{i} + \dfrac{b\;a\,E}{2\,i^2} + \dfrac{a^2\;E}{3\,i^3} - b^2\,R\log i}{K_1} \qquad \textbf{(20)}$$

If log i is taken to the base 10, the last term must be multiplied by $\dfrac{1}{\log_{10}\epsilon} = 2.31$ thus:

$$t = \frac{A + \dfrac{b\;a\;R}{i} + \dfrac{b\;a\;E}{2\,i^2} + \dfrac{a^2\;E}{3\,i^3} - 2.31\,b^2\,R\log_{10} i}{K_1} \qquad \textbf{(21)}$$

The constant of integration A is found from the initial condition when $t = 0$ $i = \frac{E}{R}$

Assume that the shunt and compensating coils have been removed and the motor supplied with a series field of 100 turns per coil. The resistance of the main circuit will be changed to 2.3 ohms, and the following constants are found from the saturation curve:

$$\phi = 56, \; a = 0.62, \; b = 0.041$$

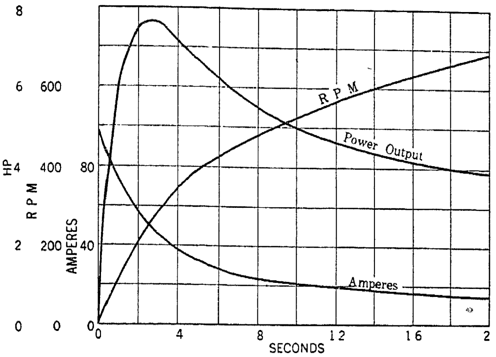

FIG. 5

Hence

$$K_1 = \frac{6 \times (111 \times 6 \times 4 \times 56)^2}{10^{11} \times 75} = 0.0177 \quad A = 0.0169$$

$$t = \frac{0.0169 + \dfrac{0.0584}{i} + \dfrac{2.92}{i^2} + \dfrac{29.5}{i^3} - 0.0089 \log_{10} i}{K_1}$$

The curves are shown in Fig. 5 and it will be noticed that compared with the shunt machine, Fig. 1, the current falls more rapidly at the start, due to the heavy torque, but the approach to the final value is more gradual as the flux decreases with the current. It will be further noticed that the speed rises above normal and, under the assumption made of no counter torque, would continue to rise. This is a condition which of course would not be likely to occur in practise, as a series motor should never be applied where there is a possibility that its load may be entirely removed. If a counter torque, corresponding to the full-load torque had been assumed, the first part of the curves would not be greatly modified, but at about 0.6 or 0.7 of a second, the speed and power curves bend rather sharply, and gradually approach their final values.

The curves also indicate that in case of compound motors with the usual amount of compounding, it is sufficiently accurate to use an average value of the flux and assume it constant over the accelerating period.

COMPOUND MOTORS

Effect of Self and Mutual Induction Considered. It has been stated that the effect of self and mutual induction is usually not sufficient to appreciably modify the curves. There are cases, however, in which the peak value of the current may be reduced considerably by self-induction, and while the mutual induction between the shunt and series coils has practically no effect on the acceleration, it is interesting to note the change in the shunt-field current due to the main current. The discussion of the self and mutual induction is best taken up in connection with the study of the compound motor, because the series field is strong enough to have marked effect on the shunt-field current.

It has already been pointed out that when the flux varies over a small range, an average constant value of the flux can be used, and the counter e.m.f. due to rotation is as before (neglecting counter torque),

$$e = K_1 \int i_a \, d\, t$$

Let L_1 = coefficient of self induction of main circuit, L_2 = coefficient of self induction of shunt field, M = mutual induction between main circuit and shunt field, r_f = resistance of shunt field.

We have for the differential equation of main circuit:

$$L_1 \frac{d\, i_a}{d\, t} + M \frac{d\, i_f}{d\, t} + R\, i_a + K_1 \int i_a \, d\, t = E \qquad \textbf{(22)}$$

and for the differential equation of shunt-field circuit,

$$L_2 \frac{d\, i_f}{d\, t} + M \frac{d\, i_a}{d\, t} + r_f\, i_f = E \qquad \textbf{(23)}$$

Differentiating equation (**22**) with respect to t, to free it from integral sign,

$$L_1 \frac{d^2 i_a}{d t^2} + M \frac{d^2 i_f}{d t^2} + R \frac{d i_a}{d t} + K_1 i_a = 0 \qquad \textbf{(24)}$$

In symbolic notation, equations (**23**) and (**24**) may be written:

$$\begin{aligned} L_1 D^2 i_a + M D^2 i_f + R D i_a + K_1 i_a &= 0 \\ L_2 D i_f + M D i_a + r_f i_f &= E \end{aligned}$$

As the operator D obeys the commutative and distributive laws of algebra, these equations can be treated algebraically thus:

$$\begin{aligned} (L_1 D^2 + R D + K_1) i_a + M D^2 i_f &= 0 \\ M D i_a + (L_2 D + r_f) i_f &= E \end{aligned}$$

eliminating i_f

$$\begin{vmatrix} (L_1 D^2 + R D + K_1) & M D^2 \\ M D & L_2 D + r_f \end{vmatrix} i_a = \begin{vmatrix} 0 & M D^2 \\ E & (L_2 D + r_f) \end{vmatrix}$$

expanding the determinants

$$[(L_1 L_2 - M^2) D^3 + (L_2 R + L_1 r_f) D^2 + (L_2 K_1 + r_f R) D + r_f K_1] i_a = 0$$

Let $L_1 L_2 - M^2 = M_1$, $L_2 R + L_1 r_f = M_2$, $L_2 K_1 + r_f R = M_3$, $r_f K_1 = M_4$ the auxiliary equation becomes

$$M_1 D^3 + M_2 D^2 + M_3 D + M_4 = 0$$

This cubic can be solved algebraically by Cardan's method, but it is usually easier to substitute the numerical values of the coefficients and solve by Horner's method of approximation.

If a_1, a_2, a_3, are the roots of the cubic, the solution of the differential equation is

$$i_a = A_1 \epsilon^{a_1 t} + A_2 \epsilon^{a_2 t} + A_3 \epsilon^{a_3 t} \qquad \textbf{(25)}$$

where A_1 A_2 and A_3 are constants of integration.

The term $M \frac{d i_a}{d t}$ can then be expressed as a function of the time $f(t)$ and if substituted in equation (**23**) it becomes

$$L_2 \frac{d i_f}{d t} + r_f i_f = E - f(t)$$

a linear equation of first degree, the solution of which is

$$i_f = \frac{1}{L_2} \epsilon^{-\frac{rf}{L_2}t} \int \epsilon^{\frac{rf}{L_2}t} (E - f(t) \, dt$$

The self induction of the main circuit of a d-c. machine when the field is excited to a point somewhat above the knee of the saturation curve, is due almost entirely to the distortion of the main field and is nearly constant over a wide range in armature current. It does, however, decrease with the saturation of the main field. With no field excitation the flux set up by the armature m.m.f. can be estimated, and the number of interlinkages of this flux with the armature conductors divided by the square of the number of circuits in parallel is the coefficient of self induction. The field excited by self induction is estimated in the same way except that the saturation must be taken into account. The self induction of the series field is usually very small compared with that of the armature.

When the armature current of a compound motor is suddenly increased, the effect on the magnetic circuit is two-fold. The series field tends to increase the flux and the cross-magnetization of the armature tends to decrease it. If the net result is to increase the flux, and it usually is in a compound motor, an e.m.f. will be set up in the shunt field of such direction as to momentarily reduce the field current or even reverse it. The flux does not increase uniformily, however, but increases rapidly at the start and becomes almost constant for large values of the current. The mutual induction is therefore not constant, but as the total change in flux is comparatively small, it is sufficiently accurate to use an average value and consider it so.

A 7 ½-h.p., 230-volt, 1150-rev. per min. motor has the following constants:

$$L_1 = 0.012, \; M = 0.13, \; L_2 = 40, \; R = 0.58, \; r_f = 410, \; \Phi = 765, \; B = 99, \; T = 4$$

Let moment of inertia $= W \times G^2 = 130$ ft-lb.

We then have

$$L_1 L_2 - M^2 = 0.463, \; L_2 R + L_1 r_f = 28.12, \; L_2 K_1 + r_f R = 265.4 \; r_f K_1 = 282$$

and the auxiliary equation is

$$0.463 \, D^3 + 28.12 \, D^2 + 265.4 \, D + 282 = 0$$

The roots of this equation are

$$-1.22 \qquad -11.5 \qquad -47.8$$

and

$$i_a = A_1\,\epsilon^{-1.22\,t} + A_2\,\epsilon^{-11.5\,t} + A_3\,\epsilon^{-47.8\,t}$$

$$M\frac{d\,i_a}{d\,t} = -\,0.158\,A_1\,\epsilon^{-1.22\,t} - 1.49\,A_2\,\epsilon^{-11.5\,t} - 6.2\,A_3\,\epsilon^{-47.8\,t}$$

substituting in equation (**23**)

$$L_2\frac{d\,i_f}{d\,t} + r_f\,i_f = E + 0.158\,A_1\,\epsilon^{-1.22\,t} + 1.49\,A_2\,\epsilon^{-11.5\,t} + 6.2\,A_3\,\epsilon^{-47.8\,t}$$

Solving

$$i_f = \epsilon^{-\frac{r_f}{L_2}t}\int\epsilon^{\frac{r_f}{L_2}t}\left[\frac{E}{L_2} + 0.00395\,A_1\,\epsilon^{-1.22\,t} + 0.0372\,A_2\,\epsilon^{-11.5\,t} + 0.155\,A_3\,\epsilon^{-47.8\,t}\right]d\,t$$

Integrating and multiplying out

$$i_f = \frac{E}{r_f} + 0.000435\,A_1\,\epsilon^{-1.22\,t} - 0.031\,A_2\,\epsilon^{-11.5\,t} - 0.00413\,A_3\,\epsilon^{-47.8\,t}$$

The e.m.f. due to rotation is

$$e = K_1\int i_a\,d\,t = E - \left(L_1\frac{d\,i_a}{d\,t} + M\frac{d\,i_f}{d\,t} + R\,i_a\right)$$

Substituting the value of i_a and i_f

$$e = 230 - 0.5663\,A_1\,\epsilon^{-1.22\,t} - 0.4877\,A_2\,\epsilon^{-11.5\,t} - 0.0257\,A_3\,\epsilon^{-47.8\,t}$$

Now when $t = 0 \quad i_a = 0 \quad i_f = \dfrac{E}{r_f} \quad e = 0$

and we have for equations of integration constants

$$A_1 + A_2 + A_3 = 0$$
$$0.000435\,A_1 - 0.031\,A_2 - 0.00413\,A_3 = 0$$
$$0.5663\,A_1 + 0.4877\,A_2 + 0.0257\,A_3 = 230$$

solving these equations

$$A_1 = 372,\ A_2 = 63,\ A_3 = -435$$

Hence

$$i_a = 372\,\epsilon^{-1.22t} + 63\,\epsilon^{-11.3t} - 435\,\epsilon^{-47.8t}$$

$$i_f = 0.56 + 0.162\,\epsilon^{-.1.22t} - 1.95\,\epsilon^{-11.5t} + 1.8\,\epsilon^{-47.8t}$$

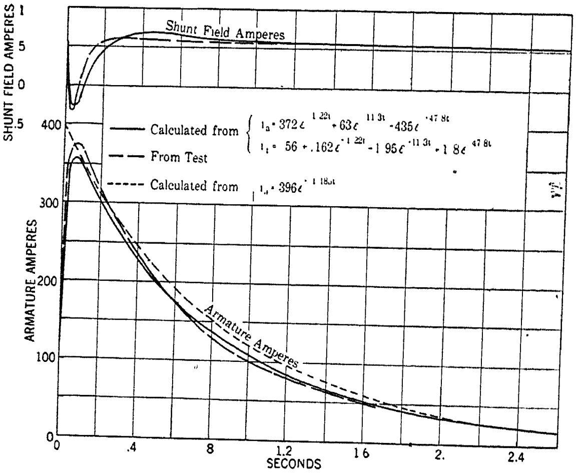

FIG. 6

The curves are shown in Fig. 6 and it will be noticed that the discrepancy between the calculated and tested shunt amperes is rather marked, due to the fact the mutual induction is not constant as was assumed.

In Fig. 7 are shown the calculated and tested curves with the flywheel effect reduced to approximately 35 and 6 ft-lb. With a starting resistance of one ohm being inserted in series with the armature, formula (1) becomes:

$$i_a = 146\,\epsilon^{-.435t}$$

The curves are shown in Fig. 8, the broken line being taken from the oscillogram.

In many applications of self-starting motors the shunt field is not excited before the armature circuit is closed, but the two are connected to the supply line simultaneously. It is evident that in all such cases the series field must be depended on to supply the accelerating torque at the start, because not only does it take the shunt-field current an appreciable time to reach its full value due to its high self-induction, but at the instant the switch is closed it rises in the opposite direction on account of transformer action of series field. An exact mathematical solution is complicated, but a composite curve can be made up considering the motor as series wound at the start and compound when approaching the final condition. The oscillograms shown in Fig. 9 were taken with the armature and shunt-field circuits closed simultaneously.

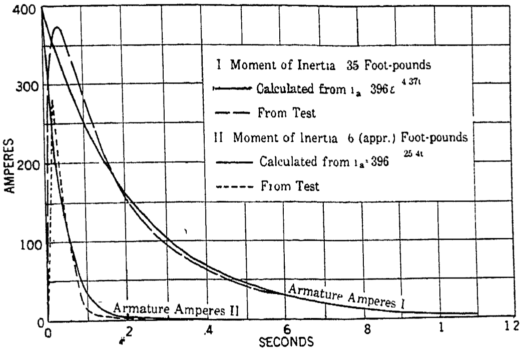

FIG. 7

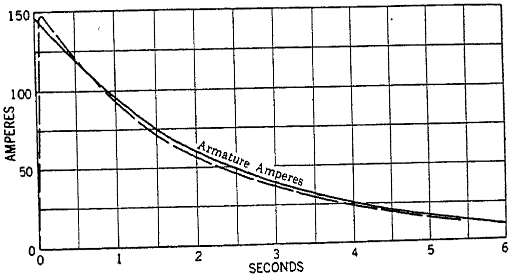

FIG. 8

A striking resemblance between these equations and those of the condenser charge is apparent, and such was indeed to be expected. In the condenser charge the potential at the condenser terminals is proportional to $\int i\,d\,t$, and the stored energy to e^2, where e is the final value of the terminal voltage. With constant flux the counter e.m.f. of a rotating motor armature is proportional to $\int i\,d\,t$ and the energy stored in the revolving parts is proportional to the square of the speed, and therefore to the square of the voltage.

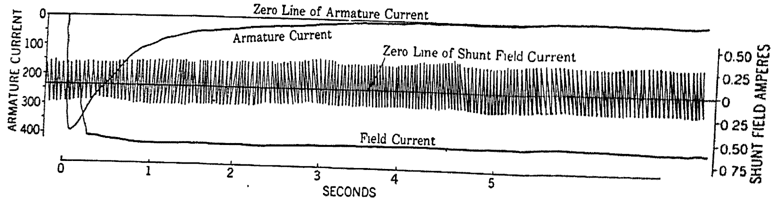

FIG. 9—STARTING TESTS ON 7½-H. P.—230-VOLT—1150-REV. PER MIN.—COMPOUND MOTOR

Shunt field—self excited—no-load condition—130-lb. flywheel effect.

The coefficients of the auxiliary cubic are in general positive and it has therefore no real positive root, having no changes of sign. It has always one root of sign opposite that of the last term that is negative. The remaining two may be negative or imaginary. In case of a pair of imaginary roots, two of the terms would appear in the form

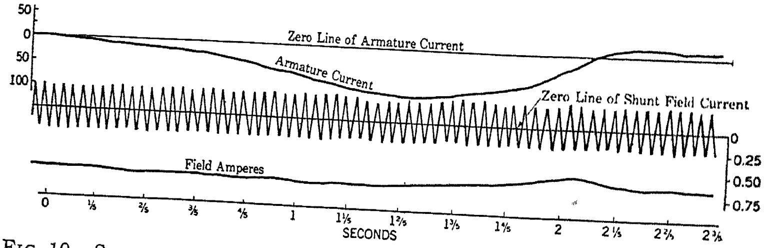

FIG. 10—STARTING TESTS ON 7½-H. P.—230-VOLT—1150-REV. PER MIN.—COMPOUND MOTOR

Shunt field excited separate 230-volt source—no series field in no-load condition—35-lb. flywheel effect.

$$A_1\,\epsilon^{-a+j\beta} + A_2\,\epsilon^{-a-j\beta}$$

From the relations existing between the exponential and trigonometric functions these terms can be written

$$\epsilon^{-a}\,(B_1 \cos\beta + B_2 \sin\beta)$$

That is, speed and current will oscillate slightly about their normal values, but will quickly settle if a constant flux is maintained. If, however, the compensating field is not strong enough to keep the flux from decreasing with load, and the resistance drop not sufficient to offset the decrease in flux, the motor has a rising speed curve. With the moment of inertia exceeding a certain value under these conditions, the current and speed will continue to oscillate and the operation is unstable. The oscillograms in Fig. 10 were taken without the series field in the circuit and the unstable condition is plainly shown. The current rises to approximately 100 amperes then decreases and reverses reaching approximately 25 amperes in the opposite direction, repeating the cycle about every 3 seconds. The speed rises to approximately 20 per cent above normal.

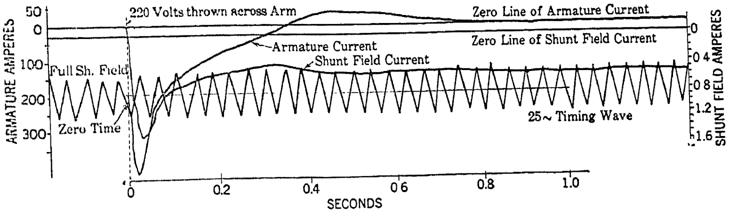

Fig. 11—Starting Tests on 7½-h. p.—230-Volt—1150-rev. per min.—Compound Motor

No-load condition—pulley removed—series field removed—shunt field excited to 230 volts separate source—220 volts applied to armature.

The oscillograms in Fig. 11 were also taken with the series field left out. In this case the flux decreases with increasing armature current and the direction of the voltage generated in the field winding is such as to increase the field current instead of decreasing it. It will be noticed that the field current rose momentarily to almost three times its normal value. It is interesting to note the effect of this on the armature current and the acceleration. The armature current decreases rapidly until the shunt field has fallen to its normal value when the weakening of the field is shown by the sharp bend in the armature current curve.

DISCUSSION ON "INDUSTRIAL CONTROLLERS—WITH PARTICULAR REFERENCE TO THE CONTROL OF DIRECT-CURRENT SHUNT MOTORS"(JAMES), "ANALYSIS OF STARTING CHARACTERISTICS OF DIRECT-CURRENT MOTORS" (HANSEN), NEW YORK, FEBRUARY 16, 1917.

C. T. Evans: You will note, as has been brought out in the paper, that on curves showing the starting with one or two steps of resistance, in nearly every case there is an interval at the beginning where the motor remains at rest, during which time the torque is building up to a value sufficient to overcome the static friction. The armature current during this time apparently increases to a final steady value, which we would naturally suppose could be calculated by Ohm's law, that is, the line voltage divided by the total resistance in the circuit. These curves in reproduction are rather too small to read accurately, but measurements made as closely as possible indicate all through that the currents obtained by means of the oscillograph are considerably less than would be expected from the line voltage and resistance in the circuit.

The explanation for this phenomenon is not self evident without some further experimental data. There may be some excess line resistance included in the circuit between the generator and motor. There is also the transformer action between the shunt field and the compensating and other series windings, for the shunt-field current is rising during this interval, which will have a reverse effect on the armature current. However, it is rather hard to say just how much effect this would have, not knowing the constants of the motor. Also, there is a slight possibility of low voltage. Mr. James states that the voltage varied between 220 and 225 or 230.

These curves then must have been made under some such conditions as these; low voltage, extra high line resistance or on a motor showing a transformer action between the windings. If any of these phenomena were not present, that is, if we had high line voltage, or low line drop, or a plain shunt motor, we are going to have higher armature peaks than are shown on these curves.

Mr. James applies these results to controllers which he calls shelf goods, or controllers for general purposes. In controllers for general purposes, we have to deal with motors of all types. It is not true that all motors are going to have exactly the characteristics of these particular motors which he has tested, although it is indeed true that the general trend of motor design in the past few years has been towards motors of such characteristics.

It has also been assumed that in general motors are started light, that is, without load. While that is true in many cases, you will find that in almost every motor control installation there are times when the motor will be called upon to start under load. Also you will find many installations where they are always called upon to start under load, as for instance, reciprocating pumps, and the like. The principal point to remember is that if we have a counter e.m.f. switch, or any type of series accelerating switch, set so as not to cut out the resistance until the counter e.m.f. has reached 75 per cent of line voltage, we must presume either a light load or high initial starting current. If such a controller, which has been adjusted for light load, is thereafter required to start a motor at full torque, it will require some readjustment of the switch in order to accomplish it, and the starting peak will be consequently much higher than those shown in the curves.

The penalty of using a single step of starting resistance is therefore higher starting peaks, in many instances even higher than those indicated in the curves. This at once suggests the necessity for time element overloads in many instances, and in some cases we may expect serious line disturbances and other troubles.

Perhaps some reduction should be made in the number of accelerating switches now used, but I don't believe that we should go so far as to use a single accelerating switch for motors as large as 15 or 20 h.p.

R. Hellmund: It has been brought out in the papers by Mr. James and Mr. Hansen that it is perfectly feasible to start certain d-c. motors with a single step in the starting resistance and that series and compound motors may even be started without any resistance by throwing them directly on the line. With very small low-voltage motors this latter procedure can usually be followed without danger with most any up-to-date motor. With motors for higher voltages, that is, for instance, motors for 500, 550, or higher voltages, and ratings of 10 h.p. and above, the same procedure is liable to lead to harmful motor flashing unless special consideration is given to this method of starting in the design of the motor. In view of the simplicity of the starting method, it is, of course, well worth some effort on the part of the motor designer to make its use possible, and it may, therefore, be in order to mention here briefly some of the principal factors influencing the flashing characteristics.

In order to obtain a flash, or in other words to establish an electric arc, it is in most cases necessary, first, to have arcing at the brushes, and second, to have sufficient voltage between commutator segments to carry this arc partly or entirely around the commutator. The establishment of an appreciable arc at the brush is largely eliminated with up-to-date interpole motors under normal running conditions. There are, however, several causes for establishing such an arc during transient conditions existing after the motor has been connected to the line. The eddy currents in the frame, solid interpoles, coil shields and other parts frequently prevent the quick establishment of the interpole flux or sudden changes in said flux so that for a short space of time the interpole cannot be depended on to fill its function properly. Furthermore, it has to be considered that sudden changes in the main flux induce a sparking voltage in the coils under the brushes by transformer action and that the interpoles are not designed to take care of such sparking voltages. The voltage between segments required for carrying the spark over is governed by a number of factors during transient conditions following changes in the motor connections. Part of this voltage is induced by rotation and governed by the strength of the air-gap flux. The maximum air-gap densities, in turn, are largely influenced by the amount of armature distortion. This distortion is usually much larger after control changes, because the armature cross field goes usually through laminated iron and is not influenced by any damping effects of the solid frame and shunt-field windings. This permits it to build up much quicker than the main field which is under the damping influence of solid core portions and the shunt field. The current peaks existing after throwing the motor on the line, as shown in the two papers, further tend to increase the distortion so that very high local densities, and voltage between segments, are possible. Other factors which temporarily increase the voltage between segments is the self-inductive voltage of the armature coils which is induced during the establishment of the current peaks and the voltage induced by sudden changes of the main flux in the armature coils near the neutral. These various factors are of different importance with the various types of motors and under different speed conditions, and are often materially affected by apparently small changes in the motor design.

In starting a series motor, for instance, from standstill with small resistance in its circuit, the field will follow the current increase quickly and induces by transformer action high voltages in the coils under the brushes and all coils near the neutral so that very severe spitting and flashing part of the way around the commutator may be obtained. The only factors reducing the rate of change of the main field are damping effects in the frame and the self induction. This condition can, therefore, be made worse by introducing a neutralizing winding which tends to reduce the self inductance of the machine; in other words, the neutralizing winding which is usually introduced for the purpose of preventing flashing actually increases flashing. It is also evident that a laminated field structure may increase the flashing tendencies in this case, while a small resistance in parallel with the fields may reduce it. A change of the material in the coil shields may make a motor that flashes, operate satisfactorily.

With shunt or compound motors, the starting from zero speed is usually less difficult because the shunt field prevents, on account of its damping effect, sudden changes of the main field. These motors, especially shunt motors, are, however, very much subject to flashing if they are suddenly thrown on the line while they are still rotating. The shunt field will then prevent the main field from building up quickly while the small self-induction of the armature permits current peaks and large cross fields, which are quickly established. The high voltage induced by rotation in the strong portions of the cross field, together with the self-induction voltage of the armature coils may cause voltages between segments sufficient to cause flashing. For this reason the control of shunt motors should nearly always be arranged to open the armature resistance switch before reclosing the line switch after a power interruption. In compound motors this flashing can largely be eliminated by the proper relation between the shunt and series field and by increasing the ratio of resistance and self inductance in the shunt fields. A large number of commutator segments, proper dimensioning of the field poles and a great many other design factors enter into this problem. A careful study of all these features for the purpose of extending the use of simple control arrangements to motors of higher voltages and larger rating seems well worth while, and while the two papers do not deal directly with this problem they give material and formulas which can be used to good advantage as a basis for investigations along these lines.

R. G. Widdows: Mr. James has presented a limited number of test curves obtained on one constant-speed and two adjustable-speed motors, and from these has made some very broad and sweeping deductions which to me do not seem justified. The constant-speed motor is connected to a load of such low inertia that full speed is reached in about three seconds from the time of starting. The normal speed of this motor is somewhat below the average practise for a motor of this horse-power size.

We all know that in closing a circuit through a motor, maximum current will not be reached instantly due to the inductance of the motor windings and if the motor speed is increasing at the same time that the current is rising, the maximum current peak will be lower than if the motor was stalled, also if the motor is accelerating rapidly, the peaks of current caused by short-circuiting the starting resistance will be less than if the motor is accelerating slowly.

The peaks of current due to the initial closing of the circuit and to stepping out the resistance will, therefore, be greater in the case of a motor driving a high inertia load than in the case of a motor driving a load of relatively low inertia.

The author states that most d-c. shunt motors designed for general purposes start with less than full load. This is, perhaps, true and while automatic controllers are rapidly being adopted as standard by progressive manufacturers for all d-c. motors regardless of the load conditions. The fact remains that at the present time an automatic controller is almost certain to be selected for a motor having a high inertia load, while a somewhat smaller proportion of automatic controllers are used with motors having loads that can be quickly accelerated.

Reducing the number of accelerating points on an automatic controller might appeal to the controller manufacturer as allowing him to cheapen his product, and while such a starter may be satisfactory in many cases, there will be a very large number of cases where dissatisfaction will result due to high peaks of current. These high peaks of current may result in flashing at the commutator and will certainly prevent giving proper overload protection to the motor as the circuit breaker must be set high enough so that it will not trip out on these peaks. Time-limit overload relays have recently been developed and these may overcome this objection in some cases, but there are other cases where instantaneous interruption of the circuit is demanded when an overload occurs.

It may be contended that automatic starters having the smallest possible number of acceleration points should be used in cases where no harmful results will follow and that starters with a greater number of accelerating points should be used for cases where they are required. This would mean that the controller manufacturer must be informed of the exact load conditions in every case, which is not practical, and the user would suffer from the lack of standardization of the controlling apparatus which would prevent switching around motors and controllers from one machine to another as desired in the plant,.

In the case of punches, presses, shears, most wood-working machinery and other high-speed drives, it has been found that three acceleration contactors are usually required up to 15 h.p., 220 volts, and four acceleration contactors from 20 to 30 h.p. and five acceleration contactors should be used on larger sizes. I believe that the controller manufacturer should endeavor to be on the safe side as no harmful results can result from having more steps of acceleration than are actually required.

The curves shown in this paper are unfortunately on such a small scale that it is hard to get an accurate conception of the current values obtained. Fig. 6 shows a 20-h.p., 750-rev. per min. motor with 1.0-ohm external resistance and this gave an initial starting current of about 140 amperes. In Fig. 7, the same motor is used with external resistance of 0.765 ohms which gives an initial peak of only 120 amperes. If we assume that the peak of 140 amperes shown in Fig. 6 is due to impressed voltage of 220 volts on a total resistance of 1.57 ohms, this leaves 0.57 ohms for resistance of the motor and the motor leads. If we substitute this value in the case of Fig. 7 it would seem that the initial starting current would be 164 amperes instead of 120 amperes as shown.

Comparing Figs. 2 and 4 we see that using two acceleration contactors, the motor was put across the line quicker than in the case of a single contactor. The acceleration period would have been still further reduced if the acceleration contactors in Fig. 2 had been adjusted to close at a higher current value. This would have been permissible since the acceleration peaks are of lower value than the initial starting peak.

In general I would say that the more acceleration contactors used, the shorter will be the time of acceleration with the same limitation of current peaks as compared with the starter having fewer steps.

Automatic control is becoming so generally used, principally because it increases production, and this is secured by rapid safe acceleration and deceleration of the motor. It does not seem a step in the right direction to retard production to make a slight saving in the cost of the controlling apparatus. Referring to Fig. 8 the author says that from this curve the second step of starting resistance is not necessary. The low-current values at which the acceleration contactors close are not usual and in the case of a motor requiring full-load torque to accelerate its load, the resistance contactors would never close, also it would appear that in this case the load was of extremely low inertia as the motor reached a speed of 500 rev. per min. in about two-fifths of a second.

With reference to the two methods shown for inserting field resistance in the shunt field of adjustable-speed motors, we are told that "By comparing these curves with the preceding curves made on a three-to-one adjustable-speed motor, it will be seen that the armature current and torque are considerably more disturbed when the relay is used than in the cases where this relay is omitted. It would seem therefore, unwise to add this complication to every small motor."

The author's deductions would be more convincing if the curves he asks us to compare were not made with two motors of different horse-power sizes, different speed ranges and with possible widely different characteristics.

Experience, based on the application of several thousand automatic controllers to all kinds of industrial drives, indicates that inserting the field rheostat resistance in a single step, will be satisfactory if the increase in armature speed follows closely the change in field strength. If, however, the load is of high inertia, the field strength will change rapidly while the armature speed will increase slowly with the result that high peaks will be had when the field resistance is inserted and this would be eliminated with the field-rheostat relay.

The author advocates a single step of dynamic braking resistance for adjustable-speed motors relying upon the gradual increase in field strength when the field rheostat is short-circuited, in order to maintain a high dynamic braking torque.

The armature current during dynamic braking will be maintained at a fairly constant value, if the armature decelerates at about the same rate as the field flux increases. If the field flux changes more rapidly than the armature speed, high dynamic braking peaks will result.

With the scheme advocated, a different value of dynamic braking resistance would be required for motors having different speed ranges, but of the same horse-power size. This would mean that a controller designed say for a 20-h.p., 500-1500-rev. per min. motor when used with a 20-h.p., 500-1000-rev. per min. motor would give a much lower value of dynamic braking current than the motor could safely stand, loss of time being the result. Also, a controller designed for the 500-1500 rev. per min. motor and to limit the dynamic braking peak to a safe value when set for maximum speed would give a lower value of dynamic braking current when adjusted for any speed lower than this.

The use of graduated dynamic braking where the accelerating contactors are used as decelerating contactors to step out the resistance during dynamic braking, permits the field current to remain unchanged and maintains the dynamic braking current and consequently the dynamic braking torque at a high average value. The same apparatus may be used equally well with constant or variable-speed motors and the armature will be brought to a standstill even when driving a high-inertia load and without any appreciable friction load, with practically no tapering off of the dynamic braking action at low speeds.

In his summary, the author suggests that in the case of machine-tool motors which are started light, the motor can be started successfully with minimum field strength and the field relay omitted. This no doubt will be satisfactory as far as protection to the motor is concerned, but the acceleration would be slower than if the field relay were used and from the standpoint of production this is not desirable. It is not necessary to omit the field relay so as to enable the manufacturer to supply the same controller for constant-speed and adjustable-speed motors as the field relay can be mounted on the rheostat panel, and this is standard practise with some controller manufacturers.

E. J. Murphy: Referring to the improvements in motor design, it is quite possible to design a motor that will stand any sort of control, that is, within reasonable limitations. The control engineer, however, is obliged to produce controllers suitable for motors having characteristics with which he is not familiar, this makes necessary various devices which might be modified or in some cases eliminated in the control of an especially well designed motor.

With reference to cutting down the number of steps in the starting resistance which Mr. James especially advocates, I would say that while there is no necessity of using a very large number of steps I would not second Mr. James' conclusions that one step of resistance is entirely satisfactory, as a general proposition for motors of 15 h.p. At the same time, two many steps would involve a waste of material. Where too many steps are used, the time required to close the contactors might be longer than the period necessary to satisfactorily accelerate the motor, hence, an excessive number of starting steps would be worse than useless. Theoretically, an infinite number of resisting steps would give the most rapid acceleration, but practically, relatively few steps are necessary or even desirable on account of the definite time required to close each contactor.

With reference to the motor starting conditions mentioned by Mr. James, I do not consider it safe to assume that in most cases motors are started without any connected load or at least with a negligible load. The controller designer must make standard equipments to meet average conditions and it would be unsafe to assume that there would be no load at starting; in many cases this would result in serious trouble.

There is one point that I do not think has been yet brought out, that is, frequently, especially in machine tool applications, it is especially desirable to accelerate the motor as rapidly as possible. This generally requires more than one starting step and in all cases, full field at starting. The very small starting torque shown in Mr. James' tests would mean an excessive time in getting the motor up to full speed. There are also many applications where frequent and quick reversal is required and this would be impossible if the field is not constantly maintained. The use of dynamic braking advocated by Mr. James, would not be easily accomplished if the field is de-energized in the "off position" as he also advocates.

I have gone over Mr. James' curves and find that there must have been a great deal more permanent resistance in the armature circuit than given in his data. In the case of the 20-h.p. constant-speed motor, the data state that this permanent resistance was 0.25 ohm. Analyses of the curves indicates that this figure was about 0.6 ohm. In the case of the 15-h.p. motor, the data show 0.34 ohm; the average value obtained by calculation from the curves indicates that this value was actually 0.9 ohm. While this increase in the permanent resistance would only slightly cut down the maximum value of the initial starting current peak, it would enormously reduce the maximum value of the last or final peak. Calculations indicate that in many cases the last peaks would have been twice as great if the permanent resistance was as shown in the data presented. Arguments based on these tests and curves could not lead to any sound conclusions.

This excessive permanent resistance would also have a very serious bearing on the question as to whether the "fluttering" field relay was necessary or not. The effect of this excessive resistance would also cut down the current peaks when the field resistance was short-circuited, as in the tests on the variable-speed motors.

I cannot agree in Mr. James' conclusions that the "fluttering" field relay is unnecessary. In the first place, the tests made on the three-to-one adjustable-speed motor without the "fluttering" relay did not include a heavy inertia load and, as I have just mentioned, the excessive permanent resistance would reduce the current peaks incident to cutting out the field resistance. Secondly, Mr. Widdows has already mentioned that it is hardly fair to assume that because a three-to-one motor could be started successfully without a field relay, a four-to-one motor could also be started without such a relay. To amplify Mr. Widdows' statement, it is well to keep in mind that the inertia of the motor armature increases with square of the speed, and therefore it is obvious that the energy required to accelerate a four-to-one motor from basic speed to full speed would be nearly twice as great as that required for a three-to-one motor. It is unfortunate that Mr. James' paper did not include tests and curves showing the acceleration of a four-to-one motor without a fluttering relay instead of drawing conclusions from the tests on the three-to-one motor.

There is one point in the paper against the use of the fluttering relay in that this relay will cause trouble in case the hand-operated field rheostat is turned too quickly, and Mr. James prescribes another special relay to cure the trouble. In my opinion, the operator is not at all liable to move the rheostat quickly enough to cause any damage to the motor or connected load.

I would comment on the conclusions tabulated in the paper under "Summary of Tests" as follows: First, the use of only one switch to short-circuit the armature resistance for motors of 15 h.p. or less is not good general practise under average service conditions. In general this may be good practise for motors up to 10 h.p. and in many cases only up to 5 h.p. It is true, that under very special conditions this practise can be followed in motors of even 75 or 100 h. p, where it is absolutely certain that there will be no load at starting. If the single resistance switch closes at a counter e.m.f. of 75 per cent of normal voltage with heavy starting load, the motor would fail to accelerate and probably burn out the starting resistance. Second, as a general proposition, starting with zero or weakened field will result in too slow acceleration. In special cases, as in certain printing presses, it is necessary to start very slowly, not for the purpose of preventing damage to the machine but breakage of the paper. The average machine tool is designed to stand very severe shocks and would certainly allow full-load torque or more at starting. Third, the reduction of the shunt-field strength in one step for adjustable-speed motors, may or may not, be good practise depending upon the speed ratio, design and size of motor and the inertia of the load. In general, it is desirable to use a fluttering field relay in all adjustable-speed motors above 10 h.p., and in all sizes of motors where the speed ratio is more than two or three to one. This holds, except in special cases where the nature of the load is accurately known. Fourth, in general, I agree with Mr. James that one step of dynamic brake resistance is good practise, except where conditions are special, and it is also good practise to short-circuit the field resistance where the dynamic brake is applied. This latter point makes it desirable to have the field permanently connected in the "off" positions, contradictory to his recommendation under heading 2. Fifth, I do not agree that the general use of the fluttering relay is bad engineering and that a "transfer" relay is necessary. Sixth, I do not agree that the time to accelerate a small motor in three seconds is sufficiently short for many applications. The very low starting torque shown in many of the tests would certainly require not only a longer time for acceleration than three seconds but would entirely fail to accelerate a motor under full load at start.

J. H. Albrecht: I think we have pretty conclusively proved most of Mr. James' contentions under actual operating conditions, which, of course, is the final answer. We have installed at the McMyler Inter-State Co. plant at Bedford, Ohio, a new munitions plant, which has been in operation possibly fourteen months, about 300 controllers which are designed along the lines which Mr. James had laid down. These motors range from 7½ to 35 h.p., they are 230-volt motors, and the speed ranges vary. Quite a few of them are four-to-one motors, the speed ranging from 400 to 1600. These motors are used on practically all classes of machine tools, such as you encounter in a munitions plant. They have been in operation twenty-four hours a day, working three eight-hour shifts, and I venture to say that that service is the equal of any service that they might get in an ordinary machine shop in from five to six years. The controllers are started by a single contactor, on counter e.m.f., they have no field relay, simply a relay for insuring full field during the accelerating conditions. When the starting switch closes, the field relay opens, and the motor is accelerated to high speed in a single step, on the field. This plant, as I say, has been in operation for fourteen months. I had occasion to inspect it the other day. The motors were in very good condition, the machines were in good condition, and the maintenance has been no greater than should be expected under the severe operating conditions, and the power plant has shown no signs of distress. I think that is a direct, conclusive demonstration of the practicability of Mr. James' suggestion.

Still a further application which might illustrate the same thing is the modern reversing-planer controller. We have reversing-planer controllers that have been in operation with this new type of control for four years, some of them operating on gear-cutting machines, making from 15,000 to 20,000 reversals in a day, very severe service. The controller is a reversing controller, simply two two-pole switches for reversing, with back contacts for dynamic braking, a single-step accelerating unit, acceleration in one step from the full-field speed to the weak-field speed. The speed variation of these motors is from 250 to 1000, and they are often accelerated under load. I have seen them many times under operating conditions where the operator would shut off the machine with the tool in the work, then throw the controller on, and the motor would pick up the load. In lathe work, and things of that sort, the men often stop with the tool in the work, and the motor must start at practically full load, and it does it.

R. E. Hellmund: In connection with what the previous speaker said, it occurred to me that in d-c. practise we can learn a whole lot from the a-c. practise. Many of the speakers have brought out theoretical reasons why a single step would not be safe. We must not forget that we have started induction motors with a single step in actual practise for fifteen or twenty years, and under torque conditions much more severe than shown in the d-c. oscillograms which Mr. James has given. I actually believe the only reason why d-c. motors have not been started long ago with one step is because the motors could not stand it; but if, with proper cooperation between controller and motor designers, a combination can be built which will stand it, there certainly should not be any more objection against using it with d-c., than there is in case of a-c. motors.

E. H. Martindale: In listening to the discussion, before Mr. Albrecht and Mr. Hellmund spoke, I had rather gathered the impression that Mr. James' proposals were a little too severe; but offsetting the theoretical reasons why single-step controllers will not work, we have the testimony of a plant which has been in successful operation for fourteen months, with the conditions apparently as good as could be expected.

The main thing that this paper brings to light, it seems to me, is the fact that there has not been the study nor the cooperation that there should have been on this very important subject. Industrial motor application is probably the biggest field of electrical engineering. We have been perhaps too much concerned—not too much concerned, but we may say out of proportion—with high-tension transmission and distribution, and power-station generation, so much so that we have neglected the other end, which is just as closely allied to successful generation and transmission as two subjects can be. The Industrial and Domestic Power Committee is this year trying to start a little work looking toward the tabulation of the bes-ttype motor, the best-type control for every motor, for every machine in every industry in the country. That is looking fifty years ahead, more or less, it will take at least ten years to complete the work, if it can be done, and if it is worth while. There must be a best d-c. motor, there must be a best a-c. motor and a best-type controller to use with each, to drive every machine in every industry. This year we are studying two small industries and a small portion of a larger industry, and are trying to get this work finished to present before the annual convention in Hot Springs. Mr. James' paper has brought out the importance of investigation along this line. If controllers can be simplified, reducing greatly the expense of controller manufacture, and get just as satisfactory operation, and if motors can be designed as they are being designed, to withstand these severe starting conditions, then it seems that we have gone a long way toward effecting engineering economy in industrial motor installation.

R. H. McLain: Everyone in discussing this paper has had something to say about the possibility of the line voltage having had a great deal to do with the favorable curves which are published. I hope very much that Mr. James can clear up this situation by adding to his paper an exact record of the line voltage at the time his tests were made. Line voltage regulation has a most decided effect on current peaks in motors at the time of starting. For instance, in a large building, where an isolated generator plant is used to supply elevators and lights, there is a decided drop in voltage as is evident by the flicker of the lights every time an elevator is started.

I have had experience in starting up very large motors where four blocks of resistance were absolutetly required when the motor was close to the power station, and, on the other hand, if the motor was 500 or 600 feet away from the power station, one of the blocks of starting resistance could be eliminated and yet the current peaks on the motor would be the same as before. I disagree with Mr. James that a one-step starter is good for what might be called standard or "shelf" goods. Shelf goods are liable to be used by those who are rather inexperienced in the art and the goods should be of such a nature as to keep those inexperienced out of trouble; to avoid this trouble it is necessary either that the shelf goods be made, in this case, with more than one starting point or else that they be labeled with proper precautionary tags. I would suggest, as a proper limitation for one-step starters, that they be used only where the motor is to be started, as a rule, without load and very infrequently under load. The machine, to be driven, should have a flywheel or WR^2 which is very low as compared to that of the armature of the driving motor and the power supply line should be such as to give a considerable voltage drop at the time the motor is being started.

It has been said that these one-step starters have worked out very successfully in munitions plants. I think that the ideal way of laying out a munitions plant is to speculate on the length of the war, and lay out the machinery so that it will last just to the end of the war. The man who guesses right will, of course, do his job exactly right. It certainly does not hurt a motor to start with an undue shock once in awhile, but if it starts with an undue shock frequently, its life will be materially diminished. If long life is not essential, it is, of course, all right to start with a shock. I would assume that the munitions plant, under consideration, was provided with a transmission system whose copper cross-section was smaller than is ordinarily the practise in plants where manufacturing operations are to be continued indefinitely. If such is the case the motors would, of course, be benefited so far as shocks, due to the one-step starter, are concerned.

In regard to the shock on machines, I think that is again where the shelf goods might lead us into trouble. The shock that comes to a machine takes into account three factors, the relation of the WR^2 of the motor to the equivalent WR^2 of the load. If that is negligible as compared with the motor, it is impossible to start the load so as to hurt it, but if it is equal it becomes a serious part of the problem, and if it is greater than that of the load, it becomes an all important part of the problem.

A. A. Gazda: In addition to its simplicity, the counter e.m.f. switch works to advantage when line regulation is poor. If the line voltage is known to remain constant at full value under all load conditions, the pull-in point of the accelerating switch may be set high without fear that it will not pull in. On the other hand, if the resistance drop in the feeders is comparatively large, the counter e.m.f. switch may be set to pull-in at a low voltage. Under this condition, when the motor is connected directly to the line in the last step, a heavy peak current will not be obtained, due to the limiting influence of the feeders. The approach to this condition seems to have been present in the tests presented by Mr. James. Although the accelerating switches close at voltages as low as 60 to 70 per cent of normal line pressure, the peaks obtained are not excessive in any case. From that standpoint, I think it is all right to make use of the line drop, wherever it is known to be present.

In regard to the time element, I believe it is becoming good practise to have time-element overload relays. They certainly are necessary on induction motors, particularly to start induction motors. Every one has become used to time-element overload relays. We don't want the circuit breaker to drop out every time a peak of a fraction of a second occurs. I think that the abnormal peaks can be taken care of in that way.

Mr. Hellmund spoke of flashing in the motor, and designing to prevent flashing when starting. When the motor is at standstill, the armature conductors are not cutting the field, or are cutting it at a very low rate, and the voltage induced across the armature bars is very low. I have found in a standard line of motors that they can safely be thrown on the line with a starting current five or six times normal, without damaging the motor; and as far as two or three times over-load current is concerned, there is no flashing or sparking at all. It seems to me that the life of the motor would not be particularly shortened by permitting such high peaks where there is no sparking. Of course, I think it is fairly well known that after a motor is started and the accelerating switches come in, flashing is not liable to occur at all.

Mr. Widdows brought out the point that we will get more rapid acceleration when we use a large number of switches. That is contrary to my experience. A very good example of that is the planer controller. I believe that all controller manufacturers use only one step of starting resistance for planers up to 75 h.p. In such installations it has been found that if you use three or four switches, the time of acceleration will be merely lengthened by the time element of the switches dropping in, and it is found that the quickest acceleration is obtained with a small number of switches.

R. E. Hellmund: I judge from Mr. Gazda's remarks that I did not make myself entirely clear on the flashing with the motor starting from standstill. I do not claim that any high voltages between segments are present in the shunt motor when started from standstill. With a series motor, however, where there is no damping effect, the sudden building up of the flux causes a very severe flash unless some damping effect is introduced; in shunt motors we always have a very strong damping effect caused by the shunt winding, and they will, therefore, not spit when started from standstill.

We make it a practise to test railway motors up to 200 h.p. by starting them from standstill across the terminals of a large generator without any external impedance in the circuit with 130 to 150 per cent of their rated voltage, and since they are series motors they always do spit quite appreciably in this case. The arc does not pull across, however, because the voltage between the segments is only large within the immediate neighborhood of the brush. The voltage induced at standstill between segments when the motor is thrown on the line may be represented approximately by the curve Fig. 1. It will be seen that starting from the toe of the brush, the voltage is first very high, but diminishes quickly, reaching zero before the other brush is reached. Therefore, the arc goes usually around the commutator only about one-third the distance between brushes.

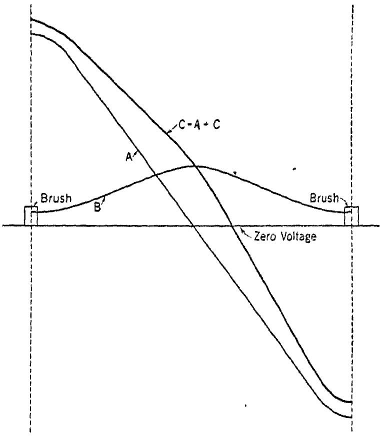

FIG. 1

A = Voltage induced between segments by change of main flux,
B = Voltage induced between segments by change of armature cross fluxes (self-inductive voltage).
C = Total voltage induced between segments. (With series motor at standstill thrown across line).

A. A. Gazda: I am very glad that Mr. Hellmund has brought out the fact that this condition exists particularly in the series motor. I must say that with the shunt motor, you will not get spitting at standstill. Am I correct in that, Mr. Hellmund?

R. E. Hellmund: Yes.

A. A. Gazda: It is only in the series motor that we have to look out for the spitting at the starting point.

J. B. Fisken: Just one word from the viewpoint of the operating man, the man who is dictated to by that modern invention, the Public Utility Commission. The idea that I gather from this paper is that motors up to 50 h.p. may be started with one-step controllers. Now, that is all right, it probably does not hurt the motor, but what I would like to have you consider is the effect on regulation.

Mr. James says, "At one time the permissible starting current was seriously limited by the danger of disturbance to the supply voltage." That is true. At one time that was perhaps more true than it is now, but it is still true to some extent. He says: "The increase in the capacity of systems supplying power now permits small and medium sized motors to start with twice full-load current, or even more, without disturbing the supply voltage." Now, in the larger system, that is no doubt true, especially in the center of the interconnected network, but very frequently you get on the limit of that interconnected network where possibly the distributing mains are of a smaller size, with the result that in starting such a motor with a one-step controller, the voltage drop in the mains would be quite serious. It is becoming more and more the custom with regulatory bodies to limit the voltage variation in incandescent lighting, and while up until now that has not been very serious—at least it has not been so out in the West with us—the tendency will be to improve that regulation from time to time, and where they allow at the present time a certain voltage variation, in two years from now that may be cut in half, and the result will be that when a motor goes on the stub end, say of a branch main, with a one-step controller, it may seriously affect the lighting.

At the beginning of his paper, Mr. James says: "The application of the controller required an analysis of the load to be driven by the motor, as well as the characteristics of the motor. The controller must cause the motor to operate through a cycle which is best adapted to the motor load or the machinery to which the motor is connected. The engineer must never lose sight of the fact that the purpose in using a motor and controller is to do certain work, and that he should select a type of motor and controller that will perform this work under existing conditions, with the least apparatus, and require a minimum of attention." That is all right. In installing a motor the utility, through its engineering department, can recommend to that customer what controller he should use, but the title to that apparatus passes to the customer, and it is his, to do what he likes with. Now, it is true that the controller he installs with that motor may be perfectly suitable for his particular purpose, but later he may find it necessary to provide a motor perhaps twice the size of the one he has, so he sells the old one to another customer, and this customer puts it to an entirely different use. I should like to see the standard controller be one which could be used for any purpose; then possibly for special purposes, where the motor is never liable to be used for anything else, have a special cotroller.

Regarding Mr. Martindale's remarks, the idea was to have a committee to establish certain standards of apparatus for certain purposes. I don't think that is wise. Again we have to come back to the regulatory bodies, and unless for each such purpose there is a specification of every possible voltage, and every possible type of motor that may be used, there will be considerable danger. The standards may provide, for instance, that for a certain purpose, a 500 or 600-volt d-c. motor with a certain controller should be used. Now, a customer goes to the utility and says that he wants that apparatus installed in his plant, the utility informs him that it cannot supply that type, but can supply him with an a-c. motor, a type of apparatus that will give perfect results. He says, "No, I want that," and when the utility refuses to furnish it he goes to the utility commission, and the commission rules that the American Institute of Electrical Engineers has established that particular type of motor for that particular purpose, and therefore the utility must put in the plant to furnish that service. There is the danger, to my mind, of doing what Mr. Martindale's committee had in mind.

E. H. Martindale: Evidently, I did not make it quite clear that the committee is going to be very, very careful not to recommend in any case a d-c. motor against an a-c. motor, or vice versa. We will, on any machines that we make any recommendations for, recommend the best a-c. and the best d-c. motor, or we will make no recommendation at all, because we know the hot water we would get into if we started anything between the alternating-current and the direct-current. It may not be possible to carry this plan through at all, it is simply to be presented at some future meeting in a series of papers for consideration, to see whether the method is possible, and whether it is worth while. Different voltages will bring up other questions which I think we had better leave for discussion at the proposed meeting when we will have data to present, to try to show the value and feasibility of such work.

F. W. Gay: One hundred and fifty controllers were installed in a munition plant for starting motors under 15 h.p. Each controller consisted of a simple knife switch which by closing into 3 clips in succession; first, connected the motor to the line through a two-ohm and a one-ohm resistance, second, short-circuited the two-ohm resistance, third, short-circuited the one-ohm resistance. Controllers were, on an average, closed and opened approximately every 3 minutes, 24 hours per day and 7 days per week for ten months. No trouble was experienced with this equipment when used with belted motors, other than the necessity of turning approximately a dozen commutators. A very heavy wiring system was installed and no serious flickering of lights occurred.

In the same company's commercial plant approximately 1000 motors are used on a multiple-voltage four-wire system. These motors are switched successively through 40, 80, 120, 160, 200 to 240 volts without the use of any buffer resistance. This control is used on motors up to 50 h. p. and has been in service over 20 years. Only ordinary maintenance has been necessary.

Alexander Gray: The point Mr. Hellmund brought out should not be overlooked, that for twenty years or more induction motors have been brought up to speed by a one-step starter, the motor drawing from three to four times full-load current at a very low power factor. If the motor does not start up with this current the auto-transformer tap is changed and the current further increased. The proposition submitted by the author is somewhat similar, except that he restricts the starting current to a value that will have much less effect on line regulation.

The larger number of motors now installed are of the squirrel-cage type and the torque required for starting is generally less than full-load torque. Many of these motors, because of patent reasons, did not have a prime number of slots in the rotor and therefore the starting torque was not as good as it might have been, and yet these motors started up the loads found in practise; induction-motor operators are therefore satisfied with a one-step starter.

Before I came to the meeting I had an unskilled operator start up a d-c. motor, which had a brake arm and a spring balance attached, by means of a face plate starter and then had him repeat the experiment with a single resistance and a knife switch, a voltmeter being connected across the motor terminals to indicate when the switch should be closed. There was little to choose between the two methods of starting and a one-step automatic starter is as good as the above combination, I therefore feel that the author should have no difficulty in defending the stand he has made.

J. P. Mallett: It has been brought out by several speakers that perhaps the reason so many steps have been used in the past for starting was due to the fact that motors would not commutate under over-load starting conditions, which I believe myself is true. With the later designs in motors, with the commutating pole, the commutating feature has been so well taken care of that I believe you can very readily cut down the starting conditions without injury, and for certain classes of work I think there is no question but what one step of resistance is quite sufficient, with a shunt motor, and none with a series motor. There is one point that has not been taken up, and that is the class of work that may be performed on the particular machine operated. It is sometimes necessary to consider that very carefully, whether it is permissible to start as quickly as the motor can start, without injuring the work.

There is one point, regarding the line voltage, and that is where the motors may be supplied from a small power-station. We are, I think, in the habit of thinking of the mere lighting question, whether the lights are going to flicker. There is another problem that is coming in more and more, in fact, it is very important today, and that is the heating load. With heat supplied on central station lines, if the voltage is going to vary seriously the heating effects may be very disastrous.

All of us who have been particularly interested in introducing the motor drive, the individual drive particularly, have run against the snag of the comparative cost of the controller. Some motors of course will not have to be started more than two or three times a day, but others as frequently as every few minutes. I cannot imagine any more severe condition than a planer motor, and if, as we have heard, the planer operated with entire satisfaction, and the motor as well, with just one step of resistance, why that very nearly proves the point.

It makes some difference whether or not the company which manufactured the motor also manufactured the controller. In other words, do you know exactly what you are putting together. Then the question of the standardization of parts enters very strongly. I am also very strongly convinced that the amount of motor application we are able to carry out will be influenced to a great extent by not only the cost of the controller but the space it will occupy. Many of us I think have found that putting a motor and controller on a piece of apparatus makes it unsightly, outside of question of expense. Where can we put the controller? That is frequently a problem that confronts us, and if by leaving out these various steps of resistance the size of the controller can be reduced, it is a point well worth considering.

H. D. James: Two or three points in this discussion have stood out. First, I think that all those present must be impressed with the necessity of giving the electric controller more serious consideration. The discussion has brought out a number of problems and has suggested further investigation. I stated in presenting my paper that I did not expect it to be a complete answer. The work done was in the hope of stimulating others to carry the investigation further.

Second, the engineers who have discussed this paper must have based their statements upon actual test data. I think it would be well if they would submit these tests, as well as their deductions.

I am sorry that it is necessary to print the oscillograph records to such a small scale. I will be glad, however, to show blue prints of the original oscillographs to anyone who is interested in seeing them.

A number of the speakers have brought out the fact that my test did not record the voltage impressed at the controller terminals during the period of acceleration. It would have been much better to have taken those data, as it would have answered a number of questions which have been brought out. I believe that further investigation should be made, in which those data are recorded.

The discussion has brought out three probable factors that reduce the starting current peaks:

(a) The resistance in the circuit external to the controller.

(b) There is always a drop in the generator voltage corresponding to a sudden increase in the current.

(c) There undoubtedly was a reaction between the armature current and the shunt-field current. This is shown very clearly at the beginning of the cycle. This reaction may have kept down the current peaks considerably.

I agree with Mr. McLain that starters installed in a power plant will have very close voltage regulation at the terminals of the starter and therefore will require at least an additional step for accelerating.

In the main, calculations seemed to show that the resistance of the average feeder is not less than half the resistance of the motor armature, so that in most installations at least this much additional resistance can be figured on.

Both Mr. B. G. Lamme and Mr. D. Hall have published data showing that when a generator is operating at no load and you suddenly connect it to full load the dip in the generator voltage approaches zero. This dip, however, is too rapid to be measured with a voltmeter. These tests would indicate that when a motor is started from a small power plant, a reasonable drop in voltage should be expected, depending upon the ratio of the size of the generator to the size of the motor.

Several speakers have assumed that the maximum value of the current on each step is equal to the line voltage divided by the resistance in the circuit. I do not agree with this deduction, as I believe that there are other factors which may materially reduce the maximum value of these current peaks. It is very necessary to make a mathematical deduction of the problem in advance of tests, so that the tests may be properly directed. No such mathematical deduction, however, should be taken as final until it has been verified by careful tests.

The National Electric Light Association rules permit the starting-current peaks to be equal to twice the full-load current. A reference to the tests in my paper show that in no case was this maximum value reached.

A number of speakers emphasized the difference between starting light and starting with an inertia load. They apparently overlooked the statement in my paper that in every case the motor was started with a load, either a prony brake, set to give full-load torque, or a 50-kw. generator belted to the motor and operated at about the same speed as the motor. The inertia of the generator was in excess to that of the motor. If the motor was started light, as often happens in practise, the current values would have been somewhat less than shown in the paper.

These tests were made in a commercial testing room, and it was necessary to use motors which were available at the time. This accounts for the fact that we used two different motors when making the tests for adjustable speed.

I trust that this is only the beginning of such tests and that we will have more investigations of this kind reported to the Institute. The curves shown in the paper for motors having different characteristics were not intended to be compared for exact values. The general shape of the curves, however, indicates relative performances.

Customers frequently have an idea that the starting torque of a shunt motor reaches its maximum value immediately. If therefore, the controller permits more than full-load current to pass through the motor, the torque will be in excess of full-load value. If the motor has four-to-one speed adjustment, the impression prevails that at starting, more than four times full-load torque may be obtained. I believe that the tests disproved this contention. They show, however, that as the motor increases in speed, the torque gradually increases, so that an adjustable-speed motor will start a heavier load using one step of starting resistance than a corresponding single-speed motor.

Mr. Widdows brings out the point that while a single-step starter might be suitable for certain applications, it is necessary for commercial controllers to meet the worst conditions. This does not seem to me to be logical. I would suggest that possibly two different starters could be used, one to meet average conditions and one for heavy starting. This would also enable us to take advantage of the location of the starter relative to the power plant, the probability of good voltage regulation and other factors of this kind.

Another objection to a single-point starter is that the overload relay when used would require a time-element device to keep it from tripping. I believe that a time-element device on all relays protecting motors is good engineering and should be used even with a large number of starting points.

Another advantage of the time-element device is to protect the starter in case of severe overloads closed to a large supply of power. In such cases, the overload protection on the controller should have a longer time element than the circuit breaker protecting feeder, so that the circuit breaker will rupture a short circuit or ground before the starter attempts to open the circuit. A time-element overload device on a controller can be set to protect the motor against continuous overload, and will have enough time element to permit the motor to take overloads for short intervals of time.

Mr. Widdows questioned the use of a single-step controller for dynamic braking resistance, his objection being that this single step must be adjusted for each particular motor. It is customary in furnishing controllers for adjustable-speed motors, to have the exact field data, so that the field rheostat may be correct. At the same time, the remaining information regarding the motor can be obtained so that the dynamic-brake resistance can be properly designed.

R. L. Goetzenberger: Short-circuited current under the brushes affecting as it does the flux of the main field; the transformer action between the field and armature which induces a high voltage at the brushes that directly influences not only the short-circuited current but also the armature output current; the residual magnetism in the commutating poles causing a variation in the demagnetization of the shunt field are all factors that demand consideration by the designer of d-c. motors. The incorporation of these together with those involved in certain construction details, over which no formulas have control, into a mathematical derivation necessarily means complication. Fundamentally therefore, in making computations from the equations, reference should always be made to the text in order to avoid indiscriminate separation of the derivation and its qualifications.

Presented at the Fifth Midwinter Convention of the American Institute of Electrical Engineers, New York, February 16, 1917.

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TRANSIENT CONDITIONS IN ASYNCHRONOUS INDUCTION MACHINES AND THEIR RELATION TO CONTROL PROBLEMS

BY R. E. HELLMUND

ABSTRACT OF PAPER

This paper discusses a number of transient conditions existing in asynchronous machines immediately after certain changes in the circuit connections are made. The advisability of considering these conditions in connection with the control layout for the purpose of eliminating bad effects caused thereby, is pointed out. The principal subjects considered are undesirable peak currents, which may be caused by the damping effect of the short-circuited rotor windings and the over-voltages, which are obtained with certain control arrangements.

DAMPING EFFECT OF CLOSED-CIRCUIT SECONDARIES

IT IS generally understood that the strong demagnetizing or damping effect of a short-circuited low resistance secondary of an inductance motor plays an important part during the starting from standstill, in so far as it counteracts and weakens the useful field of the motor to a considerable extent, resulting in very large starting current accompanied by relatively small starting torques. It is, however, not always fully realized that the same damping effect influences the motor working conditions in a number of other ways leading at times to very unfavorable effects, while serving to good advantage in other cases.

The fundamental fact to be considered in this connection is that any short-circuited low resistance winding will strongly oppose any change of flux interlinking with the winding. If the interlinking flux is of zero value, the building up of flux will be strongly opposed; on the other hand, if a certain flux happens to exist, its disappearance will be strongly opposed. This is due to the fact that any change of flux regardless of its cause or nature will at once induce a voltage in the short-circuited winding which, in turn, causes currents to flow. These currents are of demagnetizing direction if a flux is trying to build up or increase, and of a magnetizing direction if a flux is trying to disappear or decrease. This simple phenomenon should be kept in mind in connection with the design of the control with regard to every manipulation.

Let us consider, for instance, the most common method of starting induction motors by connecting the motor, first to a low-voltage tap of an auto transformer, subsequently disconnecting the motor and connecting it to the full line voltage. It is frequently assumed that the largest current surge is obtained with this method of starting when the motor is standing still and first connected to the low-voltage tap of the auto transformer, and that the current obtained when changing over to the full voltage corresponds to the current value indicated by the speed-current curve of the motor for full voltage. As a matter of fact, it is possible to obtain change-over currents far in excess of the original starting currents. The standard method for calculating the change-over current from the speed-current curve assumes that certain stable conditions exist with a rotating field and certain phase relations between primary and secondary currents fully established, a condition which, however, does not exist after the power supply to the motor has been temporarily interrupted during the change-over.

Various conditions may exist in the motor after a temporary interruption of power supply. When the low-voltage starting contacts are opened the rotating field set up in the rotor will be prevented from immediately disappearing by the previously mentioned damping effect of the rotor. The flux will continue to exist and rotate with the rotor for a certain period of time until the stored magnetic energy is used up by ohmic losses in the rotor, eddy currents, and the like. Since the rotating flux continues to exist it will, of course, continue to induce a voltage in the primary winding.

The oscillogram, Fig. 1, showing the motor voltage before and after the interruption of the primary contacts demonstrates this point. The size of the voltage maintained after interruption and the time during which an appreciable voltage is maintained, depends principally upon the amount of magnetic energy stored in the rotor at the time of interruption as well as upon the speed with which this energy is used up by losses in the motor. The losses depend largely upon the resistance in the secondary, and, therefore, the energy will be used up the quicker the higher the secondary resistance. The oscillograms, Figs. 1 and 2, were both taken on a 40-h.p., eight-pole, 60-cycle, wound-secondary motor.

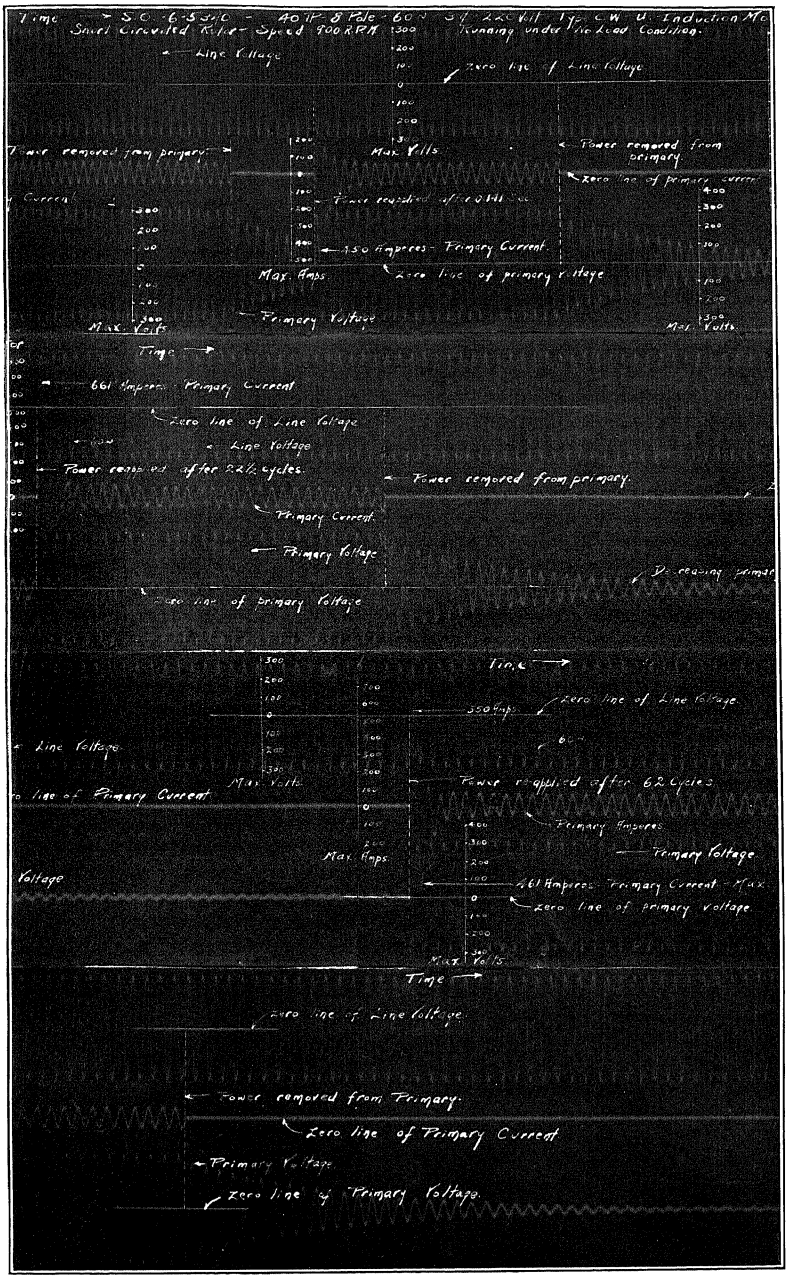
Time → S.O. 6-5340 - 40 H.P. 8 Pole - 60 ~ 3 φ 220 Volt Type C.W. U. Induction Mo
Short Circuited Rotor - Speed 900 R.P.M.
Running under No Load Condition.
← Line Voltage
Zero line of Line Voltage
300
200
100
0
100
200
300
Max. Volts.
Power removed from primary →
← Power removed from primary.
Zero line of primary current
Current
← Power reapplied after 0.141 Sec.
← 450 Amperes - Primary Current.
Max. Amps.
Zero line of primary voltage
← Primary Voltage
Max. Volts
Max. Volts.
Time →
← 661 Amperes - Primary Current
Zero line of Line Voltage
← 60 ~
← Line Voltage
← Power reapplied after 22½ cycles.
← Power removed from primary.
Primary Current.
← Primary Voltage
Decreasing primar
Zero line of primary Voltage
Time →
← 550 Amps.
Zero line of Line Voltage.
60 ~
← Line Voltage.
Max. Volts.
o line of Primary Current
Power re-applied after 62 Cycles
Primary Amperes
← Primary Voltage.
Max. Amps.
Voltage.
← 461 Amperes - Primary Current - Max.
Zero line of primary voltage.
Max. Volts.
Time →
Zero line of Line Voltage.
← Power removed from Primary.
Zero line of Primary Current
← Primary Voltage
Zero line of Primary Voltage.

FIG. 1

PLATE XIII.
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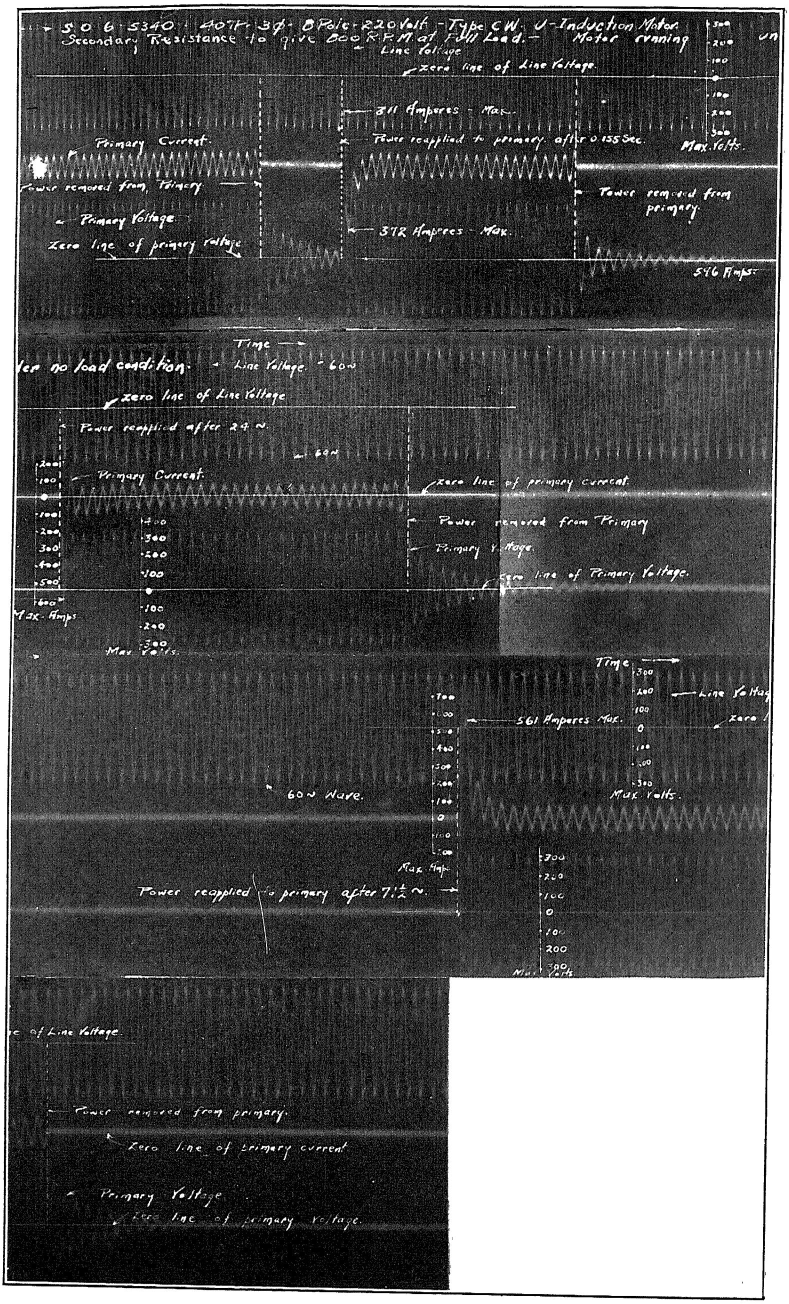

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FIG. 2

In case of Fig. 1, the rotor was short-circuited, while in case of Fig. 2, an external resistance was introduced to bring the motor speed down to 800 rev. per min.; the motor was running light in both cases. Comparision of the two figures shows plainly the effect of additional resistance upon the motor flux and voltage after the interruption. With the short-circuited rotor, the voltage drops to about 80 per cent after one cycle, to 75 per cent after two cycles and to 50 per cent after six cycles, while with the extra resistance in the rotor the corresponding values are about 60 per cent, 50 per cent and 25 per cent. With short-circuited rotor, the voltage practically dies out after about 35 cycles, or 0.6 second; with the extra resistance, only 15 cycles or 0.25 second are required. With short-circuited secondary, the motor voltage drops below half voltage after 7 cycles or 0.12 second, while with extra resistance only 1.5 cycles or 0.025 second are required.

Outside of the ohmic secondary resistance losses, there are as already mentioned other losses participating in a secondary degree in the consumption of the stored magnetic energy. The sudden changes of current in both members, and the corresponding changes in leakage fluxes are naturally accompanied by appreciable eddy losses in the conductors and the iron, hysteresis losses and the like; other core losses are caused by the rotation of the sustained field in the stator iron, etc.

The amount of stored magnetic energy is principally given by the size of the total flux in the machine, which, in turn, is the product of the magnetic densities and the magnetic sections.

A very simple consideration based* upon these fundamental facts leads to the conclusion that the flux and voltage will, in general, be maintained the longer, the larger the machine and the smaller the number of poles. While other factors, like mag-

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*A fair idea of the principal factors regarding the amount and time of the maintained flux can be obtained by the following consideration. The principal energy dissipating losses, namely, the secondary resistance losses are, in general, proportional to the length L of the conductor per pole and inversely porportional to the available space for effective conductor section S in the rotor. The length of the conductor is, in turn, proportional to the length of the core l plus the pole pitch τ times a coefficient k. The pole pitch is given by

$$\tau = \frac{D\pi}{p}$$

where D is the air-gap diameter and p the number of poles; therefore, we netic air-gap density, the copper space factor in the secondary winding, ratio of pole pitch to core width, etc., also have a certain influence, they do not vary over a wide range with up to date designs of small commercial motors. With larger motors now becoming more common in use, these points should also be given due consideration.

This general conclusion is well borne out by results obtained in practise. It has been observed, for instance, that an induction machine of about 1500 h.p., 25 cycles, 4-poles, would maintain a sufficient voltage to keep incandescent lamps luminous for seven seconds after the current supply was interrupted. On the other hand, a comparison of Fig. 3, which applies to a 5-h.p., 10-pole, 60-cycle motor, with Fig. 1 will show how a small machine maintains the flux and voltage only for a comparatively short period of time, the voltages following the circuit interruptions being about 70 per cent after one cycle, 55 per cent after two cycles and 25 per cent after six cycles, while

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have an energy dissipating constant per pole

$$E_d = \left(l + \frac{D \pi k}{p} \right) \frac{1}{s}$$

The constant of stored energy per pole E_s is given by the pole face

$\frac{D \pi l}{p}$ multiplied by the air gap density B

therefore,

$$E_s = \frac{D \pi l B}{p}$$

Assuming a ratio $r = \frac{l}{\tau} = \frac{l\,p}{D \pi}$ or $l = \frac{r D \pi}{p}$ we have

$$E_d = \frac{D}{p} (r + k) \frac{\pi}{s}$$

and

$$E_s = D^2 \frac{\pi^2 r B}{p^2}$$

It is at once evident that insofar as the stored energy is proportional to the square of the diameter, while the energy dissipating factor grows only in linear proportion, that the maintained voltage will be in time and amplitude, the larger, the larger the machine. A further study of these expressions also indicates that the maintained voltage will be in general the larger, the larger the air-gap densities, the smaller the number of pole, etc.

the corresponding values in Fig. 1 are 80 per cent, 75 per cent and 50 per cent respectively.

The load condition of the motor will also have an influence upon the maintained flux and voltage, which, in general, decrease with increased load. This is due to several reasons, for instance the fact that on account of the primary drops, the secondary flux under load is smaller than the primary flux to start with, further, that on account of the larger load leakage fluxes, and their sudden changes, more energy is dissipated by eddy currents, etc.; again, the lower load speed will, with a given rotor flux, induce less voltage in the primary than the higher no-load speed. This may be demonstrated by comparing Fig. 1 with Figs. 4 and 5. These oscillograms were taken on the same motor, with no-load (Fig. 1) and 1¼ load (Figs. 4 and 5). Fig. 6, which was taken on a 25-h.p., 6-pole, 60-cycle motor under the same load conditions as Figs. 4 and 5 on the 40-h.p., 8-pole motor, is also of interest. A comparison of these figures shows that the lower rated 25-h.p. motor maintains the voltage and flux better than the 40-h.p. motor under the same load, which is due to the relatively lower rotor resistance usually incident to the smaller number of poles.

It is evident from the previous considerations that if the primary switches are reclosed after a temporary circuit interruption, the voltage and flux conditions of the motor may be widely different under various assumptions of the motor characteristics, the load, the time elapsed, etc.

CURRENT PEAKS WITH CLOSED-CIRCUIT SECONDARIES AND ZERO FIELD

Let us consider first the case where the time of change-over from the low-voltage tap to full voltage is long enough to permit the rotor flux to reach zero value before the full voltage contacts are closed. It is evident from the fundamental facts stated in the beginning of the paper that upon closing the contacts, the damping effect of the rotor will prevent the quick establishment of the rotor flux on account of rotor currents setting up and strongly opposing the effect of the primary currents. This will also prevent all but the leakage part of the primary flux from building up quickly, so that at the first instant whatever counter e.m.f. exists has to be largely supplied by leakage fluxes no matter what the speed of the motor may be. In other words, very large primary currents are required at the first instant, no matter what the speed of the motor, to set up sufficiently large leakage fluxes to induce the proper counter e.m.f. which must always be equal and opposite to the impressed e.m.f. after the circuit is closed. The amplitude of the current peaks when connecting the motor to the line with no flux in the secondary may, therefore, be the same near synchronous speed as it is at standstill for a given motor voltage. Since with the standard method of starting, the voltage after the change-over is usually larger than when starting from standstill, it is evident that the current peak after the change-over may be larger than when starting, assuming in both cases that the rotor field is zero at the time the connection is made. This does not necessarily mean that the effect upon the line and control apparatus at the time of the change-over is worse than at starting. In either case, the currents settle down within a short time to the values given by the speed-current curve. Since these values are usually larger for starting than at the change-over, and since the continuity of current peaks plays an important part in the effect upon the system, the resultant effect depends upon the individual case. It is, however, evident from the previous discussion that in a good many cases under the conditions discussed, the effect following the change-over may be worse than during starting.

The current peak obtained depends, of course, to a large extent upon the point of the impressed voltage wave at which the switches are closed. Moreover it can easily be seen that the damping effects opposing the building up of the flux are, in general, dependent upon the same factors as the damping effects maintaining the flux after the power supply has been interrupted, namely, the rotor resistance, etc. Furthermore, the leakage fluxes will have an important influence upon the current peaks as already pointed out. In general, it will be found that the size of the current peaks relative to the sustained current values is liable to be the larger, the larger the machine and the smaller the number of poles.

Fig. 7 shows a current peak of 778 amperes obtained with the 40-h.p., 8-pole motor which has a sustained standstill current of only about 425 amperes effective (600 amperes crest value) and about 115 amperes effective (163 amperes crest value) sustained current at the speed at which the test was made.

These conditions were obtained with an external resistance in the rotor circuit and it is evident that worse conditions may be experienced with the rotor short-circuited.

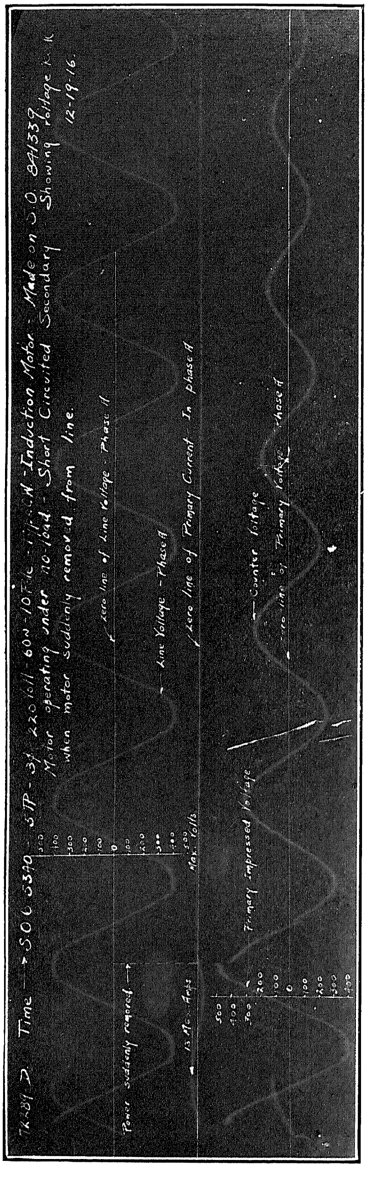

FIG. 3

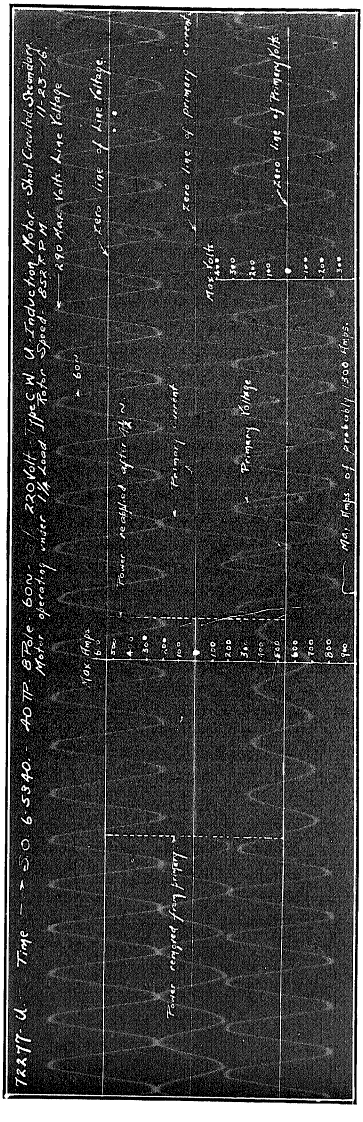

FIG. 4

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PLATE XV
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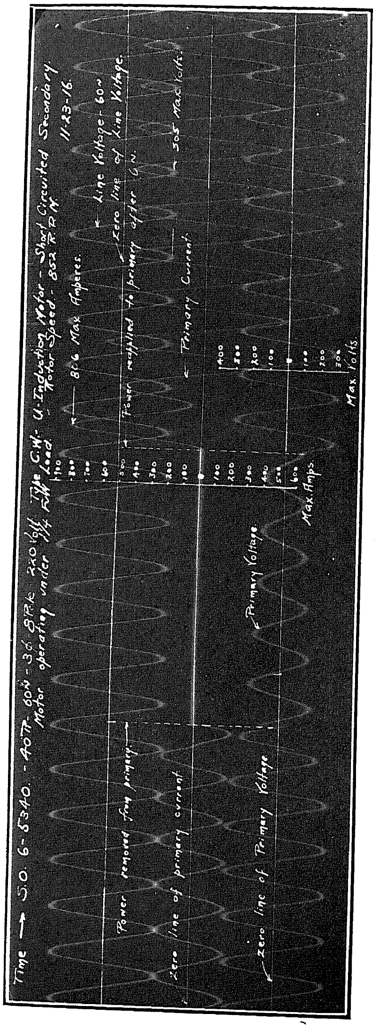

FIG. 5

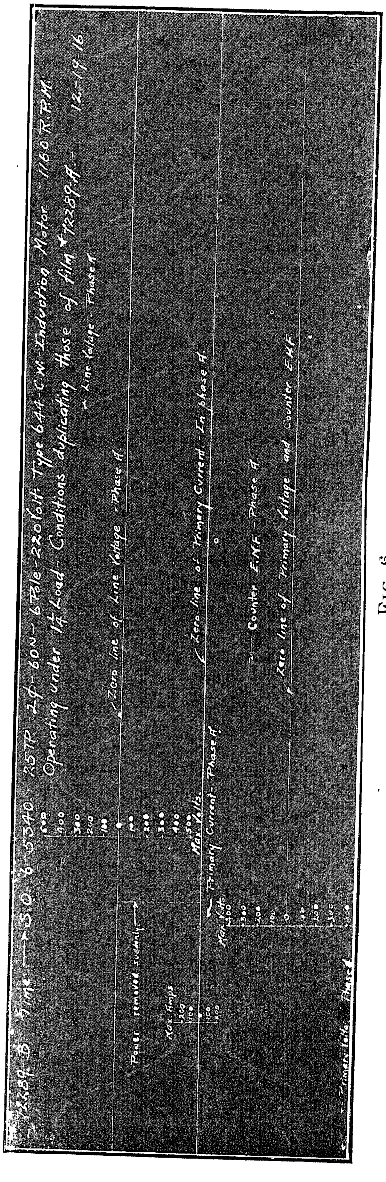

FIG. 6

[HELLMUND]

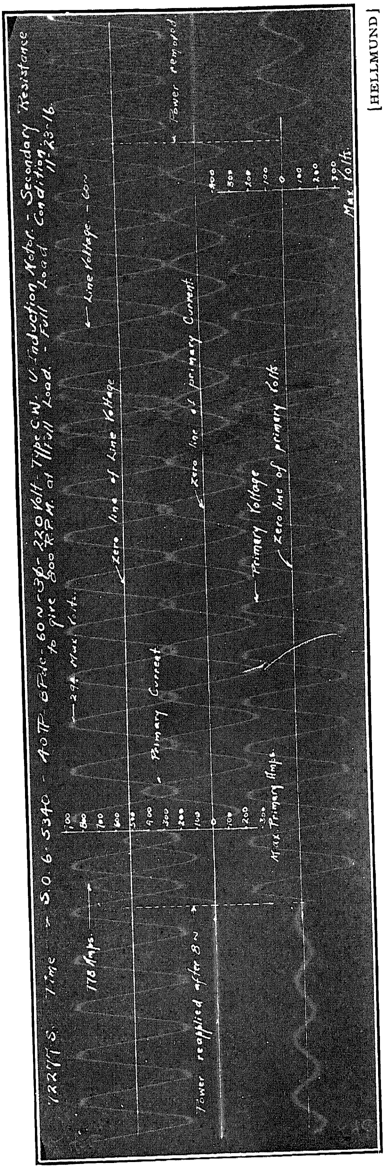

FIG. 7

[HELLMUND]

PLATE XVII.
A. I. E. E.
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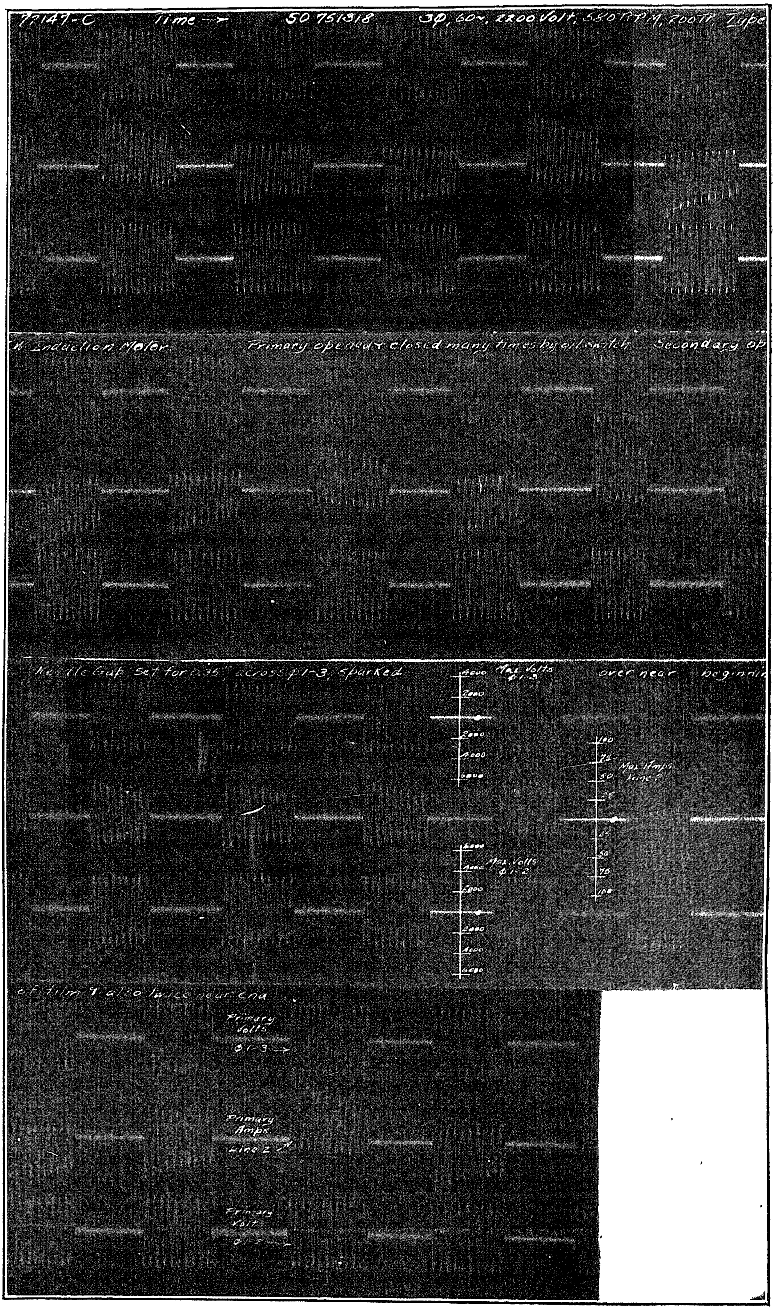

[HELLMUND]

FIG. 9

CURRENT PEAKS WITH FIELD SUSTAINED BY CLOSED-CIRCUIT SECONDARIES

Let us consider now a case in which the change-over or current interruption is accomplished very quickly so that the contacts are reclosed before the rotor field has died out. Let us also assume at present that in such a case the load of the motor happens to be light, so that the rotor rotates at practically synchronous speed and that the rotor inertia is large enough so as to keep the motor speed practically unchanged during the power interruption. With these assumptions, the counter e.m.f. of the stator induced by the flux rotating with the rotor will maintain a synchronous relation with the line e.m.f., and if the contacts are reclosed the counter e.m.f. of a certain amount and proper phase relation already exists keeping the initial current rush down to a very small value. In such cases, it may be that the initial current closely agrees with the current given by the speed-current curve of the motor. If, on the other hand, the motor is heavily loaded, so that its rotor speed is appreciably below synchronism, and if the rotor has small inertia so as to permit a further drop in speed during the current interruption, the voltage induced by the flux rotating with the rotor speed will fall quickly out of synchronism with the line voltage. Such a condition is demonstrated in Fig. 8 which has been traced from an oscillogram similar to that of Fig. 6 in order to show the conditions clearer than is possible with the oscillograms themselves, which were unfortunately taken with the voltage elements connected in opposite directions (see Fig. 6) or to different phases (see Figs. 4, 5 and 7).

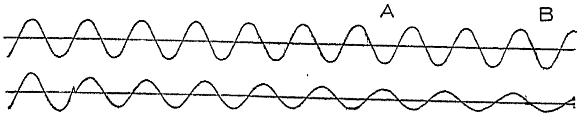

FIG 8

If we assume, for instance, that the stator voltage has slipped ½ cycle relative to the line voltage as at the point A when the contacts are closed, we have a condition very much the same as obtaining in a synchronous machine connected to the line when out of synchronism, and it is evident that very large currents may flow temporarily because it is not only necessary to build up a rotor flux against the damping effect of the secondary winding, but it is necessary to first reduce the existing flux to zero and build up the flux in the opposite direction. It is considered poor practise to connect large synchronous motors to the line before establishing synchronism, and equivalent precautions should be taken when starting large induction motors to prevent severe burning and arcing.

The author was not fortunate enough to get an oscillogram showing the closing of the circuit at the most unfavorable point. This is pretty hard to do in view of the fact that very small fractions of a second bring about important differences in the conditions. If the circuit were closed, for instance, at point *B* in Fig. 8, that is only 0.12 second later, the voltages are again in phase and fairly favorable current conditions would obtain.

Fig. 4 and Fig. 5 taken with the 40-h.p., 8-pole motor show the highest current peaks tested, giving about 1300 amperes (estimated) and 806 amperes respectively, as compared with a sustained standstill current of 450 amperes effective (635 amperes crest value), and a sustained value of 150 amperes effective (212 amperes crest value) for the speed at which the test was made.

CURRENT PEAKS WITH OPEN-CIRCUIT SECONDARIES

The previously discussed current peak values as affected by the damping currents in the rotor should not be confused with the transient current peaks obtained when connecting any inductive piece of apparatus to an a-c. voltage, even when there are no damping effects. Figure 9 shows, for instance, the current of a 200-h.p. motor, when connected to the line with open secondary. It will be seen that some of the transient peak values are about 3.5 times as high as the sustained magnetizing current. They are, however, only 150 per cent of the full-load current, or 26 per cent of the sustained standstill current of the same motor (crest values in all cases). These currents are, therefore, small in comparison with the previously discussed transient current peaks obtained with the damping effect of a short-circuited rotor, and they are, therefore, of lesser importance in connection with the design of the control arrangements. In this connection it might be mentioned that motors with open external secondary circuits, should not be always considered as entirely free from damping effects. A delta connected secondary, for instance, may develop appreciable transient damping currents under certain conditions, even with open secondary control switches, especially when temporarily only one of the primary circuits of a polyphase motor is closed, as is frequently the case on account of the fact that the switches do not all come in simultaneously.

EFFECT OF SECONDARY DAMPING CURRENTS UPON OPERATION AND DESIGN

While the damping effect of the rotor has as previously pointed out, in many cases unfavorable effects upon the system and the control apparatus, it has at least the advantage that both during starting and after the change-over with the rotor field previously reduced to zero, the torque of the motor picks up gradually because torque is impossible without a secondary field. Any harmful effects which might be caused by too sudden picking up of the torque are, therefore, avoided.

In view of the fact that with a short-circuited rotor certain large peak current conditions cannot altogether be avoided, even if the rotor flux is zero to start in with, it is evident that control apparatus, as well as all other parts of the system must be designed to stand these peak currents. It is also evident that the reclosing of the circuit before the rotor flux has died out should be avoided in control arrangements for large squirrel-cage motors, because under certain conditions this is not only liable to give current peaks much larger than otherwise obtained but also liable to give under slightly different conditions a too sudden picking up of large torques, which may result in damage to the motor and the driven machinery. When starting a motor at reduced voltage, a closed circuit transition to the full-voltage tap by means of preventive resistances or inductances is, of course, the safest, especially in the case of large motors.

In the case of wound-secondary motors, current peaks can best be reduced by always introducing a resistance into the secondary before closing or reclosing the primary circuits.

In case of the control of induction apparatus for railway work, the damping effect of the secondary of induction apparatus is of special importance. It may happen in this case, for instance, that the trolley bounces, interrupting the power for a very small interval of time, leading to very severe rushes of current when connections are reestablished. In some experimental tests conducted on a large induction phase converter, explosive arcing was repeatedly established. In actual practise arrangements are made to introduce temporarily by means of a relay an inductance into the converter circuit upon any power interruption, so that the current peaks are held down when the power comes on again within a short interval of time. Another relay interrupts the circuits completely if the power stays off long enough to permit an appreciable decrease in speed. It is of interest in this connection, that it is not possible to employ a no-voltage relay for the quick acting switch introducing the inductance, because the induction machine does not lose its voltage for sometime after the power interruption as previously discussed; it is, therefore, necessary to use a current relay in the single-phase circuit. The fact that the voltage is maintained sometime after the interruption, is used, however, to good advantage for operating the switches interrupting the circuit completely after a certain interval of time, by means of a voltage relay connected across the induction machine, which drops out about 5 to 10 seconds after the power interruption.

There is another case that should be considered in connection with the effect of the short-circuited rotor in maintaining the flux and motor voltages for a certain interval of time, after the primary has been disconnected. Certain switching arrangements are sometimes used, to change from high-speed to low-speed connections, for instance, that are not designed to operate under current and voltage. Care should be taken that none of the secondary motor circuits are interrupted by such switch arrangements, even after the primary current interruption, until the secondary damping currents have died out.

OVER-VOLTAGES WITH OPEN-CIRCUIT SECONDARIES

Another subject related to transient conditions in the machine which is worth while considering is that of over-voltages which may be caused by improper control arrangements and result in insulation breakdowns in the motor and in the control.

Let us consider the case of a wound secondary motor with the control arranged to break, first, the secondary circuit and subsequently the primary circuit. In this case the primary switches break nothing but the magnetizing current. The natural tendency of the a-c. arc is of course to rupture when the current has reached zero value, or thereabout, following the regular sine curve. Since with open-secondary circuit the flux is about proportional to the primary current, it may be assumed that the flux at the time of current interruption is zero. If this is the case, no further change of flux has to take place and no over-voltages would be liable to occur. It has been found, however, by tests that the cooling effect of the oil or air upon the interrupting arc makes the current value of the last half cycle die out quicker in line with a curve *A*, Fig. 10, instead of in line with the regular sine curve *B*. Assuming that the flux follows the same curve, we obtain voltage values as indicated by curve *C* instead of those shown by *D* corresponding to a sine flux curve. It will be seen that under the assumptions made an over-voltage of four times the normal crest voltage is obtained. These voltage surges when obtained in practise are of such short duration that their full amplitude cannot be measured by the standard oscillograph tests; voltages up to four times normal voltage have, however, been shown by spark-gap tests.

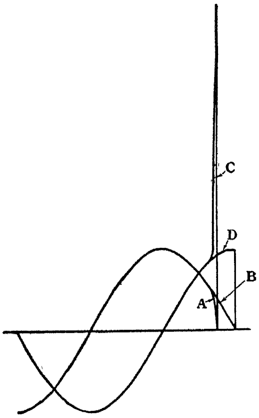

FIG. 10

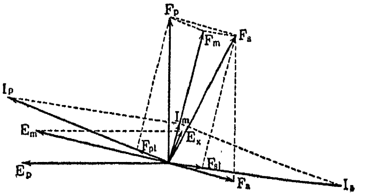

FIG. 12

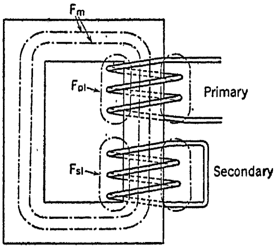

FIG. 11

OVER-VOLTAGES WITH CLOSED-CIRCUIT SECONDARIES

Since, as previously pointed out, the short-circuited secondary has a tendency of preventing any sudden changes in flux, the idea naturally suggests itself to open the primary circuit with the secondary short-circuited, thereby avoiding sudden dying out of the fluxes and over voltages. This is an excellent remedy as long as the rotor fluxes and stator fluxes are practically identical, as for instance, at no load and light loads where the leakage fluxes are practically negligible, as compared to the main flux common to both stator and the rotor.

With heavy overloads, cases are, however, possible where short-circuited rotors are not altogether a safe remedy against over-voltages because the leakage fluxes may be relatively large enough to cause trouble in spite of the favorable damping effect of the rotor. This may be demonstrated by Figs. 11 and 12. An induction motor is in all respects very similar to a transformer except that the leakage fluxes form a larger percentage of the total fluxes. For the purpose of demonstration Fig. 11 shows, therefore, a transformer with the mutual flux F_n, the primary leakage flux F_{pl} and the secondary leakage flux F_{sl}. If now the secondary winding is short-circuited and the primary interrupted, the damping effect of the secondary winding will tend to maintain constant the flux interlinking with the secondary winding. On the other hand, it is evident that the primary leakage flux F_{pl} will die out as soon as the circuit is interrupted. Not only that but by far the largest amount of the secondary leakage flux will choose the easiest path through the primary winding, there being no more counteracting ampere turns flowing in the primary winding and the path through the main iron being of exceedingly small reluctance as compared with the reluctance of the leakage path. It is obvious, therefore, that upon interruption of the primary current we will not only have the primary leakage flux disappear suddenly but add the secondary leakage flux which is practically of opposite direction to the mutual flux in the primary coil. The total flux change in the primary winding is therefore about equivalent to the total leakage and while this amount is only a part of the total flux, the change being opposed only by eddy losses, etc., in the iron takes place so quickly, that voltages of pretty high amplitude may be induced.

The change in flux is further demonstrated by the vector diagram of Fig. 12. The vector E_p may be the impressed primary voltage and I_p the primary current lagging by a certain angle. The primary current sets up, of course, a leakage field in phase with the current which may be represented by the vector F_{pl}. This vector induces a reactance voltage E_x which must be subtracted from the impressed voltage E_p to give the voltage E_m induced by the mutual flux. The vector F_m shifted 90 deg. against this voltage represents the mutual flux. By combining this flux with F_{pl} we obtain the total primary flux F_p. The resultant magnetizing current I_m is, of course, in phase with the mutual flux and the resultant of the primary current I_p and the secondary current I_s. The secondary current I_s sets up the secondary leakage flux F_{sl} which combined with F_m gives the total secondary flux F_s. If the primary circuit is now interrupted and if it is assumed for the moment that the damping effect maintains the secondary flux absolutely constant, and if we further assume that practically all of that flux swings into the primary winding, we see that the primary flux suddenly has to change from the value F_p to the value F_s. This is equivalent to the sudden addition of a flux vector F_a to the vector F_p to give the vector F_s. While this flux F_a which has to be added suddenly is smaller than the primary flux, the rate of change is so much quicker that voltages in excess of the normal primary voltage may be induced in the stator.

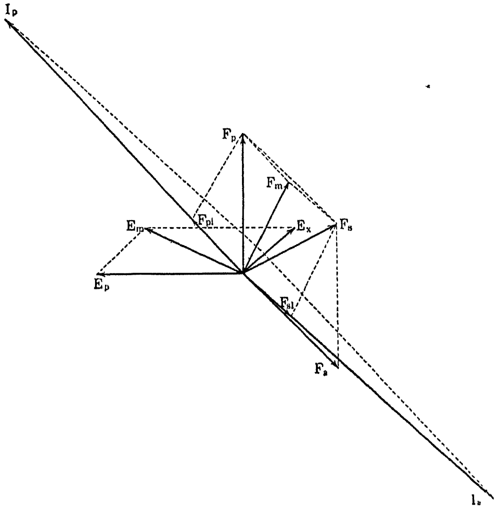

FIG. 13

The case shown in Fig. 12 is, of course, extreme, if a normal load is considered, but the relative sizes as shown may very easily obtain in standard motors with heavy overloads.

While the bad influence of leakage fluxes exists only with severe overloads, such a case is nevertheless of practical importance in so far as it is just in connection with heavy overloads that the primary circuit is frequently interrupted by overload circuit breakers while the motor is pulling out and nearing standstill conditions. Fig. 13 shows a diagram of the vector relation applying to such a case and demonstrates plainly that the sudden flux changes taking place in this case may closely approach values equivalent to the total primary flux.

Most of the oscillograms previously referred to plainly indicate the sudden phase shift of the stator flux when the power is interrupted, see for instance, Figs. 4, 5, 6 and 7.

As previously mentioned, it is not possible to show the full extent of the voltage surges by oscillograph test. Some indication of them is shown, however, in Figs. 14 and 15, which were taken at standstill when the leakage fluxes have their maximum value while the mutual flux is relatively small. The oscillograms show in this case over-voltages up to 70 per cent above the crest value of the normal voltage.

In view of this, it would seem advisable in case of large high-voltage squirrel-cage motors to use circuit breakers which interrupt the current in two or more steps by introducing resistance or inductance into circuit before the final break is made. In case of wound-secondary motors it is good practise to introduce part or all of the starting resistance into the circuit before breaking the primary circuit. This will temporarily reduce the load of the machine, because the motor speed cannot drop quickly enough on account of the rotor inertia, to give at once heavy torques with the increased resistance. Therefore, the load currents and with them the leakage fluxes, are temporarily reduced to a minimum value, and if the primary circuit is opened before the rotor speed has dropped, we have the favorable damping effect of the closed rotor circuit to a certain extent, without the unfavorable effect of large leakage fluxes.

Voltages up to double the normal peak voltage have been tested with the spark gap and appreciably higher voltages seem to be quite likely in certain cases.

The condition just described with short-circuited rotors is most likely to occur in machines with relatively large leakage fluxes, that is, principally in low-speed machines with a large number of poles. Such machines have not only relatively large leakage fluxes, but they are also most likely to pull out under heavy overloads, since they usually have rather small pull-out torques.

It appears further from the previous considerations, that over-voltages are liable to occur if a motor with short-circuited rotor is started by closing the primary switches, and if these switches are opened again, before the motor has assumed normal speed and load conditions. The reopening of starting switches before

PLATE XVIII.
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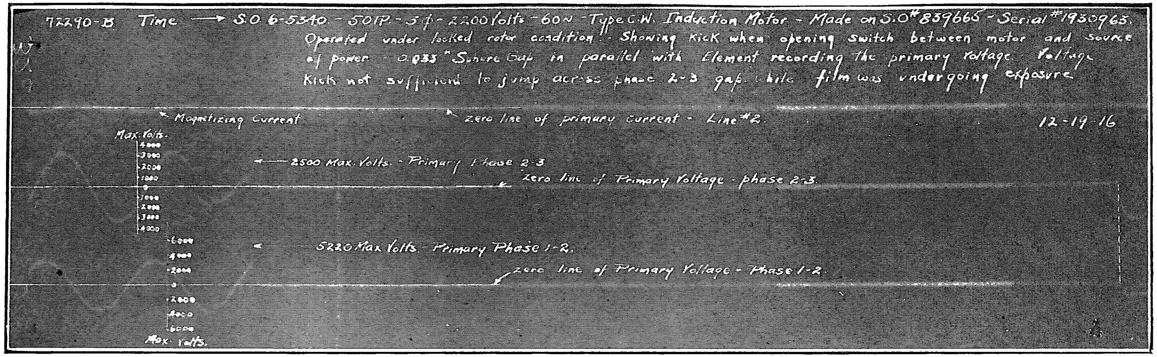
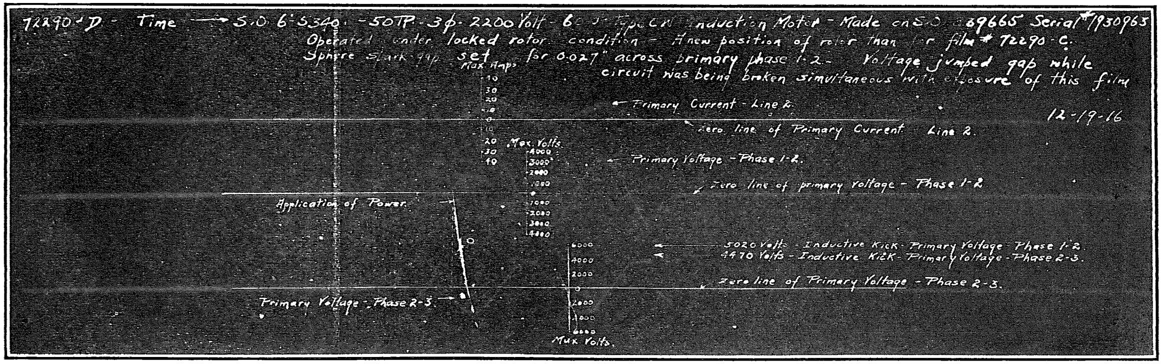


FIG. 14



[HELLMUND]

FIG. 15

the motor has speeded up should, therefore, be avoided and the switches should be designed to be positive in their action and to make bouncing impossible.

Over-Voltages with Over-Synchronous Speeds

Another instance in which over-voltages in the rotor circuit of wound-secondary motors have been experienced, is in the case of two-speed motors when the primary connections for the low speeds were established while the motor was still running at high speed. In closing the primary circuit, two contacts will always close first, in view of the practical impossibility of closing all contacts simultaneously. The motor is, therefore, temporarily running with single-phase stator excitation and double synchronous speed. It may be shown by a simple calculation that the flux in the secondary coils assumes, in this case, a shape similar to those shown in Figs. 16a and 16b, and

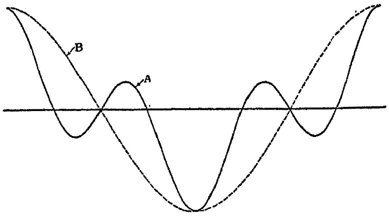


Fig. 16 a

A—Flux Interlinking with Secondary Coil—Single-phase Primary
B—Flux Interlinking with Secondary Coil—Polyphase Primary

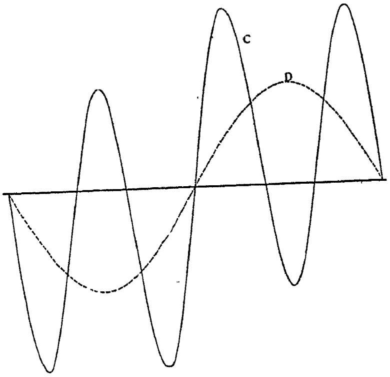


Fig. 16 b

C—Voltage in Secondary with Single-phase Primary
D—Voltage in Secondary with Polyphase Primary

17a and 17b, depending upon the phase position of the coils, and that voltages as shown in the same figures may be induced in the secondary coils. The dotted sine curves of the same figures show the double-speed secondary voltage induced by the polyphase field which is, of course, the same voltage as at standstill for which the insulation of the secondary circuits is usually designed. In other words, we obtain secondary voltages about twice as large as the normal voltage and it is either necessary to design the motors and control for this higher voltage or to avoid it by making arrangements so that the secondary is closed before the primary, which will, on account of the damping effect of the secondary, reduce the rate of change of the fluxes in the secondary coils and thereby the maximum

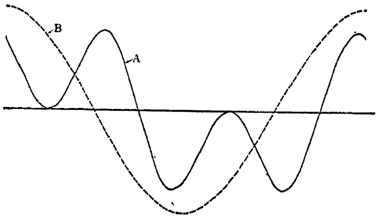


Fig. 17 a
A—Flux Interlinking with Secondary Coil—Single-phase Primary
B—Flux Interlinking with Secondary Coil—Polyphase Primary

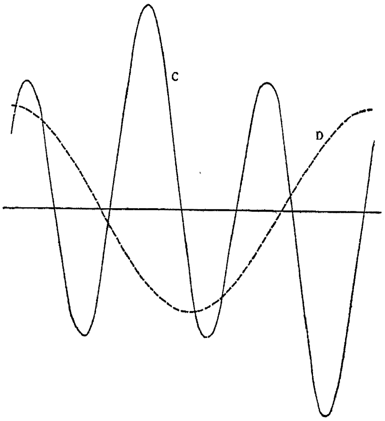


Fig. 17 b
C—Voltage in Secondary with Single-phase Primary
D—Voltage in Secondary with Polyphase Primary

secondary voltages. Over-voltages are not only possible during the short period of single-phase excitation, but also exist at the moment when the third contact of a three-phase motor is closed, because this calls for sudden re-arrangement of the fluxes which may induce high voltage in the secondary. The voltage in the primary is, of course, always limited by the line voltage as soon as the primary switches are closed.

CONCLUSIONS

Other cases where transient conditions of induction machines are of practical importance with regard to the design of control arrangements might be cited, but it is considered that the discussion of the previous examples is sufficient to explain certain difficulties which have been experienced in practise and what points have to be watched in the design of the control, either in order to avoid bad effects by the transient conditions, or to call upon the same conditions for the purpose of avoiding certain other difficulties. As has been repeatedly pointed out, allowance should be made in this connection for the size of machine to be controlled, its inherent characteristics, and the like, as it is possible to take certain risks along the lines discussed in case of small low-voltage motors, while all the points may have to be considered very carefully in connection with other motors, for instance, of large capacity and high-voltage windings.

While practical experiences have taught both control and motor designers, relatively early, to take care of most of the conditions pointed out in this paper, it is believed that a more detailed study of the conditions to be met will lead to further improvements. The induction motor enjoys the just reputation of being the simplest motor requiring the least attention and maintenance of any of the existing motor types, and there is no doubt but that this reputation can only be improved if all characteristics of the motor are more fully understood and always properly taken care of by a harmonious combination of motor and control design.

Presented at the Fifth Midwinter Convention of the American Institute of Electrical Engineers, New York, February 16, 1917.

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PERFORMANCE OF POLYPHASE INDUCTION MOTORS, UNDER UNBALANCED SECONDARY CONDITIONS

BY A. A. GAZDA

ABSTRACT OF PAPER

Continuous operation of the wound-rotor induction motor, when the external resistances in the secondary phases are not equal, is shown to be feasible. The effect upon power factor and heating is discussed. Curves showing the performance of polyphase motors with single-phase secondary are presented. The practical advantages of using unbalanced secondary connections are pointed out.

DURING the past few years, the electrical engineer has been obliged to utilize every possible expedient in order to meet the demand for control apparatus having the fewest number of parts, and simplest connections. It was with this thought, that the following study of the operation of the polyphase induction motor, with unbalanced secondary, was made. No attempt will be made to present a complete mathematical discussion of all the phenomena taking place, but rather, to give a simple explanation of the effect upon the various characteristics, and show the feasibility of continuous operation when the secondary resistance is not equal in all phases.

In order to make the investigation thoroughly practical, a series of tests was made, to cover the following points:

1. The comparative heating of various motors when operated at a constant-torque load, with (*a*) balanced secondary, (*b*) unbalanced secondary, (*c*) single-phase secondary.
2. The effect upon primary and secondary current, power factor and torque, when the secondary is unbalanced.
3. The operating characteristics of a polyphase induction motor, with single-phase secondary.

The usual method of obtaining speed control of induction motors is to vary the secondary resistance. This may be done internally in various ways, but the more common practise is to use a phase-wound secondary with slip rings, whereby external resistance may be inserted in the rotor circuit. This external resistance may be short-circuited in several steps by means of a controller, shown schematically in Fig. 1 A.

Referring to Fig. 1 B, it is evident that, although six switches are used, the resistance will be balanced on only three points, viz., No's 1, 4 and 7.

It has frequently been suggested that the unbalanced points be used for continuous running, but the objections were immediately raised that the secondary currents would be greatly unbalanced, and the heating increased to a prohibitive degree. This view seems logical at first glance, but further analysis shows that it is not justifiable, as there are several other factors to be taken into consideration. These will be discussed in turn.

I. The unbalancing of the secondary currents is not as great as the unbalancing of resistance would indicate. Some vector diagrams, showing typical conditions, will make this clear. Take first the case in which a part of the resistance in one phase is short-circuited. (See Fig. 2). For the sake of simplicity, we will neglect the inductance and resistance of the secondary winding. The former would distort the diagram in such a way as to give a slightly greater difference between the low current and high current, while the latter merely adds a constant value to all phases which tends to balance the currents. It is seen that, although the resistance in one phase is only one-half the value in the other two, the difference between the maximum and minimum current is only 30 per cent. This does not mean that the one phase will carry an overload of 30 per cent., because, under balanced conditions, full-load current would take some intermediate value.

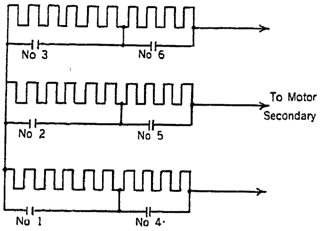

FIG. 1 A

SW	RUN						
1		O	O	O	O	O	O
2			O	O	O	O	O
3				O	O	O	O
4					O	O	O
5						O	O
6							O

FIG. 1 B—SEQUENCE OF SWITCHES

Fig. 3 is similar to Fig. 2, except that now two legs have low resistance, and one high. In this case, the ratio of maximum to minimum current is greater than before. Notwithstanding this, the per cent above normal full-load current is less than in Fig. 2, because two phases are carrying the high current. Another way of looking at this is to say that in all cases, the currents adjust themselves so that the average r.m.s. value of the three currents is approximately equal to the normal balanced full-load current. In Fig. 2, the normal balanced current would be 111 amperes, and the overload in the maximum phase 17.1 per cent. In Fig. 3, the normal balanced current would be 135 amperes and the overload 11.1 per cent.

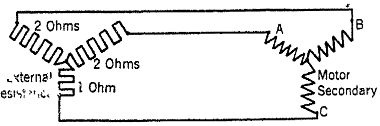

FIG. 2 A

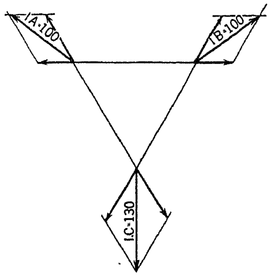

FIG. 2 B

II. The unbalanced secondary currents react upon the primary in such a way as to draw a current from the line at a frequency which is lower than normal line frequency. This phenomenon is caused by the single-phase component of the secondary field, the effect of which, in the extreme case of infinite resistance in one phase, has been explained by Mr. B. G. Lamme.[1] Fig. 4 is an oscillogram, showing this action in the case of a polyphase motor with a single-phase secondary, the motor running at half speed under this condition. Fig. 5 shows the

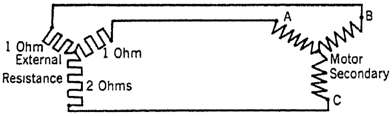

FIG. 3 A

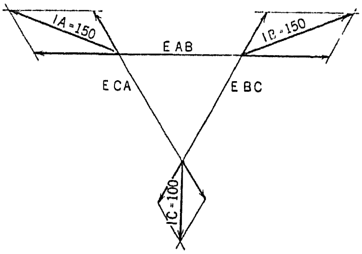

FIG. 3 B

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[1] *Electric Journal*, Vol. XIII, page 394.

corresponding action when the secondary resistance is unbalanced.

The magnitude of this low-frequency current ranges from 10 to 30 per cent of the reactive component of full-load primary current, but it does not materially affect the primary line current, or the power factor of the motor. In the first place, any currents having a frequency different from that of the line voltage, are reactive, *i.e.*, they draw no power from the line[2]. Secondly, currents of different frequencies cannot be combined algebraically.

It was shown by Bedell and Tuttle,[3] that in such a case, in order to give a vectorial representation, the third dimension must be used. Referring to Fig. 6, E represents line voltage and I_1, the current that would flow if no harmonics were present. I_2 then, is the power component, and M_1, the magnetizing, or reactive component. The low-frequency current, due to the unbalanced rotor, may now be represented by M_2 at right angles to both E and M_1, while the resultant magnetizing current will be $\sqrt{M_1^2 + M_2^2}$ or M_3. Finally, the resultant current drawn from the line is $\sqrt{I_2^2 + M_3^2}$ or I_3. Combining the two previous statements, it is seen that $I_3 =$

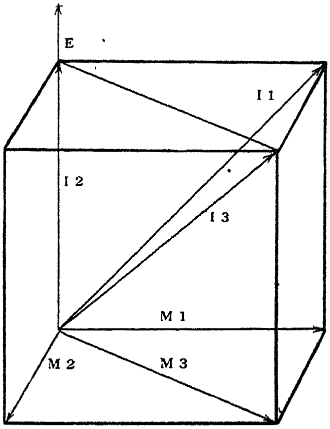

FIG. 6

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[2] This follows from the fact that $\int_0^{\pi} (\sin mt \sin nt \, dt)$ vanishes except for the case where $m = n$.

[3] TRANS. A. I. E. E., 1906., Vol. XXV., page 685.

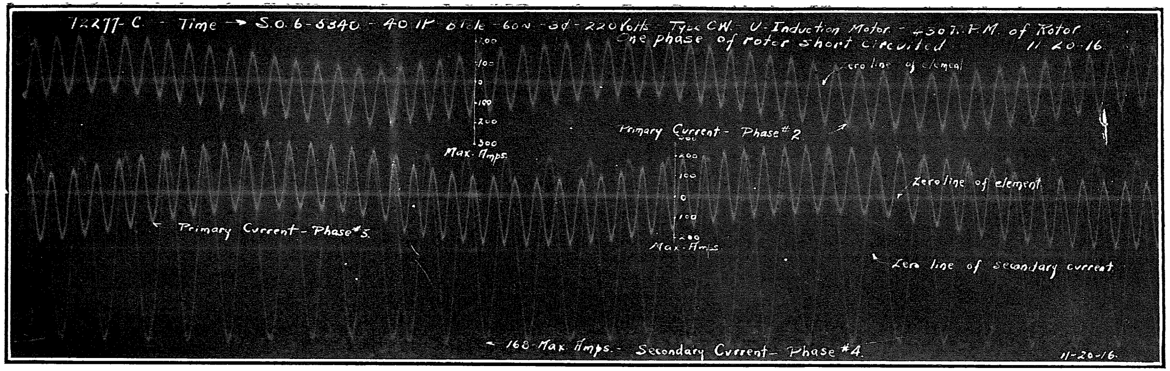
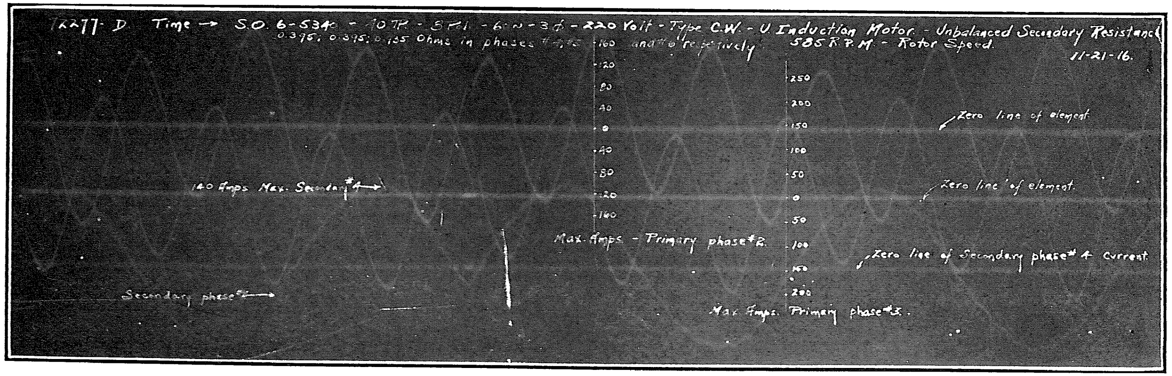


FIG. 4



[GAZDA]

FIG. 5

$\sqrt{I_2 + M_1^2 + M_2^2}$. Hence, if M_2 is as high as 30 per cent of M_1, M_3 will be only 4.5 per cent greater than M_1, which means that the increase in the reactive component, due to unbalancing, will not exceed 5 per cent. Assuming that a motor runs normally at a power factor of 80 per cent, unbalancing the rotor resistance will decrease this value to 78.6 per cent. On the other hand, the increase in line current will be 1.5 per cent under the same condition, and hence, primary copper heating will be 3 per cent greater than under balanced conditions. Since the primary draws just sufficient low-frequency magnetizing current to compensate for the pulsating action of the secondary, the flux conditions remain the same as in the case of a balanced rotor, and the iron losses are not changed.

III. The rating of a variable speed motor depends upon the heating at the lowest running speed, because ventilation is poorest at this speed. Since the fan action of the rotor varies approximately as the square of the speed, the least ventilation will occur on the first notch of the controller, which, in commercial practise, usually connects all the external resistance to the secondary circuit, in a balanced arrangement. Even though succeeding steps cut out the resistance in such a way that unbalanced conditions are obtained, the ventilation is improved so much by the higher speed, that the motor will dissipate considerably more heat, with the same temperature rise, and the rating is not diminished by the slightly increased losses.

IV. As was pointed out in Part I, the currents will not be the same in all phases of the secondary. However, it is not correct to assume that the rating will be limited by the maximum current in any phase. Experience shows that in any rotating apparatus carrying unbalanced load, the heating, broadly speaking, will be proportional to the sum of the squares of the various currents ($I_1^2 + I_2^2 + I_3^2$). This is particularly true of rotor windings. The design limitations will have a bearing on this factor; for instance, if the coils are heavily wrapped with insulation, localized heating may take place. This rarely occurs, because the designer may choose comparatively low voltage for the secondary, since it is independent of power supply. Furthermore, modern mica-insulated coils conduct the heat out of the copper very readily.

V. A large number of tests were made on motors of various sizes, speeds, etc., to check the foregoing conclusions. In the

TABLE I—OPERATION OF 40-H.P., 3-PHASE, 60~, 220-VOLT INDUCTION MOTOR WITH VARIOUS VALUES OF EXTERNAL SECONDARY RESISTANCE.

Resistance in ohms per leg			Primary Volts			Primary Amps.			Equivalent Balanced Pri. Amps. (R.M.S.)	Secondary Amps.			Equivalent Balanced Sec. Amps. (r.m.s.)	Speed in r.p.m.	Pound Feet Torque	Power Factor
0.81	0.81	0.81	220	218	220	114.8	109.2	110.8	111.8	77.2	76.8	76	76.7	415	242	78.6
0.81	0.81	0.39	220	218	220	117.7	110	114	114	64.4	90.1	79.2	78.6	520	242	77.15
0.39	0.39	0.81	220	218	220	117	110	114	113.8	90.6	62.3	79.5	78.4	600	242	77.5
0.39	0.39	0.39	220	218	220	111.7	104	110	108.7	75.2	75.2	74.8	75.15	660	242	79.7
0.13	0.13	0.39	220	218	220	114	108	112	111.4	90.7	55.4	81.7	77.2	768	242	78.4
0.39	0.39	0.13	220	218	220	114	108	111	111.2	63	92.6	72.4	77.1	723	242	78.2

load runs, particular attention was paid to the rotor copper, to detect any temperature rises greater than normal. The uniform heating of the rotor was demonstrated by both thermometer and resistance readings. The total secondary copper loss was found to run 5 to 10 per cent higher under unbalanced than under balanced conditions. This bears out the statement that $I_1^2 + I_2^2 + I_3^2 = 3\,I_n^2$, where I_1, I_2 and I_3 represent the unbalanced currents, while I_n is the normal balanced current. (See Table I.) In a series of load runs with constant torque, using a controller which gave unbalanced points, it was found that the temperature rise, with the first one or two steps of resistance cut out, was the same as on the first balanced point, while at still higher speeds, the rise was less, even though the secondary was unbalanced. Test fully justified the statements made in Part II, in regard to primary current and power factor. Very accurate readings had to be taken in order to check the small increase in current. Although it was found that the degree of unbalancing had some effect on the operation of the motor, nevertheless, with the unbalancing that is common in standard commercial controllers, the operating characteristics are well within the limits stated previously. The per cent of no-load magnetizing current in terms of full-load current, had a distinct influence upon the increase in primary and secondary currents.

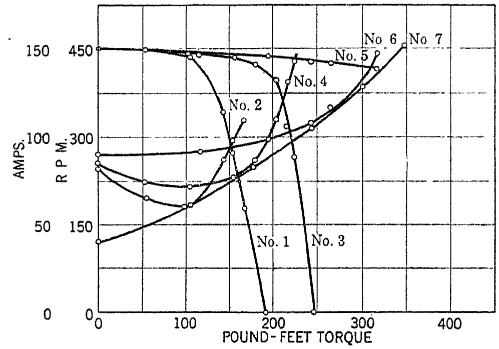


FIG. 7—FORTY-H.P., THREE-PHASE, 60-CYCLE, 220-VOLT, 8-POLE INDUCTION MOTOR—SINGLE-PHASE ROTOR

Curve 1—Speed—torque—1.6 ohms resistance in one phase of rotor
Curve 2—Primary amperes—torque—1.6 ohms resistance in one phase of rotor
Curve 3—Speed—torque—0.8 ohms resistance in one phase of rotor
Curve 4—Primary amperes—torque—0.8 ohms resistance in one phase of rotor
Curve 5—Speed—torque—one phase of rotor short-circuited
Curve 6—Primary amperes—torque—one phase of rotor short-circuited
Curve 7—Primary amperes—torque—three phases of rotor short-circuited

In general, the motors with small no-load magnetizing current, showed the best performance.

Tests were made to observe the operation of polyphase motors running with single-phase secondary. Under these conditions, the starting torque was found to be about 60 per cent of the torque with balanced conditions, the current in the secondary winding being the same in each case. With this connection, an economical method of continuous operation at half speed is obtained. An important advantage of this method is that a stable speed is maintained, independent of the load. This would mean a great saving in resistors, particularly where half speed is desired at reduced torque, as in fan or centrifugal pump application. The no-load magnetizing current is a very important factor in this case, and must be taken into account. For example, small low-speed motors may draw excessive currents with one phase of the secondary short-circuited, or if too much external resistance is inserted in the single-phase secondary, a stable condition will not be obtained at half speed. Fig. 7 shows the torque and current characteristics of a 40-h.p., 3-phase, 60-cycle, 8-pole, 220-volt induction motor, with wound secondary, when one phase of the secondary is open-circuited. These curves are plotted from test results, and perhaps are the first to show the effect of secondary resistance, under the given conditions. At no load the primary magnetizing current is approximately twice the amount required under normal bal-

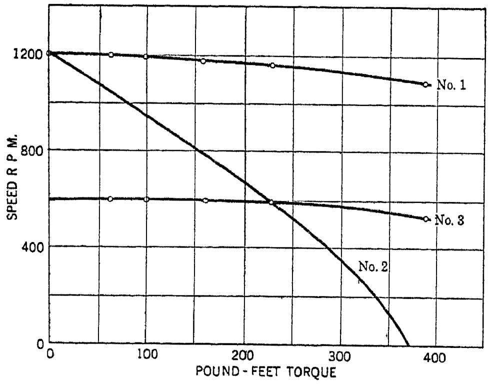


FIG. 8 A—TORQUE CHARACTERISTICS OF 50-H. P., THREE-PHASE, 60-CYCLE, 220-VOLT, 6-POLE INDUCTION MOTOR

Curve 1—Speed—torque—short-circuited rotor
Curve 2—Speed—torque—balanced secondary resistance
Curve 3—Speed—torque—single-phase rotor

anced conditions, but as the load increases, an apparent paradox takes place, in that the line current decreases to a certain minimum value before any increase takes place.

Curves in Fig. 8A and 8B show comparative characteristics of a 50-h.p. motor, (1) when operating with single-phase secondary, (2) when operating with 3-phase balanced resistance, to give half speed at full-load torque. The reduced power factor under the single-phase secondary condition is due to the great decrease in power drawn from the line, while the reactive component remains practically constant. Note that at full-load torque, the power input required is reduced almost 50 per cent. These curves show that it is quite possible to obtain a half-speed running point by this method.

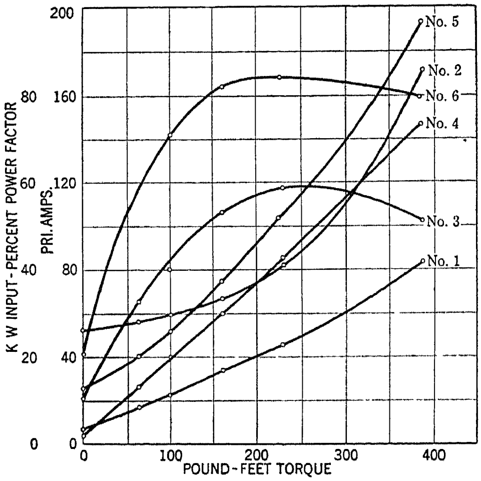


FIG. 8 B—TORQUE CHARACTERISTICS OF 50-H.P., THREE-PHASE, 60-CYCLE, 220-VOLT, 6-POLE INDUCTION MOTOR

Curve 1—Input—torque—single-phase rotor
Curve 2—Primary amperes—torque—single-phase rotor
Curve 3—Power factor—torque—single-phase rotor
Curve 4—Input—torque—polyphase rotor
Curve 5—Primary amperes—torque—polyphase rotor
Curve 6—Power factor—torque—polyphase rotor

CONCLUSION

The practical significance of this investigation may be summarized as follows:

1. With the proper design of resistor steps, it is safe practise to operate polyphase induction motors at full-load torque, with

unbalanced secondary resistance. Under these conditions, the power factor will be slightly reduced, but the increased copper loss will be amply compensated for by the better ventilation of the motor at the higher speeds.

2. A limited range of motors which have a low percentage of magnetizing current, may be operated under full-load torque at half speed, with single-phase secondary, while practically any motor may be operated under these conditions when connected to a variable torque load, such as blowers, centrifugal pumps, etc.

By taking advantage of the principles brought out in this paper, it will be possible to obtain much simpler control apparatus than has been common in the past. Control schemes, including single-pole resistance switches, may be used, which will result in fewer parts to maintain, less control wiring, and smaller size controller. The amount of resistance may be reduced, and floor space saved.

DISCUSSION ON "TRANSIENT CONDITIONS IN ASYNCHRONOUS INDUCTION MACHINES AND THEIR RELATION TO CONTROL" (HELLMUND) AND "PERFORMANCE OF POLYPHASE INDUCTION MOTORS UNDER UNBALANCED SECONDARY CONDITIONS" (GAZDA), NEW YORK, FEBRUARY 16, 1917.

W. B. Kouwenhoven: I should like to ask Mr. Gazda to explain the operation of the polyphase induction motor with a single-phase secondary at half speed. The paper does not make it clear how this half-speed characteristic is obtained from the ordinary induction motor. I have known of cases where one of the secondary windings burned out, or opened circuited, and the machine kept on running at full speed.

A. A. Gazda: Let curve 1, Fig. 1, represent the normal speed-torque curve of a polyphase wound-rotor induction motor. If one secondary terminal is disconnected, the resultant single-phase secondary winding sets up an unbalanced e.m.f. that reacts upon the primary and causes the current to flow into it through the circuit completed by the transformer or generators, to which the motor may be connected. If f represents the line frequency and s slip frequency, the frequency of the current, which is pumped back into the line, will be $2s - f$. It is evident that at half speed, this frequency and therefore the current will be zero. Below that point, a motor action will take place, and above a generator action. Curve 2 shows the component torque, as set up by the action of the single-phase secondary. The resultant external torque of the motor is shown in Curve 3. The motor will have two pull-out points, so that when it is operating at high speed under this condition, it will carry a load which will approach the normal pull-out point more or less closely, depending upon the characteristics of the particular

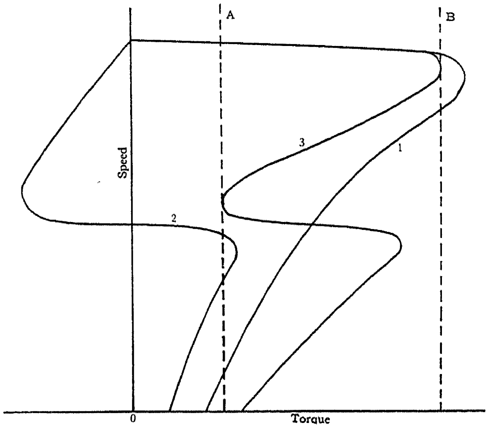


FIG. 1

motor. In this case, the secondary windings will take more current than in the normal three-phase motor because only two-thirds of the windings are working and it will give about 60 per cent starting torque. Line *A* of Fig. 1 represents the minimum load which the motor will hold at half speed. This line may coincide with zero torque or it may not. I have found with some motors that if there was high resistance in the secondary, this would not come back to zero; that is, with a small load, say 10 per cent of full load, the motor would go right through the half-speed point and run at full speed, but in the case of larger motors, stable operation will be obtained at half speed from zero torque to some pull-out point which is less than the normal pull-out point of the motor with polyphase secondary.

R. B. Williamson: There is one feature in connection with the unbalanced secondary resistance to which I would like to call attention. In case a motor operates on the same circuit with lights, the pulsations caused by the unbalanced secondary currents may be sufficient to cause undesirable fluctuations in the lights due to periodic variations in line drop. I have in mind one case where this has occurred and where it will probably be necessary to change to the balanced type of secondary resistance.

H. Maxwell: It has long been the practise to furnish drum controllers with collector-ring type of induction motors so arranged as to cut out the steps of resistance in the different phases of the secondary in successive order. It is very interesting to have results of actual tests and mathematical deductions therefrom, which prove that the unbalancing thus produced does not materially increase the heating of the motor.

Mr. Gazda mentions the possibility of utilizing the stable half-speed which can be obtained with a low resistance single-phase rotor. There is one very serious objection to operating an ordinary induction motor with one phase of the rotor open, which should be pointed out.

An operator occasionally switches a loaded motor on or off the circuit when it is running near synchronous speed, and as contact is not made simultaneously in all phases of the primary the motor frequently operates for a few cycles with single-phase primary circuit. If the rotor also has a single-phase secondary circuit during this period, a very high voltage is induced in the open phase of the secondary.

We took some oscillograms of this induced voltage a few years ago on a 10-pole, 60-cycle, 75-h.p. motor of standard design, and found it to be an alternating wave of approximate double-line frequency with a very sharp peak in one direction reaching a maximum value seven times the normal standstill open-circuit voltage of the rotor.

The explanation of this lies in the fact that with a single-phase primary and single-phase secondary there are certain positions of the rotor where the m.m.f. of the load currents in the primary and secondary coils do not directly oppose each other, but combine to produce a very high resultant value of m.m.f. forcing a flux sufficient to saturate the magnetic circuit at right angles to the main flux. This peak of the flux is enclosed by the open phase of the secondary and its high value coupled with the double frequency of the secondary produce the very high induced voltage.

Campbell Macmillan: The case of polyphase motors with single-phase secondary resistance has been analyzed by Arnold in his book on induction motors (page 186). The formulas given can be confirmed by tests on commercial motors within close limits.

The torque curve when plotted against slip is analogous to the curve for a cascade-connected set consisting of two similar motors. It consists of a motor portion with synchronous conditions at one-half the synchronous speed of the normal motor. For some distance above half speed the torque is negative but changes again to positive before reaching full synchronous speed. By inserting a large resistance in the third phase of the rotor and gradually reducing it, the curve is transformed by the elimination of the negative torque into the normal curve of the motor with balanced polyphase rotor.

Until the balanced condition is approximately attained there is a dip in the torque curve near half speed, and with most types of load, this dip will cause the speed to be unstable in the region just above half speed and with certain types of load, hunting actually occurs. The amount of unbalance which will give rise to this condition for any given type of driven load will depend on the extent to which the iron of the motor is saturated.

The cascade characteristics of single-phase rotor operation are indicated in the following figures for a standard 6-pole, 25-h.p. motor.

	Three-phase rotor	Single-phase rotor operation	Ratio
Current running light at rated e.m.f. (220 volt).	19.2 A (1200 r.p.m.)	40.3 (600 r.p.m.)	2.1
Current at standstill (220 volts) (tested at lower e.m.f. and assumed proportional)............	360 A	254	0.707
Maximum torque (220 volts)...............	326 lb. at 1 ft.	282 lb. at 1 ft.	0.866
Slip at maximum torque	22 per cent	12.6 per cent	0.573
Starting torque........	160 lb. at 1 ft.	160 lb. at 1 ft.	1.00

In order to follow the operation of the motor with single-phase rotor it is convenient to consider the single-phase rotor current as equivalent to two systems of balanced polyphase currents of half maximum amplitude rotating in opposite directions relatively to the rotor. One of these locks with the primary rotating field, the other acts as primary system for which the stator short-circuited through the supply system acts as a secondary system for currents of all frequencies other than that of the generator. These currents are not strictly reactive although they draw no further energy from the line. On the contrary by the absorption of energy they contribute to the torque of the motor.

A. E. Averrett: A short time ago I made some tests similar to those Mr. Gazda has outlined, on a rather small motor, a 5-h.p., with the secondary resistance rather high, and obtained some rather unusual results. The three-phase torque was about like curve *A* Fig. 2. The single-phase rotor torque started a little below, ran through half speed, down, and came back like curve *B*; the cascade action when above half synchronous speed seemed to take out a portion of the single-phase torque. It showed 140 lb. for the three-phase, 130 lb. for the single-phase, 160 lb. was the maximum, and 300 lb. negative torque, which is very unusual. This motor, as you can see by the shape of the torque curve, was of rather high resistance. We used to make it a practise in our testing department to start a 5-h.p. induction slip-ring motor with single-phase secondary, and sometimes they would run to speed and sometimes would not. It was almost entirely a question of brush contact resistance. In a motor of perhaps 10 h.p. the carbon-brush contact resistance would be enough to eliminate the dip but in a motor of perhaps 100 h.p. we could not reach full speed unless we ran all three phases. We continued this test by adding in resistance and

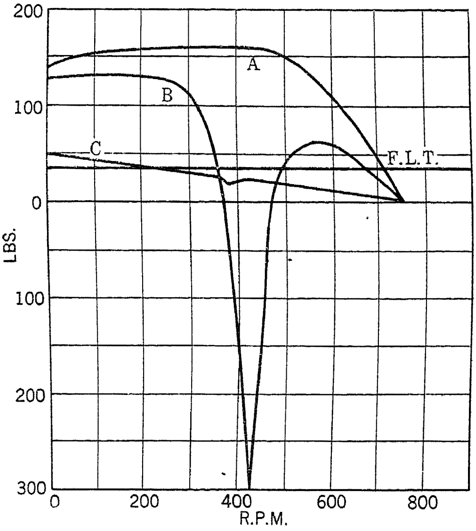


FIG. 2

starting at a lower value torque, and we obtained a curve that came down like curve *C*. By putting in secondary resistance to give full-load torque, as a three-phase motor, or half full-load torque single-phase motor, it would come almost to a straight line, so that it seems to be safe to start the motor at half-load torque, single-phase secondary and run it up to speed; that is, it would be safe if it were not for the danger that Mr. Maxwell speaks of, a rise in secondary potential, but for small motors that would not be serious,—for motors of perhaps 20 h.p.to 30 h.p.

R. E. Hellmund: Mr. Gazda has treated in his paper a subject of rather marked practical importance, namely, the use of unbalanced resistances in the secondary of induction motors, and has shown that the unbalancing of the currents is not appreciable in most cases. He has given two specific cases in Figs. 2 and 3 of his paper. and vector diagrams relating thereto, Since a knowledge of the unbalanced conditions over the entire range used in practise may be of interest, I have worked out a simple diagram for this purpose and a curve showing the results obtained therefrom.

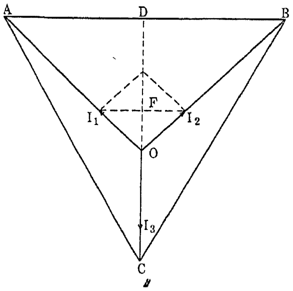


Fig. 3

If we assume that the primary impressed voltages are of a true polyphase system with balanced voltage and correct phase relation, it follows at once that the primary field must be a fairly uniform rotating field no matter what happens in the secondary, because such a field is necessary to induce the proper counter e.m.f. in the primary. It is, of course, further assumed in this connection that the ohmic and reactive drops in the primary are small, and negligible. Since the primary fields must be fairly uniform, and if we further neglect the inductive effect of the secondary, it is at once evident that no matter what the speed, currents and resistances of the secondary are, we will always induce fairly balanced three-phase voltages in the secondary. The voltages between phases will, therefore, be represented by the 60 deg. triangle *A B C* of Fig. 3. Since in practically all control systems two of the secondary resistances are balanced while the third resistance may be different, it is at once evident that two of the currents and two of the voltage drops in the reactances are alike. The neutral point of the resistances must, therefore, always be located upon the perpendicular *C D* upon one of the sides *A B* of the triangle. If we assume any point *O* on this perpendicular, we know at once that the lines *O A*, *O B* and *O C*, must represent the ohmic drop in the resistances. If we further assume any current *I*3 in the one resistance, we know at once that the sum of the two other currents combined must be equal and opposite to this current, that is, we can find the other current by making *O F* equal to one-half of $O I_3$ and find the intersection point of a line parallel to $A B$ through F and the lines $O A$ and $O B$. Having the ohmic drops and the currents, it is easy to find the corresponding resistances for each leg. The curve, Fig. 4, was obtained in in this manner by assuming the point O at different locations between C and D. The curve shows that for the usual range of unbalanced resistances the increase of current over the equivalent balanced current is not appreciable, and indicates that the increased losses are in most cases very small, as pointed out in Mr. Gazda's paper. It should not be overlooked that the figures given apply only if the ohmic drops are relatively large as compared with the reactive drops, so that the reactive drops can be neglected. This is only the case when the external resistance is appreciable as compared with inductance. With the external resistance approaching zero, conditions are essentially changed from those given in the diagram and curves.

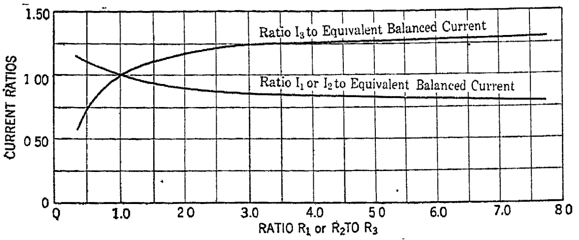


FIG. 4

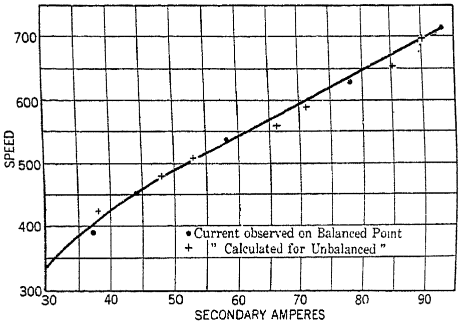


FIG. 5—SPEED-AMPERE CURVE

Arthur Simon: I was glad to see that Mr. Hellmund has given a method of calculating the currents which actually flow in the unbalanced resistance, or in the unbalanced phases, due to the unbalanced resistance in the induction motor. This method can be extended to a system under which all three currents have different values. We have used such a system for several years, and we find that it is only, obviously, an approximate method. If the first approximation does not seem to be sufficiently accurate, we can introduce a second approximation, and that usually will lead to results which are more than sufficiently close to actual conditions. By using this method, we can precalculate the speed curves of an induction motor very carefully. Fig. 5 shows the behavior of an induction motor under unbalanced conditions, having the resistance in all three phases different. The actual speeds observed under operating conditions at no point vary more than two and half per cent from the calculated speed.

I wish to point out what perhaps may be an error. It is not very serious, but it misled me, and it took me a long time to understand Table I in Mr. Gazda's paper. You will find the first three columns give the resistance in the rotor circuit. I drew the conclusion that the secondary amperes which are given in three columns about the middle of the page, fell in the same rotation as the resistance, that is, the first column of currents belonged to the first column of resistance and so on, but apparently that is not the case. The second column of currents apparently corresponds to the third column of resistance. If you analyze the table that way, it seems to check up.

A. A. Gazda: Mr. Hellmund has brought to light some actions, concerning which very little data have been available. The current peaks, obtained when the line voltage is interrupted for only a few cycles, give a hint as to the reasons for violent explosions of switches, which have occurred in the past, particularly when changing connections on rotating induction motors. I was interested to know what maximum possible peaks could be obtained under this condition. On account of the numerous variables involved, it is practically out of the question to get the worst case by means of oscillographs. The value of the applied voltage wave and the phase relation of the counter e.m.f. to it, are the two main factors. In addition to these may be mentioned the external load, the number of cycles of voltage interruption, the stored energy and dissipating constants, as brought out by Mr. Hellmund. By making some assumptions in regard to these factors, it seems that under the worst possible conditions, a peak current of approximately four times locked-motor current may be obtained.

In Figs. 6 and 7, instantaneous values of line voltage, current and flux are shown, and in Fig. 8, the motor or counter voltage is shown in addition. As has been brought out, we will assume that it takes several cycles to build up the main motor flux; hence, only leakage fluxes will be set up and the flux curves in the diagrams refer to these. The effect of motor resistance is small and may be neglected. With the dead motor running at full speed, the line switch is closed on the maximum point of the line voltage wave. The rate of change of flux is maximum at this point and a symmetrical wave of flux and current is obtained, the maximum value of the latter being equal to the motor current under locked conditions. Fig. 6 illustrates this case. In Fig. 7 the line switch closes at the zero point of the voltage wave and the rate of change of flux must be positive (or negative, as the case happens to be) for half a cycle. Remembering that at zero time the flux was zero, it is seen that an unsymmetrical peak of about twice locked current, will flow. Now, take the case in which the motor voltage has not decreased appreciably during the switching process; assume that the impressed voltage is equal and opposed to the motor voltage and the switch closes on the zero point of the wave. The flux now starts at zero and must balance double line voltage over a period of half a cycle. In comparison with Fig. 7, evidently four times normal flux must be set up with a corresponding unsymmetrical current peak of four times normal locked value. This is shown in Fig. 8. Other factors, such as resistance, building up of the main field, and decay of motor voltage, may prevent the practical realization of this large current, but at all events, it gives an indication of the limiting or worst condition.

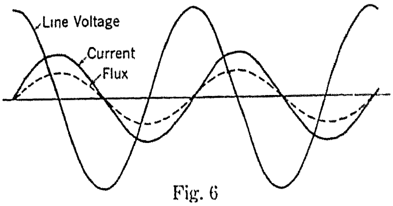


Fig. 6

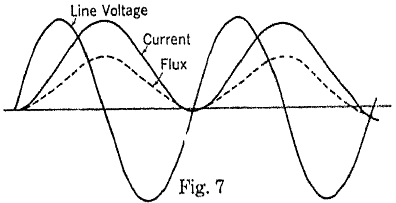


Fig. 7

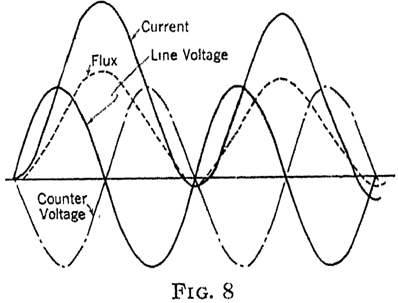


FIG. 8

H. D. James: We have been starting induction motors for twenty years and have experienced little trouble from the causes set forth in this paper. This is largely due to the fact that the phenomena described by Mr. Hellmund are not of sufficient magnitude in small motors, or even large low-speed motors, to cause serious difficulty. Several accidents have been observed in starting large two and four-pole squirrel-cage induction motors. At first, these troubles were attributed simply to an accident, but the fact that they reoccurred several times under similar conditions, indicated that something was wrong with the method of handling the motor. The investigation that was made resulted in the paper which Mr. Hellmund has presented.

This paper is of particular significance in connection with turbo-blowers and similar applications where large high-speed squirrel-cage induction motors are used. It has not been until recently that motors of this kind have been placed on the market. The demand, however, is rapidly increasing and it will be well for engineers to note the particular features brought out by Mr. Hellmund.

This emphasizes the necessity of the engineer designing the motor being familiar with the controller design also. There is little danger of trouble if the conditions are understood and the motor and controller designed to avoid the difficulties brought out in the paper.

Mr. Gazda's paper brings out the possibility of operating a slip-ring motor at half speed, where the motor is driving a fan or blower. The tendency of the motor to run at practically constant speed over a considerable variation of torque is a very desirable feature, as it is sometimes difficult to predetermine the exact torque of the fan when operating at half speed. Very frequently, the resistor has to be changed after the fan has been installed, in order to get the proper speed.

It has been pointed out that surges may take place with single-phase secondary operation, due to the reaction of a single-phase primary on a single-phase secondary. I believe that this can be easily avoided by introducing resistance into the secondary circuit before the primary switch is opened.

A. E. Averrett: I happen to recall a little incident related to Mr. Hellmund's subject. Several years ago we had a large induction motor, an 18-pole machine, and through some accident, there was force enough exerted to bend the ends of the barrel winding clear back against the end flanges. No one could explain it. Undoubtedly it was something of this same type. It has been known for some time that at the instant of throwing in transformers there is a heavy surge of current. With the tendency of economically designed induction motors to reach high saturation, the exciting current required comes up very much faster than proportional to the magnetization and it takes several times current to get twice the magnetism; under these conditions an enormous surge of current for a cycle or two may be produced. I think that is what occurred in this particular motor. It was simply electro-magnetic force bending the conductors back against the frame, and ruining the motor windings.

H. D. James: It has been the general opinion of engineers that the maximum current taken by an induction motor will not exceed the lock current of that motor. This paper shows that the maximum current may be at least four times this value. In starting an induction motor, it is the usual practise to connect the motor to low voltage, usually obtained from auto transformers. After the motor has accelerated to its maximum speed on this voltage, it is disconnected from the transformers and connected to the full line voltage. In passing from the starting to the running position, the motor is disconnected entirely from the line, and it may be reconnected with the phases 180 deg. out of step. This would give a very severe rush of current.

R. E. Hellmund: I would like to add a few remarks with regard to the best combination of high-voltage motors and control, which I have touched upon only briefly in my paper. We know that an oil switch is one of the best means for interrupting a current quickly, and it is a switch that does not need much renewal of contacts. But in looking at the question of over voltages in the motor, it may be found that the switch which takes the best care of itself is not by any means the best switch for the motor. It is an old practise to interrupt a d-c. machine field circuit slowly, possibly by drawing out a long arc. A similar practise should perhaps be considered in connection with induction motors. Among other things the use of air switches for high-voltage motors is worth while considering. While an air switch may not be as good a switch as an oil switch, it might greatly assist in preserving the insulation of the motor.

In this connection I wish to mention that the little figure I have given, (Fig. 10), to demonstrate how an over-voltage may be induced, is only given as an example, and does not by any means cover this field. Investigations have shown for instance that a circuit-opening arc in oil will break off and then suddenly break through the oil again, and in doing so will build up the current and field very quickly. The same investigations have shown that over voltages frequently occur when this happens. Studies along such lines should be useful in bringing about the best all around combination of high-voltage motors and their control apparatus.

A. A. Gazda: Mr. Williamson asked a question in regard to the effect of pulsating current in the primary. I can say that in most cases it will not have any effect upon the power system. Trouble may arise when the size of the motor is comparable with the size of the generator. The primary of the motor acts as a secondary for the pulsating current, and it pumps back through a circuit, which consists of the network, but in general the resistance of the circuit on which it is feeding is very much less than the resistance or inductance of the motor windings itself; therefore, you have practically a short circuit which wipes out all effect upon apparatus on the line.

The test data that were presented, giving the results of actual tests on different sized motors, are very interesting and I believe that they corroborate the conclusions in the paper.

Mr. Maxwell brought out the point that high transient voltages have been recorded when switching takes place on a polyphase motor running with a single-phase secondary. With ordinary connections of controllers, the single-phase secondary connection is made when the motor is at a standstill. Under this condition voltage surges will not be obtained when the primary switches are closed, even though two should close before the third. On the other hand, the way of preventing high voltage when the motor is running would be to arrange the controllers so that when the low speed, or half speed is desired, it is necessary to come to the off position first, because in stepping back from a three-phase to a single-phase secondary, the motor may continue to run near synchronous speed. Returning to the off position and starting all over again will eliminate any danger of the high voltage, and prevent the accidental operation of the motor at high speed with single-phase secondary.

In regard to hunting, and the action of different motors under different conditions, we are making a detailed investigation of the effect of all the variables, including primary resistance and reactance. This has not been completed as yet and I am not prepared to say what will happen in every case. In a motor which has a high primary resistance, the unbalanced voltage reacting upon the primary will create a greater torque than if the resistance is very low and give more stable operating characteristics at the half-speed point.

I am very glad that Mr. Simon has a method of calculating the unbalanced current, and that he will give us the benefit of his investigations. Mr. Hansen, whose paper was presented this morning and I have independently derived methods of calculating unbalanced current. The methods are based on a very simple fundamental law; that is, we assume that the inductance and the resistance of the motor are known, and then add the external resistance to each phase, finally treating the problem as if a balanced voltage was applied to a three-phase circuit with a variable amount of resistance and inductance in each phase. The method has been found to give a fairly accurate predetermination of the speed-torque curve, and of the secondary currents. The greatest difficulty we have had is to determine the actual amount of resistance there is in the circuit. For instance, we found that the resistance of the external circuit varied with the frequency, and we could not take the normal resistance as obtained with d-c. measurements, as being the true resistance when alternating current is applied to it.

Presented at the 330*th meeting of the American Institute of Electrical Engineers, Chicago, Ill., March* 9, 1917.

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RELAYS FOR HIGH-TENSION LINES

BY PHILIP TORCHIO

Abstract of Paper

The author considers the following arrangements desirable: For *radial feeders not in parallel*—inverse-time-limit overload relays, with selective ground relays. *Radial feeders in parallel*—reverse-power relays in combination with reverse-time-element relays, or the balanced system. *Single lines in tandem*—definite and inverse-time-limit relays with progressively longer time settings near the generating station. *Two or more parallel lines in tandem*—inverse-time-limit relays at the points where the power leaves the station or substation, and overload relays in combination with reverse-power relays at the points where the power enters the substation; also, the balanced system preferable, when possible. *Single line ring systems*—inverse-time-limit relays, combined with reverse-energy relays, or, preferably, the balanced system. *Parallel line ring systems*—inverse-time-limit relays on the outgoing feeders at the generating stations, and a balanced interconnected combination of reverse-energy and inverse-time-limit relays at the receiving ends of the supply feeders. *Tie lines*—some balanced arrangement. *Generator relays* seldom used, but arrangements are suggested. *Transformers*—differential relays. Special grounding relays for *extra high-tension cable feeders.*

Appendices A, B, C describe connections of reverse-power relays, relay testing and a special installation.

I. Introduction

During the past two or three years important papers and discussions have been published describing the latest developments and operating results regarding relays for high-tension lines.

These developments have narrowed down to certain specific arrangements based on different fundamental principles and each requiring characteristically different types of relays. I shall not here describe the structural and operating details of all these types of relays. Full descriptions can be found in the references and other literature. I shall, however, attempt to review the most prominent systems of relay protection and their fields of application, pointing out for a few practical cases the combinations used to obtain the maximum degree of protection with minimum expenditure, especially for the conditions of existing lines.

II. DEFINITIONS

Certain terms and expressions have assumed, in practise, specific meanings. Those used in this paper are defined as follows:

Radial feeders or independent feeders. Lines transmitting power from a generating station (or main source of supply) to a substation or a customer's installation.

Radial feeders are operated (or not operated) *in parallel* at the substations according to whether their high-tension terminals are *metallically* connected to the same high-tension bus bars or to independent high-tension bus bars (which may feed, through transformers, the same set of low-tension bus bars).

Lines in tandem—feeders from a generating station supplying two or more substations in series.

Lines connected in ring, or *closed feeder circuits*—two or more substations connected in series with the two outside substations, each connected by separate feeder or feeders to the generating station.

Tie lines—connections between generating stations or between substations not in the order of the ring. The latter are often called *connectors.*

III. RELAY PROTECTION

The object of relay protection is to disconnect any part or section of the system on which a fault occurs, but leave the rest of the system in operation without being further affected by the faulty section.

It is to be noted that the function of the relay is not any longer correlated to the idea of protection of apparatus against overloads. The relay is intended to operate only on breakdowns and not on overloads, though its setting is usually given in per cent overload of the rated capacity of the circuit.

IV. RADIAL FEEDERS NOT IN PARALLEL

For radial feeders not operated in parallel at the substation, overload relays of the inverse-time-limit type have generally and successfully been used in this country. Fig. 1 shows a one-line diagram together with the relay connections and relay characteristics that one of the large operating companies has used successfully.

In addition, for over ten years several large companies have operated a system of selective-ground relays to indicate a ground on the affected feeder. * The long reliable experience which this system has given seems to warrant further applications in this country; extensive use has already been made abroad in the past five or six years. † These relays should be particularly valuable for high-tension service customers in conjunction with overload relays.

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*TRANS. A. I. E. E., 1903, Vol. xxi, p. 417.

Generally speaking, radial feeders not operated in parallel at the substation can be protected with overload and selective-ground relays and, on large systems, protective feeder reactors to obtain 100 per cent protection to the system.

V. Radial Feeders in Parallel

Radial feeders operated in parallel at the substations present greater difficulty for their relay protection. The best practise utilizes two different methods, namely: (a) the reverse-power relay in combination with overload relays and (b) the balanced or Merz-Price system. The balanced system is one where the current, when flowing in the same direction and of the same relative magnitude, in parallel conductors or in different portions along the length of the same conductor, will not produce any force on the relay even under heavy short-circuit conditions. The relays are only operated when the currents are of different relative magnitude or direction at these points.

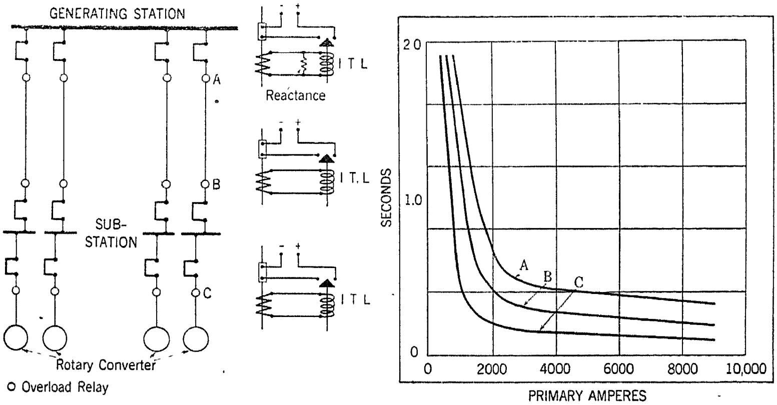

Fig 1

Radial feeders not paralleled—one-line diagram

(a) *Reverse-power relays* have, in isolated cases, given very satisfactory results. The recent developments in the induction-type reverse-energy relays have made it possible to use this form of protection more extensively as these relays are made so that they will properly select down to ½ per cent voltage.

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†The Journal of the Institution of Electrical Engineers (London) p. 159, Jan. 15, 1915.

In cases where a number of feeders are paralleled in the substation, a combination of balanced and reverse-energy relay connections, as shown in Fig. 2 gives very satisfactory results.*

Appendix C outlines the plans of the New York Edison Company for two of its principal substations where this combination is used in connection with selective-ground relays.

(b) The *balanced system* of which the *split-conductor system* is an illustration, has given such satisfactory results abroad that it has lately been introduced in this country by several large companies, like the Boston Edison Company and the Public Service Electric Company of New Jersey. It is especially suitable for new installations or where existing duplicate overhead lines can be coordinated to give a resultant split-conductor circuit.†

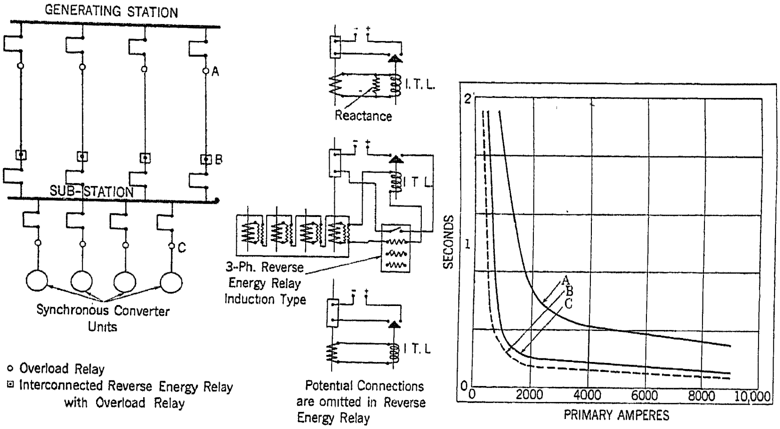
GENERATING STATION
A
B
SUB-STATION
C
Synchronous Converter Units
○ Overload Relay
⊡ Interconnected Reverse Energy Relay with Overload Relay
I.T.L.
Reactance
I T L
3-Ph. Reverse Energy Relay Induction Type
I T L
Potential Connections are omitted in Reverse Energy Relay
SECONDS
2
1
0
0
2000
4000
6000
8000
10,000
PRIMARY AMPERES
A
B
C

FIG. 2
Parallel feeders—one-line diagram

VI. LINES IN TANDEM

For single lines in tandem, extensive use has been made of definite and inverse-time-limit relays, with progressively longer time settings on relays near the generating station.

For two or more parallel lines in tandem, the inverse-time-limit relays at the points where the power leaves the station or substation, and overload relays in combination with reverse-power relays at the point where the power enters the substation, have given very satisfactory results. In Fig. 3 a one-line diagram is shown of a system that has been in operation for some time with excellent results.

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*Journal of the Institution of Electrical Engineers (London) p. 159, Jan. 15, 1915.

†TRANS. A. I. E. E., 1916, Vol. XXXV., Part I, p. 724.

In laying out a system of parallel feeders in tandem, it has been found very advantageous to make a careful analysis of the constants of the circuits and short-circuit conditions, to set the relays to give the best selective action. As an example, the equivalent 7800-volt constants of each circuit shown in Fig. 3 are:

Generating station to substation No. I—Reactance 1.45 ohms, resistance 0.53 ohms per leg.

Substation No. I *to No.* II—Reactance 0.33 ohms, resistance 0.31 ohms per leg.

Substation No. II *to No.* III—Reactance 4.3 ohms, resistance 3.7 ohms per leg.

The settings of the various relays are given in the curves of Fig. 3 and from the maximum short-circuit currents that can pass through any two relays in series, there is a selective time difference of half a second, even under the most severe conditions.

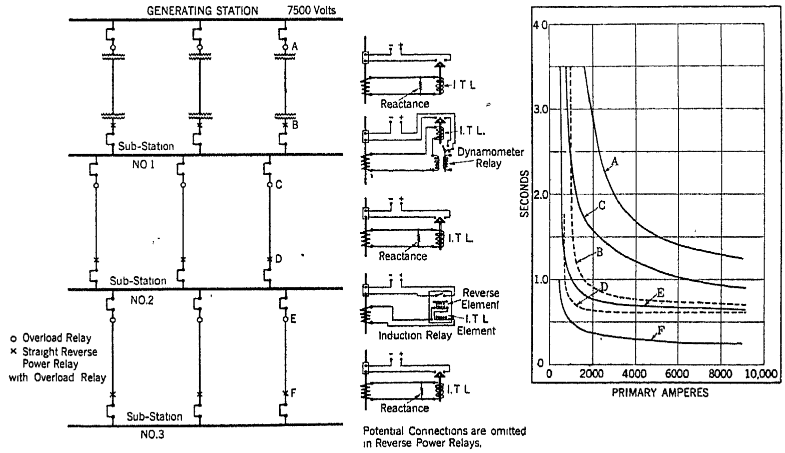

FIG 3
Parallel feeders in tandem—one-line diagram

The balanced system, as explained in Section 5, is also especially adapted for parallel lines in tandem.

VII. SINGLE LINE RING SYSTEMS

On single line ring systems, inverse-time-limit relays combined with reverse-energy relays may be used with the settings on the inverse-time-limit element made progressively longer near the generating station when feeding in the outgoing direction. The balanced system might be preferable to the above especially in cases of new installations.

VIII. PARALLEL LINE RING SYSTEM

In a parallel line ring system, the system of relays shown in the one-line diagram Fig. 4 gives promise of very good results.

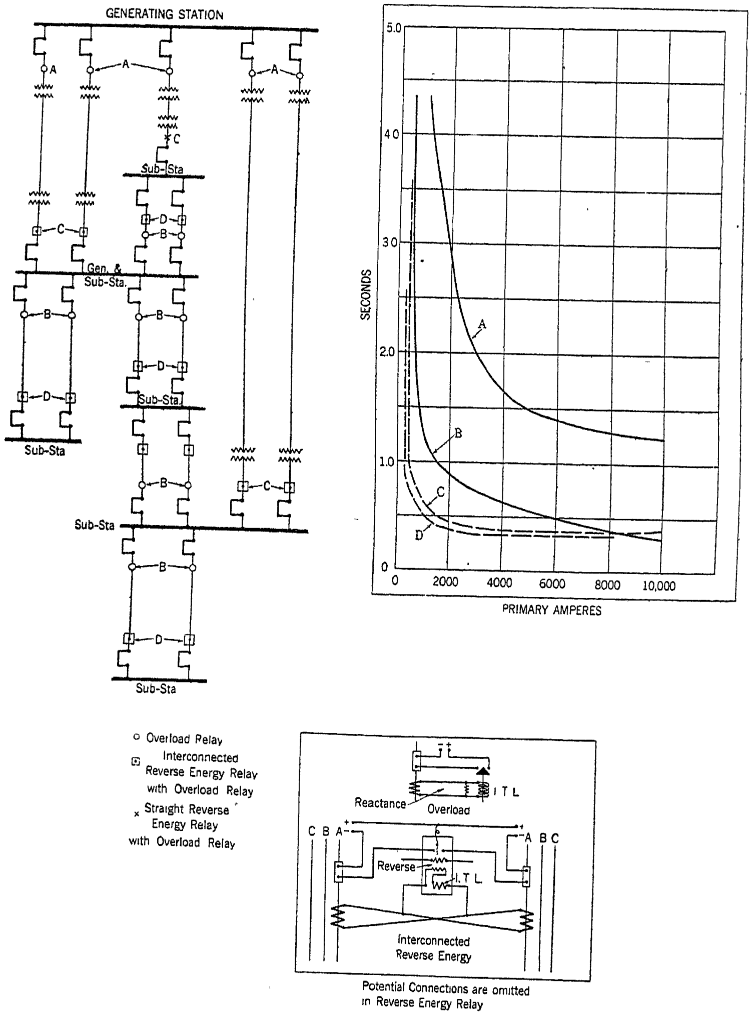

FIG. 4
Parallel feeders—ring network—one-line diagram

In this the inverse-time-limit relays are used on the outgoing feeders at the generating stations and a balanced interconnected combination of reverse-energy and inverse-time-limit relays at the receiving ends of the supply feeders.

In the case shown, the supply is furnished to several parts of the system, which parts are interconnected through several substations by duplicate feeders. Here the overload relays are used on the outgoing feeders at each of the main receiving substations to eliminate the possibility of a complete system shut-down due to the failure of any relay to function properly or in case of a short circuit on a bus.

IX. TIE LINES

With tie lines between generating stations where power can pass in either direction, the Merz-Price or balanced system of relays are probably most advantageous. The balanced arrangement may be on the split-conductor principle, the combination interconnected reverse-power and inverse-time-limit relay as shown in Fig. 2 or a mechanical interlock between the plungers of the several relays on the parallel feeders.

X. GENERATOR PROTECTION

Generators are seldom protected by relays; however, protection in cases of short circuits in generator windings may be best obtained by means of differential relays connected to current transformers in the two ends of each individual winding. It is also possible to protect against this trouble by means of reverse-power relays in connection with overload relays with their tripping contacts in series and set to trip out the switch at a high value of current, when the power is flowing in the direction of the generator.

As in most cases the generator breakdowns start with a ground, a good protective arrangement consists in adopting the selective relay ground principle, as in the case of feeder protection. For this arrangement precaution must be taken to ground the system either by a ground placed outside of the generator neutral, or by grounding the neutral of one or more generators and passing these grounding leads through the transformer of the selective ground relay system.

XI. TRANSFORMER PROTECTION

Transformer protection has been best obtained for a very long time by means of differential relays connected to transformers on both the high-tension and low-tension sides of the power transformer.

XII. GROUNDING RELAYS

Special grounding relays have been found very important for use on extra high-tension feeders. The accompanying Figs. 5 and 6 show the method that has been used by The New York Edison Company where the high-voltage feeders are metallically isolated at both ends with transformers.

While the experience is not conclusive, it is interesting to note that in the case of Fig. 5, on 15 miles (24.1 km.) of 25,000-volt cable, eleven cable breakdowns occurred in the first seven months of service without the grounding relays, whereas only five cable breakdowns have occurred in the past year with the grounding relays. Of the first eleven breakdowns, six occurred in service and five under test after a service breakdown. Of the latter five breakdowns all occurred in service and none under test. The theory is held that the grounding relay clears the

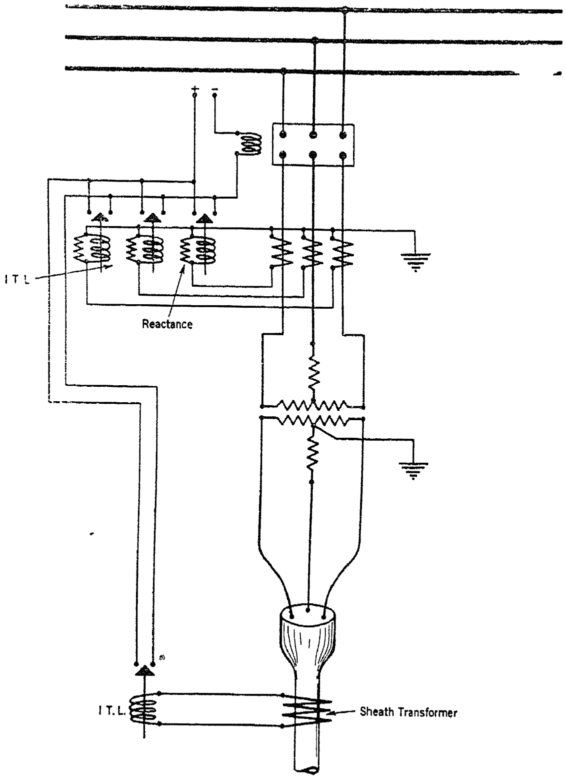

FIG. 5

trouble before the short circuit through the impaired insulation has sufficient time to develop a large flow of current creating heavy surges and mechanical stresses which may weaken other points on the system.

APPENDIX A

The reverse-power relay cannot be considered to give any selective time feature as the moving torque is dependent upon the power factor and the voltage of short-circuit conditions. In order to obtain the best results, the reverse-power relay should be set as sensitive as possible, with an inverse-time-limit relay for time and current adjustments. The tripping contacts of the two relays should be in series so that the circuit will only trip out with a short-circuit current flowing in a given direction and of a certain magnitude. The time required to trip is then dependent upon the magnitude of the current alone.

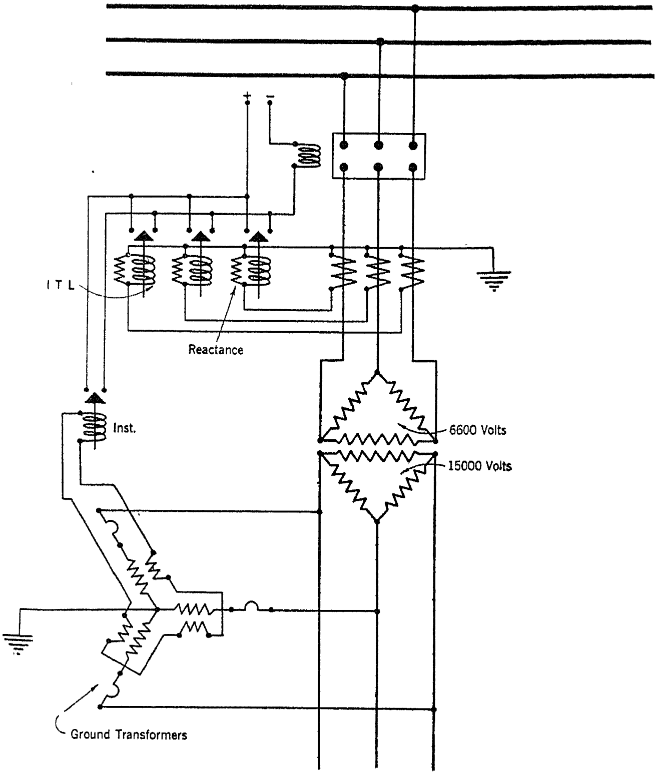

FIG. 6

The power factor of a short circuit may vary from practically zero to unity, and in addition there is a second possible shifting of phase relation from Y voltage of 90 electrical degrees, due to voltage distortion on single-phase short circuits. This necessitates the reverse-power relay selecting through a range of 180 degrees.

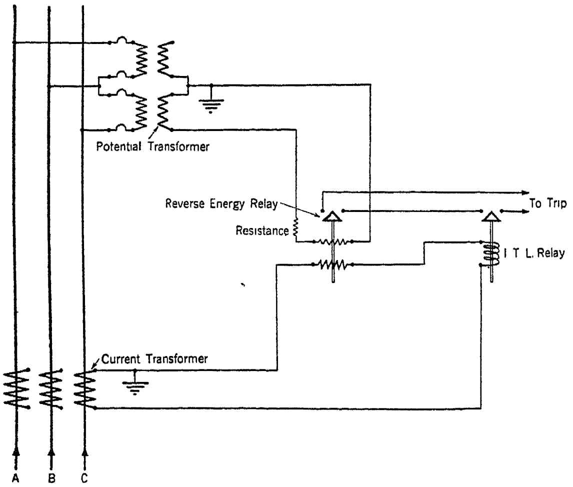

Fig. 7—Phase Rotation A B C

It is therefore very important that the relays have their potential and current coils properly connected to select under all of these conditions. Fig. 7 shows the method of connection that The New York Edison Company has employed for the last four years with very satisfactory results. Fig. 8 shows another method of connection that is proposed by one of the manufacturing companies and gives promise of good results.

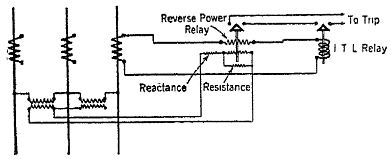

Fig. 8—Reverse-Power Relay Connections

In both of the above connections the current in the current coil leads the current in the potential coil with unity power factor three-phase reverse power. Hence the currents in the two coils are in closer phase relations with the lagging current of short circuits. Also the potential elements are connected in delta which limits the distortion on single-phase short circuits to 30 electrical degrees in place of 60, as in the case of a Y connection.

APPENDIX B

Relay Testing To insure selective operation when carrying short-circuit currents, the relays should be tested under conditions approximating, as nearly as possible, short-circuit conditions. This involves heavy testing currents, and correspondingly short timing intervals, both of which present certain difficulties.

In keeping with the foregoing considerations, marked changes have taken place in the testing of series inverse-time-limit relays in the generating stations of The New York Edison Company. A brief outline of the present method employed is given, together with sketches of the testing circuits, etc.

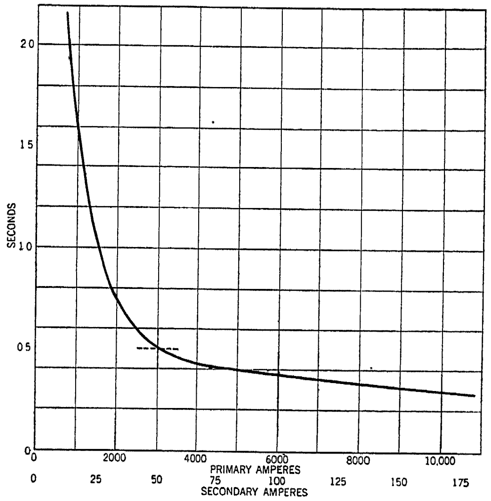

FIG. 9

Although the general scheme remains the same as heretofore, the testing is now done at considerably higher currents and an accurate automatic timer has replaced the stop watch and signal lamp. Furthermore additional testing circuits have been provided, and the test connections are made to give a more complete check on the condition of the relay circuits.

The extent to which the testing currents could be increased was necessarily limited by the size of wire on the relay circuits.

These circuits are all of No. 10 A. W. G. copper wire and a testing current approximating 50 amperes was considered to be a safe maximum. A circuit breaker which is automatically opened when the relay closes is used in the testing circuit to prevent injury to the wiring by leaving the test current on for too long a period.

A secondary test current of 50 amperes corresponds to about 10 times the normal capacity of the circuits and is well down near the flat part of the inverse-time-limit relay curves (see Fig. 9.). It will be noted that at this part of the curve an error in current measurement will have a considerably smaller effect on the timing than at a point of the curve corresponding to two or three times the normal circuit capacity.

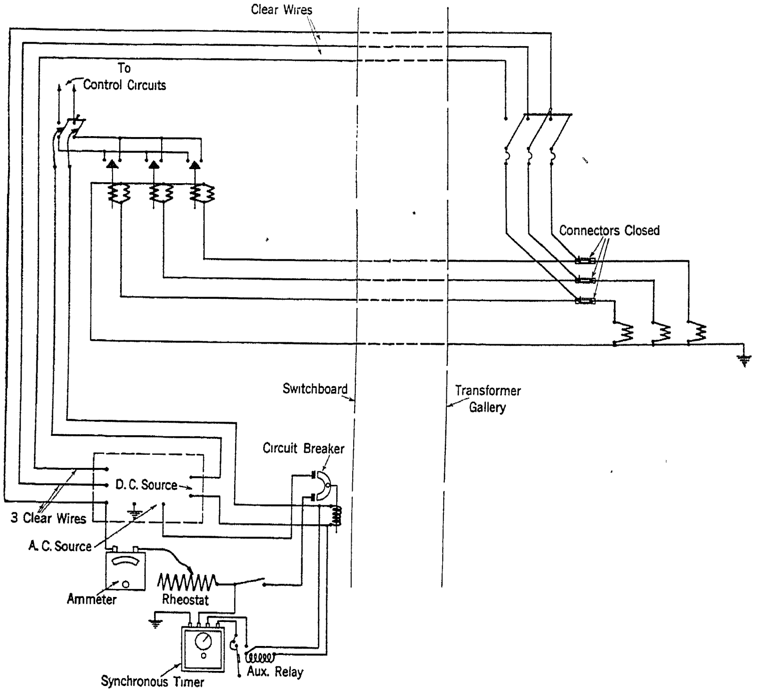

FIG. 10

The curve, Fig. 9, also shows that the time at 50 amperes secondary is about 0.5 second. It is, of course, impossible to measure this interval with sufficient accuracy by use of a stop watch, and a small synchronous timer is used. This instrument is accurate to 3 or 4 per cent at 0.5 second and good results have been obtained with it. The operation is entirely automatic, eliminating all personal errors as shown in the method of connections in Fig. 10.

The connectors on the current-transformer test blocks are not removed under tests, which leaves the secondary of the current transformer in shunt with the relay circuit while testing. Under these conditions and with test currents not exceeding 50 amperes, only a very negligible current is shunted out by the transformer secondary. If, however, a high resistance is present in the relay circuit or if the impedance of the transformer is low on account of short-circuited turns, the balance of current between the relay and transformer will be disturbed and the defective circuit will be found on periodic retest of the relay. This method has already proven very successful in locating defective current transformers.

If it were possible to test with still higher currents, then, with the transformer shunting the relay circuit, an appreciable part of the test current will pass through the transformer secondary. This would be a big advantage as it would account for the high magnetizing current which changes the ratio of the transformer when working under short-circuit conditions.

APPENDIX C

The New York Edison is installing the protective relay system for the generating station and some of its substations where all the radial feeders are operated in parallel, as shown in Fig. 11.

This company has used the system of selective-ground relays for nine years with excellent results. This system was arranged to ring a bell and locate a defective grounded feeder by means of a relay drop which was in turn connected to a current transformer that enveloped the three conductors.

By this indication the operator was in former years able to disconnect the defective cable before it developed into a short circuit in about 80 per cent of the cases.

With the increase in size of the underground system the grounding current capacity has increased so much that it gives the operator very little time to disconnect a grounded feeder before it develops into a short circuit. It was, therefore, decided to make this operation automatic and a zigzag three-phase grounding transformer is being connected to the generating station bus for establishing a more positive ground current.

The resistance is connected in series with these transformers to limit the current to 200 amperes.

A plunger-type bellows relay is being connected to the ground-sheath current transformer and this relay is being connected to automatically trip out the oil switch at the station when a ground occurs on the feeder.

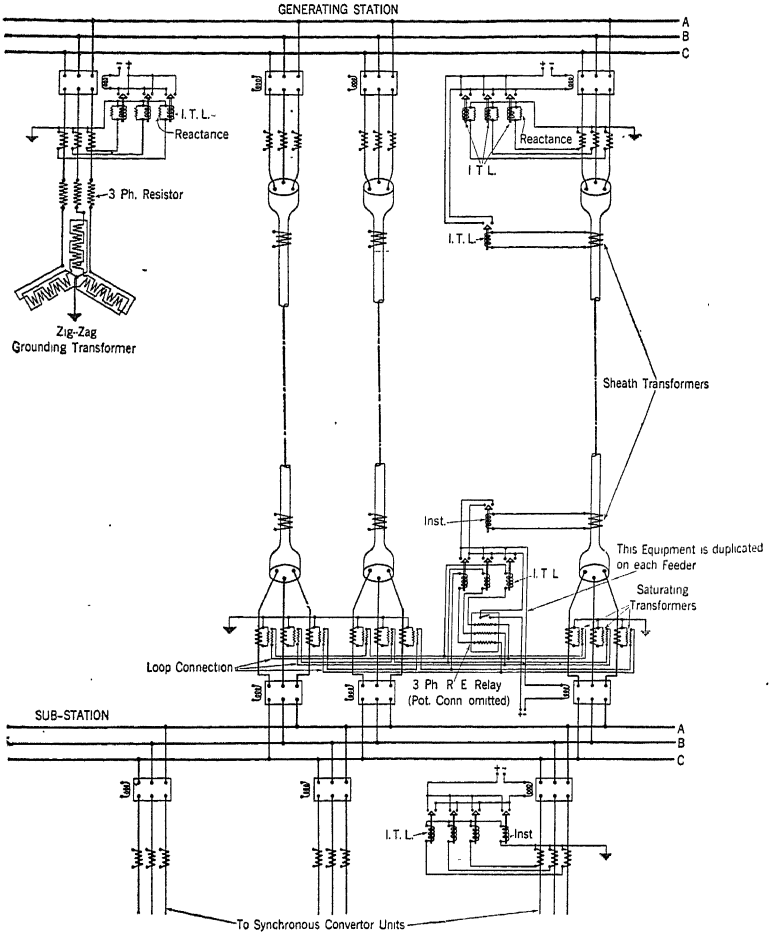

Fig. 11

In cases where the feeders are paralleled in the substation a grounding relay is used in conjunction with the balanced interconnected reverse-energy relay arrangement of Fig. 2 on the sub-station ends of the feeders. On these feeders the bellows grounding relays in the generating station are given a short time setting to permit the substation switches to operate first.

All feeders outgoing from the substation bus are also equipped with grounding relays.

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THE PROTECTIVE EQUIPMENT ON THE SYSTEM OF THE COMMONWEALTH EDISON COMPANY

BY R. F. SCHUCHARDT

ABSTRACT OF PAPER

The use of protective devices is based primarily on the desire to maintain a high continuity of service rather than to protect apparatus. The tendency in the development of large systems is toward interconnection, and to make this practicable, satisfactory relays are necessary.

The paper describes briefly the system of the Commonwealth Edison Company of Chicago with special reference to the protective devices installed thereon.

Generators are equipped with balanced relays, lines with induction-type inverse-time-element relays, some of them of the uni-directional type, and balanced relays, while substation apparatus has the usual equipment of overload relays, speed-limit devices, etc., and outgoing feeders the instantaneous type of relays. The arrangement of these relays is shown and some of the settings described.

INTRODUCTORY

PROTECTIVE relays on a central station system may be defined as devices or apparatus used primarily to maintain continuity in the supply. The protection is with reference to the service rather than to the generator, line, or transformer, etc., to which the relay is connected. The ideal protective relay is one that immediately disconnects a faulty member of the system before the disturbance caused by the fault has spread beyond that member. Unfortunately, development to date has not yet evolved such an ideal device. Synchronous apparatus has a tendency to drop out of step promptly whenever an unusually heavy shock occurs on the system, thus at times causing disturbance in other than the faulty member. However, the relays now available will serve properly in the great majority of cases and we may hope ultimately to have synchronous apparatus designed with a higher degree of stability.

At the present time the conditions met with in different systems vary considerably, and no one scheme of protection will fit them all. Each system must be studied and the individual solution applied. There are, however, many points of similarity and the solution in one case will serve as a partial guide, at least, in others.

The tendency in the development of large systems is toward the simplicity of the low-tension Edison network, which is giving such excellent results. Unfortunately, the protective problem on these large high-tension systems has not the simplicity of the Edison network. We seem, however, to be in a fair way of making satisfactory progress toward this goal. The devices now obtainable and commercially within reach make it practicable to interconnect quite freely and thereby to gain considerably in the use of the copper invested in lines and, in general, to localize faults.

It is only fair to the manufacturer to state that his problem has not been an easy one, since the ideal action of protective relays would require them to have almost human intelligence. They must invariably discriminate between conditions under which they ought to operate and those under which they must remain inoperative, and this discrimination must be made instantaneously. The present apparatus made by the manufacturers reflects credit on their engineers and gives promise of continued progress along the desired lines.

In the following paper will be found a description of the protective schemes recently adopted by the Commonwealth Edison Company of Chicago as a result of considerable experimentation with the newer relays. With the continued growth of the system the earlier relays were frequently found inadequate and it was necessary to seek relief in a better development in order to continue the maintenance of a satisfactory service.

Brief Description of the Plant

The plant of the Commonwealth Edison Company contains at present three main high-tension generating stations with a number of auxiliary peak stations, having a total generating capacity at this time (March 1917) of approximately 400,000 kw., with 800 miles (1287.4 km.) of high-tension transmission lines feeding sixty-six distribution substations and about forty industrial customers' substations. The energy is generated at two frequencies, 25 cycles and 60 cycles. The bus connections of the units and the direct ties between stations are shown in Fig. 1. Summarized substation data are shown in Table I.

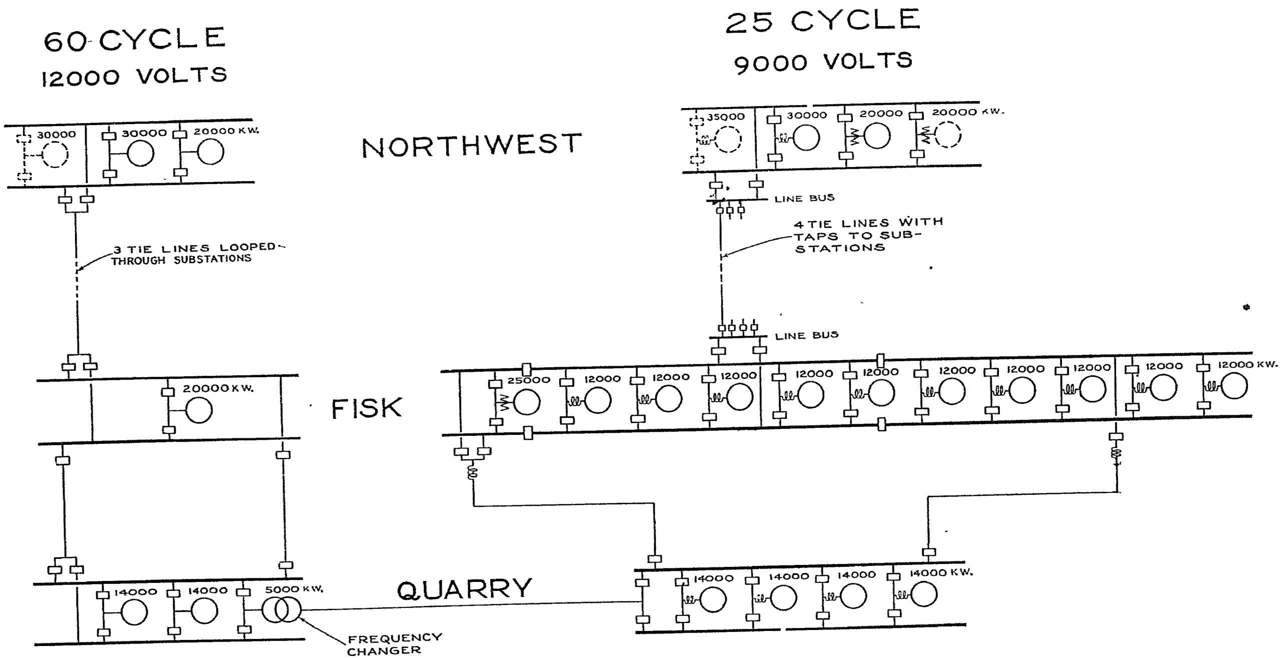


Fig. 1—Diagrammatic Sketch of Units and the Bus Connections—March, 1917

The size of the system and its growth are indicated in the curves in Fig. 2, which show the annual maximum loads since 1900. The total coincident maximum during the past winter was 370,000 kw.

TABLE I—SUMMARY OF SUBSTATION CAPACITY DATA

	25 Cycle		60 Cycle	
	Total kw. capacity	Range of sizes	Total kw. capacity	Range of sizes
250-volt synchronous converters on Edison system...........	101,300	100-4200		
600-volt synchronous converters for railway service..........	*226,000	1000-4000	9,000	3000
12,000-volt to 4,000-volt transformers....................			102,500	100-3000
Frequency changers...........	21,500	500-2000		
Total....................	348,800		111,500	

* 140,000 kw. of this is in substations of the traction companies.

Fig. 3 is a diagram of the transmission lines. The length of these lines at the various voltages is as follows:

481 miles (774 km.) of 9000-volt, 25 cycles; cable size 4/0, 250,000 cir. mils and 300,000 cir. mils.

215 miles (345.9 km.) of 12,000-volt, 60 cycles; same cable as for 9000 volts.

92 miles (148 km.) of 19,000-volt, 25 cycles; cable size 2/0 and 250,000 cir. mils.

12 miles (19.3 km.) of 22,000-volt, 60 cycles; cable size 250,000 cir. mils.

Elements of the System and their Protection

All of the 25-cycle generators in these three stations are provided with external reactors except in the case of two of the units which generate at half pressure and step up to 9000 volts through auto-transformers. The total reactance of generators and external coils (or auto-transformers) is as follows:

12,000-kw. capacity generators...............	8 per cent
14,000 " " "	8 " "
20,000 " " "	10 " "
25,000 " " "	11 " "
30,000 " " "	12 " "
35,000 " " "	12.5 " "

The 60-cycle generators have sufficient inherent reactance, so no external reactors are required, this reactance being,

14,000-kw.	(18,670-kv-a.)	capacity generators...	7 per cent
20,000 "	(25,000 ")	" " ..	12 " "
30,000 "	(35,300 ")	" " ..	12.5 " "

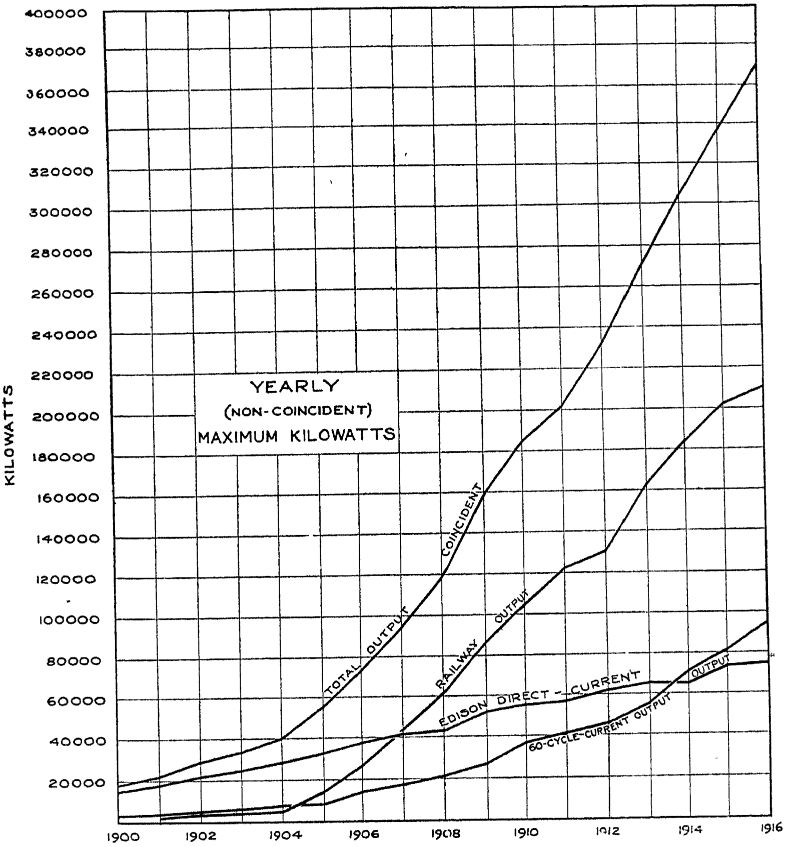


Fig. 2—Maximum Loads since 1900

With one exception the vertical generators, which include all those under 20,000-kw. capacity and two of this capacity, have no automatic relays at present. All of the newer units and one of the 20,000-kw. vertical units are equipped with balanced relays of the Merz-Price type. These are connected so that in the event of a fault in the generator or in its leads, they will promptly open the main oil switch, the neutral switch

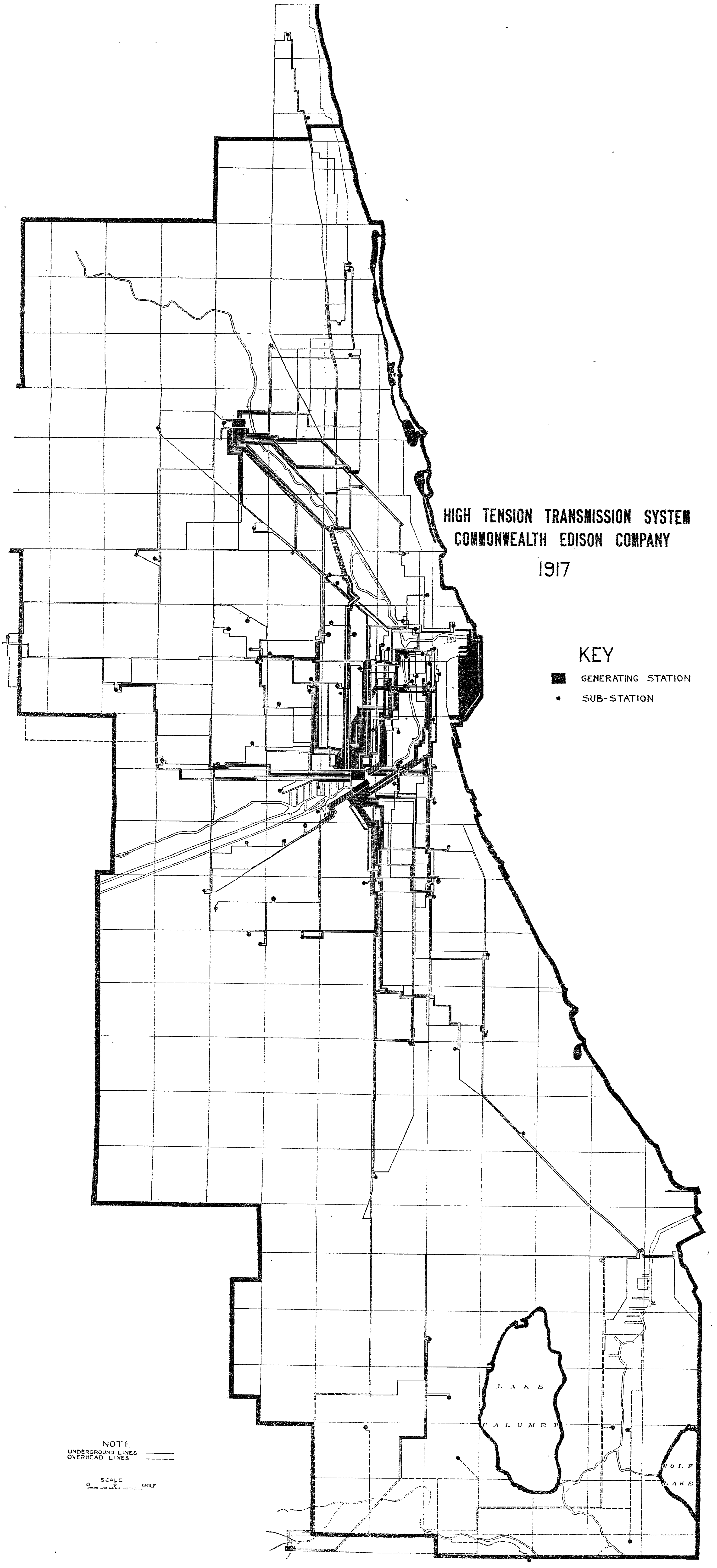


Fig. 3—Diagram of Transmission Lines

(if closed), and the field switch of the faulty unit and also disconnect the blower motor where there is one. A diagram of this relay connection is shown in Fig. 4.

Fisk Street station is normally operated in two sections which, however, are tied together through the Quarry Street bus, sectionalizing reactors being placed in each tie line. (See Fig. 1.) These reactors are designed to absorb 20 per cent of the pressure at the full-load current of the tie line, which consists of three 250,000-cir. mils cables in parallel. These tie lines are protected with balanced relays, the connections of which are shown in Fig. 5. It will be noted that the connections are such that the relays will operate only in the event of ground current, but experience has indicated that the vast majority of cable failures are to ground. Incidentally this could always be assured if cable of the Hochstaedter type—that is, with grounded sheaths around each conductor of the three-conductor cable—were used. Tie lines between Fisk Street and Northwest station which are about eight miles (12.8 km.) long are treated like ordinary lines at each station. On each of the sections, both for the 25-cycle and the 60-cycle units, the neutral switch of one of the running units is closed to the neutral bus. Each section has its own neutral bus and this is connected to ground through a $2\frac{1}{2}$-ohm non-inductive resistance.

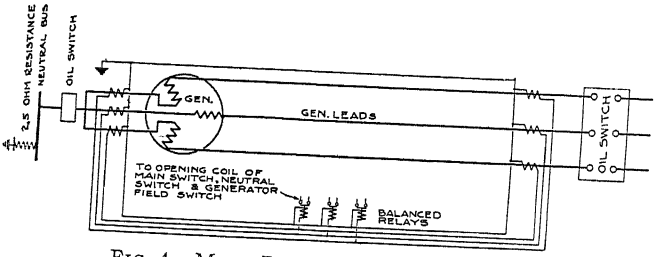


FIG. 4—MERZ-PRICE RELAY ON GENERATOR

At the Quarry Street station there is a 5000-kw. frequency changer serving as tie between the 25-cycle, 9000-volt busses and the 60-cycle, 12,000-volt busses. The bellows overload relays now on the units are to be replaced by uni-directional relays at either end, and in addition, an adjustable definite-minimum, inverse-time-element relay will be installed to trip the 60-cycle switch in the event of a sustained excessive overload such as would result when either of the systems receives a shock and excessive synchronizing energy passes between them

through this frequency changer. The uni-directional relays will be set to operate instantaneously on energy feeding inward and equal to about 15,000 kv-a., while the inverse-time relay will be set for approximately 10,000 kw., three seconds.

On the transmission lines in the generating stations the bellows-type relays are being rapidly replaced by an induction-type, adjustable definite-minimum, inverse-time-element relay. Where the line is part of a ring and the time setting of the relay at the station end is more than one and one-half seconds, an additional relay of the solenoid type is used but set to operate instantaneously. Its current setting, however, is slightly higher than that current

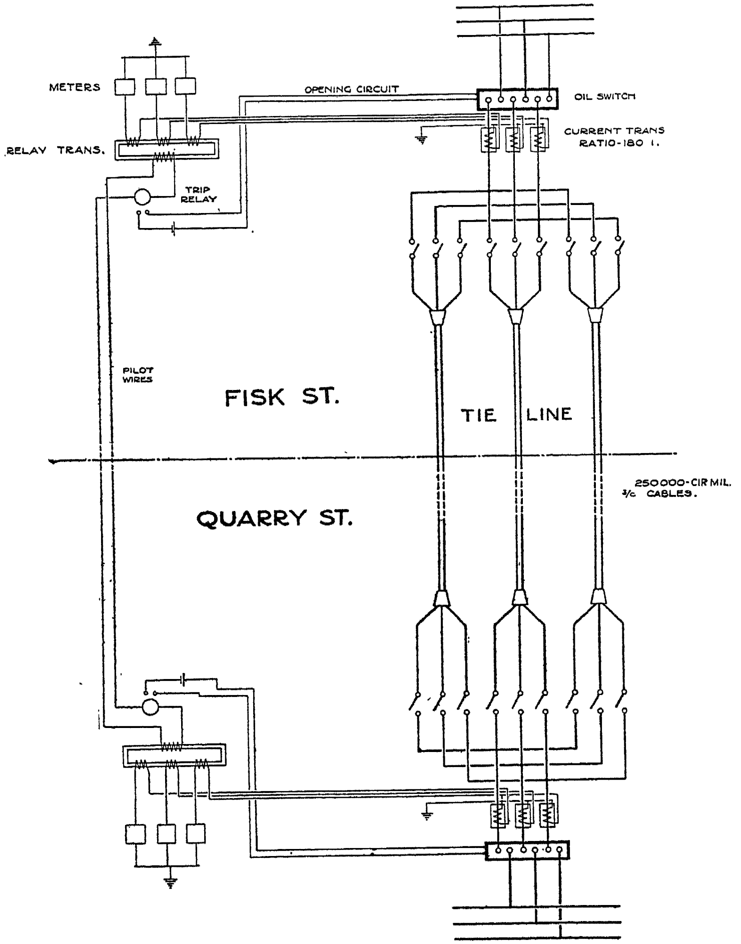
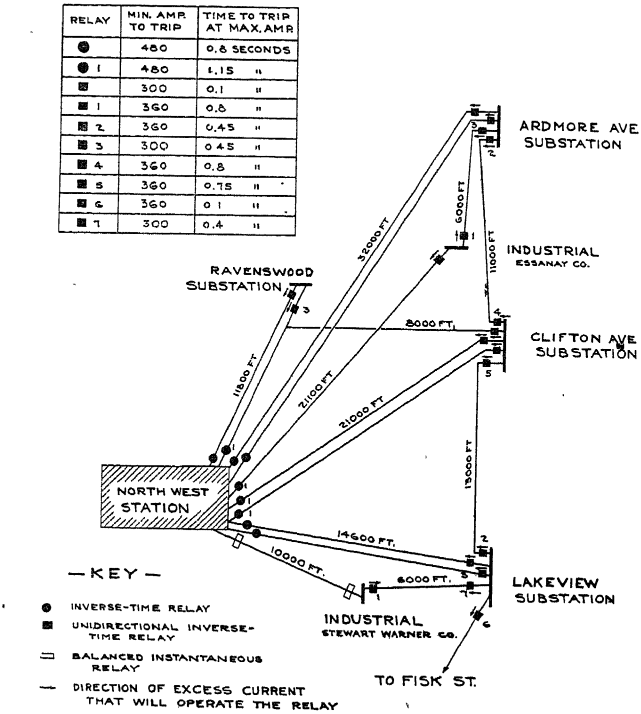


FIG. 5—DIAGRAMMATIC SKETCH OF RELAYS ON FISK-QUARRY TIE LINES

which could flow over the resistance of this line and into the next link of the ring. It is planned ultimately to operate lines in parallel, wherever practicable and consistent with the sectionalizing at the generating stations.

At the substation end of these lines as well as on tie lines between substations there are connected uni-directional inverse-time-element relays of the induction type, or in some instances on rings, balanced relays, depending on the relation of the particular line to the other elements of the ring of which it is a part.



RELAY	MIN. AMP. TO TRIP	TIME TO TRIP AT MAX. AMP.
●	480	0.8 SECONDS
● 1	480	1.15 "
■	300	0.1 "
■ 1	360	0.8 "
■ 2	360	0.45 "
■ 3	300	0 45 "
■ 4	360	0.8 "
■ 5	360	0.75 "
■ 6	360	0 1 "
■ 7	300	0.4 "

FIG. 6 A—TYPICAL PROTECTIVE SCHEME—TRANSMISSION LINES

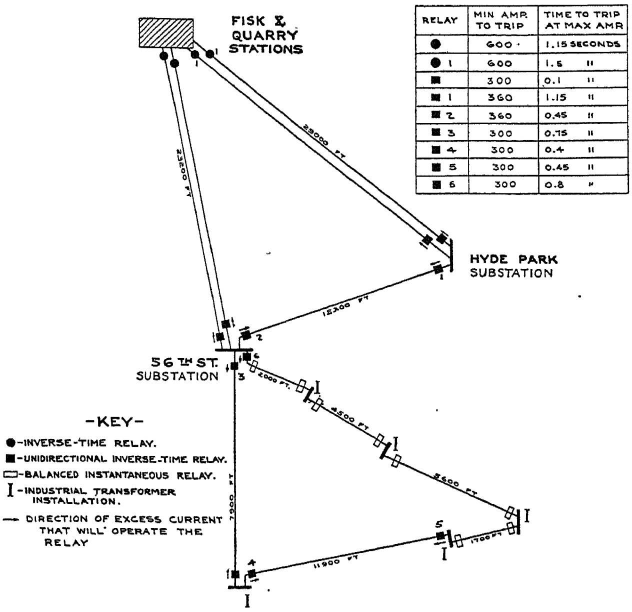
Protective apparatus in use on substation units is as follows:

On 250-Volt Synchronous Converters of the Edison System. Inverse-time-element overload relays of the bellows type on the high-tension supply to the transformers and operative on the oil switches; speed-limit devices operating on the d-c. circuit breakers; and on all down-town substation units in addition reverse-current relays operating on the d-c. circuit breakers.

On 600-Volt Converter Units for Railway Service. Inverse-time-element overload relays operating on the oil switch; overload attachment on the positive circuit breaker; speed-limit relays operating on the positive circuit breaker; and in addition a special leakage-current relay connected between the frame of the machine and earth and set to operate both the oil switch and the d-c. breaker in the event of potential in excess of 100 volts existing on the frame, such as might result from a flash-over.

On 60-*Cycle,* 12,000-*Volt to* 4000-*Volt Transformers.* Bellows overload relays operating on the 12,000-volt switch are being replaced by differential relays operating on both the 12,000-volt and the 4000-volt oil switches. The 4000-volt outgoing-feeder circuits are provided with straight overload relays.

On 9000-*Volt,* 25-*Cycle, to* 4000-*Volt,* 60-*Cycle Frequency Changers.* Bellows overload relays on both the motor and the generator, each operating its own oil switch.



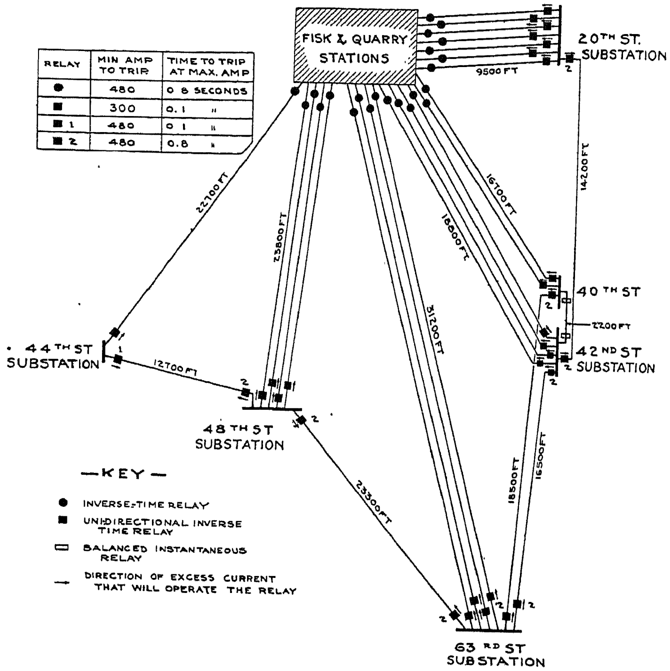
RELAY	MIN AMP. TO TRIP	TIME TO TRIP AT MAX AMP.
●	600 ·	1.15 SECONDS
● 1	600	1.5 "
■	300	0.1 "
■ 1	360	1.15 "
■ 2	360	0.45 "
■ 3	300	0.75 "
■ 4	300	0.4 "
■ 5	300	0.45 "
■ 6	300	0.8 "

FIG. 6 B—TYPICAL PROTECTIVE SCHEME—TRANSMISSION LINES

SETTING OF RELAYS AND EXPERIENCE

Generators. The balanced relays on generators are, like all the other balanced relays on the system, set for instantaneous operation. Since they have been installed, the relays on one of the units have been called upon to operate twice, in each case promptly disconnecting the unit. The benefit of generator reactors and bus-sectionalizing reactors has been apparent a number of times in limiting the extent of the disturbance resulting from a line break-down close to the station. Previous to the installation of such reactors, cable breakdowns occurring within half a mile (0.8 km.) of the station usually caused fairly widespread disturbance and often considerable damage to generators.

Lines. The adoption of the line protective devices described resulted from a desire to make the large line investment more productive by operating in parallel as far as practicable, as well as to localize more fully the disturbance resulting from a cable failure. In the case of the station tie lines, the straight inverse-time-element relays frequently opened when either station got a shock due to a line failure. The installation of balanced relays on these two lines has entirely removed this trouble.



RELAY	MIN AMP TO TRIP	TIME TO TRIP AT MAX. AMP
●	480	0 8 SECONDS
■	300	0.1 "
■ 1	480	0 1 "
■ 2	480	0.8 "

FIG. 6 C—TYPICAL PROTECTIVE SCHEME—TRANSMISSION LINES

On the transmission lines, the aim is to use the induction type of relays in all cases where their satisfactory use does not require the time setting at the station to be too high, and it was generally considered that this maximum setting should not materially exceed two seconds. In the case of ring-connected lines, where the succeeding relays in each link, counting toward the station, must have an increasing time element, the use of these relays only, would in many cases, require a setting in excess of the two-second limit. It is obvious that the difference in time setting between any two successive relays in the series must be equal to the time required for the oil switch to fully break the circuit after the relay contact has closed, plus a safe margin, and this, with the speediest apparatus, is approximately three-tenths of a second. In order, then, to keep down the total number of such relays in series in a ring of many links and thus to prevent the above time limit from being exceeded, the shortest lengths in the ring are provided with balanced relays and pilot wires. The illustrations in Figs. 6A, 6B and 6C show typical installations of relays on a few sections of the system.

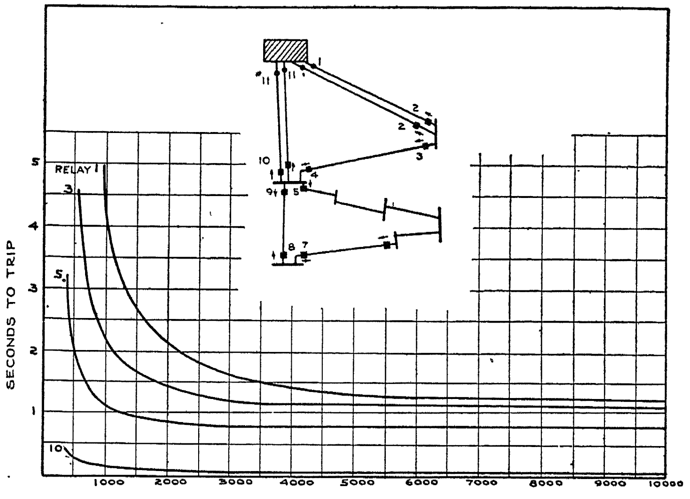


FIG. 7 A—CURRENT-TIME CURVES OF RELAYS

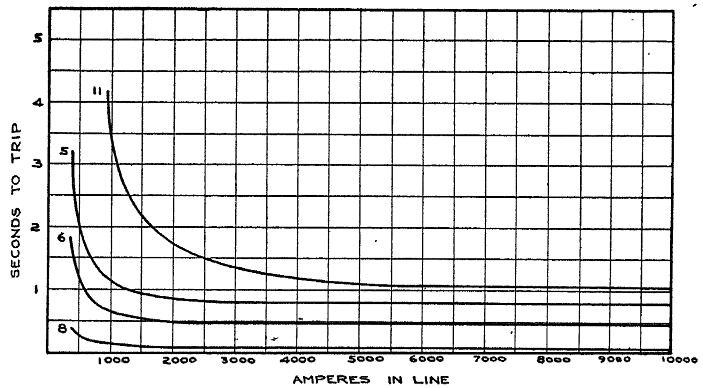


FIG. 7 B—CURRENT-TIME CURVES OF RELAYS

The data of the settings are also given. The current-time curves of sets of uni-directional relays in a ring are given in Figs. 7A, 7B, 7C and 7D and show clearly how the selectiveness of operation is obtained at all current values. It will be noted in Fig. 6A that in the case of the line to Lake View substation which loops through an industrial installation, the rule of applying the balanced relays to the short length has not been followed. In cases like these the preference for the balanced relay is given to the section nearest the generating station, even though this section may not be the shortest, the reason being that the balanced relay, having a practically zero time setting, would clear a fault close to the station more promptly than would a relay with a definite-minimum time setting

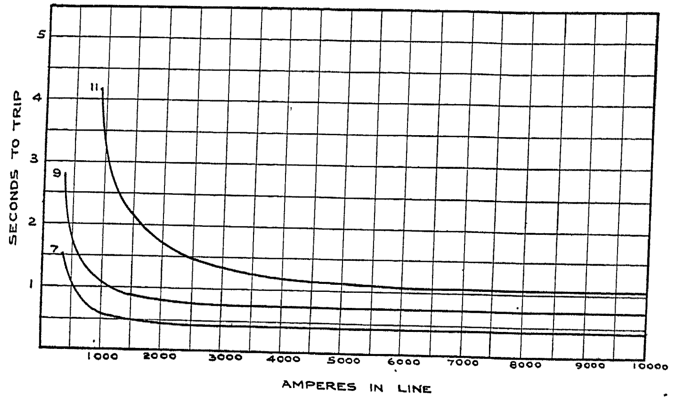


FIG. 7 C—CURRENT-TIME CURVES OF RELAYS

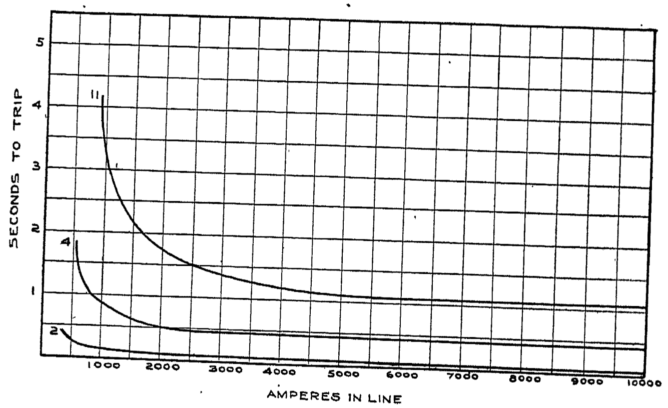


FIG. 7 D—CURRENT-TIME CURVES OF RELAYS

Substations. The bellows overload relays on the high-tension side of substation units are set to operate at a minimum flow of approximately two and a half times rated current, to operate in two seconds on three and a half times, and instantaneously on five times rated current. On synchronous converters the speed limit is set at 15 per cent over speed. Differential relays on substation transformers are arranged for instantaneous operation and are shown diagrammatically in Fig. 8. The 12,000-volt primary coils of these transformers are delta connected while the secondary 2300-volt coils are star connected giving a 4000-volt delta pressure. The relays on the 4000-volt feeders are of the instantaneous type and set to trip at three times normal load.

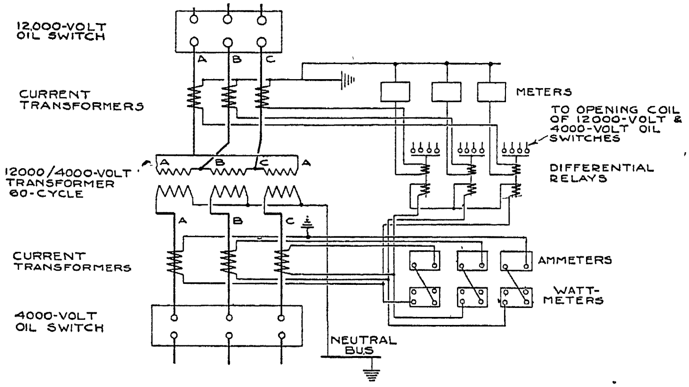


FIG. 8—DIFFERENTIAL RELAY ON SUBSTATION TRANSFORMER

There is a considerable number of minor installations of protective relays on the system not covered in the above, but for sake of brevity and because they have no features of special interest they have here been omitted.

Discussion on "Relays for High-Tension Lines" (Tor and "The Protective Relay Equipment of the Sy of the Commonwealth-Edison Company" (Schucha Chicago, Ill., March 9, 1917.

H. R. Summerhayes: We may be sure that any relay sc which is widely adopted on the two systems with whic authors are connected has not only been scientifically ana but has withstood the acid test of experience.

A scheme which works beautifully on a smaller system fails when subjected to the enormous rushes of current due short circuit on one of these great systems.

These papers show that modern relay practise is approac standardization.

Both engineers use on the generating-station end of transmission lines some form of inverse-time-limit overload r protected from excessive currents in the relay by some for saturating reactance or magnetic shunt. At the substation of parallel feeders both use a unidirectional or reverse-cur relay with inverse-time overload element in series, which t care of most of the trouble. Both admit that this syste not perfect by using on their shorter and more important lines the balanced or pilot-wire protection where the extra in ment is justified.

Neither shows a leaning toward the twin-conductor c system. It would be interesting to learn why this much heral foreign scheme has not been used either by Chicago or New Y

Both started with, and have for years maintained, a stri radial system, preferring continuity of service to reduced vestment; and both are taking advantage of the modern re developments to work towards an inter-connected net w with its great saving of copper.

The remote contingency of a short circuit on the generat bus or substation bus is left to the operators to take care except for tie-line relays between generator-bus sections, a certain overload relays mentioned by Mr. Torchio in Section which prevent a substation-bus short circuit from affecting whole inter-connected system.

Commenting on Mr. Torchio's paper, from the relay cur Fig. 4, it would appear possible for the overload relays *B* trip in case of a short circuit between them and the generati station, if it exceeds 6000 amperes through these lines. T introduction of these overload relays would appear to make t system less selective for cable shorts than the system, Fig. of Mr. Schuchardt, but to give better protection for a substati bus short circuit.

I strongly recommend for generator protection the balanc or differential-relay protection mentioned by Mr. Torchio, a already used by Mr. Schuchardt. I do not see why the latt calls this the Merz-Price scheme, since it is only the ancient a honorable differential relay used since many years for tra former protection.

There is a certain tradition that says that no automatic generator protection shall be used. I think this is really more of a tradition from the old days when the relays were very unreliable and they did not want to shut down the generator because the relay operated. The generators of that time were open machines, and if anything happened to them the operator could see it and cut it off and put out the fire. The new machines are enclosed; the operator cannot see the trouble until too late. If the field is opened the generator may run for a long time, the draft of air keeps on and trouble in one coil may cause the destruction of all the coils. It therefore appears that differential relay protection should be installed to cut off the machine instantly on the appearance of internal trouble. It also appears that steam pipes should be installed, steam openings by which the operator can put steam into the machine to put out the fire, and arrangements for shutting off the draft.

Ground protection relays are mentioned by Mr. Torchio in Appendix C. This development will be watched with great interest, as it is very important, either on grounded or ungrounded systems, to disconnect a grounded cable immediately, to avoid a short circuit destructive to the cable and its neighbors, and damaging to the system.

I am very much interested in Mr. Schuchardt's plan for protecting frequency changers tied in between two large systems.

Since the old overload relays protect the machine itself, it must be that the proposed changes were found necessary because the switches opened too often, due to violent fluctuations of load.

As these systems increase in size there are certain dangers of tying in multiple on substation bus bars a number of heavy high-tension cables from one or more large generating stations. The duty of the substation switches may be thus made heavier than those at the generating stations.

Substation bus bars in such cases should be sectionalized by reactors.

Referring to the maximum time limits of one and one-half or two seconds, mentioned in both papers, I understand this depends on the thermal effect of the current on the cable conductors. This is a very important factor in large stations.

I have recently made some calculations which show that for fifty times full-load current, the temperature rise is at the rate of 55 deg. cent. per second, and for one hundred times current, 220 deg. cent. per second.

Since it takes a conductor several minutes to cool from a temperature of 220 deg. to 100 deg. cent., the effect is not merely momentary, and the cable insulation may be permanently injured if the conductor rises to over 200 deg. cent. A short circuit starting at one hundred times full load and sustained at forty times would produce a temperature rise of from 180 to 200 deg. cent. at the end of two seconds.

Such high values are possible in these large plants when feeder reactances are not used, and the injury resulting to the insulation may account for subsequent break-downs.

William H. Cole: Mr. Torchio made brief reference in his paper to the Boston Edison Company as being a user of split-conductor cables. While I cannot tell very much about it in five minutes, I may say we expect this summer to have in service at least sixty miles of such cable protected by balanced relays. Some of it is used with the English type of switch gear; the bulk of it, however, is used with gear we have developed on our own system. We have also applied the balanced-current principle to two installations of parallel feeders; overhead lines in one case and in the other case ordinary three-conductor cables. The operating experiences we have had in the last three or four years lead us to believe that it is an admirable system of protection for any type of feeder or inter-connector. We believe, furthermore, that it lends itself to the growth of our system, that is to say, we do not have to re-arrange such protective arrangements from time to time, as would be necessary with some of the other types that have been described here tonight.

I wish to point out to anybody who may be particularly interested in split-conductor cables, that they must consider the whole installation as a unit, when they adopt a scheme of protection, both the cable and the operating gear, because both must be considered jointly. Some mistakes have been made in the application of balanced protection to split-conductor cables, because the two elements were not considered jointly. This point is one of the most important things to be considered in any application of this system.

Mr. Torchio mentions in one or more places that he believes the balanced system might be preferable to some of the methods described, particularly in new installations. In a large system having a great many cables, or a great many lines, it is not at all necessary to use split-conductor cables in order to secure the benefits of balanced protection. In fact, in many ways, it is preferable not to invest in split-conductor cables, for if existing cables of the ordinary type can be combined with balancing gear, then if a fault occurs in one, the other one is available for use. If you lose a split-conductor cable you lose the entire capacity between the two points as represented by that line.

I want to call attention to a type of relay that has been recently brought to our attention, a relay called by various names —biased relay, percentage-differential relay, or ratio-balance relay—which is designed to be used in connection with lines which are operated in parallel at both ends, but which differ in their constants. Such lines may be different in length, cross-section, reactance or otherwise. In such cases these lines themselves set up relatively large differential currents under normal conditions, and it is necessary when lines of this type are so operated, that the relay system be non-operative under the maximum normal differential current that can be set up on the occurrence of the most severe through short circuit, but without change of adjustment be operative on a fault current, even though the fault differential current is less than the normal differential current. It may look as though it might be an impossible thing to do; however, we have one installation in our system protected with this type of relay which operates with entire satisfaction.

Edward B. Meyer: I think it would be well to say just a few words in connection with the split-conductor cable as applied to the balanced system of protection.

During the past year, the Public Service Electric Company, of New Jersey, has installed approximately seven miles of 350,000 cir. mil. sector-type split-conductor cable, for operation at 13,200 volts; this will be increased to ten miles during 1917. The cable is manufactured under a specification which provides for 6/32 of an inch of paper insulation over each conductor and 6/32 of an inch paper-belt insulation. The insulation between the inner and outer split of each conductor is 5/64 of an inch, which thickness was determined more or less arbitrarily on account of the fact that it is still an open question as to the maximum potential that may exist between the inner and outer conductor under short-circuit conditions, and as to what is best practise both for voltage and mechanical considerations.

In the Public Service system, reactances are placed one in each split, at opposite ends. These reactances are designed to give a reactance drop of about twelve per cent of the resistance drop at rated current. With this arrangement, the maximum voltage difference between splits during an end fault, it is believed, should not exceed 1000 volts, which difference is constant over the entire length of the cable.

It is possible with other schemes to have higher voltages than that just stated.

The cost of the cable is from ten to fifteen per cent higher and the cost of the jointing twenty-five per cent higher, than for the ordinary type of cable, in which the individual conductors are not split, but this additional expense appears warranted, particularly on important tie feeders.

It is usual in installing split-conductor cables to transpose the inner and outer splits of each conductor at intervals throughout the length of the line, making the resistance and reactance of the two splits substantially equal.

It appears that information is still lacking as to the carrying capacity of split-conductor cable as compared with the usual type of cable; from one point of view the carrying capacity should be greater on account of the larger over-all diameter, and the larger conductor radiating surface; but on the other hand, due to the fact that the heat generated in the inner split must pass through the insulation between splits, as well as the over-all insulation, the carrying capacity of the cable may be reduced.

Since the current carrying capacity of any cable depends entirely on the conditions under which it operates, the Public Service Electric Company, in order to be conservative, has given split-conductor cables the same rating as solid-conductor cables.

L. N. Crichton: In discussing protective relays, it would be interesting to hear from the operating engineers present, as to what they consider an interruption, that is, how long they feel that the power can be off a piece of apparatus or off the system, or, in other words, how long a short circuit can be held on without causing an unreasonable amount of load to be lost. It seems to be the prevailing opinion that two or three seconds is the maximum time which can safely elapse before a short circuit is cleared. Several years ago, a number of tests were made to see how long the power could be cut off induction motors without causing them to slow down to such a point that they would not readily come back to full speed when the power was restored. The motors were rated at about 200 h.p., and were driving a mixed load consisting principally of rock crushers. It was found that the power could not safely be removed for more than three seconds, the danger being not that the motors would stall, but rather that the belts and clutches between the motor and the load would not stand the sudden increase in speed, which resulted when the power was restored at full voltage.

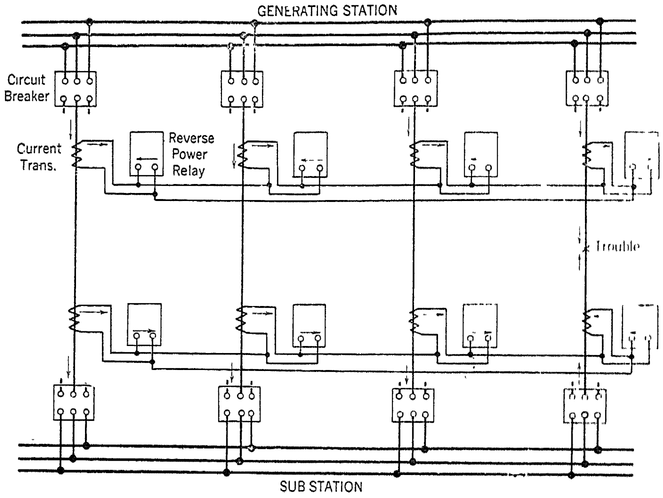


Fig. 1

The necessity for clearing trouble within a short time has been given as an objection to the use of time-limit relays, and partly because of this objection, two balanced schemes have been introduced quite extensively in England, and one or two modifications of these schemes are in use in this country. I refer to the pilot-wire system and the split-conductor system. Both of these schemes appear to be giving good service, the one objection to them being that they will not operate in case of trouble on the bus bars at the switching stations. There has recently been introduced in this country a balanced scheme which does not require the use of special cables or pilot wires, and which in its results is essentially the same as either of these two schemes, when it can be applied to a system having a number of duplicate feeders between each switching station. It consists in the use of reverse-power relays which are connected as shown in Fig. 1.

It will be observed that when the current is balanced in all the parallel feeders, there will be no flow of current through any of the relays. Whenever a short circuit occurs on any of the cables, the relays at both ends of the defective cable operate to clear the cable, by reason of the fact that they are carrying an excessive current, and also because the flow of power is away from the bus bars. A similar scheme might be applied to two parallel cables, but it is more convenient to use a reverse-power relay having double contacts, and arranged as shown in Fig. 2. Either of these schemes can be applied to parallel feeders which form a part of a complicated net work, and experience for the past two years has shown that they will give excellent service under practical operating conditions. It is theoretically possible to set the relays so lightly that they will operate on very small unbalanced currents, but operating difficulties are liable to be encountered if such a setting is made, and there is in general no object in adjusting relays to operate on currents which are smaller than those normally obtained during times of short circuit.

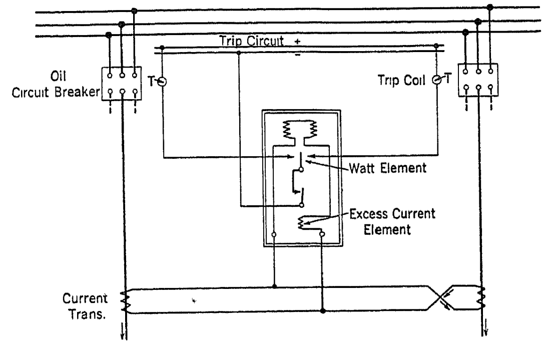


FIG. 2—DOUBLE CONTACT *C R* RELAY APPLIED TO DUPLICATE FEEDERS —CURRENT TRANSFORMER SHOWN IN ONLY ONE PHASE—POTENTIAL CIRCUIT OMITTED

The description of this method of balancing relays immediately brings up for discussion the general question of the construction and application of reverse-power relays. Until comparatively recently, there was no satisfactory reverse-power relay on the market, and furthermore, the conditions necessary for the correct application of such relays were not fully understood. The practise which has been recently established in relay construction is to use a sensitive device of the nature of a contact making wattmeter which will indicate the direction in which the power is flowing towards a short circuit, and to connect an excess current relay in series with this device so that the circuit breaker can not be tripped except during times of trouble. It is customary to apply time limit to the excess-current device so that the relays, therefore, have three separate and independent adjustments; namely, direction, magnitude of current, and time. The problems connected with the excess-current element are comparatively simple, but the watt element is still the subject of considerable discussion, and, therefore, an explanation of some of the phenomena connected with the application of reverse-power relays seems to be in order. The conventional diagram which is used to illustrate the application of reverse-power relays is shown in Fig. 3. It is usually stated that when a short circuit occurs at the point marked *X*, the power will flow away from the substation bus bars into the bad line. As an actual fact, however, the total polyphase power flow will frequently not be reversed when an unbalanced short circuit occurs. In other words, power may be reversed only in the affected phases. Assume that the relays on the lines at the substation end are capable of accurately measuring the magnitude of the power flowing, *i.e.* that they are true polyphase wattmeters. If two of the conductors are short-circuited, the relay will be influenced by that power which is flowing in the reverse direction equal in amount to the loss in the conductors between the relay and the

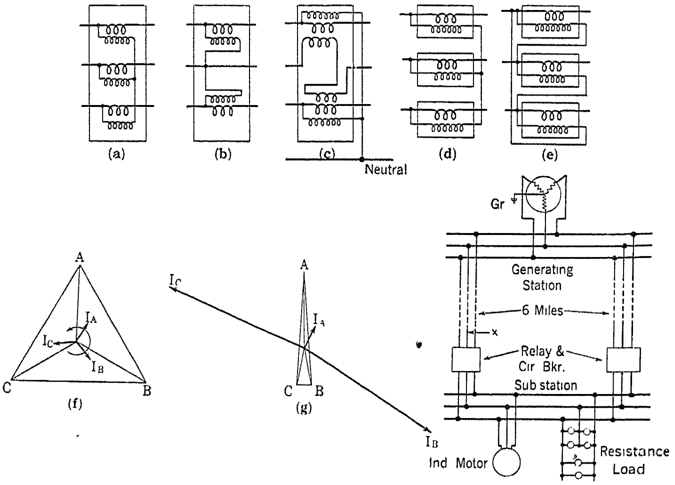


FIG. 3—REVERSE-POWER RELAYS—METHODS OF APPLYING

trouble. On the other hand, the load at the substation is still drawing power over the bad line as well as the good one, and the relay will, of course, be also acted upon by this power. The result of this condition is that, although the short-circuit current may be very heavy, the actual power loss occurring in the conductors may be small whereas the load on the substation may be large. Any polyphase relay which is acted upon by the total power in the circuit cannot, therefore, be expected to give satisfactory service.

Several suggested methods of connecting reverse-power relays are shown in Fig. 3, and tests have been made which show their relative merits. A polyphase relay having three elements is shown at *a* and can not operate satisfactorily for the reasons just mentioned. A polyphase relay, consisting of two elements, arranged and connected according to the practise ordinarily followed in connecting up polyphase meters cannot, strictly speaking, be said to measure the total power flowing in a line when there is an unbalanced short circuit, because the sum of the currents in three wires does not necessarily equal zero. The action of a relay of this type, when only two wires are short-circuited, depends upon which pair of wires is involved. The curves in Fig. 4 show the action of the relay under various kinds of loads, and with the short circuits placed at varying distances from the substation bus bars. The tests were made on an artificial system which represented a pair of lines each consisting of a 4/0 cable six miles long. These tests were made by placing the short circuit on the line at the point shown in Fig. 3, and then varying the load until the polyphase wattmeter read zero.

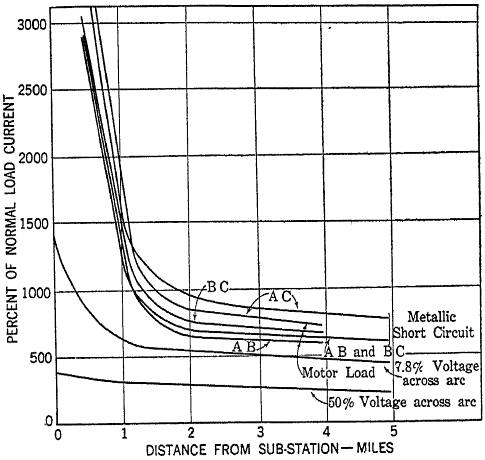


Fig. 4—Two-Element Polyphase Reverse-Power Relay—Ungrounded Neutral System—Two Wirfs Short-Circuited

The short circuit was then removed and the load current measured. The vertical ordinates on the curve indicate the number of times the short-circuit current exceeded the load current to make the wattmeter read zero. If the short-circuit current is less than this value, the wattmeter will not reverse; therefore the relay system will not clear the trouble. It will be observed that when the trouble occurs close to the substation, the short-circuit current must reach an excessive value before the relay can be certain to operate. With the test arrangement available, it was impossible to determine the values of some of the curves with trouble placed nearer than one-half mile to the substation, but it appears that on most actual installations of relays, it would be impossible to expect such an arrangement to operate, if trouble is nearer to the substation than one-quarter mile. It will be observed that the conditions are sometimes more favorable towards relay operation when the load consists of induction motors. This is because the motors attempt to balance up the voltage on the various phases, and as a result, actually furnish power to the short circuit. On the other hand, the motors, in their attempt to balance up the voltages, draw an excessive amount of power from the good phases which has a tendency to prevent the relays from operating. The unequal effect of these two circumstances is the probable cause of the crossing of the two sets of curves. It is obvious that if the short circuit has a high resistance such as might be caused by an arc, the relays would be better able to operate. Two curves are shown where the short circuit consisted of an arc between carbon electrodes.

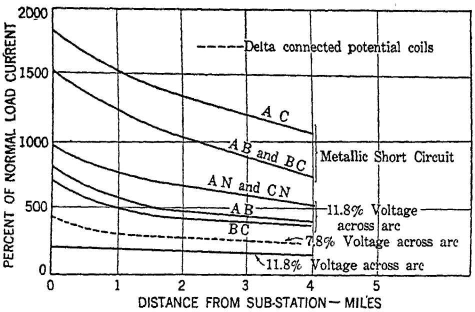


Fig. 5—Two-Element Polyphase Reverse-Power Relay Star-Connected Potential Coils—Grounded Neutral System

Fig. 5 shows a similar set of curves made with a wattmeter having two elements, but with the current coils split as shown at *c* in Fig. 3. This is the type of wattmeter which is frequently used in measuring power on a four-wire circuit. It will be seen from the curves that a relay constructed on this principle will give better service, but will nevertheless fail under many practical conditions.

In the past, it has been customary to employ single-phase relays with the potential coils connected in star, as shown in *d* on Fig. 3. The objection to this scheme is that when two of the wires are short-circuited, one of the relays is liable to operate backwards. This may result in tripping out the good line as well as the bad line at the substation end. The reason for the reversal of one of the relays is shown by the vector diagrams at *f* and *g* in Fig. 3. This diagram represents the conditions in the good line and it will be observed that the distortion of the voltage triangle has caused the current in the *C* phase to lag more than 90 degrees behind the voltage, with the result that the wattmeter or relay which utilized this current and voltage will operate in the wrong direction. It, therefore, appears that a polyphase reverse-power relay is liable to fail by not opening either of the lines and that single-phase relays connected in star are liable to fail by opening both of them. This is the result which has been experienced in practise. Both these troubles can be overcome by using single-phase relays and connecting them with their potential coils in delta. The potential vector is chosen, which at unity power factor lags thirty degrees behind the current, and therefore, any lag in the current which is sure to occur during short circuit will be in the proper direction to give maximum torque on the relay.

Reverse-power relays are now manufactured which will operate positively on one or two per cent of normal voltage and recent experience, as well as numerous tests, has shown that this degree of sensibility is sufficient to insure the correct operation of the relays under practically all operating conditions. Whenever a short circuit occurs between only two wires, or between one wire and ground, low voltage cannot exist on all the relays, and when all three wires are short-circuited, there is invariably sufficient voltage across the arc so that the relays can discriminate even if the trouble is right at the bus bars. The only possible case of failure is, therefore, a three-phase metallic short circuit at the bus bars, which condition cannot be obtained on any except small systems having insufficient capacity to burn off the short circuit and start an arc.

In conclusion, I would like to point out the fact that the phrase "normal direction of power" and the term "reverse-power relay" have proved stumbling blocks to many engineers who were contemplating a complete system of automatic sectionalizing. In treating such problems it should be borne in mind that the direction in which power normally flows through the circuit has very little to do with the selection and adjustment of relays. The question which must be determined is in what direction does the power flow when a short circuit occurs? If the source of power is fixed, the system can ordinarily be sectionalized by means of reverse-power and definite-time-limit relays. On the other hand, if the source of power is not fixed, that is, if there are a number of generating stations on the system which may be cut into or out of service, it may be necessary to resort to a balanced scheme.

J. R. Craighead: In Mr. Torchio's paper he made reference to the performance of current transformers used with selective relays when subjected to high overloads. This has in some cases an important influence on selectivity.

The ratio curve of a good current transformer under normal conditions is ordinarily of the general shape shown in Fig. 6. The reason for its comparative flatness is that for ordinary metering and instrument purposes we do not use the full range of which the transformer is capable, on account of accuracy requirements.

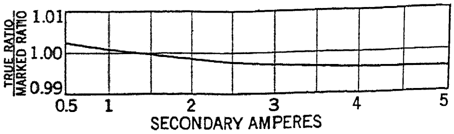


FIG. 6—RATIO OF CURRENT TRANSFORMER UNDER NORMAL CONDITIONS

When we push the current on the transformer to higher points, this curve turns upward after the density in the core passes the point of maximum permeability, and when it does turn upward it goes at a rather rapid rate, eventually reaching a point of complete saturation, at which the secondary current hardly increases at all for further increase of primary current, but at which the phase position of the secondary current recedes further and further from the primary current. (See Fig. 7.) In transformers that are pushed to the absolute limit, phase angles as large as 40 to 50 degrees between the position of the primary current and the position of the reversed secondary current are readily obtained. The point where the curve commences to turn upward is determined by the amount of secondary load, which is connected to the transformer. If the transformer has a very low secondary connected load the ratio curve will start somewhat lower, and will not turn up until it gets out to a very

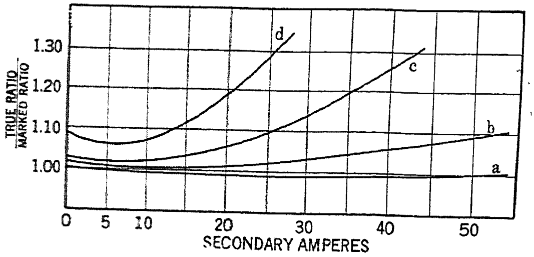


FIG. 7—RATIO OF CURRENT TRANSFORMER UNDER HEAVY OVERLOAD CURRENTS

a, b, c and d represent successively increasing steps of volt-ampere load extreme current. It is possible to load good transformers as high as twenty to thirty times normal current and still have only from five to ten per cent change in ratio by making the secondary connected load very low. If the secondary connected load is very high, the curve may be rising at the full-load current, and turn upward sharply beyond, so that, even at twice or three times the load it may show changes of from ten to one hundred per cent in ratio.

Now, as applied to selective relays, this means that the shape of the curve of the selective relay when you get to the point where this happens is decidedly affected by the transformer. In general, the tendency is to flatten the curve of the selective relay. It does not reach the point of turning up again due to the transformer, for the reason that the secondary current of the transformer never diminishes; no matter how far out you push the increase of primary current, the secondary current is still slightly increasing, but only slightly.

Mr. Schuchardt gave some illustrations, showing that the definite-time-limit relay on a series of stations tends to pile up the time delay necessary in the generating station, and he showed specific means in use on his lines for keeping the time down by the use of balanced systems to cut out certain parts of the feeder.

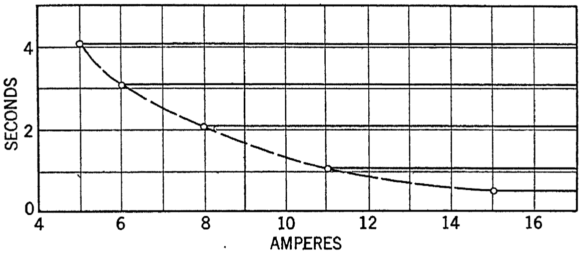


Fig. 8—Relation of Inverse and Definite-Time-Limit Relays

There is another means by which in general the time can be kept somewhat lower on these arrangements. By the substitution of a suitable inverse-time-limit relay for the definite-time-limit relay, the time of the relay nearest the fault tends to be relatively less and that of those further removed from the fault relatively greater, particularly where much synchronous apparatus is connected. Thus a safe differential may be secured between the successive relays with closer settings than when the time of each relay is definite.

An inverse-time-limit relay is equivalent to a combination of a number of definite-time-limit relays set for varying currents and different times. Referring to Fig. 8, suppose five definite-time-limit relays at the same point on a system are set for amounts of 5, 6, 8, 11 and 15 amperes, and for times of 4, 3, 2, 1 and 0.5 seconds respectively. Evidently with any current above 6 amperes and below 8, tripping will occur in 3 seconds; between 8 and 11 amperes, in 2 seconds, etc. Evidently an inverse relay which has a time-current curve represented by the dotted line accomplishes the same function as an infinite number of definite-time-limit relays with settings such as indicated.

When the study of a system shows a certain time-current curve to be desirable, it is not always possible to secure a relay which will give the desired result. In the curves given in the paper the slope of the first part is so sharply downward that an undue share of the possible selective time difference is wasted in a very small increase of current. A curve approximating more nearly a straight line maintains selectivity through a greater range of currents with less total time setting. It can be applied to practically any of these circuits we have been speaking of and can somewhat diminish the actual time to be set without diminishing the separation between the various successive switches that are to be tripped.

O. C. Traver: It is very gratifying to note that both the New York Edison and the Commonwealth Edison companies are devoting considerable time and energy in order to properly apply relays to the lines of their systems. I believe it would pay every large operating company to keep a well paid expert continually at work on this problem. Generally speaking, by proper investigation and application, the most unattractive and low priced relays can be made to do as good or better work than the highest priced devices on the market are capable of doing as applied in some cases. Greater investment along these lines will yield good dividends.

Mr. Schuchardt spoke of relays having almost human intelligence. Our president tonight used the same reference. Human intelligence is available for correct application of relays. I believe it advisable to use it here as well as to expect it in the relay as suggested by Mr. Schuchardt. For, gentlemen, if we could endow a device with human intelligence and then add animal instinct and woman's intuition, all three of the highest order, we would yet come far short of what is generally desired of a relay. If we would allow a relay as much time as is required by human intelligence to act, it would do its work with a fraction of the errors occasioned by human action. In fact, a relay's work lies in the superhuman class.

I am very glad to see that Mr. Torchio has defined his terms at the beginning of his paper. This eliminates misunderstanding. As an example, many of us are accustomed to terming the load feeder connecting the substation with a customer or actual load, as a radial feeder. That is, the last link in the line to the load or customer, would be called a radial feeder. As you remember, Mr. Torchio referred also to a trunk feeder or the connection between busses back to the generating station as a radial feeder. Mr. Torchio's care to forewarn us completely obviated any difficulty. Some standardization of these terms, however, might well be undertaken by the Institute.

I believe both Mr. Torchio and Mr. Schuchardt are relaying particularly for break-downs on their system. I am sorry to say that a large number of operating companies have not yet come to this way of protection. I believe it to be the proper one wherever overload relays are used.

Mr. Torchio referred to the Merz-Price system in a way which leads me to believe that he had also in mind a system fathered by Mr. Hunter, also of England, although the description by Mr. Torchio would apply equally well to either one.

I notice in both papers the use of the terms reverse-power relays and reverse-energy relays and unidirectional relays. Here, again, I would suggest that it would be worth while for all to get together and use the same terms so there would be no misunderstanding. Sometimes I feel it would be preferable to use the simple term directional relay, on account of the fact that the relay may be called upon to operate in case of power flowing in either direction, such, for instance, as in the case just shown by Mr. Crichton on the screen, viz: a relay having two contacts, closing one in the event of power in one direction and the other in the event of power in the opposite direction.

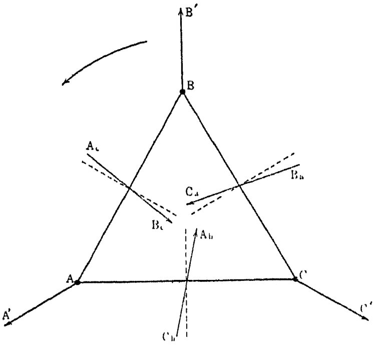


FIG. 9

With reference to the question of polyphase relays, what Mr. Crichton has said tonight I agree with completely. So far as I know, all the large companies have given up the use of the in-phase connection shown by him on the screen.

I would like to bring out in a little more detail the advantages of one of the comparatively new methods of directional or reverse-power relay connection.

Referring to Fig. 9, A, B and C, represent the voltage triangle of a three-phase circuit, in which circuit, at unity power factor

A-A′ is the current in phase *A*
B-B′ " " " " " *B*
C-C′ " " " " " *C*

In most wattmeters and directional relays maximum torque is obtained when the current and potential are in phase—I will confine myself herein to this type of relay, and for the purpose of brevity, I will omit all discussion concerning the failure of earlier forms of connections.

If now we consider the current $A-A'$ it is desirable to obtain for use with it a voltage of approximately the same phase direction as $A-A'$ For the proper results this potential phase direction should remain reasonably fixed even with the most severe distortion of the voltage triangle under conditions of single-phase short circuit. Potential $B-C$ is 90 deg. out of phase with $A-A'$, therefore if we displace $B-C$ 90 deg. the displaced potential will be in phase with $A-A'$. The use of such a potential would result in maximum torque at unity power factor.

Serious faults, however, almost invariably result in lagging current. Instead of displacing $B-C$ 90 deg. then, if we displace it as shown by B_a-C_a our maximum torque will be available with a certain degree of lagging power factor. So—

B_a-C_a is the displaced potential $B-C$ and it is used in connection with current $A-A'$.

C_b-A_b is the displaced potential $C-A$ and it is used in connection with current $B-B'$.

A_c-B_c is the displaced potential $A-B$ and it is used in connection with current $C-C'$.

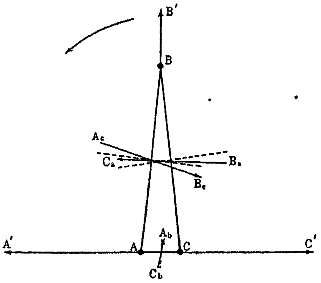


FIG. 10

This is what I would call "connected in quadrature".

We will now investigate the condition resulting from a single-phase short circuit.

Fig. 10 represents the vector relations of the various currents and potentials in such a case. For simplicity and clearness unity power factor will be assumed.

It will be observed that the displaced potential B_a-C_a again slightly lags behind the current $A-A'$. Not only is the vector relation good but the potential is nearly at full value, a very valuable combination. The same is also true concerning the short-circuit current $C-C'$ and its companion potential A_c-B_c.

Even in the case of the third phase which is not greatly affected by the fault, the relation of potential and current is still correct.

If we consider a dead single-phase short circuit between A and C, then potential $A-B$ will be superimposed on potential $B-C$. The phase relationship of the connections to the relay coils is still correct and operation thereby safeguarded even with zero potential across the short-circuited phase.

For a single-phase load and no distortion of the voltage triangle, a maximum relative variation of 30 deg. between the current and potential is possible. The amount of distortion under such conditions is entirely harmless.

This method of quadrature connection is now used exclusively by one of the large manufacturing companies for both dynamometer and induction types of directional relays.

In the polyphase relay as it is now being exploited, three elements are used; three absolutely separate and individual elements excepting that they are all working together on one common shaft. It consists then, of three single-phase relays working on the same shaft. Current and potential are again connected in quadrature as described above. From these diagrams it is seen that each individual phase of that relay will operate properly so that there is no particular tendency—there seems to be no tendency whatever—for any one of the three phases to work incorrectly. So, if you can get power enough to operate a single-phase relay you will have, in general, three times as much power to operate the polyphase relay. In the case of a single-phase short circuit we have, in two elements, large currents working in good phase relationship with large potentials. These two elements provide many times more power than is needed, so that there seems no chance under the sun for the relay to go wrong in such a case.

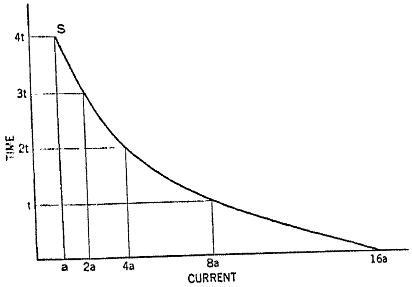


FIG. 11—THEORETICAL TIME-CURRENT CURVE

Now, do not misunderstand me when I say no chance to go wrong. We have to be particularly careful if we use the polyphase relay on a system with grounded neutral. That is because of a tendency, sometimes in the case of a ground, to cause power to flow in one direction over one wire, and in the opposite direction over the other two wires. This difficulty in a large number of cases, can be overcome by proper connection of the relays, as Mr. Torchio has brought out in his paper, viz., by inter-connecting them. By such inter-connection, the power current which would cause the trouble, is balanced out so that simply the fault current is left to work in the relays.

I would like to say just a word in regard to inverse-time-limit relays. Mr. Craighead mentioned the fact that the shape of the curve was limited. I think that I am perfectly safe in saying that the shape of the curve is limited by the price that you are willing to pay for the relay. I do not think that it would be at all impossible or difficult to obtain a relay which will do the work you want, if you want it badly enough.

I would like to give one example of how an inverse-time-limit relay would be of greater value than a definite-time-limit relay.

If you take, for instance, two links of one circuit which may be in tandem or in series with one another; the one farther from the generating station, is equipped with a current transformer whose ratio is one-half of that of the one nearer by, or if the current setting of the relay farther from the station is one-half that of the relay near the station, it is perfectly evident that the actual current in the farther relay, in case of a short circuit beyond it, will be proportionately twice as great as the one near the station. Now supposing we lay out a theoretical curve, as in Fig. 11.

For this theoretical curve we will assume that each time the current is doubled, the time will be reduced *t* seconds, which is that amount of time necessary to give selective action between two circuit breakers.

Inasmuch as the relay nearest the fault always gets the equivalent of two times the current in that relay next farther back toward the source, the difference in time of the two relays will always be *t* seconds and selective operation will result in every case providing, of course, proper setting is made so that time can never be less than *t*, excepting for trouble in the immediate section.

If there is enough current through the system to be seriously dangerous, the time would be down to so low a value that there would be no necessity of using the extra instantaneous overload relay spoken of by Mr. Schuchardt.

A. A. Meyer: Summing up Mr. Torchio's paper, I find that in all of the schemes of connection, overload relays, usually reverse-time-limit with graded settings, are used in all outgoing feeders, and overload relays in connection with reverse-power relays on incoming feeders. This is true in both radial and parallel operated lines.

He has made a number of classifications which apparently are only combinations of the two fundamental schemes of operating lines, viz. radial or parallel. I cannot appreciate the value of his classification and I believe if each operating engineer will take his own system and try to fit it into one or the other of the divisions, he will find that his particular system may be the same in many respects, yet considerably different than any one of the types indicated in Mr. Torchio's paper. Also, I do not think that Mr. Torchio meant to infer that the examples which he cited under those headings are particularly the best schemes of relay connections.

On incoming lines in some cases he uses straight reverse-power relays, without any overload feature; and in other cases a combination of the two. I have reference particularly to Figs. 2, 3 and 4 and wonder why in some instances the overload feature is omitted.

Of the various schemes of protecting parallel operated cables I favor the balance scheme and I wonder why so much attention is being paid to the adaptation of reverse-power relays. These, in combination with the overload relays, require, usually in very extensive or heavily connected systems a scheme of finely graded settings which in practise, for various reasons, do not always give the desired sequence of operation. Then also the direction of energy may not always be fixed and it may be desirable at times to send power in a direction reverse from normal. The possible complications seem much more numerous with the reverse-energy relay combinations than with any of the balance schemes.

Many of the operating companies have cable systems still working radially which might be changed advantageously to parallel operation. The scheme of protection of course depends largely on local requirements, but I want to point out how easily the change from radial to parallel operation was made in the city transmission system of the Detroit Edison Company by using the same cables and introducing a balanced scheme of protection. This company is operating two main generating plants, viz. Delray and Connors Creek Power Houses, located on the Detroit River and at opposite ends of the city of Detroit. From each of these generating stations the 24,000-volt cable lines were feeding radially to the various substations and between the latter were interconnected tie-line cables. The lines going to the same substation were selected with care and paired up, two cables in a pair acting as one unit. These paired cables were then permitted to tie to a common bus in each substation. This resulted in a system of several loops extending from the one power house through the various substations and then to the other power house.

Each pair of cables serves in a manner similar to a split-conductor cable and is protected by means of a balanced scheme of relay protection. This consists primarily of a simple overload relay connected between like phase conductors of the two cables, its tripping function depending upon a reasonable out-of-balance in the currents of the two cables. In case of trouble in one cable, both cables of that pair would be caused to trip, thus isolating that pair from the system. The faulty cable might be tested out and, if required, its good mate might be put back in service. In such case the single cable is protected by an overload relay, substituted for the differential.

This scheme of operating and protecting parallel cables has been in operation over a year and has been very effective in isolating trouble and also in reducing the number of interruptions of service. The necessary reserve-cable capacity to any substation is considerably less in the new parallel scheme and with the new system there is also a greater flexibility in proportioning the load between the two power houses.

J. B. Taylor: Mr. Meyer wants to reduce all systems to two general classifications, and its seems to me that the idea is quite sound. I want to suggest that all this protective idea, protective development, is along two general lines; the first is that no matter how complex the system we can look it over and say that when a fault comes it will do thus and so, power will feed this way or that way, current will be greater here or there, voltage will drop here or rise somewhere else, phases will do this, that or the other.

Now, in so far as our assumptions are correct and our analysis is right, we can apply relays more or less effectively, and by far the greater amount of experimental work, development work, has been along this line. But I think if we look at the problem a little more from the inherent points that we are trying to accomplish it will shape itself up this way: No matter what the system is, no matter whether we are considering cables, transformers or generators, any fault which is the starting point for clearing switches means that the current flows in at one end and does not flow out at the other. That is the foundation of this differential idea, whether you want to call it by Merz-Price or something else. In the papers tonight we have seen that the Chicago people have applied this principle to the generator. Any fault in the generator means that the current goes in at one end of the winding and does not come out at the other. We have seen the same idea in transformers where current going into the primary does not balance, that the current in the secondary upsets the balance. These applications are fairly easy. The two ends of the circuit, transformer and generator, are close together. The leads in a differential connection are short and inexpensive. The idea of applying this to cables and lines was thought of later.

It seems to me that the use of the split-conductor, or the two cables in parallel, which Mr. Meyer has just described as used in Detroit, is an extension of the pilot-wire system, in which, in order to avoid the investment, the trial investment of the pilot wire, we simply take another conductor, use it as a working conductor and also as a pilot wire. So, if that is the principle applied, we seem to be on a very simple fundamental basis for isolating any faulty piece, no matter how extensive, and only by carrying out this principle can it be followed to the limit, and of course the limit is far more complex than any of the diagrams shown tonight, because just so soon as we can reliably disconnect a faulty feeder we can effect great economies in copper by interconnecting wherever we wish. There are, of course, limitations to this idea. We must not forget that the very transformers which we put on to determine this differential are themselves pieces of apparatus, so that we may get to the point where we have to put transformers on other transformers, and in many connections the switches are outside of the differential link. So, there are still a few problems to be worked out before we can say that there is available 100 per cent protection.

A paper presented last month by the Chicago Edison man at New York, referred to where a cable failed and the switch did not clear because the very protective devices, the current transformers put in to clear that cable, were mechanically disconnected by the large forces developed.

H. L. Wallau: In our experience in Cleveland we have found a satisfactory solution for our problems to date by the use of the overload relay at the sending end, and the reverse-energy relay at the receiving end. We operate tandem with cables in parallel. The balanced scheme suggested where two cables are used practically as one transmission line at first blush seems like a wasteful thing, because when your relays operate two lines instead of one go out; but I think if we study the problem a little bit we will find that the resultant economies due to the inter-connection value of all the lines in one net work will give us more extra emergency capacity than we lose by using two lines at once, so that the net gain will be greater than the apparent loss.

The problems as we have had to meet them in Cleveland,—the particular locations of our substations and the cable lines already installed and the subways, which to a large extent had to be used—have apparently mitigated against our developing any system of balanced protection. Nevertheless, we have had it constantly in mind, and I think that the day will come when we shall go over to that system. I think the system is much more easily applied when there are two generating stations located at a considerable distance apart, with substations scattered in between. If there is but one generating station and the substations are mostly located in one general direction from the plant, it is sometimes hard to justify a large expenditure in subway and ring feeds to bring about the conditions necessary for a balanced system of protection.

J. S. Jenks: I will give a little experience to show what developed in the application of the various types of relays we have heard discussed this evening. We started with a system of transmission connecting a smaller number of substations with radial feeders. The system so grew that it finally consisted of over 600 miles of transmission lines connecting over eighty substations. When we came to loop the system with twelve or fifteen substations in a loop, you can see that the undertaking of applying any time-element, selector-type or inverse-current relay was almost an impossible job; our experience had shown us that it was impossible to get a satisfactory relay that could be depended upon to operate closer than a one-half second. Hence, in graduating the time, we would have to have at least a half second between two stations, and with half a dozen stations it meant an abnormal time and with ten stations an impossible time; in one case where we had fifteen stations it made a setting of seven one-half seconds. This would not give satisfactory protection. After giving the matter considerable study and looking particularly to the financial end of the problem, we found that about the simplest and most effective system of relays we could devise is one, as shown in Fig. 12 consisting of a three-phase primary relay which is connected to a secondary of potential transformers on the high-tension substation buses and a secondary relay actuated from the substation lighting transformer and controlled by the primary relays controlling the locking relay on the switches. The one primary and secondary relay will control any number of locking relays on any number of circuit breakers.

The operation of this system of relays is as follows: When the line potential drops below a certain predetermined value, the primary relays drop and short-circuit the secondary coil through a resistance which in turn opens the secondary contact, thereby interrupting the potential to the locking relay, which then unlocks and leaves the circuit breaker in condition to operate from an overload. So long as the potential remains within a certain percentage of normal no breaker can be opened by the automatic feature on same. As soon as a fault of any kind develops which will lower the potential to a predetermined value at any station, it unlocks the circuit breakers in that station, and if two stations happen to be very close together, it may unlock them in both stations, and then the section of the line which is in trouble is tripped out by the overload trip. This arrangement not only has reduced our interruptions to a minimum, but has very materially reduced the service of the switches, there being a certain time element due to the fact that the primary and secondary relays and the lock on the switch have to operate before the circuit breaker opens; and before that procedure starts there has to be a sufficient drop in the line to reduce the voltage to some predetermined value. Such an

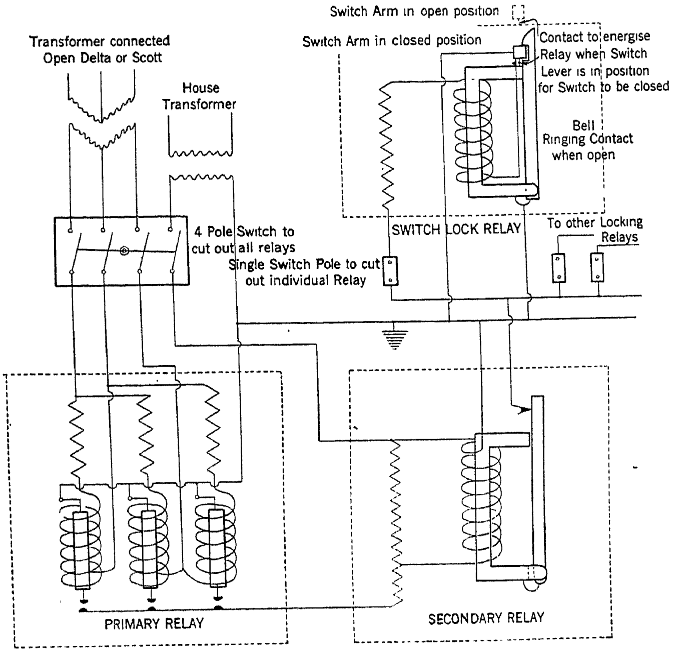

Switch Arm in open position
Transformer connected Open Delta or Scott
Switch Arm in closed position
Contact to energise Relay when Switch Lever is in position for Switch to be closed
House Transformer
Bell Ringing Contact when open
4 Pole Switch to cut out all relays
Single Switch Pole to cut out individual Relay
SWITCH LOCK RELAY
To other Locking Relays
PRIMARY RELAY
SECONDARY RELAY

FIG. 12

arrangement has proved, as far as our experience is concerned, more satisfactory than any other arrangement of relays which we could work out on account of the great number of stations in circuit and the large number of sources of power, the system which I have reference to having six power stations and 600 miles of transmission line and about eighty substations, all looped together and all operated as one unit. In addition, we have made a number of connections with lines which are parts of other systems, so that at the present time we are operating not only all our own system in parallel but are in parallel with systems of three other companies, and all our lines protected by the simple, inexpensive system of relays.

One of the speakers mentioned interruption. We adopted our plan with the idea of limiting interruption. He wanted to know what we called an interruption. We call any cessation of power an interruption, if it only be for a fraction of a second. If the power is off the line, the breaker opens and is immediately closed, we term that an interruption. Some of our customers call a fluctuation in voltage an interruption.

Philip Torchio: With the exception of the very interesting contribution of Mr. Jenks, about whose system of protection I am sure all of us would like to know more, the main questions submitted by the speakers in reference to my paper are principally calling for explanations of statements rather than criticisms of the subject matter.

Mr. Summerhayes has commented upon the statement that generator protection is seldom used. I did not mean to endorse that practise. On the contrary I think that generators, especially large generators, should be protected, and I agree with Mr. Summerhayes.

Mr. Cole has misunderstood my reference to one of the preferable arrangements. I did not refer to the split conductor as the preferable one. I said that the balanced system is preferable, and I was rather careful in describing what a balanced system is. Under letter (b) I gave a definition generic enough to include the Merz system, the split-conductor system and all kinds of balanced systems.

I am glad that Mr. Meyer of Detroit has asked why, as in Fig. 2, I did use only a reverse-energy relay. I had already discovered that the diagram showed only the reverse-energy relay, though alongside in the diagram of equipment connections both the overload relay and the reverse-energy relay are shown. The matter will be corrected by rewording the references.

The reverse-energy relays are never calibrated for overload. In all arrangements with reverse-energy relays the overload calibration is secured by the other element in series, which is an overload relay. This applies to all cases where a reverse-energy relay is shown.

I have been asked by Mr. Craighead why in Fig. 11 the sheath transformer was used instead of using the neutral of the three current transformers. If the grounding relay is connected to the neutral point of the standard transformers, as shown in the connections to the converters in the substation, the third harmonic produced by saturation of the current transformers causes a current to flow through the grounding relay under conditions of heavy balanced overloads. We found from tests that this current amounts to 0.2 ampere with 34 amperes secondary current and runs up as high as 5 amperes when the balanced three-phase current reaches a value of 70. This makes the installation of the grounding relay objectionable in the generating station where very excessive currents may be obtained under short-circuit conditions.

R. F. Schuchardt: Mr. Summerhayes asked why the split-conductor cable is not more generally used. Mr. Cole has already given several reasons for this. In addition, the system is not as simple as it seems on paper. It costs more than other systems giving equally good results and it is not an easy matter to apply it to an existing system. The transmission system must have a high degree of flexibility so that industrial installations may readily be added at any points without requiring a complete rearrangement of the protective system. The scheme outlined in my paper permits such additions readily. If we were to start with a clean slate we might, to advantage, use split-conductor cable throughout.

Mr. Summerhayes also asked my personal opinion regarding the proper size of frequency changer to be used as a tie between two 50,000-kw. systems. A great deal more information regarding the systems is necessary before this can be answered definitely. However, it may be of interest to note that the particular installation referred to—that is, the 5000-kw. frequency changer at Quarry Street—connects a total of 56,000 kw. of 25-cycle capacity with 28,000-kw. capacity of 60-cycle units, though each of these groups of units is connected through heavy tie lines with other units at Fisk Street station. With the present relays the frequency changer switch opens at times on relatively light disturbances on either section. We expect that the new relay installation will prevent this and cause the unit to drop out only when there is a fault within it or when the disturbance on either section is so great that the synchronizing energy passing through the frequency changers is more than they can safely carry.

Mr. Crichton asked our ideas regarding the maximum safe time for holding on to a faulty line. The answer for the Chicago system is given in my paper as 1½ seconds. In New York a somewhat longer period is permitted. In his question Mr. Crichton undoubtedly had in mind the effect on the service delivered. It will be noted that in the scheme for the Chicago system the opening up of a line in the interconnected system does not affect the remaining lines and therefore does not interrupt any service. That is the fundamental idea of the entire scheme.

Mr. Crichton's discussion of the various combinations for the connections of relays is very interesting indeed and we are pleased to have his assurance that the particular connection adopted in Chicago is the one that will always work.

Mr. Craighead showed a curve illustrating what could be obtained by a combination of relays, which curve I agree is a desirable one. Mr. Traver stated that a single relay having a characteristic curve as drawn by Mr. Craighead could be built if we want it badly enough. We certainly have wanted it badly enough for many, many years, but the success of the relays we are now using has changed this. By using an induction type of relay together with an instantaneous relay of the simple solenoid type we can very nearly obtain the same results, and Mr. Traver's instrument now has this combination to compete with commercially.

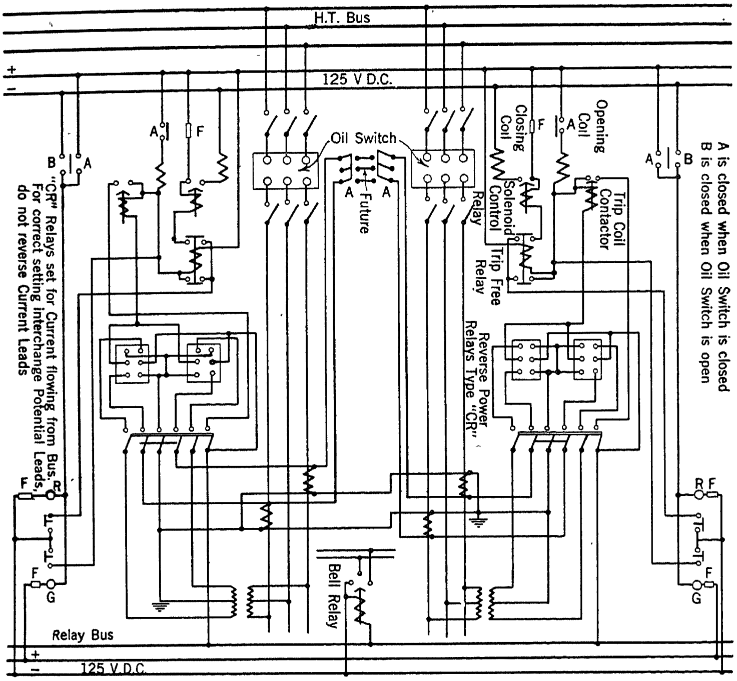

H.T. Bus
+
–
125 V D.C.
Oil Switch
A
F
B
Future
Closing Coil
Opening Coil
Solenoid Control Relay
Trip Coil Contactor
Trip Free Relay
Reverse Power Relays Type "CR"
A is closed when Oil Switch is closed
B is closed when Oil Switch is open
"CR" Relays set for Current flowing from Bus.
For correct setting interchange Potential Leads,
do not reverse Current Leads
R
T
G
Bell Relay
Relay Bus

FIG. 13—GENERAL REVERSE-POWER RELAY CONNECTIONS, TIE FEEDERS

N. L. Pollard (communicated after adjournment): Mr. Torchio mentions several combinations of relays covering different conditions on the New York Edison Company's System that apparently have been giving good protection.

Fig. 2 of this paper shows four feeders in parallel between a generating station and a substation. These feeders are protected by a combination of balanced and reverse-energy relays. It is not clear to me from the diagram, how the secondaries of the current transformers on any one of the circuits may be automatically cut out or disconnected. I imagine this may be accomplished by means of auxiliary switches operated by the oil switch. I also note that two sets of current transformers are used, one set probably having a ratio of 1:1. This is evidently an additional safeguard to prevent the high-tension current from reaching the low-voltage relay circuits.

The Public Service Electric Company have had in successful operation in South Jersey for nearly a year, an arrangement similar to the above, with the exception that there are only two feeders in parallel and we use only one set of current transformers. Each oil switch is equipped with a D. P., S. T. auxiliary switch which breaks the connection between the two sets of current transformers. In case one line is automatically cut out, the second line is left with a straight overload protection. The connections are as shown in Fig. 13. We have had a number of cases of trouble and the relays have to date, always operated successfully.

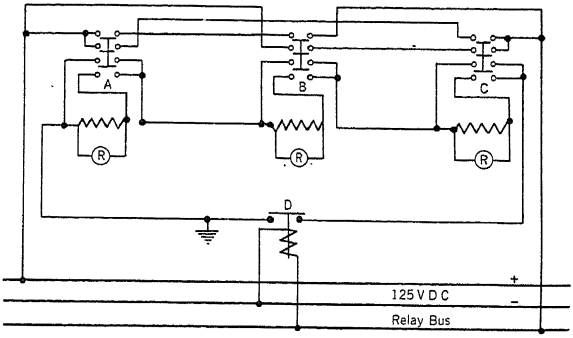


Fig. 14—Connection Diagram for Selective Relay Scheme

R = Reverse-Power Relay Contacts *A*, *B* and *C* are up when oil switch is open and down when switch is closed.

D is open when *AB*, *AC*, or *BC* are up.

Fig. 14 shows an arrangement of auxiliary switches for cutting out of circuit, the secondaries of the current transformer of the feeder in trouble, when three feeders are operated in parallel, with the same relay connection as mentioned above.

I wish Mr. Torchio would explain a little more in detail regarding the use of the "Zig-Zag" 3-phase grounding transformer, shown in his Fig. 11.

In regard to the balance system, using split-conductor cables, which we expected to put into operation last fall I would say that due to trouble in getting deliveries on the differentially-wound current transformers and small reactances, we are operating the split-conductor cable without the balance protection.

We feel that the split-conductor cable, operated on the balanced principle, will give us 100 per cent protection.

We expect, this year, wherever possible, to pair a number of our overhead lines and operate them as split-conductor cables. This, we feel, will give us better-protection than any other scheme, providing of course, that the power can be fed into the substation by some other line or cable, in case these two lines, operated as a split-conductor cable, should be automatically cut out of service.

Mr. Torchio's paper has made me optimistic on the relay situation, and I feel that almost any situation can be taken care of by using one of the schemes illustrated in his paper.

Chester Lichtenberg (communicated after adjournment): The shape of the time-current characteristic of a protective relay is an important feature of its economic application.

The time delay of the direct-connected solenoid and induction designs is affected at low currents by the responsivity of the relay. Suppose, for example, that the relay is set to start operating at 5 amperes. If a current of 5.1 amperes is sent through the relay, it will start to operate, but its time delay will be long, and for successive trials may vary several hundred per cent. If the test is repeated at 5.2 amperes, the range of variation will be found to be less, and at 5.3 amperes it will be found to be still less. An investigation of this characteristic has indicated that the most consistent results were obtained when the first point of the time-current characteristic was determined with a current equal to 110 per cent of the current at which the relay was set to start operating.

The time delay of direct-connected solenoid or induction relays is also affected at high currents by the characteristics of their time-delay features. The time delay of the bellows-equipped solenoid relay becomes irregular and may vanish at currents from eight to ten times the rated current of the operating coil. This is caused by the forces, produced by the relatively high currents, straining the time-delay feature beyond its capacity, and prematurely closing the auxiliary contacts. The ordinary induction relay has a similar critical current point, above which, however, the time delay is lengthened. It varies from 25 times rated operating current in one design to over 80-times operating current in another design. It is caused by the leakage flux from the core of the operating windings counter-acting the effect of the flux of the circuit normally controlling the time-delay feature.

The critical point in the time-current characteristics of these relays can be fixed beyond the maximum operating current by the addition of a shunt reactor. The reactor is so designed that it shunts practically no current from the relay at low currents, but shunts out 50 per cent or more at high currents. It not only permits less current to enter the relay at high currents, but also acts as a stabilizer by smoothing the current rushes into the relay.

The time delays given in Mr. Torchio's paper range from about 0.1 second to about 4.5 seconds. A stop watch, such as ordinarily used for measuring time, will not give accurate determinations through this range. Its indicating hand moves in steps of 0.2 second. An error, therefore, of only one step, will, for a time of say two seconds, give a result 10 per cent too high or too low. A larger error is introduced, if, as is customary, the watch is manually operated. The responsivity of the operator then enters, and may affect the results 10 per cent to 20 per cent or more.

The manually operated stop watch is now, however, being rapidly superseded by the automatically operated synchronous timer. This device has an indicating hand or scale which is automatically started when the relay is energized, and automatically stopped when the auxiliary contacts meet or part. It is moved in units equal to or double the frequency of the circuit to which it is connected. It will therefore indicate time intervals in the one-sixtieth or the one one-hundred-twentieth part of a second, if energized from a 60-cycle source.

The time delay of oil circuit breaker mechanisms must also be taken into account when setting relays to operate at short-time intervals. The contacts of ordinary oil circuit breakers will part about 0.25 second after the tripping circuit is energized. This time, however, may vary from twenty to sixty per cent for different designs of breakers, and from ten to twenty per cent for different breakers of the same design. For accurate setting, therefore, relays should be timed and set with the oil circuit breakers which they will operate.

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PROBLEMS OF OPERATION AND MAINTENANCE OF UNDERGROUND CABLES

BY JOHN L. HARPER

ABSTRACT OF PAPER

The scarcity and high cost of transmission materials demands that the transmission capacity of electric underground cables be increased to a maximum.

The causes of failures of cables are analyzed and the conclusion drawn that overheating is the most important for consideration.

Methods of removal of heat from the cables are considered, and recommendations made for flooding the conduit with water, in order that the copper in underground lead-covered cables may approach the transmitting capacity of aerial cables.

THE COST of underground cables for power distribution has doubled within the past two years and operating companies feel loath to meet increased demands by the purchase of new cables at the present inflated prices, when it is not known that either demands for power or the increased prices of cables will be permanent. It, therefore, now often devolves upon the operating engineer to devise ways and means for working present installations to a capacity much greater than they were supposed to have.

Cable manufacturers have endeavored, from their experience, to give the purchaser some idea of the carrying capacity of the cables sold, but the first hand experience of the manufacturers usually ends with the delivery of the cable; hence the data and information given out have about the same relative reliability as those furnished by generator manufacturers for the capacity of generators in the days when such capacity was rated on the temperatures of the outflowing air instead of the temperatures of the inside hottest parts of the coils.

The fundamental causes of cable break downs have not been differentiated from local causes, as data and information acquired by individual cable manufacturers have usually been influenced by local conditions of installation and use, with the result that the fundamentals have not been clearly emphasized and users with varying local conditions have received much misinformation.

For instance, many of the largest distributing systems using underground cables are in large cities where a very low load factor exists and there is a relatively small concentration of power delivery through any one conduit. Under these conditions, cables would naturally be given a higher rating than would be considered for places where they operate under a much higher load factor, as in the case of power distribution having the characteristics of the one with which the writer is connected, where the yearly load factor is in excess of 91 per cent and the daily load factor is in excess of 98 per cent. It is apparent that with this high load factor, cables have no chance to use off peak periods to dissipate the heat accumulated during peak periods. The concentration of a large number of cables in one conduit also has a direct influence on their total capacity.

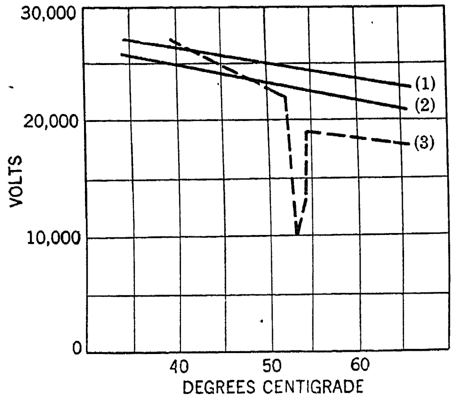

FIG. 1—(1) JOINT COMPOUND, (2) CABLE OIL, (3) MIXTURE OF 1 TO 1 JOINT COMPOUND AND CABLE OIL

Assuming cables to be made of good material and with good workmanship and design, I venture to propose the following as the fundamental causes of failure, viz.: Joint troubles, mechanical injury to lead sheaths, and overheating.

JOINTS

Under this heading may be noted:

1. Those troubles which arise from improper making of the joint, due to improper workmanship, poor material, inexperience, etc., the remedy for which is principally accumulation of knowledge by actual experience and practise.

2. The incorporation of a joint filling compound that has not the same characteristics as the insulating compound used in the cable itself.

Cable insulation compounds are nearly always of a light color, while joint compounds are usually black. The mulatto combination of these two materials (each of which may be of the highest quality in its own clear unadulterated state) may produce a mixture having very different insulating characteristics from either of the original compounds. In the writer's experience, one whole system of cables had to be replaced on account of break downs in the vicinity of the joint wherever the cable insulation and the joint compound mixed. At that time a test was made of the insulating characteristics of the mixture for different temperatures and it was found that, while neither of the compounds in their clear form showed an insulation break down within the operating temperatures, the mixture did, and the curves in Fig. 1 show the relative conditions found to exist.

In a case like this, the operating engineer, being unable to wrench the secrets of the nature of the component parts of the insulating materials from the manufacturing company, was obliged to make further requirements of the cable manufacturer to the effect that he furnish a joint compound which would not show this objectionable effect. As the reinstallation of the cables under this method of procedure proved satisfactory and lasting, it has been assumed that in the privacy of the manufacturer's laboratory such changes were made in the joint or cable compounds that they maintained the high insulating characteristics even in the mixture.

Some expert joint makers contend that, if, in the handling of cables either at the factory or in the manhole, they are cut and allowed to hang down and bleed slightly, a bubble of air is liable to get into the cable, replacing the fluid which has bled. This bubble causes no disturbance until the cable is in operation and thoroughly heated up, when it may, by expansion, displace insulating compound and decrease the insulation at some special point, causing a slight leak of current with the resultant charring of the insulating materials and the production of a hot spot, whose temperature soon passes beyond the current resisting characteristics of the insulating material. This form of trouble often proves very elusive, as, in the case of the hot spot, as noted, if the cable is taken out of commission, cooled and again started up, the trouble may have moved several feet from the first location, or may still be present in the cable even if the original hot spot is cut out.

MECHANICAL ABRASIONS

In regard to mechanical abrasions of lead sheaths, these troubles are positive and must show themselves sooner or later. The writer is not aware of any means of overcoming these troubles in cases where due care has been taken in the insulation of the cable, except to let nature take its course. If the conditions surrounding the operation of the cable are such as to force the early determination of these mechanical faults, the assurance of future continuous operation is greatly increased.

OVERHEATING

The production of heat within the cables is a direct function of the square of the current transmitted as represented by the I^2R losses; and also of any dielectric loss, which is a function of the voltage and temperature of the insulating materials. Any heat produced in the cable must be dissipated through the insulating materials, the protective sheath, the conduit structures, and the surrounding medium, if the cables are to be kept at a temperature within the range necessary to maintain their insulating characteristics.

The paper of paper-insulated cables belongs to that class of materials whose insulating value decreases with the rise of temperature; therefore, within working limits, the cooler the cable is kept the less the dielectric loss.

An instance of the destruction of paper insulation under high temperatures was noted in taking out a length of cable in which a section, at least fifty feet from where any known trouble had existed, was found where the paper insulation had practically disappeared. The three conductors holding their relative positions for about two feet, the lead sheath being fully intact but only half filled with ashes from the destroyed insulation. This cable, while in operation had given no trouble at this place on a 12,000-volt working circuit.

The problem, therefore, resolves itself into a control of the dielectric loss and into an even and rapid dissipation of all heat produced within the cable. The retention of a constantly produced heat, no matter how small, would result in the building up of temperatures not only such as to destroy the insulating characteristics of the cable structures, but to destroy the cable itself.

It is apparent that the area of obstruction to rapid dissipation of heat from the cable or conduit may not be in the cable itself or in the conduit structures, but may be a factor of the quality and condition of the earth surrounding the conduit; thus, a cable system installed under the sod at the side of a paved street may have a very different radiation coefficient from a similar conduit under the pavement of the same street.

In a paper by Mr. L. E. Imlay in 1915 this matter was brought up, and the suggestion made of dampening the earth on the outside of the conduit by means of porous tile laid in the ground outside, which, undoubtedly, is a step in the right direction.

In the writer's experience, a 12,000-volt, 3-conductor, 4-0 cable was found to transmit 190 amperes per leg easily when the conduit contained but one cable, while this same cable could transmit only 140 amperes per leg with the same rise of temperature through the same conduit when other similar cables had been added and were all connected in parallel. This would indicate that, in the particular instance noted, which was through clay soil and under a paved street, the obstruction to the dissipation of heat was in the earth surrounding the conduit structure. It was also noted that in this conduit, which was made up of nine ducts, an installation of eight cables in the outside ducts would transmit practically the same amount of power for the same rise of temperature as would be transmitted when the ninth cable was installed in the central duct. This indicates that any conduit has a total carrying capacity dependent upon the exterior surface of the conduit structure in contact with the surrounding earth. In Niagara Falls, under paved streets, in clay soil, and without artificial cooling, the capacity of any one conduit system is restricted to approximately 30,000 horse power.

That a cable may for a short time, without permanent injury carry power far in excess of its ordinary capacity was shown by a case where seven cables were furnishing power to a customer. A short circuit occurred that opened the breakers on six of these but the seventh failed to open. The needle of the dial on the instrument of the switchboard not being in sight, the operator did not notice that it had disappeared at the upper end of the scale, and he, therefore, left the cable in circuit while waiting advice from the plant to again put on the power. This not being received in the ordinary time, he called up and found that the customer was not cut off. An examination of the recording wattmeter indicated that the seven cables were delivering approximately 22,000 kw., that when the six were opened the power dropped to 18,000 kw. and gradually ran down during a time of 17 minutes to 9,000 kw., at which time it was taken out of service by the station operator. On examining this cable in the conduit it was found to be extremely hot and with several of the joint sleeves broken from internal pressure of compounds or gases. Upon cooling it down, filling up and resealing the joints, the cable was again put in operation and has been doing its work satisfactorily for four years.

In a recent construction of a hydroelectric power plant a saving of $500,000 was made on the first cost by adopting a speed of 300 rev. per min. instead of 180 rev. per min.; therefore, why should not a similar effort be made to decrease the first cost of conduit and cable distributions of power by speeding up the dissipation of the heat formed by the I^2R losses, at least to the point where in any cable system, the power value of the I^2R losses may be made to balance the operating expense of increased construction and installation.

It is, therefore, suggested that the dissipation of heat by water be applied to new and permanent construction and that, in places where such construction is possible, conduit systems should be built so that the ducts can be flooded with water and cables operated under practically submarine conditions. Bare copper wires in aerial transmission lines now carry approximately twice the current that is carried in the corresponding conductor of an underground paper-insulated cable and, if the lead sheath of the cables were kept at a temperature below 60 deg. cent. by the presence of water, the carrying capacities of underground cables could be made to nearly approach that of bare aerial conductors.

The rapid and definite dissipation of the heat from the cables with water immediately surrounding them would not only greatly increase the carrying capacity of any particular cable or of a particular conduit system, but would decrease possibilities of cable trouble from all other causes except mechanical injuries of the lead sheath.

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THE FUNDAMENTALS OF SUCCESSFUL HIGH-TENSION CABLE JOINTS

BY D. W. ROPER

ABSTRACT OF PAPER

The specifications are intended to apply in general to joints on cables operating at voltages above 10,000 and in particular to joints operating above 20,000 volts. The specifications include the following points:

1. The cable sheath should be cut back from the ends of the conductor a sufficient distance to allow the application of the insulation to the joint without injuring the insulation around the conductors.
2. The copper splicing sleeves should have a carrying capacity at least equal to that of the conductors.
3. The ends of the copper splicing sleeve should be carefully tapered, rounded and finished so as to eliminate all sharp points or edges.
4. The conductors should be separated from each other and from the enclosing lead sleeve by some form of solid insulation that will hold them in a fixed position relative to each other and to the enclosing lead sleeve.
5. The filling compound should have sufficient dielectric strength, which should not be materially reduced at the maximum operating temperature of the cable.
   - a. It should be non-hygroscopic.
   - b. It should not form cracks in cooling nor during cold weather.
   - c. It should be sufficiently fluid when poured that it will flow into all air spaces before becoming chilled.
   - e. It should be capable of being readily removed when necessary without injury to the insulation around the conductors.
   - f. It should not have an excessive coefficient of expansion.
6. The insulation should be applied and the compound poured in such a manner that the joint will be without air pockets or voids.
7. All moisture should be carefully excluded from the joint.
8. If filling compounds with low melting point are used precaution should be taken to prevent the compound from running back into the cable at the maximum operating temperature.
9. The initials, or some other symbol, should be stamped on the joint to indicate the name of the splicer.
10. The joint should be so located, supported and protected that it cannot be readily injured by men working in the manhole.

DURING the past few years there has been a considerable increase in the number of companies installing cables for operation at voltages from 20,000 to 25,000. In some cases the companies in making the cable joints have attempted to use methods similar to those which have been found adequate on cables operating at materially lower voltages. The results secured in this manner were frequently disappointing. As the other extreme, some companies have adopted a type of cable joint that required three men a day and a half to complete. In some companies one splicer and a helper make up two joints per day, although three can be made by the same crew under favorable conditions.

It is probable that no two operating companies use the same methods and materials in making up their 20,000-volt cable joints. It is thought that if the engineers engaged on such work would agree upon the fundamental requirements of successful high-tension cable joints, some progress could then be made on standardizing such joints to the advantage of all concerned. The labor cost on the various joints being used by different companies for the same purpose probably ranges from $5 to $30 and the time required ranges from a half day or less to one day and a half. The more expensive joints, which take so long to make, may in some cases be considerably stronger electrically than the cable, and perhaps somewhat stronger than necessary. It is a very serious disadvantage in case of repairs following burnouts to take a day and a half to make up the joints necessary for the purpose. If a temporary joint is made up to reduce this delay, then the joint must be replaced later by the standard type of joint, and in the meantime there is a possibility of joint failure. If the nature of the burn-out has required the use of the cut and try method for locating the trouble it will probably be necessary to stop all other cable splicing work in order to secure enough cable splicers to make up the joints on the cable undergoing repairs.

The following specifications are intended to apply to joints on cables operating at about 20,000 volts, or upwards, but in general they will be found of importance, though in somewhat lesser degree, on cables operating above 10,000 volts.

1. The lead sheath of the cable should be cut back from the exposed conductors so as to provide sufficient space for the application of the insulation around the conductors without danger of cracking the insulation near the end of the lead sheath.

If this point is left to the judgment of the cable splicer he is liable to cut the lead back an insufficient distance so that in order to apply the insulation he will bend the individual conductors so sharply near the end of the lead sheath that the insulation will be cracked. In order to prevent this trouble, the distance from the exposed conductors to the lead sheath should be about six or seven inches, depending upon the thickness of the insulation around the conductors. If the joint is to be insulated with wrappings of tape the ends of the insulation around the conductors should be tapered.

2. The copper sleeves or tubes used in splicing the conductors should be of such material and so proportioned that they will have a carrying capacity at least equal to that of the conductors.

As the thickness of insulation in the joint will be greater than in the cable there will be a greater resistance to the radiation of heat from the joint than in the cable. If in addition the copper sleeves have a carrying capacity less than that of the conductor, the temperature of the joint will be further increased. Experience with paper insulated cables shows that the dielectric loss increases with the temperature, and that all forms of insulation have a critical temperature above which the dielectric loss is so great that the temperature will continue to rise due to the dielectric losses until burn-out occurs. This critical temperature is lower for cables impregnated with a resinous compound than with those in which a mineral compound is used. In order to avoid trouble from this cause the splicing sleeve should have ample conductivity. The copper sleeve, as well as the ends of the cable, should be thoroughly tinned, and the soldering of the joint should be carefully done so as to insure that the sleeve is entirely filled with solder. It is essential to have the sleeves heated to the proper temperature before filling with solder. Rolled copper tubing of a proper diameter, and with a length of about four times the diameter of the conductor, makes a very satisfactory sleeve. If the splicing sleeves are made of brass they should be carefully tested to insure proper conductivity.

3. The ends of the copper sleeve should be carefully tapered and rounded, and the outer surface should be finished so as to eliminate all sharp points or edges.

Sharp points on the exterior of the copper sleeve or the conductors will cause an increase in the dielectric stress at these locations, and such points should therefore be very carefully avoided. For this reason the exterior surface of the splicing sleeves should be carefully finished and all small drops or points of solder should be removed with a file or emery cloth so as to leave a smooth, well-rounded surface.

4. The conductors should be separated from each other and from the enclosing lead sleeve by some form of solid insulation so that they will retain a fixed position relative to each other and to the enclosing lead sleeve.

The insulation of the joint may consist of

a. Wrappings of insulating tape, or

b. Formed or moulded insulation, such as tubes of paper or other solid insulating material.

If layers of insulating tape are used the splicer should be furnished with gages for determining when the proper amount of insulation has been applied. After this work is completed the three conductors should be bound together in such a manner that they are equally spaced, and at the same time mechanically firm and substantial. This may be done by wrapping bands of tape at several points along each of the conductors, together with some bands around all three conductors at the same locations.

When mechanically formed insulation is used care should be taken to see that the parts are so shaped, placed and fastened that all air will be driven out by the filling compound. Spacers of porcelain, or other suitable material may be found desirable for supporting and holding the conductors at a point between the mechanically formed insulation and the ends of the lead sheath.

5. The filling compound should have the following qualities:

a. It should have sufficient dielectric strength for the purpose, which strength should not be materially reduced at the maximum operating temperature of the cable.

In general there is no great difficulty in securing a compound having sufficient insulating strength at ordinary temperatures. Resinous compounds which decrease rapidly in dielectric strength with increase in temperature do not appear to be as satisfactory as those mineral compounds which have a lower temperature coefficient. This is particularly important for the compound which is used for applying the wrappings of insulating tape around the conductors.

b. It should be non-hygroscopic.

Moisture is one of the greatest enemies of a good high-tension joint and all possible precautions should be taken to exclude it from the compound, or any of the insulating material which enters the joint.

c. It should not form cracks in cooling after the joint is poured, nor during cold weather.

This result is best obtained with those compounds which remain somewhat plastic at the ordinary temperatures and do not become hard or brittle.

d. When heated to the proper temperature for pouring, it should be sufficiently fluid so that it will flow into all air spaces before becoming chilled.

To insure that the filling compound is at a proper temperature for pouring, thermometers should be used, or some simple form of test should be devised which will indicate the proper temperature.

e. The compound at ordinary temperatures should be capable of being readily removed, when necessary, without injury to the original insulation around the conductors.

This is frequently necessary when locating trouble or making alterations, and in such cases it should be possible to open the joint and make the necessary tests, and later re-make the joint without cutting off any of the conductors.

f. It should not have an excessive coefficient of expansion.

After the first filling, time must be allowed to permit the compound to cool before sealing. Ordinarily a small amount of additional compound must be added to fill the void left by the cooling of the compound. One such filling following the original filling should be sufficient.

If the filling compound is liquid or soft at ordinary temperatures, it may be necessary to use a cast metal sleeve in place of the ordinary lead sleeve, or to wrap the insulation with some form of porous tape to act as a cushion between the insulation and the lead sleeve.

6. The application of the insulation and the pouring of the compound should be done in such a manner that on completion the insulation will be solid; that is, without air pockets or voids.

In applying wrappings of tape, the conductors, the splicing sleeve and the insulation around the conductors should first be given a generous coating of some thin insulating compound, applied, preferably, with a brush. The tape should have a lap of about one-half and the compound should be in sufficient quantity and of such consistency that when the wrappings of tape are tightly drawn, all air between the several layers of tape and between the tape and the conductors will be entirely excluded. Additional applications of the compound should be made before applying each layer of tape. Suitable gages should be furnished the splicer so that he can determine when the proper amount of tape has been applied.

If some formed or molded insulation is used it should be held in position temporarily by the form of the insulation, or by tying with string or tape. The solid insulation should be so made and held in position in the joint that when slightly tilted at the time of filling, all of the air within the enclosing sleeve will be driven out by the filling compound.

7. All moisture should be carefully excluded from the joint.

For the purpose of excluding moisture it may be necessary to install some temporary shelter so as to prevent drippings from the roof, or contact with the adjacent wall of the manhole. Moisture in the insulating tape should be driven out by dropping it in a vessel containing filling compound heated considerably above the boiling point of water. Rubber gloves should be worn by the cable splicer in warm weather, and at other times if the hands are naturally moist. The splicer should also take suitable precautions to insure that no perspiration drops from the face into the joint.

8. If filling compound with a low melting point is used, precautions should be taken to prevent the compound from running back into the cable at the maximum operating temperature.

If the impregnating compound in the cable does not entirely fill the interstices between the strands of the conductors, as well as the insulation, then the filling compound as it becomes fluid may run back into the cable. This would leave voids in the upper part of the cable sleeve and would give a distribution of dielectric stresses that would probably later cause trouble. If such compounds are used the soldering of the splicing sleeve should extend beyond the ends of the sleeve, and in addition, several wrappings of impervious tape should be applied to the conductors where exposed between the splicing sleeve and the original insulation around the conductor.

A thin filling compound may also flow back into the jute filling between conductors if it is not thoroughly impregnated. The use of a filling compound with a low melting point should, therefore, be avoided unless the insulation, the jute, and the conductor are all very thoroughly filled with the impregnating compound.

9. The initials, or some other symbol, should be stamped on the joint to indicate the name of the splicer.

This stamp can be made in the wipe without danger of damaging the joint. Good cable splicers, like other first class workmen, take considerable pride in their work, and it makes them more careful if they know that in case of trouble the maker of the joint can be readily identified.

10. The joint should be so located, supported and protected in the manhole that it cannot be readily injured by men working in the manhole.

For this purpose the cable should be supported at each end of the lead sleeve. If a rope and cement protective coating is applied to the cables in the manholes, the same coating should be extended over the joint. If this is done the joint cannot be injured by being accidentally struck with tools or ends of cable, or by workmen bruising cable with the ladder in entering or leaving the manhole.

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THE INFLUENCE OF DIELECTRIC LOSSES ON THE RATING OF HIGH-TENSION UNDERGROUND CABLES

BY A. F. BANG AND H. C. LOUIS

ABSTRACT OF PAPER

This paper describes a series of tests made to determine the dielectric losses in various kinds of 13,000-volt cable. These losses were found to increase greatly with the temperature and thus become an important factor in the rating of such cables. A method is developed whereby the influence of these losses on the rating can be approximately calculated for duct lines with uniformly loaded cables.

IN THE early part of 1916 a series of tests on 13,200-volt cable under conditions closely approximating operating conditions was conducted in Baltimore, Md. The object was to obtain information relative to the rating of cables at their highest safe capacity, and to analyze the various factors affecting this, such as the influence of dielectric losses.

It was, therefore, attempted:

1. To measure the dielectric losses of cables at different temperatures.

2. To compare cables of various makes and from different manufacturers with each other in this respect, and if essential differences were found to investigate the possible causes of this, such as,

3. Different compounds used for impregnating the paper,

4. Percentage of moisture in cable,

5. Thickness of insulation, etc.

Assembly. A nine-hole duct line was constructed in the basement of Sharp Street Station, Baltimore, where the temperature conditions are fairly constant. The layout and dimensions of the duct system are shown in Fig. 1. It consisted of a nine-hole duct, seven ducts being occupied with cable. The duct was 48.5 ft. (14.7 m.) long. The duct was surrounded on sides, top and bottom by sand, which was held in place by a wooden framework. The ends of the cables outside of the ducts were made as short as possible, and were wrapped with sheet asbestos and covered with sand, in order to have these at approximately the same temperature as the part of the cables in the ducts.

Temperature Measurements. As conditions made the use of mercury thermometers impractical in the ducts, we used a differential galvanometer-type thermometer with temperature coils, twenty-seven coils in all being used, each duct containing three coils. As it was important that these temperature coils touch neither the duct nor the cable, they were mounted in a little wooden cradle. Temperatures of the copper conductors and lead sheath were determined by rise of resistance method.

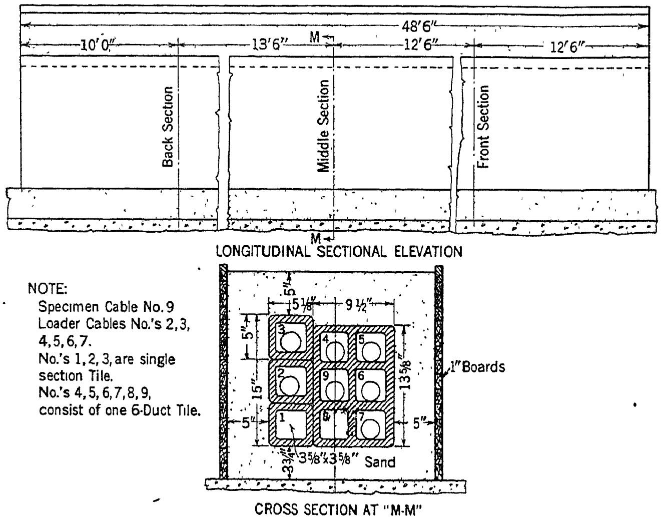

FIG. 1—ARRANGEMENT OF DUCTS AND CABLES IN TEST

Cable Connections and Wiring. Cables in ducts numbered 1, 2, 3, 4, 5 and 6 were all connected in series and carried current only. Cable in duct number 9 carried both current and potential, this being the specimen cable; the different specimens were tried in this duct, the same cable, of course, being used in the other ducts through the test.

Fig. 2 shows the wiring diagram.

Application of Current and Potential. Three-phase current and voltage was applied to the specimen cable. Each phase was short-circuited on itself through current transformers run inverted, and excited from the 5-ampere side; separate current transformers were used to measure the current. High voltage was obtained by using three potential transformers inverted, rated at 200 watts each, 13,200/110 volts; primary and secondary windings were both connected in star. The neutral of the high tension was grounded and connected to the sheathing; consequently, the dielectric strains and losses corresponded with those which occur under practical operating conditions. The dielectric losses were determined by measuring with an ordinary wattmeter the input of the potential transformers on the low-voltage side, and subtracting from this the input with the high-tension cable disconnected, and also the copper losses in the potential transformers, which could be readily calculated. The heater cables carried current only.

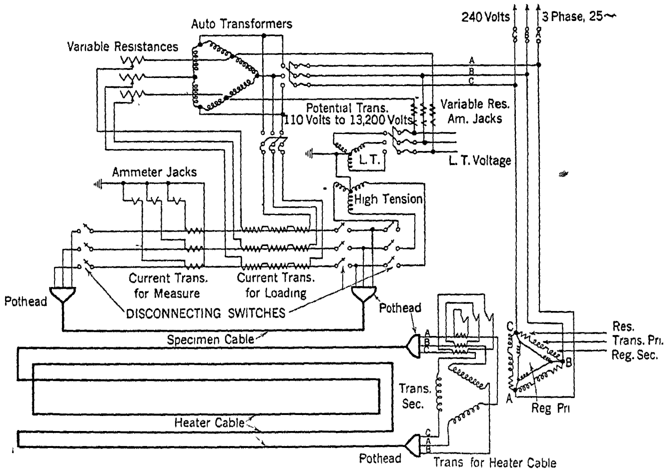

FIG. 2—WIRING DIAGRAM OF TEST

Potheads. Standard potheads were used on the cable. In order to correct for the dielectric losses occurring in these, a special test was conducted on a very short piece of cable with two potheads; this test proved these losses to be practically negligible, at least for temperatures not exceeding the actual pothead temperatures occurring during the main test.

Procedure of Tests. Some runs were made with C^2R losses present only, that is, with current only applied.

Dielectric runs were made with current on all cables, and high-tension voltage applied to the specimen cable. Practical conditions precluded the application of potential to the heater cables, but the lack of these dielectric losses was compensated for as follows: Starting with current in the heater cables of the same value as in the specimen, dielectric loss of specimen cable was measured when constant temperatures were reached. The current in the heater cables was then increased so that the increased C^2R loss equalled the dielectric loss of the specimen cable. This process was repeated until stable conditions were obtained. On account of the lagging effect of the duct line it was often necessary to continue each test a number of days in order to reach constant conditions.

RESULTS

Specimen Cables. Tests were made in all, on four different specimens of cable from three different manufacturers; all of them were three-conductor, No. 4/0 paper-insulated copper cable; the insulation was distributed as follows:

Specimen A—6/32 in. by 6/32 in. (4.7 mm. by 4.7 mm.)
" B—8/32 in. by 2/32 in. (6.3 mm. by 1.5 mm.)
" C—6/32 in. by 6/32 in.
" D—8/32 in. by 2/32 in.

Dielectric Losses. The curves shown in Fig. 3 show the dielectric losses for these cables expressed in watts lost per foot of cable and plotted against the temperature of the copper conductors. These curves show the marked effect of temperature on the dielectric losses. At low temperatures corresponding to a cold condition of the cables, the losses are low, and about the same for all the specimens, while at higher temperatures the dielectric losses run up rapidly in all cables except specimen D.

These curves show specimen D cable to be much superior in this respect to the others, the dielectric losses being very much lower at high temperatures than the others, and the rate of increase smaller. This rate of increase is important, as it effects the final temperatures, due to the cumulative effect as brought out later on.

Thickness of Insulation. Specimen B—8/32 in. by 2/32 in., (6.3 mm. by 1.5 mm.) shows up better than specimen A. This may be due to the greater thickness of insulation between conductors, since both samples are of the same make and otherwise supposed to be alike. However, it will be noticed that specimen C, which is 6/32 in. by 6/32 in. (4.7 mm. by 4.7 mm.), and of another make shows up slightly better than specimen B, 8/32 in. by 2/32 in. (6.3 mm. by 1.5 mm.).

Material of Cables. This markedly superior performance of specimen D shows that there must be some radical differences in material. Samples about 10 in. (25.4 cm.) long were cut from the specimens and sent to a chemist for analysis. The analysis showed that in specimens A, B and C, a rosin-oil base is used for impregnating the paper, whereas in specimen D, a mineral-oil base is used. Puncture tests made by us on these oils showed but little difference in dielectric strength at high and low temperatures for the mineral oil, whereas in the case of rosin oils the strength decreased greatly with increased temperatures.

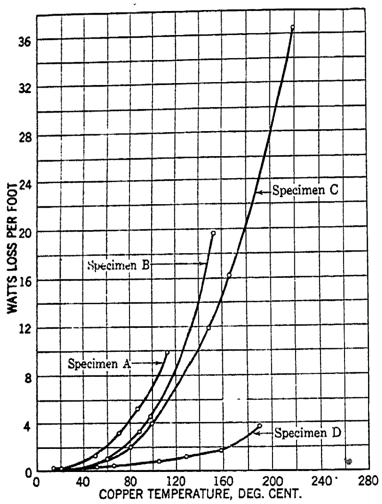

FIG. 3—DIELECTRIC LOSSES IN WATTS PER FOOT LENGTH OF CABLE AS DEPENDING ON THE TEMPERATURE OF THE CONDUCTORS

The difference in impregnating compound we believe is, therefore, the greatest cause producing the marked difference in performance.

Analysis for moisture was made also on 5 in. (12.7 cm.) pieces of the specimen cables. Specimens A, B and C using rosin-oil compound showed the same relative order of moisture as dielectric losses. Specimen D showed a high percentage of moisture, and yet has the lowest dielectric losses; this fact can only be taken as a proof that the nature of the compound, which here is a mineral oil, has a greater influence than the per cent moisture.

Relative Temperatures. The test on the Sharp Street duct line shows comparatively small differences between copper, lead and duct temperatures, under constant conditions. In the run made on specimen D cable with 175 amperes on the heater and specimen, and 13,200 volts on the specimen, the total watts lost per foot on the specimen was, for instance, 6.5 watts and the temperature as follows:

Copper Temperature	89 deg. cent.
Lead	86 deg. cent.
Duct	82 deg. cent.
Room	20 deg. cent.

The reason for the close proximity of these temperatures to each other in spite of the high rise above the room temperature is essentially the fact, that the main temperature rise in our case took place in the dry sand surrounding the duct line and that this rise was secured by moderate loads on the cables. This condition imitates very well actual duct conditions where ducts pass under streets through dry soil or ashes.

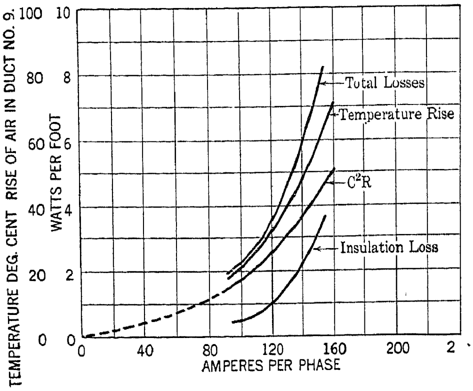

FIG. 4—CURVES SHOWING RELATION OF C^2R AND DIELECTRIC LOSSES IN SPECIMEN *A* IN SHARP STREET TEST

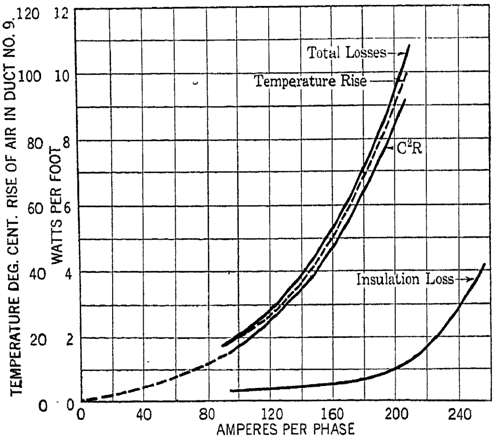

FIG. 5—CURVES SHOWING RELATION OF C^2R AND DIELECTRIC LOSSES IN SPECIMEN *D* IN SHARP STREET TEST

Relative Losses. A comparison of C^2R and dielectric losses can only be made for specific cases, as the C^2R losses depend mainly on current carried, and slightly on temperature, and the dielectric losses for a given cable and voltage only on temperature. Curves shown on Figs. 4 and 5 show this relation for specimens A and D, which constitute the two extreme cases of the test. The conditions were such that 100 amperes produced no excessive temperatures, so that all the specimens had a dielectric loss of less than 25 per cent of the C^2R at 100 amperes. It will be noticed that for specimen A (See Fig. 4) the dielectric loss runs up very rapidly at currents above 100 amperes, a condition which would be far more aggravated if the initial room temperature were higher than the one at which the test was made (18 deg. cent.). With higher currents the ratio of dielectric to C^2R loss increases rapidly and approaches 100 per cent. Only specimen D (Fig. 5) shows no great increase of dielectric loss as the load and

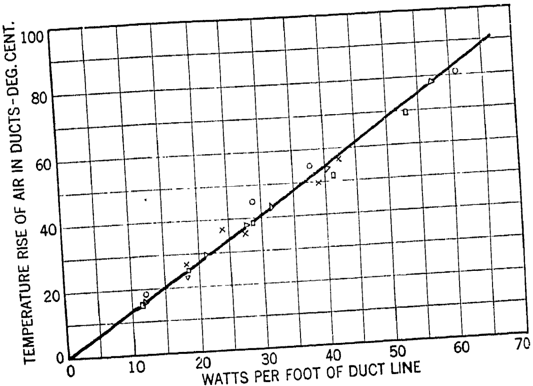

FIG. 6—DISSIPATING CURVE OF SHARP STREET EXPERIMENTAL DUCT LINE

temperature go up, and the ratio of dielectric to C^2R even decreases with increased current and temperature, this holding good for a range of current well within practical working limits; even an increased room (or soil) temperature would not affect this to any great extent.

Heat Dissipating Curve. From the amperes per phase, the resistance, and the dielectric loss measured we can calculate the total energy lost in the duct line and from this plot a curve between watts lost per foot of the duct and the temperature. This is done in Fig. 6. Since this curve represents constant conditions, the watts loss are a measure of the heat dissipated by the duct system at various temperatures. The curve shows that this relation is practically a straight line. In Fig. 6, this temperature plotted refers to the air in the duct. Other experiments made by the writers proved, that also when we plotted the temperature rise of the copper conductor above soil or any one of the cables against the watts lost per cable, per foot, the result would be a curve very close to a straight line. In the following calculations this relation is therefore always taken to be a straight line. It is also assumed that the relation is independent of whether the heat dissipated is generated in the copper by the current or in the insulation by the voltage.

Any duct line will of course have its own particular dissipating curve.

Critical Point (*Temperature and Current*). The effect of dielectric losses on the heating of cables is cumulative, *i.e.*, when a cable has reached constant conditions with low-tension currents only applied and high voltage is then added, certain definite dielectric losses will occur and produce a temperature increase; but, on account of the higher temperature thus reached an increase of the original dielectric losses will take place, which fact again increases the temperature and so on indefinitely; there is, therefore, always a possibility that a critical point may be reached where the heat generated in this way is larger than the heat that can be carried away; if this point has been reached and passed the temperature of the cable will evidently rise indefinitely until break down occurs and will do so even for a constant current load. That such a critical point exists for any high-tension cable surrounded by a poor heat conductor has been repeatedly pointed out. (See especially the N. E. L. A. Cable Report—1913). In our tests we were able to produce this condition on specimen cables A, B and C, by means of the "compensated" runs mentioned above. In these runs it was found that a point was reached in the loading where it was not any longer possible to compensate, as the dielectric losses ran up faster than the heat dissipating power of the duct line. In the dielectric curves, Fig. 3, the lower points are plotted from values measured in the compensated runs; when compensation was no longer possible a constant current was maintained on the heater cable *and the dielectric losses on the specimen cable would then gradually raise the temperature of this cable until destruction*. The higher points in Fig. 3 are plotted from this last part of each test. Only on specimen D did we not determine the critical point. While it doubtless exists, it would evidently in this case be found at a higher temperature than reached in our test. (190 deg. cent.)

As it is evident that very unstable and dangerous conditions exist in a high-tension duct line, where this critical point has been reached and as it will be found in many practical cases, that this point, therefore, rather than any special high temperature, constitutes the danger line for the rating of high-tension cables, we have, in the following, developed a method for the determination of this critical point for any duct line, with uniformly loaded cables, and on the whole calculate the temperatures that will be reached for given loads under due regard to the dielectric losses. To do this it is necessary to know only the following few data, which can be found experimentally.

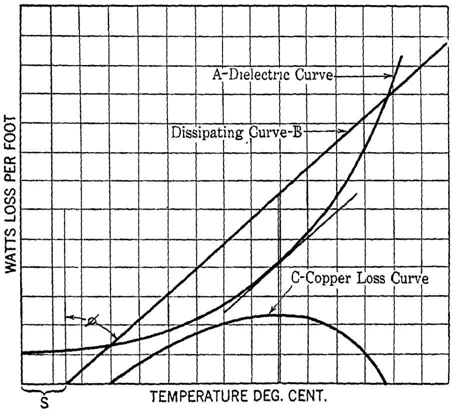

FIG. 7—GRAPHIC DETERMINATION OF THE PERMISSIBLE CURRENT AND THE CRITICAL TEMPERATURE FOR A DUCT LINE, WHEN ITS DISSIPATING CURVE B, ITS DIELECTRIC CURVE A, AND THE SOIL TEMPERATURE S ARE KNOWN

Calculation of the Influence of Dielectric Losses. 1. The dielectric characteristics of the cable, expressed in a *dielectric curve* as for instance those shown on Fig. 3.

2. The heat characteristics of the duct line and the cable expressed by the heat *dissipating curve* for same.

3. *The soil temperature* at a distance sufficiently far away from the duct line to be unaffected by the heat produced in the duct.

Let curve A in Fig. 7 represent the dielectric loss curve for the cable installed.

Let curve B represent the dissipating curve for the duct line at a given soil temperature S, If we now assume that we have reached constant conditions where no heat is used for increasing the temperature of cable and duct, the watts dissipated as indicated by curve B are equal to the total losses generated which consist of dielectric and C^2R losses.

If we therefore deduct the dielectric losses (as shown in curve A) from the total losses (curve B) we get the C^2R loss. This subtraction is done graphically in Fig. 7. The result is curve C which represents the copper losses. From this curve we can calculate the corresponding current for any number of points from the formula

$$C = \sqrt{\frac{W}{3R}}$$

Where W = Watts taken from curve C.
R = the resistance of one conductor (at a temperature taken from the corresponding point of curve B).

In Fig. 7 it is assumed that the soil temperature is a constant S. A change in soil temperature which is the base for the duct and cable temperatures means a parallel shifting of the dissipating curve and therefore produces an entirely new copper loss curve C.

Determination of the Critical Temperature by Tangent Method. It will be seen from Fig. 7 that curve C, which represents the copper losses, has a maximum point. This means that for temperatures beyond this point, less current can be carried than at lower temperatures and if the current is not reduced the temperatures will evidently continue to rise indefinitely. This point corresponds therefore to the *critical point* as defined above.

A glance on the same drawing, Fig. 7, will also show geometrically that this maximum ordinate in curve C (copper loss curve) is determined by drawing a tangent to the dielectric curve parallel with the dissipating curve. In other words if we only know these two curves for a given duct line, we can determine the critical point by drawing such a tangent; the abscissa to the point of tangency will then indicate the critical temperature for this particular case. Physically one can express the same fact by saying that at this point the rate of increase in the dissipating power of the duct line equals the corresponding rate of increase in the dielectric losses generated (curve A), while beyond this point the latter rate will be the larger; hence the unstable condition.

The fact that we can determine the critical point in this way is important since it allows us to grasp, at a glance, the essential facts about the critical temperature for a given case.

Conditions Which Influence the Critical Temperature. It shows for instance—1. That for a given duct line with a definite number and make of cables, the critical point will always be reached at the same actual temperature (critical temperature) independent of the original soil temperature.

2. That this critical temperature will be the lower, the steeper the dielectric curve is, and the greater the angle ϕ of the dissipating curve; cables with a low and flat dielectric curve are, therefore, greatly preferable.

3. The angle ϕ of the dissipating curve depends on the character of the soil surrounding the duct line. If the surroundings consist of poor heat conductors, like dry sand or ashes, it will result in a greater temperature rise for the same watts lost; this means a greater angle ϕ of the dissipating curve and therefore a lower critical temperature.

4. In a similar way will the critical temperature be influenced by the number of cables installed in a duct line. With the same energy lost per cable we get a total loss in a duct line directly proportional with the number of cables installed and since the heat dissipating surface of the duct line only increases at a slower rate, the result is a higher temperature rise of the duct for the line with the many cables. Hence the greater angle ϕ of the dissipating curve and the lower critical temperature.

These facts are important since they show that dry soil surrounding a duct line and the installation of too many cables in one line should be avoided, as far as possible. Not only do both conditions tend to produce much higher temperatures for the same energy lost, but since the critical temperature is lower, it means also that cables in such a line can only stand a lower temperature.

Example of Application. In the theory developed above, it is clearly pointed out, that each duct line represents its own individual problem in rating. The examples given below are therefore to be taken only as illustrations of the application of the principles involved and not as giving definite ratings—always applicable.

Fig. 8 shows a section of a duct line in Baltimore, Md. It consists, as shown, of twenty ducts of vitrified clay, arranged in two sections. It contains at present fourteen current carrying cables of which thirteen are operated at 13,000 volts and one at 4000 volts. All of them are number 0000 copper cable, three-phase, lead-covered, and with an insulation 6/32 in. by 6/32 in. (4.7 mm. by 4.7 mm.).

For this duct line the dissipating curve was calculated from the known load, the dielectric characteristics of the cable and a large amount of measurements of the copper, duct and soil temperature at the very hottest sections of the line.

In Fig. 9 this dissipating curve is shown as B_3. It should be understood that in this case it is calculated so as to represent temperature rise of the copper of the hottest cable above the soil surrounding the duct line.

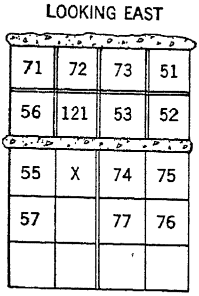

FIG. 8—SECTION THROUGH FAYETTE STREET DUCT LINE

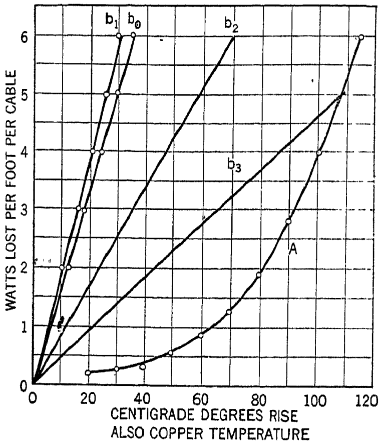

FIG. 9—DISSIPATING CURVES FOR VARIOUS DUCT LINES

By means of this dissipating curve and the dielectric curve we can now calculate the final copper temperatures for various loads and soil temperatures by means of the procedure described above.

A number of such calculations has been made and the results shown in Fig. 10 for initial soil temperatures of 0, 10, 20, 30 and 40 deg. cent.

It will be seen that at about 65 deg. cent. the curve starts bending backwards; this is therefore the critical point for this particular duct line.

In order to play safe it is evident that the average temperature on a steady load run should never be allowed to reach this critical point. Let us assume that 10 deg. cent. below this point, *i. e.*, a final copper temperature of 55 deg. cent., is the highest temperature we will permit. It is then evident that the curves in Fig. 10 give us a cable rating as depending on the soil temperature. This rating is shown in Fig. 11 (curve marked specimen C). As abscissas are used the initial soil temperatures in 0 deg. cent., while as ordinates are used both amperes, *i. e.*, continuous carrying capacity and also the kw-hr. output per day, corresponding to such a continuous load at power factor of 1. From this curve it may be noted, for instance, that at freezing temperature of the soil, the cables can carry safely 106 amperes continuously, or 58,500 kw-hr. per day, while at a soil temperature of 40 deg. cent. or above, no load whatever can be carried.

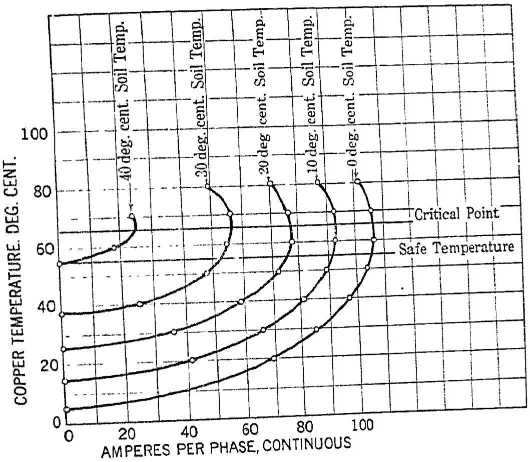

FIG. 10—TEMPERATURES OF CONDUCTORS REACHED IN A HIGH-TENSION DUCT LINE WITH 13 CABLES INSTALLED AS DEPENDING ON THE LOAD AND SOIL TEMPERATURE—SPECIMEN *C* DIELECTRIC CHARACTERISTICS ARE ASSUMED

Cable Rating with Cables of Different Make. The cable rating thus determined for the above described duct refers to cables of dielectric characteristics as specimen C in Fig. 3. The rating of the same duct line with cables of dielectric characteristics as specimens A, B and D has also been figured out and is shown in Fig. 11. For the specimens A and B the safe copper temperature has been assumed to be 10 deg. below the critical temperatures (60 deg. and 50 deg. respectively) in the same way as was done for specimen C above. For specimen D where the dielectric losses increase so very much slower with increasing temperature than they do on the other specimens, it was found, however, that the critical point would not be reached before the copper temperature was 160 deg. cent. Hence the rating of this cable would not be limited at all by the critical temperature of the duct line, but would rather depend on whatever temperature could be considered safe for the insulation itself. If we follow the A. I. E. E. Rules laid down in section 677 and adopt 72 deg. cent. as a safe temperature for this cable, we get the rating shown by curve *A*, Fig. 11. As it is probable that the temperature limits set by the Institute Rules in its section 677, are made so low on account of a vague fear of reaching a critical temperature for the cables and as it has been shown that for specimen D cable, no such fear need be entertained, it would, perhaps, seem more reasonable to use as a permissible temperature for this cable the figure (105 deg. cent.) given in the A. I. E. E. Rules, section 377 for Class A insulation. Corresponding to this we get curve *B*, Fig. 11.

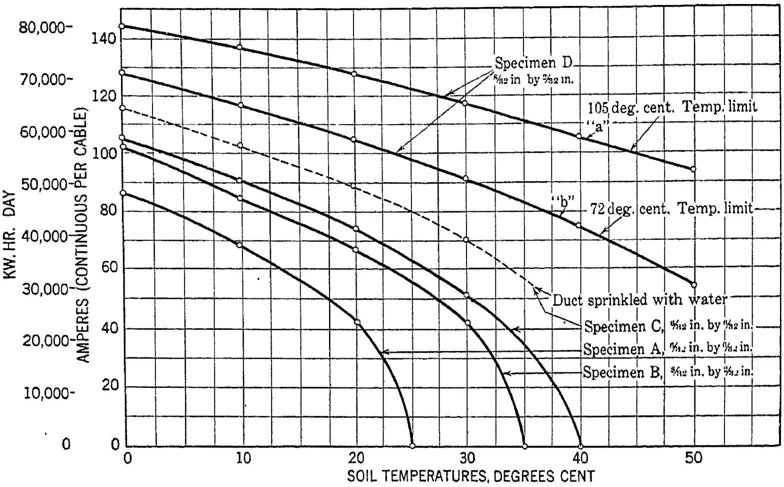

FIG. 11—CURVES SHOWING HOW THE SOIL TEMPERATURE AFFECTS CABLE RATING IN FAYETTE STREET DUCT LINE FOR CABLES OF VARIOUS DIELECTRIC CHARACTERISTICS

A comparison of all these curves shown on Fig. 11 gives a very clear impression of the extreme limitation the dielectric losses may put on cable ratings, especially in hot summer weather. As an illustration let us consider a summer condition with a soil temperature of 21 deg. cent. (At the average depth of the duct line.)

To this corresponds the following ratings:

Specimen	A	38	amperes	21,000	kw-hr.	per	day
"	B	65	"	36,000	"	"	" "
"	C	72	"	40,000	"	"	" "
"	D	127	"	71,000	"	"	" "

Or a rating more than three times greater for specimen D than for A.

In winter time, of course, more load can be carried on all types of cables, but the *difference* in rating is less striking.

A number of other practical cases with the number of cables in a duct line varying from 2 to 12 were also calculated. With dielectric curves of the characteristics found for specimens A, B and C the critical temperature varied from 50 deg. to 90 deg.

In view of these facts it seems reasonable to propose,

1. That high-tension cables always should be purchased under certain guarantees as to dielectric losses at various temperatures.

2. That for determining the load carrying capacity of a given high-tension duct line the critical temperature should be investigated and the rating made so that the highest copper temperature (under steady load conditions) always would remain a definite number of degrees centigrade (say 10 deg. cent.) below this point. If the critical temperature on the other hand is found to lie so high that temperatures lower than this may be destructive to the installation these naturally would govern; but in such a case the same temperature should be considered permissible for high-tension cable as for a low-tension cable.

In this paper only constant load conditions have been considered.

The above investigation has been conducted under the auspices of the Consolidated Gas, Electric Light and Power Co., of Baltimore and the Pennsylvania Water and Power Co.

Presented at the 332nd Meeting of the American Institute of Electrical Engineers, New York, June 27, 1917.

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INSULATION CHARACTERISTICS OF HIGH-VOLTAGE CABLES

BY W. S. CLARK AND G. B. SHANKLIN

Abstract of Paper

The insulation characteristics of different types of single- and three-conductor cable, determined largely by dielectric energy loss measurements, are discussed in detail. Particular attention is given to paper insulated cable, both new cable and cable that has been in service being considered. Varnished cambric and different grades of rubber insulation, as well as cable compounds are also dealt with briefly.

THE experience of operating companies here in America over a period of years shows that practically all failures on high-voltage underground cable systems can be attributed to heating. The cable line on which failure occurs is usually not heated over its whole length to a dangerous temperature, but only at certain points, generally known as "hot spots."

The "hot spot" temperature plays an important part in all sections of an electrical system, but in no part of a system is it so much the limiting feature as in the underground cables, and in consequence, in no other part of the system does dielectric energy loss at normal frequency and voltage play such a predominating part. This is so because of the rapid rise of dielectric loss with temperature and its cumulative tendency at high temperature.

Discussion of Dielectric Energy Loss Measurements

The purpose of this paper is to show the value of dielectric energy loss measurements in studying cable designs and as an assistance to the manufacturers and users of cable. To obtain the greatest usefulness, the study of energy loss should, of course, be paralleled with other tests and observations, and this we have attempted to do.

These data were taken with the object of representing operating conditions as closely as possible. A constant frequency of

60 cycles was used throughout. There are so many variables to be considered that it requires numerous data to include them all, and this complicates things quite enough without bringing in frequency variations. In every case the voltage range used gives readings well above and below the operating voltage of the sample.

In duct operation, the copper temperature is only a few degrees higher than that of the lead sheath and the temperature is graded through the thickness of insulation. It is not desirable to represent this condition in the laboratory, even if it were possible to do so with any degree of accuracy. The temperature gradient would not be the same in samples of different cross sections and the measurements on these would not be comparable. A very complicated variable is eliminated by heating the cross section to a uniform temperature. This represents conduit conditions fairly closely. The greatest loss per cm^3 occurs in the insulation next to the conductor, where the stress is greatest, and the increase brought in by having the outer wrappings at a slightly higher temperature than they would be in the ducts, is not important. At any rate the losses measured are on the safe side, that is, higher than would occur in operation.

METHOD OF TEST

The readings were taken by the "compensated dynamometer wattmeter method," fully described in an article published last year. * The principle of this method is simply a correction of all phase-angle errors by reference to a standard high-voltage variable air condenser having a negligible phase difference from 90 deg. The possible errors that can be introduced and checks upon the accuracy of the method are discussed in detail in the article.

A smooth-core generator and special testing transformer were used and gave a sine wave under all conditions of test as shown by oscillographic records. This is not of great importance, however, as distortion of wave shape within reasonable limits does not affect energy loss measurements. Monasch † brought this out in an interesting way.

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* "Compensated Dynamometer Wattmeter Method of Measuring Dielectric Energy Loss," by G. B. Shanklin, *General Electric Review*, Oct. 1916.

† See "The Loss of Energy in the Dielectric of Condensers and Cables" by Dr. Bruno Monasch. London *Electrician*, Vol. 59, p. 416.

Paper-Insulated Cable

It is an interesting fact that paper-insulated cable, the most generally used of all types of cable, is susceptible to the widest variation in its insulation characteristics. The reason is not hard to find but it is hard to put on paper in condensed form.

In practically all of the standard types of impregnated or treated fibrous insulations, with the exception of paper cable insulation, treatment is given after it is thoroughly filled to render the filler solid and firmly "set." This is called "curing" and it is well known that a cured insulation is more stable and uniform than an "uncured" one, although it does not necessarily have inherently better electrical characteristics. Now, in paper insulated cable the filler remains in a semi-liquid or viscous form and the material operates throughout its life in an "uncured" or "green" condition. It is not an inert mass but is chemically, electrically and physically "alive." It will not be active in any of the above senses if the ingredients and processes are properly chosen, still it is "alive" and every detail of its manufacture must be carefully watched.

The paper and jute serve as mechanical spacers and the characteristics of the insulation as a whole are determined to a great extent by the quality of the filling compound. In making this statement it is assumed that the paper, jute and compound are initially free of moisture and other conducting impurities. In choosing the compound we must not only be sure of its having the best insulating properties when first put in the cable but we must also be sure that it is chemically stable and that it will not react chemically with either the paper, jute or copper under the temperature and voltage stress met with during its years of operation.

Tests on Compound

The 60-cycle dielectric energy loss data on a standard type of mineral hydro-carbon base cable compound are given in Table I.

Space will not allow the presentation of curves showing all the different characteristics. They are all interesting and well worth studying from the table but only those curves will be given that bring out particular points of interest. This plan will have to be followed throughout the paper. The tables are submitted with the readings arranged consecutively in the same order in which they were taken.

Fig. 1 shows the variation of power factor and permittivity with temperature. The relative permittivity decreases with temperature and there is a very decided hump in the curve between 50 and 75 deg. cent. It is over this range that the compound is gradually changing from a solid to a semi-liquid state, with a corresponding expansion of volume and decrease in specific gravity.

The permittivity curve is replotted in Fig. 2 for comparison with a corresponding curve of a typical heavy, mineral insulating oil. The permittivity values of the oil were measured at low voltage, 750 cycles, by the Anderson bridge method. Fig. 2 also shows the variation of the specific gravities * of these two materials with temperature. It is interesting to note that the corresponding values of permittivity and specific gravity vary

TABLE I.

MINERAL HYDROCARBON BASE CABLE COMPOUND NO. 1

Frequency 60 cycles

Temp. deg. cent.	kv./cm.	W/cm.^{3} 10^{-3}	I/cm.^{2} 10^{-6}	P.F. per cent.	K	ρ 10^{10}
105	6.06	0.776	0.486	26.3	2.322	4.71
	8.05	1.410	0.641	27.3	2.295	4.58
	10.08	2.260	0.800	28.0	2.283	4.52
	12.08	3.280	0.960	28.2	2.280	4.44
	14.94	5.040	1.182	28.3	2.284	4.44
	18.10	7.520	1.442	28.6	2.292	4.40
75	6.06	0.202	0.492	6.75	2.421	18.10
	8.05	0.356	0.651	6.74	2.415	18.20
	10.08	0.557	0.812	6.86	2.412	18.30
	12.08	0.817	0.972	6.95	2.406	17.80
	14.94	1.282	1.194	7.08	2.400	17.40
	18.10	1.910	1.445	7.27	2.394	17.60
48	6.06	0.0410	0.493	1.44	2.436	90.3
	8.05	0.0702	0.652	1.34	2.427	92.4
	10.08	0.1045	0.815	1.27	2.421	97.6
	12.08	0.1532	0.982	1.30	2.412	94.8
	14.94	0.2370	1.195	1.32	2.406	94.2
	18.10	0.3570	1.450	1.35	2.406	94.2
32	6.06	0 01260	0.494	0.422	2.442	295
	8.05	0.01982	0.653	0.377	2.433	320
	10.08	0.02916	0.814	0.351	2.430	335
	12.08	0.04260	0.979	0.359	2.433	341
	14.94	0.06490	1.212	0.424	2.436	344
	18.10	0.9460	1.472	0.362	2.439	355

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* Thanks are due Mr. J. Dantsizen who made the specific gravity measurements on the compound.

in exactly the same way. The gravity curves show that the total expansion of the solidifying cable compound is no greater than that of an oil which remains liquid over the given range of temperature.

The effective a-c. resistivity vs. temperature is shown in Fig. 3, which also shows the d-c. resistivity measured at low voltage under entirely different conditions and several weeks later. The corresponding curves for a special grade compound of the same type, not included in the tables, are also given in Fig. 3.

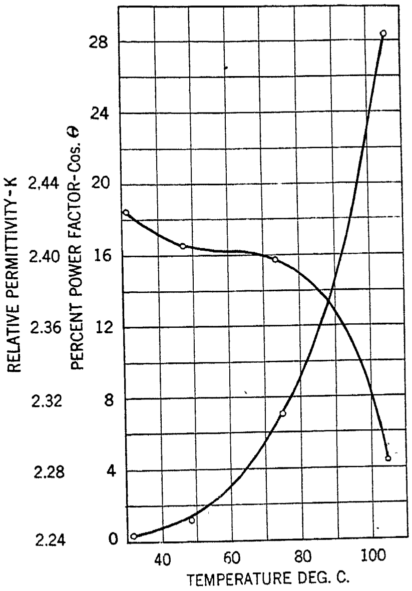

FIG. 1—RELATIVE PERMITTIVITY AND POWER FACTOR OF MINERAL HYDROCARBON BASE CABLE COMPOUND AT 16 KV. PER CM.

The relative difference between the two compounds is approximately the same under the two methods of test, one carefully made with elaborate apparatus, and the other, under entirely different conditions with comparatively simple apparatus. This is important for it supplies an easy way of determining the value of a compound. To appreciate this fully an outline of the two tests is given below.

The a-c. measurements were taken between plane disks spaced one cm. apart with the edge of the grounded plate protected by a guard ring, 1/32 in. (0.794 mm.) spacing between guard ring and plate. The active area was 40 cm. in diameter. Every care was taken to obtain a uniform temperature and shield from all sources of exterior loss.

The d-c. measurements were taken at 500 volts between disks 5 cm. in diameter and spaced 0.33 cm. apart. No guard rings were used and no special precautions were taken to eliminate surface leakage. The spacing was accomplished by placing a small glass rod 0.33 cm. diameter between the disks.

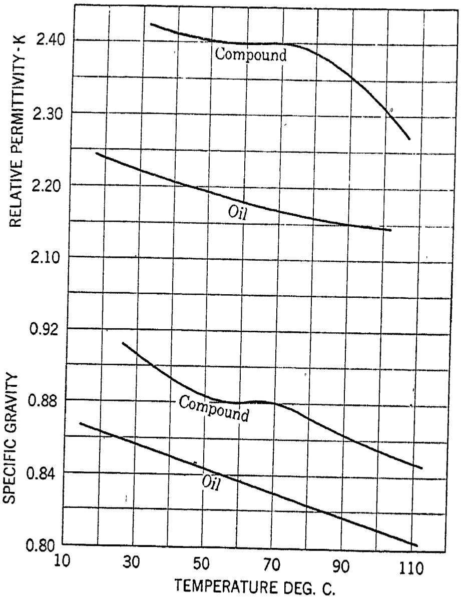

FIG. 2—RELATIVE PERMITTIVITY AND SPECIFIC GRAVITY OF A HEAVY MINERAL INSULATING COIL AND MINERAL HYDROCARBON BASE CABLE COMPOUND

Considering the difference in the methods, the a-c. and d-c. readings check quite closely. Theoretically the d-c. resistance should be slightly the highest. The tests on compound are always started at high temperature so that any moisture that may have been absorbed during exposure to air will be expelled.

SINGLE-CONDUCTOR CABLE

The data given in this paper have become generally known as dielectric energy loss measurements. This is only part of the truth, since all of the electrical constants and characteristics can be calculated for the particular conditions of temperature, frequency, etc., under which they are taken.

The nomenclature and formulas for single-conductor cable as used throughout are given in *Appendix A*.

Data on Single-Conductor Paper Cable

Cable No. 1. The data in Table II were taken on a six-ft. (1.83-m.) length of single-conductor paper-insulated cable, made up special for test and tested a few weeks after being made. The data on only one 6-ft. sample are given but three, cut from the same length of cable, were tested and checked almost exactly. The specifications were:

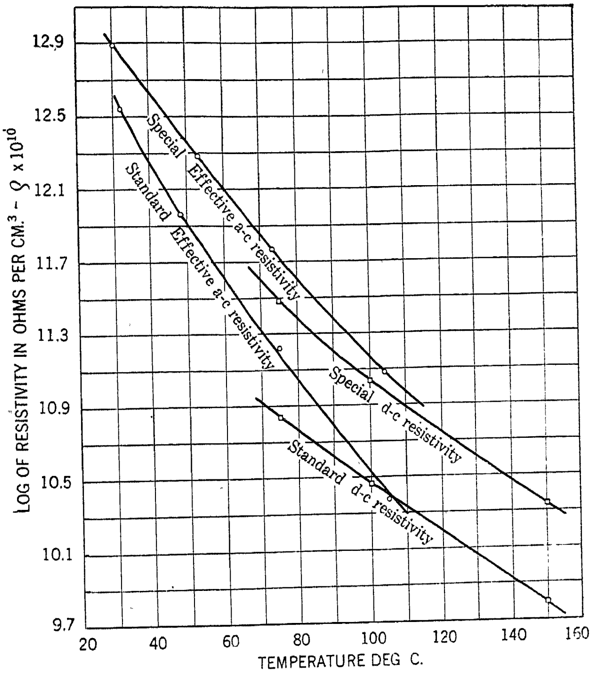

FIG. 3—LOG OF RESISTANCE PER CM.^{3} OF TWO TYPES OF MINERAL HYDROCARBON BASE CABLE COMPOUND

2/0 stranded conductor, 0.412 in. (10.45 mm.) max. diameter.

9/32-in. (7.144 mm.) paper, vacuum treated and filled with a mineral hydrocarbon base cable compound.

3/32-in. (2.38-mm.) lead sheath.

TABLE II. (CABLE NO. 1)

2/0 single conductor, maximum diameter 0.412 in.
9/32 in. paper filled with mineral hydrocarbon base compound.
3/32 in. lead sheath.
Frequency 60 cycles.

T deg. cent	E 10^3	W/ft.	I/ft. 10^{-3}	P.F. per cent.	C/ft. 10^{-9}	K_{av}	ρ_{av} 10^{10}
25	6	0.00882	0.1455	1.01	0.0645	3.257	91.0
	10	0.0250	0.244	1.024	0.0648	3.280	89.40
	14	0.550	0.344	1.142	0.653	3.300	79.50
	18	0.140	0.445	1.742	0.0651	3.320	51.60
	22	0.308	0.548	2.552	0.0661	3.347	35.00
	26	0.542	0.654	3.19	0.0667	3.377	28.15
	30	0.8125	0.760	3.563	0.0672	3.402	24.50
50	6	0 0450	0.147	5.10	0.0650	3.286	17.82
	10	0.1360	0.246	5.53	0.0651	3.300	16.40
	14	0.280	0.3455	5.79	0.0655	3.312	15.60
	18	0.505	0.448	6.26	0.0660	3.340	14.30
	22	0.837	0.557	6.905	0.0671	3.398	12.89
	26	1.268	0.676	7.22	0.0692	3.485	11.90
75	6	0.214	0.155	23.13	0.0667	3.380	3.76
	8	0.382	0.207	23 30	0.0668	3.382	3.74
	10	0.600	0.259	23.50	0.0669	3.386	3.72
	12	0.883	0.312	23.76	0.671	3.396	3.64
	14	1.218	0.367	24.09	0.675	3.418	3.58
	16	1.646	0.426	24.49	0.0686	3.472	3.48
	18	2.130	0.486	25.05	0.0694	3.515	3.39
100	5	0.292	0.150	39.05	0.0735	3.713	1.982
	7	0.587	0.211	39.78	0.0736	3.720	1.860
	10	1.224	0.304	41.10	0.0738	3.722	1.820
	13	2.127	0.398	41.10	0.0740	3.750	1.775
	14	2.483	0.432	41.10	0.0748	3.790	1.764
	15	2.850	0.468	41.10	0.0752	3.850	1.760
75	6	0.239	0.1585	25.07	0.0682	3.44	3.36
	8	0.432	0.211	25.60	0.0677	3.43	3.33
	10	0.688	0.264	26.07	0.0677	3.43	3.24
	12	1.000	0.318	26.22	0.0680	3.436	3.215
	14	1.378	0.374	26.32	0.0685	3.467	3.18
	16	1.852	0.4333	26.72	0.0694	3.507	3.06
	18	2.412	0.496	27.02	0.0705	3.567	3.00
50	6	0.059	0.147	7.83	0.0650	3.286	11.40
	10	0.211	0.246	8.58	0.0651	3.300	10.58
	14	0.442	0.3455	9.13	0.0655	3.312	9.87
	18	0.805	0.448	9.985	0.0660	3.340	8.96
	22	1.321	0.557	10.27	0.0671	3 398	8.15
	25	1.860	0.646	11.51	0.0692	3.485	7.49
25	6	0.0105	0.1455	1.202	0.0645	3.257	76.40
	10	0.0305	0.244	1.250	0.0648	3.280	73.00
	14	0 0762	0.344	1.582	0.0653	3.300	57.30
	18	0.1935	0.445	2.418	0 0651	3.320	37.40
	22	0.383	0.548	3.175	0.0661	3.347	28.20
	26	0.627	0.654	3.692	0.0667	3.377	24.05
	30	0.934	0.760	4.100	0.0672	3.402	21.50

TABLE II. (CABLE NO. 1)—*(Continued)*

2/0 single conductor, maximum diameter 0.412 in.
9/32 in. paper filled with mineral hydrocarbon base compound.
3/32 in. lead sheath.
Frequency 60 cycles.

T deg. cent	E 10^3	W/ft.	I/ft. 10^{-3}	P.F. per cent.	C/ft. 10^{-9}	K_{av}	ρ_{av} 10^{10}
50	6	0.056	0.147	6.35	0.0650	3.286	14.32
	10	0.158	0.246	6.425	0.0651	3.300	14.10
	14	0.331	0.3455	6.84	0.0655	3.312	13.20
	18	0.592	0.448	7.345	0.0660	3.340	12.20
	22	0.960	0.557	7.83	0.0671	3.398	11.24
	26	1.508	0.676	8.59	0.0692	3.485	10.03
75	6	0.238	0.157	25.28	0.0670	3.40	3.38
	8	0.433	0.209	25.90	0.0670	3.40	3.32
	10	0.678	0.2615	25.90	0.0671	3.404	3.28
	12	0.990	0.3155	26.12	0.0673	3.417	3.24
	14	1.367	0.3715	26.28	0.0679	3.447	3.20
	16	1.837	0.4305	26.68	0.687	3 493	3.12
	18	2.410	0.492	27.22	0.0696	3.540	3.00
100	5	0.363	0.164	45.76	0.773	3.920	1.535
	7	0.723	0.230	45.00	0.0778	3.943	1.512
	10	1.484	0.330	44.85	0.0783	3.969	1.500
	13	2.510	0.4323	44.50	0.0784	4.005	1.500
	14	2.910	0.468	44.30	0.0791	4.036	1.500
	15	3.340	0.5055	44.00	0.0804	4.07	1.500
75	6	0.238	0.1605	24.72	0.0688	3.486	3.38
	8	0.428	0.214	25.00	0.0687	3.485	3.33
	10	0.675	0.2673	25.25	0.0686	3.482	3.30
	12	0.980	0.3215	25.41	0.0688	3.486	3.27
	14	1.354	0.378	25.60	0.0692	3.508	3.24
	16	1.820	0.4375	26.00	0.0702	3.553	3.14
	18	2.360	0.500	26.22	0.0712	3.606	3.06
50	6	0.063	0.147	7.15	0.0650	3.286	12.75
	10	0.185	0.246	7.525	0.0651	3.300	12.08
	14	0.371	0.3455	7.672	0.0655	3.312	11.79
	18	0.676	0.448	8.39	0.0660	3.340	10.70
	22	1.144	0.557	9.34	0.0671	3.398	9.44
	26	1.770	0.676	10.06	0.0692	3.485	8.55
25	6	0.0105	0.1455	1.203	0.0645	3.257	76.40
	10	0.0305	0.244	1.250	0.0648	3.280	73.00
	14	0.0725	0.344	1.504	0.0653	3.300	60.40
	18	0.1805	0.445	2.255	0.0651	3.320	40.10
	22	0.3525	0.548	2.923	0.0661	3.347	30.60
	26	0.5825	0.654	3.430	0.0667	3.377	25.90
	30	0.8830	0.760	3.877	0.0672	3.402	22.70

The length was immersed in an oil bath. In this way the temperature could be carefully controlled and leakage losses eliminated.

Starting at 25 deg. cent., readings were taken through two complete heat cycles as shown in Table II. The temperature at each step was held constant and watt readings taken every five minutes until constant, (these readings are not given in the table). In this way we were assured of an even temperature through the cross section.

The heat-cycle curves in Figs. 4, 5 and 6 show the variation of watts, power factor and permittivity with temperature at 10 kv. The loss and permittivity are both higher in the second heat cycle than in the first at temperatures above 70 deg. cent. We can give no clear explanation of this. It is probably due to a combination of things such as, a slight temperature lag, physical and chemical changes and consequent change in the inherent conductivity of the material from the high temperature and voltage stress. (The over-voltage was only left on a few seconds, while the highest points on the curves were being taken and could not have caused an appreciable rise in temperature.) All

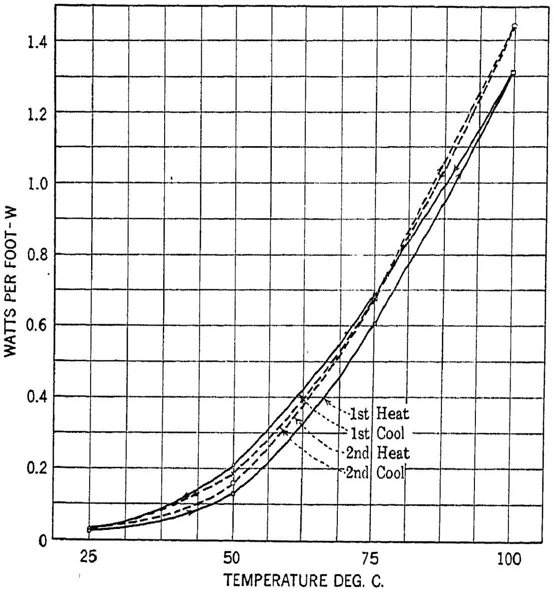

FIG. 4—WATTS PER FOOT CABLE 1 (PAPER) AT 10 KV.

cable engineers have noted this same characteristic "fatigue" effect in another form when applying a high-potential test to lengths of cable. The d-c. resistivity is much lower just after the high potential test, and when this is severe it never regains its former value. Middleton and Dawes * gave data on this in a paper read before the Institute in 1914.

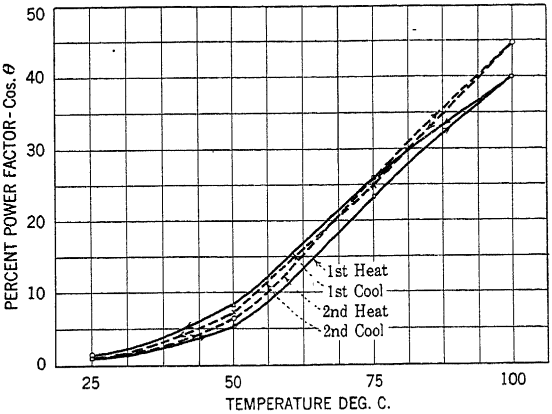

Fig. 5—Per cent Power Factor Cable 1 (paper) at 10 kv.

S. Evershed * formulated a very interesting theory of moisture conduction in capillary passages which could account for the "heat cycles" and also the increase of the second cycle at higher

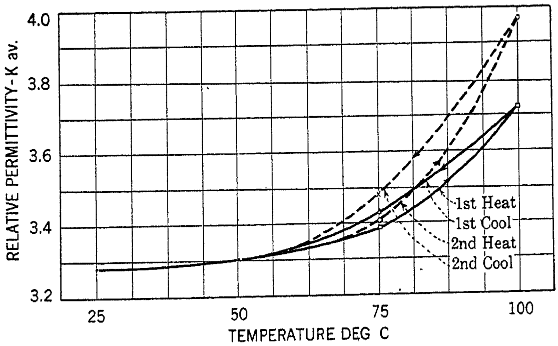

Fig. 6—Relative Permittivity Cable 1 (Paper) at 10 Kv.

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* *Voltage Testing of Cables*, by W. I. Middleton and C. L. Dawes, Trans. A. I. E. E., Vol. 33, p. 1185.

* "The Characteristics of Insulation Resistance" by S. Evershed, *Journal* I. E. E., Vol. 52, 1914, p. 51.

temperatures (both effects must be, in truth, caused by the same phenomena). After a careful study of this and many other characteristics of insulating materials we are convinced that Evershed's theory does not entirely account for or fit in with the things noted and which will be given in this paper.

Cable manufacturers have paid a great deal of attention to moisture and developed very effective ways of eliminating it and it does not seem possible that moisture could play an important part in present day cable insulation as long as the lead sheathing remains intact. We have never yet found a case where trouble experienced in operation could be attributed to initial moisture under the above condition. Moisture is sometimes entrapped in cable joints but it does not, as will be shown later, (tests on Cable No. 4), travel along the length of cable to a great extent.

The same characteristic "heat cycles" were experienced with rubber insulated cable sealed in lead sheathing. There was no moisture in this rubber but there was a chemical change due to heating during test. It is chemical instability under temperature and voltage stress that is the predominating question in present day cable insulation problems and not moisture. What is needed is working theory of chemical changes and intermolecular phenomena under high temperature and voltage stress.

The similarity between the watts and permittivity cycles (Figs. 4 and 6) is also worthy of note. This same similarity has been experienced with rubber, varnished cambric and other types of insulation.

For a long time we were puzzled by the erratic behavior of the resistivity vs. voltage curves on paper cable insulation at low temperatures. The 25 deg. cent. curves in Fig. 7 illustrate this peculiar characteristic. At higher temperatures the curves are stable but invariably the room temperature curves behave as shown.

The conclusion was finally reached that this was due to the number of times the samples were bent while cold in preparing them for test. This bending would tend to slightly distort the cross section and cause minute voids or vacuum spaces, which would in a short time be filled with gases from volatilization of the compound. When the cable is again straightened the gases prevent the space being entirely filled up; and, when voltage stress is applied and reaches a value sufficiently high these low-pressure gases are ionized.

Referring to Fig. 7 the 25 deg. cent. curves begin to decrease rapidly between 11 and 12 kv., that is, internal ionization begins here. Gradually a point of saturation is reached and the energy consumed by ionization approaches more nearly a true ohmic loss.

The 50 deg. cent. curves are more stable and the 75 deg. and 100 deg. cent. curves entirely so. The specific gravity of the compound alone, Fig. 2, shows that it expands appreciably with

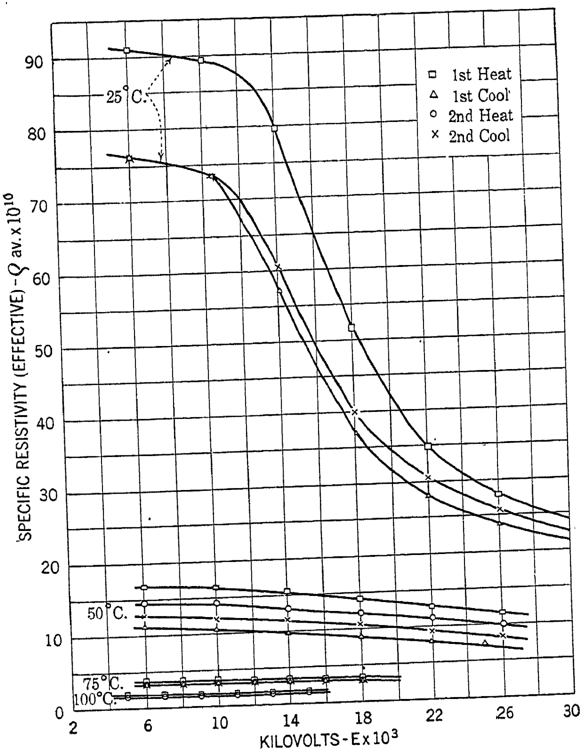


FIG. 7—SPECIFIC RESISTIVITY (EFFECTIVE) CABLE 1 (PAPER)

temperature. The same thing must occur in cables and the small spaces are gradually filled up as the temperature increases, with a corresponding increase in gas pressure. There is still slight ionization at 50 deg. cent. but the inherent loss in the material, itself, predominates. At 75 deg. cent. expansion of the compound has probably reduced these air spaces to an almost infinitesimal size, and in addition, the gas pressure must be so high that ionization is practically eliminated.

A test made later on the sample previously discussed shows the effects of bending paper cable to small radii while cold. Curve *A* in Fig. 8 was taken at 24 deg. cent. under the same conditions as the 25 deg. cent. curves in Fig. 7, that is, with the length bent U-shape to a radius of 9 in. (22.8 cm.) The length was then wound into a coil having a diameter of 4 in. (10.16 cm.) Curve *B* in Fig. 8 shows that considerably more energy was consumed by ionization and there must, therefore, have been more space to be ionized. It is interesting to note that ionization began at the same voltage in both cases.

An unfilled sample length of cable was made up and the effects of internal ionization studied more closely. This will be discussed later.

We have never discovered a case where trouble in operation could be attributed to this low temperature characteristic. Cables used in installation work are handled more carefully than the test samples were and in addition there is little likelihood of continuous internal ionization under operating conditions. The total length of time a cable is operated cold is only a small per cent of its total life and even when the conditions are favorable it is only in cables of small conductor diameter or very high-voltage rating that internal ionization is probable at normal operating voltage. That this ionization must be of a weak nature is evident by the extremely small loss at 25 deg. cent. The same effect has been studied in highly stressed armature coils over long periods of time and it is surprising how little damage internal ionization causes in comparison with external corona at points along the surface of the coil.

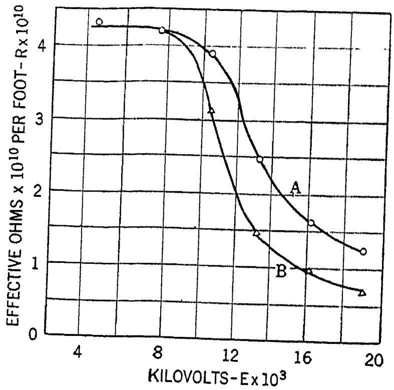


Fig. 8—Effect of Bending on Internal Ionization at 24 Deg. Cent. A—Bent to Large Radius. B—Bent to Small Radius

Here also, Evershed's moisture theory could be made to apply and even better than in the case of the "heat cycles." The 25 deg. cent. effective a-c. resistivity curves in Fig. 7 have much the same characteristic shape as the d-c. resistivity curves given in his paper. But, Evershed's measurements were taken on materials exposed to air and known to contain appreciable amounts of moisture. The typical d-c. moisture curve drops rapidly from the start and the saturation point is reached at comparatively low-voltage stress, while the a-c. resistivity curve on "dry" materials, that is, materials reasonably free of moisture, do not begin to drop until the materials are well stressed. As the paper progresses other points will be brought out confirming the conclusion that internal ionization accounts for the peculiar shape of these curves.

The rapid decrease in effective resistance and corresponding increase in watts loss with increasing temperature as shown in Figs. 4 and 7 is not due to secondary effects such as moisture or internal ionization of entrapped gases, but is inherent in the material itself and is probably some sort of ionic conduction, with consequent change in the chemical structure. We would naturally expect, then, that the material with the highest effective a-c. resistance at high temperature would be the least liable to deterioration at "hot spots" in a line and is therefore the most desirable one to use.

The power factor at 100 deg. cent. in Fig. 5 is 45 per cent, much higher than any measured on samples of three-conductor cable in which the same materials are used. In looking into this it was found that in making up the short length (30 ft. or 9.1 m.) for test purposes the regular process was not followed as closely as it should have been. The table and curves are given, however, because they are complete and bring out a number of interesting points. Another test length was made up under more favorable conditions and the readings on this are given below.

Cable No. 2. Three samples were cut from the second special length of cable. The readings on these checked very closely and only one will be considered. This same plan was followed throughout in testing lengths of new cable, the samples always checking within three or four per cent. Cable No. 2 was similar in every respect to cable No. 1, except that care was taken to represent the regular three-conductor process more closely.

Only one heat cycle was taken and the values are given in Table III. The resistivity vs. voltage curves are given in Fig. 9. The same characteristics drop in resistivity noted in Cable No. 1 is again in evidence, starting at 12 kv. and reaching saturation at about 26 kv. The resistivity is much higher than that of cable No. 1 and the power factor lower. At 100 deg. cent., as shown in Fig. 10, the power factor is 13.4 per cent and this

TABLE III. (CABLE NO. 2)

2/0 single conductor, maximum diameter 0.412 in.
9/32 in. paper filled with mineral hydrocarbon base compound.
3/32 in. lead sheath.
Frequency 60 cycles.

T deg. cent	E 10^3	W/ft.	I/ft. 10^{-3}	P.F. per cent.	C/ft. 10^{-9}	$K_{av.}$	$\rho_{av.}$ 10^{10}
22	6.06	0.0040	0.132	0.505	0.0577	2.910	205.6
	8.08	0.00711	0.176	0.500	0.0578	2.915	205.2
	10.00	0.0110	0.2155	0.520	0.0577	2.910	202.5
	15.20	0.0349	0.329	0.696	0.0575	2.895	147.0
	18.10	0.0832	0.392	1.172	0.0575	2.895	87.5
	21.00	0.1970	0.456	2.050	0.0576	2.900	49.8
	25.95	0.6420	0.582	4.240	0.595	2.995	23.3
51	6.06	0.0162	0.136	1.409	0.0584	2.940	70.05
	8.08	0.0205	0.178	1.426	0.0586	2.955	71.0
	10.00	0.0320	0.220	1.454	0.0590	2.969	69.7
	15.20	0.0975	0.336	1.910	0.0586	2.955	52.20
	18.10	0.1850	0.397	2.575	0.0581	2.930	39.40
	21.00	0.3380	0.462	3.480	0.0581	2.930	29.10
	25.95	0.8740	0.590	5.700	0.0625	3.155	17.12
73.5	4.34	0.0241	0.093	6.36	0.0567	2.860	16.90
	6.06	0.0480	0.132	5.92	0.0569	2.868	16.82
	8.08	0.0852	0.175	5.80	0.0568	2.862	16.74
	10.00	0.1340	0.214	5.94	0.0565	2.852	16.60
	12.10	0.1926	0.258	6.10	0.0564	2.840	16.90
	15.20	0.3280	0.325	6.64	0.05645	2.845	15.70
	18.10	0.5200	0.387	7.41	0.05645	2.845	14.00
100	4.34	0.0568	0.097	13.78	0.0589	2.970	7.54
	6.06	0.1094	0.138	13.60	0.0590	2.978	7.46
	8.08	0.1960	0.182	13.50	0.0591	2.980	7.40
	10.00	0.303	0.2255	13.41	0.0591	2.980	7.35
	12.10	0.447	0.272	13.60	0.0591	2.980	7.29
	15.20	0.720	0.342	14.20	0.0591	2.980	7.13
	18.10	1.116	0.408	15.10	0.593	2.985	6.54
73	4.34	0.0254	0.095	6.16	0.0580	2.925	16.50
	6.06	0.0499	0.136	6.14	0.0581	2.930	16.46
	8.08	0.0886	0.177	6.20	0.0581	2.932	16.40
	10.00	0.136	0.220	6.22	0.0584	2.940	16.36
	12.10	0.1990	0.267	6.38	0.0584	2.940	16.32
	15.20	0.3340	0.336	6.78	0.0584	2.940	15.40
	18.10	0.5540	0.400	7.50	0.0585	2.950	13.19
52	6.06	0.02135	0.137	2.57	0.0594	2.995	38.20
	8.08	0.0410	0.181	2.81	0.0594	2.995	37.40
	10.00	0.0600	0.224	2.78	0.0594	2.994	36.70
	15.20	0.1520	0.339	2.95	0.0592	2.980	33.70
	18.10	0.2684	0.405	3.55	0.0594	2.995	27.20
	21.00	0.4580	0.468	4.66	0.592	2.980	21.50
	25.95	1.024	0.594	6.65	0.0605	3.050	14.60
21	6.06	0.00497	0.135	0.607	0.0577	2.912	164.0
	8.08	0.00882	0.179	0.610	0.0580	2.922	165.0
	10.00	0.0139	0.219	0.635	0.0580	2.920	160.5
	15.20	0.0425	0.334	0.837	0.0580	2.920	121.1
	18.10	0.0906	0.397	1.262	0.0581	2.925	80.5
	21.00	0.203	0.462	2.090	0.0581	2.925	48.5
	25.95	0.692	0.590	4.510	0.0604	3.040	21.6

corresponds with measurements on samples of similar three-conductor cable. We can give no explanation for the peculiar behavior of the permittivity curves in Fig. 10. This is the only length of cable that showed a decrease of permittivity with temperature.

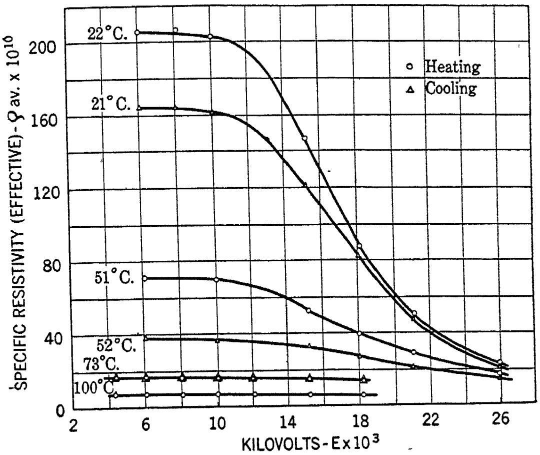


Fig. 9—Specific Resistivity (Effective) Cable 2 (Paper)

Although the high temperature losses and power factors are representative of the materials and process used in making three-conductor cables, the low-temperature characteristics are not, and indicate that this sample length was not sufficiently well filled with compound. This will be brought out more clearly as the paper progresses.

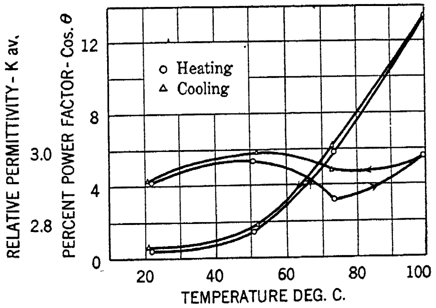


Fig. 10—Power Factor and Relative Permittivity Cable 2 (Paper) at 10 Kv.

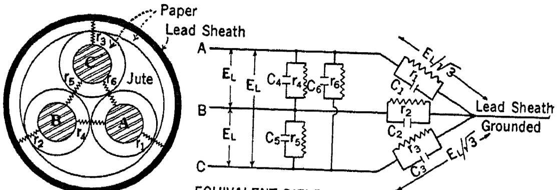
THREE-CONDUCTOR CABLE

It is difficult to measure three-phase dielectric losses in short samples with a reasonable degree of accuracy, and elaborate corrections are necessary. For this reason the measurements on three-conductor cable were taken single-phase. Three-phase values can be approximated from single-phase readings.

Fig. 11 is a cross-sectional sketch of a three-conductor cable and also shows the equivalent dielectric circuit. It is assumed that

$$r_1 = r_2 = r_3, r_4 = r_5, = r_6, c_1 = c_2 = c_3 \text{ and } c_4 = c_5 = c_6$$

By applying single-phase voltage to all three conductors and grounding the lead sheath through the instrument, readings can be taken from which r_1 and c_1 can be calculated as shown in Appendix B. In the next set of readings voltage is applied to conductor A, and conductors B and C are connected to the lead sheath. From these readings r_4 and c_4 can be determined. Following the same method the three-phase values can be calculated. The nomenclature and formulas are all given in Appendix B.



CROSS SECTION 3ϕ CABLE

EQUIVALENT DIELECTRIC CIRCUIT OF 3ϕ CABLE

FIG. 11

Data on Three-Conductor Paper Cable

Cable No. 3. This length was cut from an unused reel and was never in service. Specifications are given below:

2/0 three-conductor, max. diameter 0.412 in. (10.45 mm.)
9/32-in. (7.14 mm.) paper on each conductor.
7/32-in. (5.5 mm.) paper over-all.
Filled with a mineral hydrocarbon base compound.
1/8-in. (3.1 mm.) lead sheath.

The test samples each had an active length of lead sheath of 9 ft. (2.7 m.) bent into a half circle with the spread ends properly sealed and immersed in crocks of oil. The whole was placed in a large electrically heated oven.

To be sure that no error was introduced by "end effects" and exterior sources of loss a 20-ft. (6.1-m.) length was first tested at 28 deg. cent. and then cut into 9-ft. (2.6-m.) lengths. The watt readings per foot on the two halves checked within 1.0 per cent of those on the total length.

It required such a long time to reach uniform temperature at each step, (4 to 5 hours, whereas it only required one-half hour in the case of single-conductor cable immersed in oil), that a complete heat cycle was not taken but only steps of 28, 48, 74 and 98 deg. cent. The calculated single- and three-phase values are given in Table IV. Fig. 12 shows the total calculated three-phase watts ($W_{3\phi}$) and three-phase power factor (cos θ_ϕ) for different temperatures at 25 kv. The corresponding single-phase measurements (W_2 and cos θ_2), taken by applying high voltage to one conductor and connecting the other two to lead sheath, are also given.

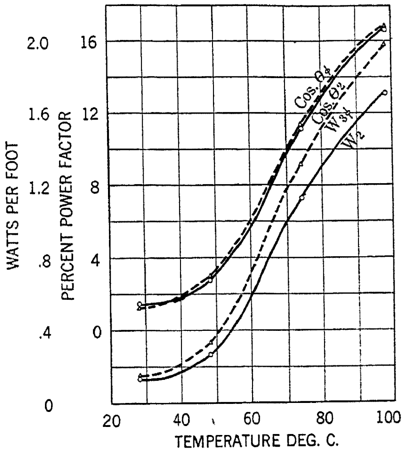


FIG. 12—WATTS PER FOOT AND POWER FACTOR CABLE 3 (PAPER) AT 25 KV.—NEW CABLE, NEVER IN SERVICE

Cos θ_ϕ and cos θ_2 agree almost exactly, and there is a close comparison between $W_{3\phi}$ and W_2. For samples having the same thicknesses of conductors and jacket insulation, $W_{3\phi}$ and W_2 would probably not check so closely but there would still be a good agreement between cos θ_ϕ and cos θ_2, and a study of power factor readings is sufficient; in fact, it is the only practical method

TABLE IV. (CABLE NO. 3) NEVER IN SERVICE

2/0 three-conductor cable
9/32 in. paper on each conductor
7/32 in. paper overall
1/8 in. lead sheath
Frequency 60 cycles

T deg. cent.	E 10^3	W_1	W_2	$W_{3\phi}$	I_1 10^{-3}	I_2 10^{-3}	I_ϕ 10^{-3}	$\cos\theta_1$ per cent.	$\cos\theta_2$ per cent	$\cos\theta_\phi$ per cent.	r_1 10^{10}	r_4 10^{10}	r'' 10^{10}	C_1 10^{-9}	C_4 10^{-9}	C'' 10^{-9}	C_ϕ 10^{-9}
28	10	0.0090	0.0185	0.0262	0.0960	0.160	0.1154	0.937	1.155	1.313	1.11	2.106	0.540	0.0254	0.0085	0.0424	0.0308
	14	0.0169	0.0362	0.0485	0.1300	0.221	0.1549	0.930	1.193	1.290	1.160	2.030	0.542	0.0246	0.0082	0.0410	0.0295
	18	0.0282	0.0605	0.0775	0.1640	0.281	0.1952	0.955	1.227	1.277	1.150	2.007	0.535	0.0242	0.0081	0.0404	0.0287
	22	0.0465	0.0965	0.1170	0.1985	0.341	0.2340	1.064	1.324	1.314	1.041	1.936	0.502	0.0239	0.0080	0.0399	0.0281
	26	0.0783	0.1510	0.1673	0.2325	0.402	0.2725	1.294	1.497	1.364	0.864	1.860	0.447	0.0237	0.0079	0.0396	0.0278
	30	0.1190	0.2210	0.2319	0.2675	0.463	0.3125	1.482	1.650	1.430	0.757	1.760	0.407	0.0236	0.0079	0.0395	0.0276
48	10	0.0162	0.0395	0.0514	0.0970	0.151	0.1024	1.670	2.615	2.905	0.617	0.859	0.253	0.0258	0.0071	0.0400	0.0272
	14	0.0325	0.0785	0.1005	0.1363	0.211	0.1402	1.701	2.660	2.920	0.603	0.852	0.249	0.0258	0.0071	0.0400	0.0272
	18	0.0541	0.1318	0.1704	0.1752	0.272	0.1849	1.712	2.690	2.940	0.598	0.833	0.246	0.0258	0.0071	0.0401	0.0272
	22	0.0834	0.2015	0.2572	0.2140	0.335	0.2266	1.767	2.730	2.970	0.580	0.817	0.240	0.0258	0.0071	0.0401	0.0272
	26	0.1202	0.2880	0.3630	0.2527	0.392	0.2675	1.830	2.820	3.010	0.563	0.807	0.235	0.0258	0.0071	0.0401	0.0272
	30	0.1637	0.3890	0.4879	0.2925	0.452	0.3072	1.865	2.860	3.050	0.550	0.800	0.231	0.0258	0.0071	0.0401	0.0271
74	10	0.091	0.160	0.2055	0.0990	0.154	0.1042	9.20	10.46	11.27	0.1100	0.286	0.0625	0.0259	0.0073	0.0406	0.0275
	14	0.180	0.323	0.3945	0.1390	0.217	0.1479	9.25	10.62	11.02	0.1088	0.274	0.0606	0.0262	0.0073	0.0408	0.0278
	18	0.302	0.548	0.6720	0.1790	0.280	0.1900	9.31	10.87	11.33	0.1071	0.261	0.0592	0.0263	0.0073	0.0410	0.0278
	22	0.465	0.850	1.0185	0.2190	0.342	0.2330	9.66	11.30	11.50	0.1038	0.251	0.0569	0.0263	0.0073	0.0410	0.0279
	26	0.685	1.222	1.4335	0.2580	0.405	0.2780	10.21	11.60	11.35	0.0987	0.242	0.0553	0.0263	0.0074	0.0410	0.0281
	30	0.968	1.735	1.9785	0.2980	0.470	0.3230	10.85	12.30	11.80	0.0932	0.235	0.0518	0.0262	0.0075	0.0413	0.0284
98	10	0.130	0.230	0.2940	0.1000	0.158	0.1073	13.00	14.55	15.80	0.0769	0.200	0.0435	0.0263	0.0075	0.0415	0.0281
	14	0.260	0.487	0.6015	0.1410	0.222	0.1510	13.17	15.63	16.39	0.0753	0.1726	0.0403	0.0264	0.0075	0.0415	0.0282
	18	0.440	0.840	1.0200	0.1810	0.286	0.1951	13.50	16.28	16.73	0.0737	0.162	0.0386	0.0264	0.0075	0.0416	0.0284
	22	0.680	1.280	1.5300	0.2215	0.350	0.2385	13.95	16.62	16.81	0.0712	0.161	0.0377	0.0264	0.0076	0.0416	0.0285
	26	0.990	1.817	2.1405	0.2615	0.415	0.2838	14.56	16.84	16.79	0.0685	0.1534	0.0361	0.0264	0.0076	0.0416	0.0286
	30	1.370	2.580	3.0300	0.3015	0.478	0.3270	15.14	18.00	17.83	0.0656	0.149	0.0349	0.0264	0.0076	0.0416	0.0286

of comparing a number of three-conductor samples having different cross-sectional dimensions.

The three-phase calculations in Table IV require entirely too much time so the "two conductor to lead sheath" connection (W_2, $\cos \theta_2$, etc.) was adopted as a standard in testing the other three-conductor samples.

The 98 deg. cent. power factor, Fig. 12, is 16.7 per cent while that of cable No. 2, made up by the same process is 13.5 per cent. This is a very fair agreement when it is considered that one is single- and the other three-conductor, and that they represent different batches.

It is to be expected that a single-conductor cable would have a slightly lower power factor than a similar three-conductor cable. Referring to Fig. 13 and assuming a single-phase voltage E impressed between conductor A and the other two conductors, B and C, which are grounded to the lead sheath, the potential at points 2 and 3 would be 0.5 E. The potential at point 1 would be a little above ground, say 0.1 E, then the potential between 2-1 and 3-1 would be 0.4 E. This tangential stress causes a leakage or equalizing current to flow along the surfaces between the paper and jute filler, and thereby adds to the total loss. The value of the excess loss would be determined by the value of the effective a-c. resistance in the surface leakage path. If this is very high, the excess loss will be only a small part of the total, but if it is comparatively low, this loss may be an appreciable part. It is important then, that materials be used here that have the highest possible "surface resistance," especially at high temperatures.

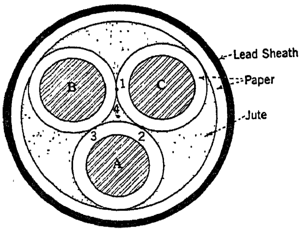


FIG. 13—CROSS SECTION OF THREE-CONDUCTOR CABLE

Three-phase tangential stress between the points 1, 2 and 3 is higher than single-phase, and appears in a different way. Let the three-phase line voltage be E_ϕ. Point 4, being the center of a symmetrical arrangement, will be at ground potential and the potential between this point and conductors A, B and C will be $E_\phi / \sqrt{3}$. An electrostatic polyphase field will rotate about point 4 and M. Höchstadter * has shown by diagrams that, assuming the surface resistance to be high, the difference in potential between the points 1, 2 and 3 is 0.87 E_ϕ.

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* "Three-Core Cables" by M. Höchstadter, *Electrotechnische Zeitschrift*, Heft 47, 1915. Abstract in the London *Electrician*, 1916, p. 209.

The actual three-phase loss will, therefore, be higher than the calculated loss in Table IV (depending upon the value of surface resistance), since the stress between 2-1 and 3-1 will be about twice as great.

These tangential stresses, occurring as they do in the hottest part of the cross section are a weak point in three-conductor cable design and will probably be the limiting feature of the maximum voltage rating that will be reached in the future.

At present 25,000-volt, grounded *Y*, is about the highest rating here in America, where conduit transmission is used. In Germany a 30,000-volt system has been installed *, and one in England, rated at 33,000 volts. † These two systems, however, make use of buried cable and operate at relatively low temperatures.

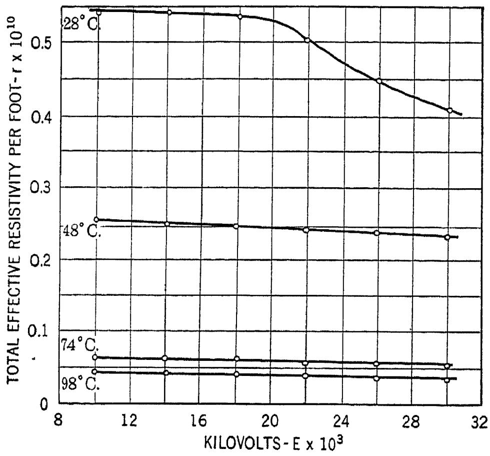

FIG. 14—TOTAL EFFECTIVE RESISTIVITY PER FOOT CABLE 3 (PAPER)

That the present maximum voltage rating for three-conductor paper-insulated cable in conduits is reliable under reasonable conditions is shown by the experience of one of the operating companies. They have lines rated at 24,000 volts, grounded *Y*, that have been in service continuously for five years and no more trouble has been experienced with these than with lower voltage lines in the same system.

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* "30,000-Volt, Three-Phase Cable System, Berlin" by W. Pfaunkuch, E. T. Z., Vol. 33, 1912.

† "Recent Practise in the Manufacture, Laying and Jointing of 33,000-Volt, Three-Phase, Underground Cable" by C. Beaver, London *Electrician*, June 23, 1916.

A sample length, cut from one of their lines was obtained and the tests on this are discussed under the following heading

Cable No. 4.

Length of sample, 22 ft. (6.7 m.)
1/0 three-conductor.
9/32-in. (7.1 mm.) paper on each conductor.
7/32-in. (5.5 mm.) paper over-all.
Filled with mineral hydrocarbon base compound.
1/8-in. (3.1 mm.) lead sheath.
In service approximately five years at 24,000 volts.

When preparing this length for test we found that only one end was sealed up. It had evidently been cut from a longer length which was properly sealed when taken from the line and, through an oversight, sealing of the sample omitted. The reel on which this sample was wound had lain out doors in the rain and snow for four weeks after receiving and it was feared that moisture absorption had spoiled it.

Energy loss measurements showed, however, that although this sample had absorbed a slight amount of moisture it did not affect the insulation characteristics to a great extent.

The whole length was first tested at 24 deg. cent. and then cut in halves. The sum of the loss in the two halves checked the total, but the half towards unsealed end had 6 per cent greater loss than the other, thus indicating that it had absorbed the most moisture. Even this amount must have been extremely small for the power factors were quite low.

The single-phase energy-loss data of the half towards the sealed end are given in Table V. Fig. 15 shows the variation of watts and power factor with temperature. At 25 deg. cent. the 25-kv. power factor is 2.3 per cent and at 100 deg. cent. it is 26.3 per cent. The corresponding power factors for new cable of the same kind are 1.9 per cent and 16.7 per cent. (Fig. 12). Allowing for possible variation in different batches, and any slight amount of moisture that may have been absorbed during exposure, this is a good agreement, and together with the operating experience, shows that this type of cable had not deteriorated to an appreciable extent, if at all. After the tests were completed Cable No. 4 was cut open and closely examined. It was found to be in good condition.

The resistivity r'' curves are given in Fig. 16. Internal ionization begins at about 20 kv. in the 24 deg. cent. curve, and there is also a slight break in the 51 deg. cent. curve. The corresponding low temperature (28 deg. cent.) curve of Cable No. 3 (Fig. 14) shows ionization starting at the same voltage, but at 48 deg. cent. there is none at all. Ionization is not very pronounced in either sample.

TABLE V. (CABLE NO. 4) IN SERVICE 5 YEARS.

Specifications same as for Cable No. 3 with exception that conductors were 1/0. Frequency 60 cycles.

T deg. cent.	E 10^3	W_2	I_2 10^{-3}	P.F. per cent.	r'' 10^{10}	C'' 10^{-9}
24	10.08	0.0238	0.1420	1.66	0.427	0.0374
	12.92	0.0386	0.1786	1.678	0.430	0.0366
	15.80	0.0584	0.2170	1.705	0.428	0.0363
	18.10	0.0771	0.2480	1.720	0.425	0.0362
	23.10	0.1450	0.3140	1.999	0 368	0.0361
	25.00	0.1850	0.339	2.200	0.338	0.0360
	30.00	0.3230	0.4040	2.670	0.279	0.0357
51	10.08	0.1103	0.1462	7.51	0.0919	0.0385
	12.92	0.1820	0.1859	7.57	0.0916	0.0380
	15.80	0.2700	0.2270	7.46	0.0914	0.0379
	18.10	0.3600	0.2615	7.63	0.0911	0.0379
	23.10	0.5940	0.3300	7.76	0.0900	0.0378
	25.00	0.728	0.3560	8.15	0.0860	0.0377
	30.00	1.129	0.4280	8.77	0.0799	0.0377
74	10.08	0.202	0.1512	13.26	0.0503	0.0396
	12.92	0.334	0.1930	13.39	0.0502	0.0393
	15.80	0.496	0.2340	13.45	0.0501	0.0390
	18.10	0.655	0.2640	13.52	0.0500	0.0384
	23.10	1.063	0.3370	13.70	0.0499	0.0384
	25.00	1.287	0.3670	14.20	0.0487	0.0384
	30.00	1.970	0.4370	15.00	0.0466	0.0384
77	10.08	0.219	0.1512	14.37	0.0472	0.0394
	12.92	0.356	0.1930	14.26	0.0469	0.0392
	15.80	0.528	0.2350	14.31	0.0468	0.0390
	18.10	0.704	0.2670	14.55	0.0466	0.0388
	23.10	1.143	0.3380	14.69	0.0465	0.0384
	25.00	1.400	0.3680	15.18	0.0446	0.0385
	30.00	2.110	0.4400	15.92	0.0426	0.0385
101	10.08	0.463	0.162	28.4	0.0220	0.0408
	12.92	0.740	0.204	28.1	0.0225	0.0404
	15.80	1.079	0.248	27.5	0.0231	0.0401
	18.10	1.400	0.283	27.3	0.02034	0.0398
	23.10	2.215	0.360	26.7	0.0241	0.0395
	25.00	2.635	0.392	27.0	0.0238	0.0402
	30.00	3.920	0.477	27.4	0.0229	0.0406

TROUBLE EXPERIENCED IN OPERATION

The most interesting case encountered was the trouble experienced by another one of the operating companies.

Several years ago it installed a considerable length of new cable in its 13,800-volt, *Y*-connected, grounded-neutral system. It had the following specifications:

4/0 three-conductor.
7/32-in. (5.5 mm.) paper on each conductor.
7/32-in. (5.5 mm.) paper over all.
Filled with vegetable hydrocarbon base compound.
1/8-in. (3.1 mm.) lead sheath.

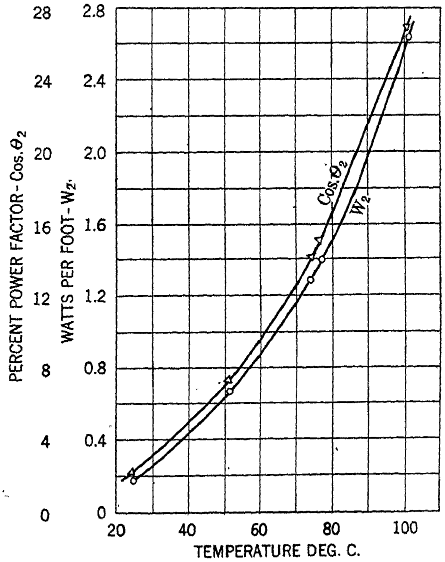

FIG. 15—WATTS PER FOOT AND POWER FACTOR CABLE 4 (PAPER) AT 25 KV.—IN SERVICE FIVE YEARS AT 24 KV.

The failures encountered in this lot of cable gradually increased with time and finally reached such serious proportions that a special investigation was made to determine the trouble.

By referring to the records it was found that the cables had never been heavily overloaded and there were no indications of unusual high-frequency disturbances.

Most of the trouble was experienced at points known to be heated above the average duct temperature from external sources.

The cables were in a number of cases quite warm even when lightly loaded, indicating that the dielectric energy loss must have been so high that it was comparable with copper losses.

In one instance, after a cable had failed at a known "hot spot" in the system, sections of the remaining cables at this point were removed from the ducts, cut open and examined.

Referring to Fig. 13, the triangle of jute filler and the surfaces of the paper wrappings between the points 1, 2 and 3 were badly scorched. There were spots in some of the sections where the paper wrappings were charred almost down to the copper. The wrappings next to the lead sheath and those next to the copper (except at the spots where charring was very bad) showed no signs of deterioration and this would indicate that the charring mentioned was not caused directly by burning from overloads or from exterior sources of heat.

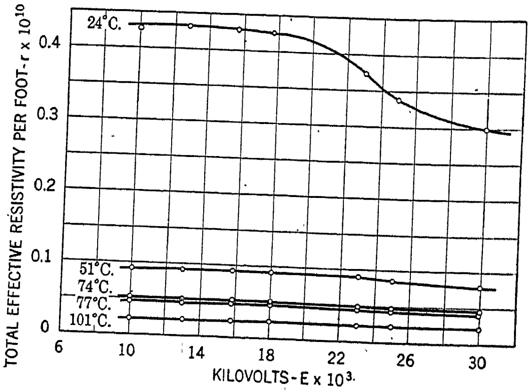

FIG. 16—TOTAL EFFECTIVE RESISTIVITY PER FOOT CABLE 4 (PAPER)

Mr. Höchstadter described this same characteristic charring in the article previously referred to.

By arrangement, whenever failure occurred a sample 12 ft. (3.6 m.) in length was cut from the line near the point of failure, sealed up and sent to the laboratory. Twelve samples were collected in this way and in addition a new one of the same kind was cut from a reel in the store room.

Cable No. 5a. The energy loss data on the new sample referred to above are given in Table VI. As there were so many samples to be tested readings were taken at only two temperatures, 21 deg. and 100 deg. cent.

The power factor and loss curves are given in Fig. 17. The 13,800-volt power-factor is 1.5 per cent at 21 deg. cent. and increases to 54 per cent at 100 deg. cent., showing that this lot of cable had poor temperature characteristics even when new.

TABLE VI.

4/0 three-conductor
7/32 in. paper on each conductor
7/32 in. paper over-all
Filled with vegetable hydrocarbon base compound
1/8 in. lead sheath
Frequency 60 cycles

T deg. cent.	E 10^3	W_2	I_2 10^{-3}	P.F. per cent.	r'' 10^{10}	c'' 10^{-9}
CABLE NO. 5-A NEVER IN SERVICE						
21	4.05	0.00542	0.0860	1.552	0.3025	0.0554
	6.06	0.01187	0.1280	1.529	0.3100	0.0560
	8.89	0.02530	0.1894	1.500	0.3120	0.0566
	11.95	0.04510	0.2550	1.480	0.3150	0.0567
	14.80	0.07145	0.3168	1.522	0.3070	0.0568
	17.88	0.1300	0.3836	1.896	0.2460	0.0569
100	4.05	0.2888	0.1365	52.27	0.00567	0.0763
	6.06	0.6000	0.2020	49.00	0.00614	0.0768
	8.89	1.350	0.2990	50.85	0.00585	0.0769
	11.95	2.555	0.4070	52.50	0.00560	0.0770
	14.80	4.220	0 5170	55.16	0.00519	0.0773
	17.88	6.763	0.6500	58.22	0.00476	0.0784
CABLE NO. 5-B IN SERVICE 5 YEARS						
22	4.05	0.00864	0.0803	2.655	0.1900	0.0524
	6.06	0.01875	0.1210	2.550	0.1960	0.0527
	8.89	0.0416	0.1787	2.620	0.1900	0.0533
	11.95	0.0899	0.2432	3.095	0.1592	0.0539
	14.80	0.1790	0.3060	3.955	0.1222	0.0549
	17.88	0.3322	0.3750	4.955	0.0963	0.0558
100	4.05	0.947	0.2850	82.10	0.001730	0.1064
	6.06	2.185	0.4400	81.95	0.001684	0.1100
	8.89	4.963	0.6760	82.00	0.001592	0.1157
	11.95	9.55	0.9725	82.15	0.001498	0.1230
	14.80	17.23	1.3700	84.56	0.001272	0.1282
	17.88	30.62	1.9580	87.50	0.001043	0.1402
CABLE NO. 5-C IN SERVICE 5 YEARS						
29	4.05	0.03925	0.10865	8.90	0.0418	0.0718
	6.06	0.0870	0.1650	8.68	0.0422	0.0720
	8.94	0.1881	0.2412	8.74	0.0423	0.0717
	12.07	0.3673	0.3256	9.35	0.0396	0.0714
	14.93	0.6025	0.4025	10.02	0.0367	0.0711
	18.01	1.0050	0.4935	11.30	0.0321	0.0710
111	4.05	0.9000	0.2900	76.70	0.001825	0.1219
	6.06	1.9940	0.4338	74.50	0.001848	0.1223
	8.94	4.3170	0.6613	73.10	0.001845	0.1314
	12.07	8.5700	0.9350	76.00	0.001699	0.1338
	14.93	14.9100	1.2870	77.70	0.001492	0.1432

TABLE VI. (*Continued*)

4/0 three-conductor
7/32 in. paper on each conductor
7/32 in. paper over-all
Filled with vegetable hydrocarbon base compound
1/8 in. lead sheath
Frequency 60 cycles

T deg. cent.	*E* 10^3	W_2	I_2 10^{-3}	P.F. per cent.	r'' 10^{10}	c'' 10^{-9}
		CABLE NO. 5-E IN SERVICE 5 YEARS				
20	4.05	0.00705	0.0950	1 830	0.2309	0.0622
	6.06	0.01532	0.1440	1.755	0.2421	0.0627
	8.89	0.0322	0.2112	1.715	0.2450	0.0634
	11.95	0.0625	0.2855	1.832	0.2290	0.0640
	14.80	0.1098	0.3550	2.090	0.1995	0.0639
	17.88	0.2110	0.4300	2.747	0.1517	0.0638
100	4.05	0.3546	0.150	58.30	0.00462	0.0797
	6.06	0.747	0.225	54.80	0.00493	0.0844
	8.89	1.516	0.333	51.25	0.00521	0.0854
	11.95	2.842	0.450	52.80	0.00503	0.0849
	14.80	4.450	0.559	53.75	0.00493	0.0842
	17.88	6.840	0.677	56.50	0.00468	0.0816

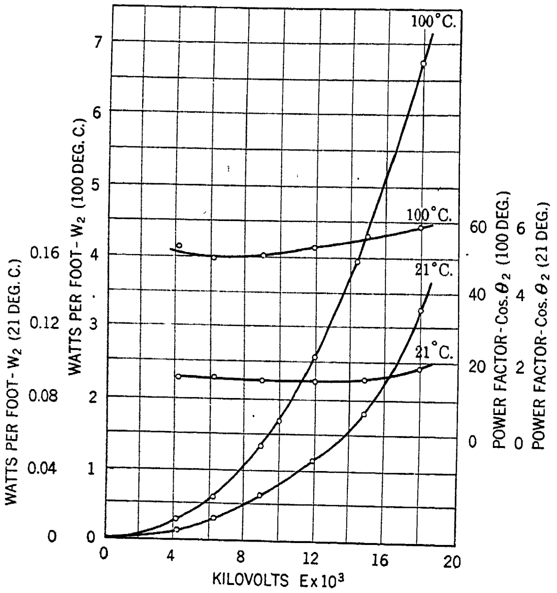

FIG. 17—WATTS PER FOOT AND POWER FACTOR CABLE 5a (PAPER)—NEW CABLE NEVER IN SERVICE

This is an important point and it is safe to say that any new cable with such a high power-factor at 100 deg. cent. will in time cause trouble at "hot spots."

Cable No. 5b. The measurements on this sample, one of those cut from a line near point of failure, are also given in Table VI. The 13,800-volt power-factor in Fig. 18 is 3.7 per cent at 22 deg. cent. and increased to 85 per cent at 100 deg. cent. It was thought that this sample must have been partly broken down and charred to give such an extremely high power factor at 100 deg. cent., although the 22 deg. cent. power-factor is within reasonable limits. But, when it was cut open no burning or scorching of any kind was found. It was well filled with compound and the surface of the paper between the points, 1, 2 and 3 (Fig. 13) showed evidence of trouble just beginning. The compound had coagulated here, a characteristic sign of distress. One other sample checked the above one closely in every detail.

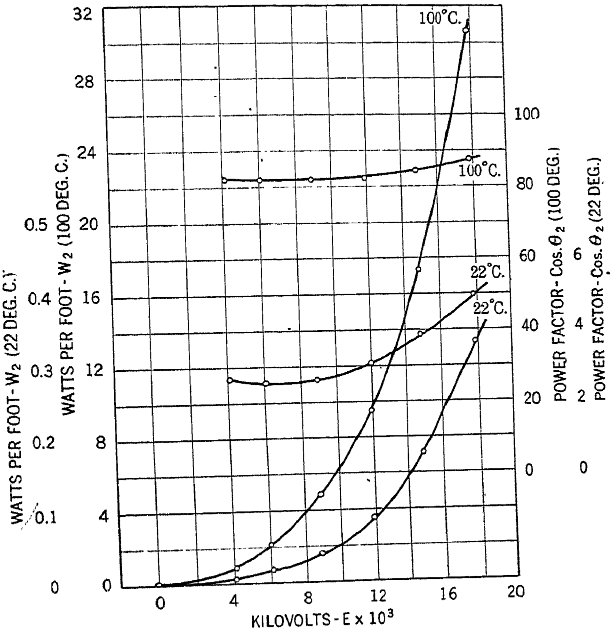

FIG. 18—WATTS PER FOOT AND POWER FACTOR CABLE 5b (PAPER)—IN SERVICE FIVE YEARS

Cable No. 5c. The data and curves in Table VI and Fig. 19 show this sample to be either saturated with moisture or partly broken down. The 13,800-volt power factor at 29 deg. cent. is 10 per cent. No insulation gives power factors this high at low temperature unless conducting impurities are present. The 111 deg. cent. power-factor is 77 per cent, not quite as high as that in Cable No. 5*b*.

When cut open three spots, each about one foot in length were found where the characteristic charring of the central section had eaten almost down to the conductors. In the rest of the length this section was either scorched or the compound had coagulated as previously described.

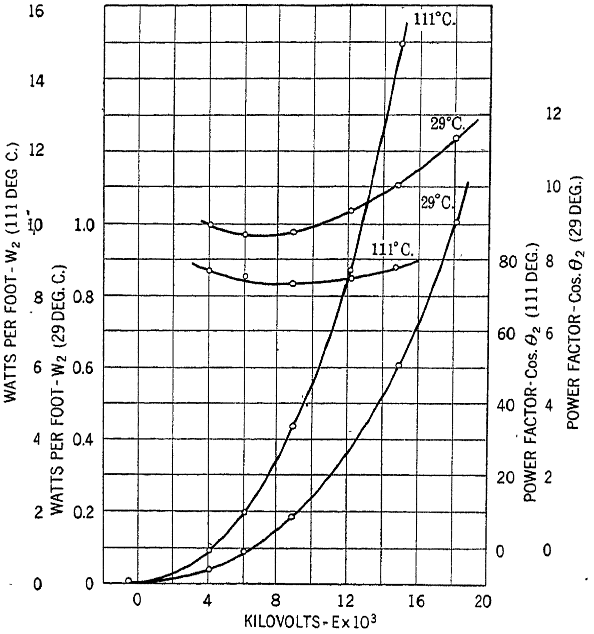

FIG. 19—WATTS PER FOOT AND POWER FACTOR CABLE 5c (PAPER)—IN SERVICE FIVE YEARS

Cable No. 5*d*. The energy loss readings on this length checked those on cable No. 5*c* very closely and are therefore not given in the table. They indicated that the sample was partly broken down, so an endurance run at 15 kv. and 100 deg. cent. was made to get some idea in regard to how a cable actually fails in operation. A log of this run is given in Fig. 20. It is interesting to note that it required two hours and 40 minutes to reach complete breakdown.

Examination showed the same characteristic charring at four places. One of these had charred completely through to the conductors, (the point where break down occurred.) It is probable that the endurance run had little to do with the damage at the other spots. It requires a number of days to cause such complete carbonization in an air tight cross section, as we have often observed in aging tests at 200 deg. cent. on different kinds of materials.

Cable No. 5e. The energy-loss data are given in Table VI. The 13,800-volt power factor is 2 per cent at 20 deg. cent. and 53 per cent at 100 deg. cent. thus comparing with the new sample, (No. 5*a*). This sample (5*e*) had the lowest power factor and losses of any of the old samples tested.

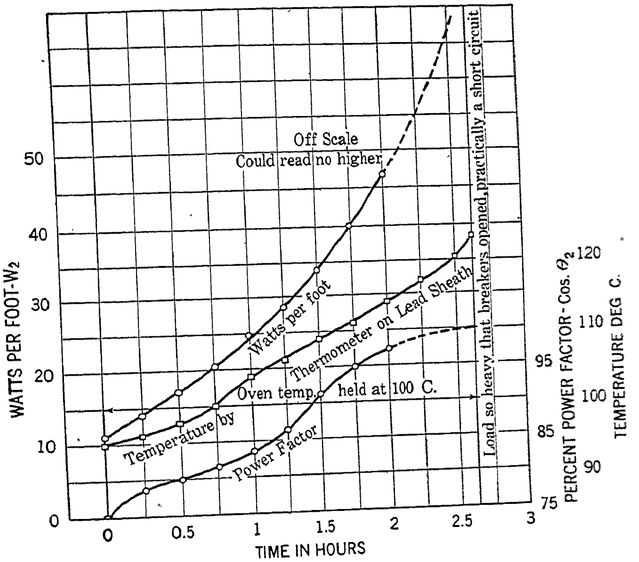

FIG. 20—ENDURANCE RUN, CABLE 5d (PAPER)—IN SERVICE FIVE YEARS

When cut open it was found that the greater part of the length was comparatively "dry," that is, only partly filled with compound. No signs of deterioration were in evidence.

Several of the samples were in this "dry" condition and in none of these were there signs of deterioration, in addition the power factor was always lower than in the well filled samples. This indicates better than anything else, that the compound filler was at fault, and that no moisture was present, for moisture would naturally accumulate in the "dry" sections.

Resistivity Curves.—The low temperature resistivity curves r'' are shown in Fig. 21. The voltage was not carried up to complete saturation in any case. It was necessary to avoid over-stressing and thereby be sure that the samples when examined after test were in the same condition as when cut from the line. Ionization started at 9 kv. in all the samples except the new one (5*a*).

Variation of Capacity with Temperature. A study of the capacity C'' values in Table VI shows that there is considerable increase at high temperatures and that this increase is greater the higher the loss in the sample; in other words, the lower the resistance of the material the higher the capacity. At zero resistance the capacity would be infinity.

Tests on the Compound. Compound was pressed from one of the lengths that had been in service, and a short section of the new length (5*a*). Neither length had been given an electrical test and they remained sealed until ready for the extraction of compound.

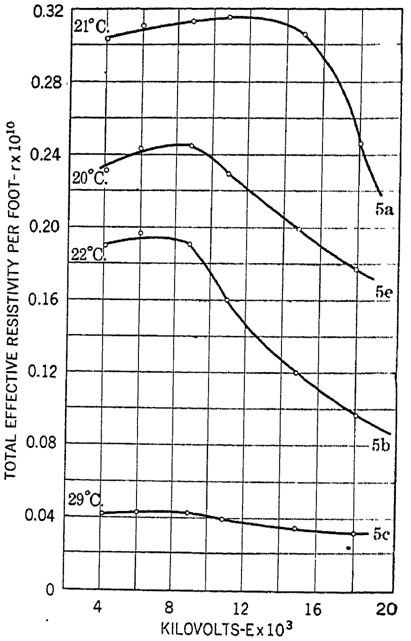

Fig. 21—Total Effective Resistivity per Foot Cables 5a, 5b, 5c, 5e (Paper)

Vacuum treatment gave no evidence of an appreciable amount of moisture in the compound. A chemical analysis showed the presence of free organic acid and this was more marked in the compound which had seen service.

The d-c. resistivity at different temperatures, taken between 5 cm. terminals as described in the first part of the paper, is given in Fig. 22. The resistance of both is very low and that of the old sample shows considerable deterioration, which must be due to chemical change under the conditions of voltage stress and temperature met with in operation.

VARNISHED CAMBRIC AND RUBBER INSULATED CABLE

Numerous tests have been made on different kinds of varnished cambric and rubber insulated cable as well as paper insulated cable other than those already discussed.

Space will allow the characteristics of varnished cambric and rubber insulation to be dealt with but briefly. Later we hope to present results showing the value of energy-loss measurements in determining different grades of rubber.

Representative samples of 2/0 single-conductor cable having varnished cambric and rubber insulation have been selected for comparison with paper cable as follows:

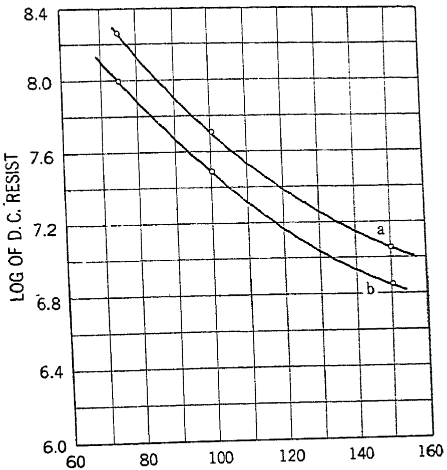

FIG. 22—LOG D-C. RESISTIVITY OF TWO CABLE COMPOUNDS FROM CABLES IN GROUP 5. A—FROM NEW CABLE NEVER IN SERVICE. B—FROM OLD CABLE IN SERVICE 5 YEARS

Cable No. 6. 9/32-in. (7.1 mm.) black varnished cambric.

Cable No. 7. 9/32-in. (7.1 mm.) 33 per cent pure grade para rubber.

Cable No. 8. 9/32-in. (7.1 mm.) 30 per cent good commercial grade rubber compound.

The permittivity and resistivity values of these samples are included in Figs. 24 to 27 which will be discussed under the following headings.

RELATIVE PERMITTIVITY AND GRADING

It would perhaps be well to combine the three standard types of cable insulation in one group and study their relative characteristics more closely.

Variation of Permittivity with Voltage Stress. The voltage gradient curve for the cross section of single-conductor cable considered is given in Fig. 23 at a total impressed voltage, E = 10 kv.

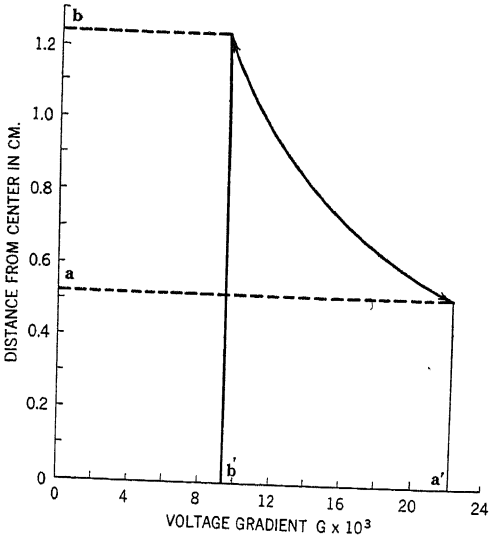

FIG. 23—VOLTAGE GRADIENT AT 10 KV. FOR 2/0 SINGLE-CONDUCTOR CABLE—9/32 IN. THICKNESS OF INSULATION

$$g = \frac{de}{dx} = \text{voltage gradient}$$

$g_{max.}$ = maximum gradient (a, next to conductor)
g_{av} = average gradient.

The relative permittivity values, K_{av}, in Tables II and III show that K_{av} varies slightly with voltage E. It is evident, then, that K varies with g through the thickness of insulation, and K_{av} represents the value of K at a point approximately coinciding with g_{av}.

In Fig. 24 the 50 deg. cent. values of K_{av} for the different materials are plotted against g_{av} instead of E. The section of the curves between the perpendicular lines shows the variation of K with g through the thickness. The largest variation occurs in Cable No. 8 (rubber) and is only 1.6 per cent. The variation in Cable No. 6 (varnished cambric) is 0.7 per cent. The actual value of g_{max} will therefore be slightly less than the theoretical value given in Fig. 23; in other words, the insulation tends to grade itself automatically, but only to a slight extent.

The above grading of K considers a uniform temperature. In operation, the distribution of temperature will cause still further grading, since K increases with temperature in most materials.

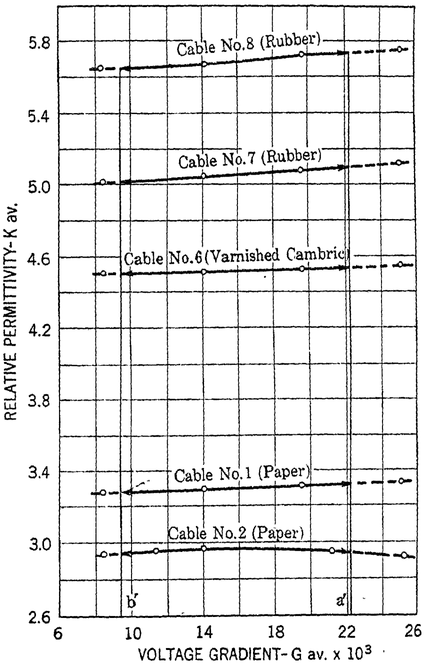

FIG. 24—RELATIVE PERMITTIVITY OF DIFFERENT MATERIALS AT 50 DEG. CENT. AND 10 KV.

Variation with Temperature. Fig. 25 shows the change in K_{av} with temperature for the different materials at $E = 10$ kv. The slope of the varnished cambric curve (Cable No. 6) is very noticeable. The K of good paper insulation (Cable No. 2) does not increase much with temperature but that of medium paper (Cable No. 1) increases appreciably above 75 deg. cent. An idea of the rapid increase of K with temperature in poor paper insulation can be obtained from the capacity c'' values in Table VI.

Assume that the conductor is at a temperature of 50 deg. cent. and the lead sheath 40 deg. cent. Referring to Fig. 25, the largest variation of K, 4 per cent, occurs in varnished cambric (Cable No. 6).

Combining the automatic grading due to voltage stress and that due to temperature the total effect in Cable No. 6 will cause approximately a 3 per cent decrease in g_{max}. In other words, automatic grading is hardly appreciable and the theoretical gradients in Fig. 23 are approximately correct. The 3 per cent decrease in g_{max} discussed above is probably entirely neutralized by the distortion at the surface of stranded conductors tending to raise g_{max} slightly above the theoretical value.

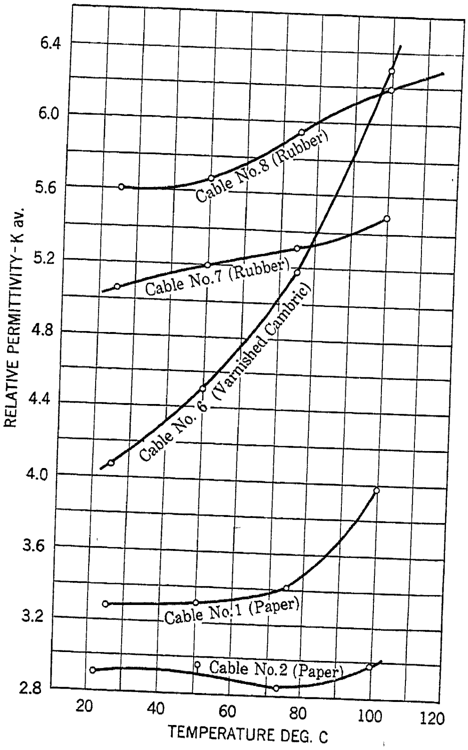

FIG. 25—RELATIVE PERMITTIVITY OF DIFFERENT MATERIALS AT 10 KV.

EFFECTIVE RESISTIVITY

Variation with Temperature. The 10-kv. curves of effective specific resistivity versus temperature for the different cable samples are given in Fig. 26.

A comparison of the slope of these curves shows that paper insulation has the poorest temperature characteristics and must be of the highest grade to bring the 100 deg. cent. resistivity well up in value (Cable No. 2). The resistivity of Cable No. 1, a medium sample, drops very rapidly with temperature and it can be imagined how rapidly the resistivity of a *very* poor cable, such as No. 5b, will drop. It is this effect at "hot spots" that is of such great importance in cable work.

Varnished cambric follows after paper and it is evident that the resistivity characteristics are not particularly good. Varnished cambric has this advantage, though; it is very stable and uniform. Its good mechanical qualities are another point in its favor.

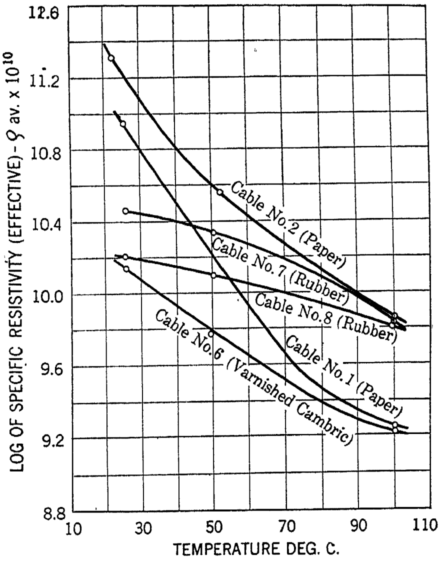

Fig. 26—Log of Specific Resistivity (Effective) at 10 Kv.

Rubber has the best temperature characteristics. It is to be regretted that these cannot be better utilized, but the physical changes that take place in rubber prohibit its use at high temperatures.

Variation with Voltage Stress. The variation of ρ through the thickness should be dealt with in the same way as the variation of K.

In Fig. 27 the 50 deg. cent. values of ρ_{av} for the different samples are plotted against g_{av} instead of E. The section of the curves between the perpendicular lines shows the variation of ρ with g through the thickness. The greatest variation, 26 per cent, occurs in Cable No. 8, and the least, 8 per cent, in Cable No. 6.

Referring again to Fig. 26 and assuming temperatures of 50

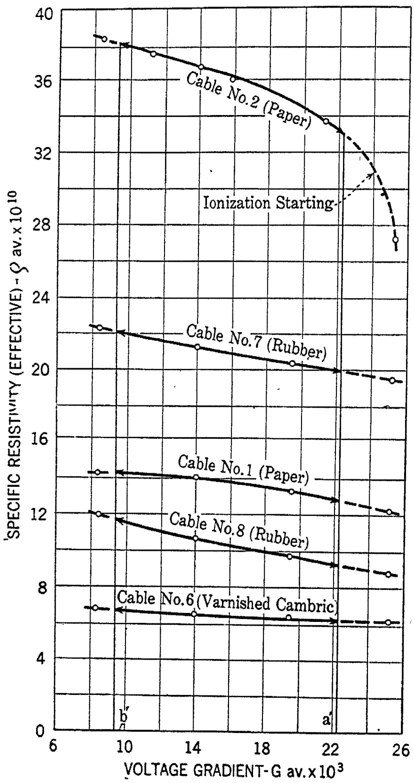

FIG. 27—SPECIFIC RESISTIVITY (EFFECTIVE) AT 50 DEG. CENT. AND 10 KV.

to 40 deg. cent. for the conductor and lead sheath respectively, the greatest variation of ρ through the thickness, 50 per cent occurs in Cable No. 1, and the least, 5 per cent in Cable No. 7.

When the above two effects are combined it is evident that the value of ρ is much less, and consequently the watts per cm.^{3} next to the conductor is much more, than the values further out in the thickness.

INTERNAL IONIZATION

As mentioned in the first part of the paper, coils have been overstressed for long periods of time and the effects of internal and external ionization noted. Examination, after a continuous run of a year or more at two times normal operating voltage, shows deposits on the walls of the gas spaces; but it is surprising how little actual damage is done. The materials used, however, were known to be quite stable chemically.

Overstressing to a point where internal ionization begins is, of course, to be avoided in practise; it is obvious however, that materials should be used that are chemically the least affected by it, for ionization may sometimes be met with; for instance, when high-potential test is applied to lengths of cable in low-temperature conduits.

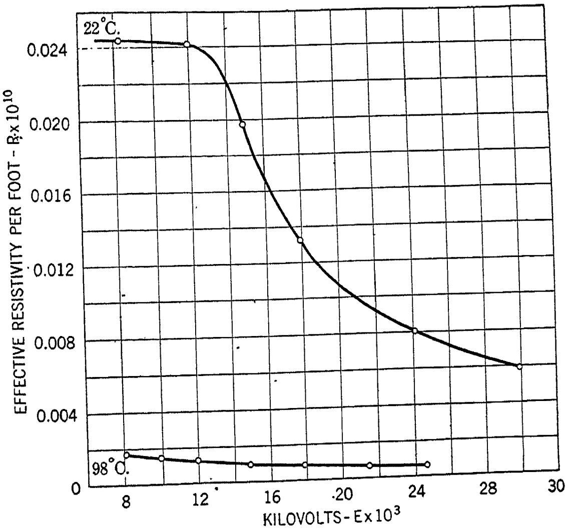

FIG. 28—INTERNAL IONIZATION IN A HIGH-VOLTAGE ARMATURE COIL

Fleming and Johnson* have dealt to some length with the effects of internal ionization in overstressed coils insulated with chemically unstable materials. They found these effects very harmful in some cases.

The resistivity curves of an armature coil whose thickness of main insulation, (0.22-in. or 5.6 mm.) was known to contain small

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*Chemical Action in the Windings of High-Voltgage Machines", by Fleming and Johnson, *Journal* I. E. E., Vol. 47, p. 530.

gas spaces are given in Fig. 28. The ionization at 22 deg. cent. starts at 12 kv. and is quite pronounced.

Ionization in Unfilled Cables. Special lengths of cable (Nos 9a and 9b) were made up for the purpuse of studying internal ionization.

The specifications are as follows:

2/0 single conductor
9/32 in. (7.1 mm.) varnished paper tape.
3/32 in. (4.7 mm.) lead sheath.

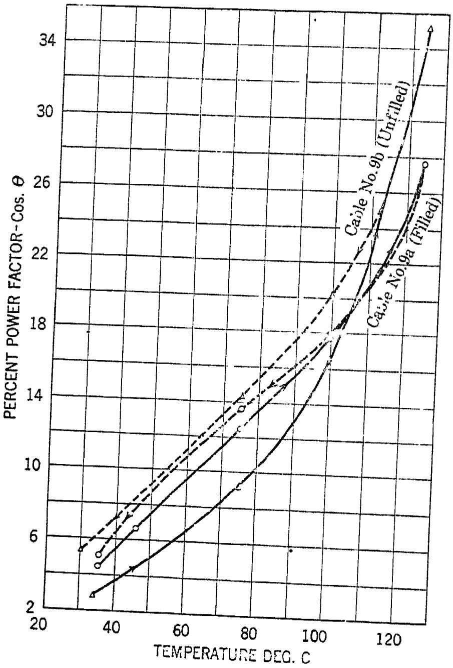

Fig. 29—Per Cent Power Factor Cables 9a and 9b (Varnished Paper) at 10 Kv.

The tape in No. 9a was first run through compound and then applied half lap. Compound was copiously used and a thoroughly filled insulation resulted. The tapings in No. 9b were applied dry without filling of any kind, thus leaving small air spaces between layers at the overlapping edges of the tape.

Three samples of Cable No. 9a were tested and checked exactly. None of the four No. 9b samples checked and one broke down at 24 kv. while taking readings. The No. 9b curves given are for the average sample.

Fig. 29 shows the 10-kv. power factor heat cycles. That of No. 9a is very stable but there is a big difference between the heating and cooling curves of No. 9b. The permittivity cycles

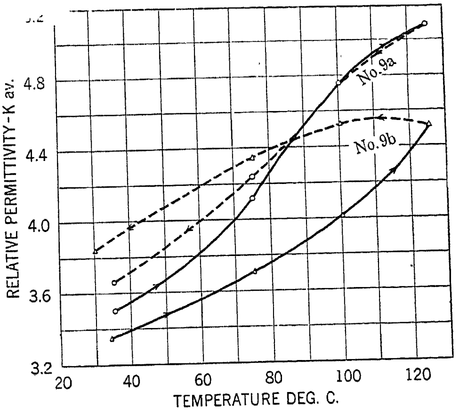

FIG. 30—RELATIVE PERMITTIVITY CABLES 9a AND 9b (VARNISHED PAPER) AT 10 KV.

in Fig. 30 show much the same characteristics. Some very appreciable change must have taken place in No. 9b during the cycle.

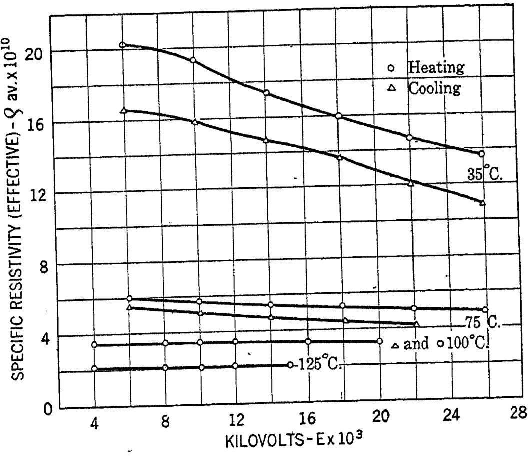

FIG. 31—SPECIFIC RESISTIVITY (EFFECTIVE) CABLE 9a (VARNISHED PAPER, FILLED)

The resistivity curves of No. 9a are given in Fig. 31. The inherent resistance decreases noticeably. There are only traces of ionization in the 35 deg. cent. curves.

The corresponding curves for No. 9b in Fig. 32 show pronounced ionization in all of the heating curves, beginning at 6 kv. It has almost entirely disappeared in the cooling curves. The permittivity values in Fig. 33 and 34 follow the same characteristics in an even more pronounced way.

From the general trend of the test it was evident that some sort of partial breakdown occurred in Cable No. 9b at 125 deg. cent., so it was cut open for examination. The wrappings especially those nearest the conductor, were perforated with small "pin pricks" in a distinct way. The majority of them followed a straight line lengthwise down the middle of the taping where the adjacent overlapping edge had been in contact.

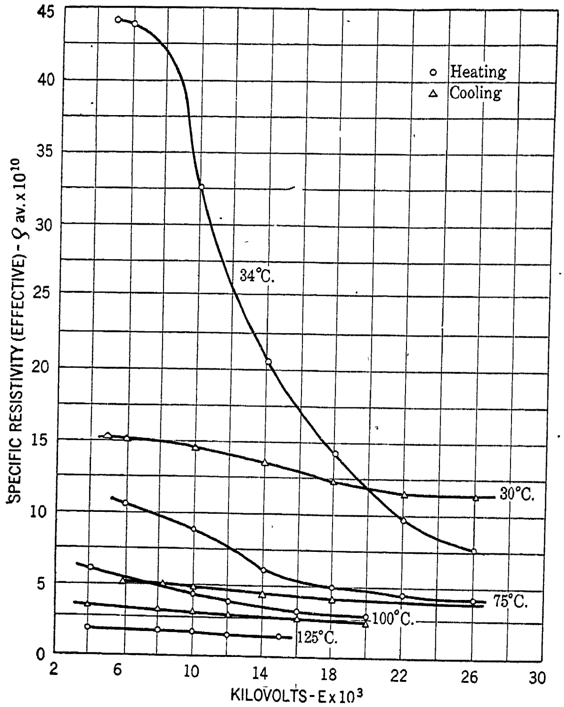

FIG. 32—SPECIFIC RESISTIVITY (EFFECTIVE) CABLE 9b (VARNISHED PAPER, UNFILLED)

The air space along this line was ionized and since ionized gas is a good conductor it acted like a sharp metallic edge, that is, concentrated the stress to such a degree that partial failure occurred. Osborne* first noted this characteristic perforation.

The walls of the perforations must have been conducting and this tended to short-circuit the air spaces, for Figs. 32 and 34 show that they were eliminated in some such way in the cooling curves.

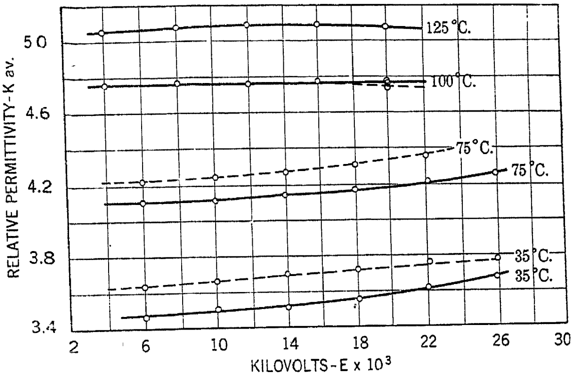

FIG. 33—RELATIVE PERMITTIVITY CABLE 9a (VARNISHED PAPER, FILLED)

The permittivity characteristics in Fig. 34 are of particular interest. They illustrate step by step what happened. In the 34 deg. cent. heating curve the effective permittivity increases

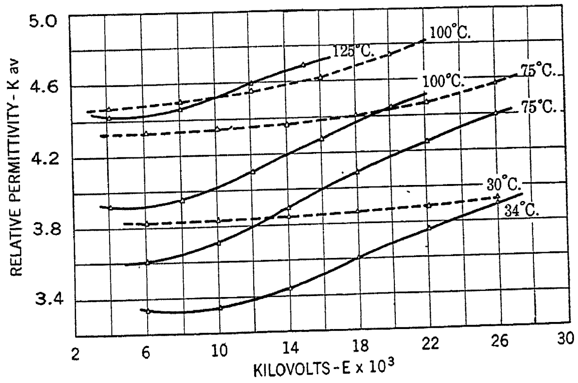

FIG 34—RELATIVE PERMITTIVITY CABLE 9b (VARNISHED PAPER, UNFILLED)

with voltage stress as the low permittivity air spaces are gradually ionized, in other words, eliminated from the effective permittivity. In the 30 deg. cent. cooling curve the air spaces have been short-circuited by perforation at 125 deg. cent. and consequently the curve does not depend upon voltage stress but is almost flat, meeting the 34 deg. cent. heating curve at 27 kv.

**Potential Stresses of Dielectrics* by H. S. Osborne, TRANS. A. I. E. E., Vol. 29, p. 1553.

It would be interesting to determine at various pressures and spacings the stress necessary to ionize the gases filling spaces in different kinds of insulation, and see if it is not possible to predetermine by calculation the starting point of internal ionization. This would furnish very useful information to designing engineers regarding the proper voltage rating of their apparatus. The investigation is still in its first stage, however, and eventually it is hoped to reach this point.

Acknowledgment. In closing we wish to express our appreciation and thanks to Dr. C. P. Steinmetz and J. L. R. Hayden for their interest and able advice; the operating companies who placed at our disposal the opportunity to study cable from their lines; Mr. J. J. Matson for his invaluable assistance in preparing this paper; and Messrs. H. R. Summerhayes and E. W. Hawley whose cooperation was of great help.

APPENDIX A

SHOWING METHOD USED IN CALCULATING SINGLE, CONDUCTOR CABLE DATA

The following nomenclature and formulas are used throughout the paper:

Let E = impressed voltage
I = total amperes per foot length of cable
W = watts loss (dielectric) per foot length of cable
$\cos\theta$ = dielectric power factor
C = capacity in farads per foot.
$K_{av.}$ = relative permittivity (average through thickness)
K_a = capacity per cm³. of air = 0.08842×10^{-12} farads
R = total effective a-c. insulation resistance per foot.
$\rho_{av.}$ = effective resistance per cm³. (average through thickness)

$$g = \frac{d\,e}{d\,x} = \text{voltage gradient.}$$

$g_{av.}$ = average voltage gradient through thickness
l = length in cm. = 1 ft. = 30.48 cm.

b = radius in cm. inner side lead sheath
a = " " " of conductor.

The dielectric circuit is considered as a parallel circuit, that is, the resistance and capacity are considered as being in parallel.

$$\cos \theta = \frac{W}{E\,I} \tag{1}$$

$$C = \frac{I \sin \theta}{2\,\pi f E} \tag{2}$$

For a single-conductor cable in which the thickness of insulation consists of one kind only:

$$\frac{1}{d\,c} = d\,s = \frac{1}{K_a\,K_{av.}} \cdot \frac{d\,x}{2\,\pi\,l\,x}$$

therefore:

$$C = \frac{k_a\,k_{av.}\,2\,\pi\,l}{\log\ b/a}$$

$$\frac{I \sin \theta}{2\,\pi f E} = \frac{k_a\,k_{av.}\,2\,\pi\,l}{\log_\epsilon\,b/a}$$

therefore:

$$K_{av.} = \frac{I \sin \theta \cdot \log_\epsilon b/a}{(2\,\pi)^2\,l f E K_a} \tag{3}$$

$$d\,R = \rho_{av.}\,\frac{d\,x}{2\,\pi\,l\,x}$$

$$R = \frac{\rho_{av.}}{2\,\pi\,l}\,\log_\epsilon b/a = \frac{E^2}{W}$$

therefore:

$$\rho_{av.} = \frac{2\,\pi\,l\,E^2}{W \log_\epsilon b/a} \tag{4}$$

For the voltage gradient:

$$g = \frac{d\,e}{d\,x} = \frac{C\,E}{K_a K_{av.}\,2\,\pi\,l\,x}$$

therefore:

$$g = \frac{E}{x \log_\epsilon b/a} \tag{5}$$

A = area of gradient curve between a and b

$$A = \int_a^b g\, d\,x = \frac{E}{\log_\epsilon b/a} \int_a^b \frac{d\,x}{x}$$

threrfore: Area, $A = E$ (6)

$$g_{av.} = \frac{E}{b-a} \tag{7}$$

APPENDIX B

SHOWING METHOD USED IN CALCULATING THREE-CONDUCTOR CABLE DATA

Referring to Fig. 11 it is assumed that,

$r_1 = r_2 = r_3$, $r_4 = r_5 = r_6$, $c_1 = c_2 = c_3$ and $c_4 = c_5 = c_6$

For Single-phase Readings:

Let E = variable test voltage
W_1 = loss due to r_1 per foot
W_2 = loss due to r_1 r_4 and r_6 per foot
W_3 = loss due to r_4 per foot
I_1 = current due to Y_1
I_2 = current due to Y_1 Y_4 and Y_6
I_3 = current due to Y_4
$c'' = 2\,c_4 + c_1$ = capacity in farads per foot.
$r'' = \frac{r_1\, r_4}{2\, r_1 + r_4}$ = effective resistance in ohms per foot.

By applying high voltage to all three conductors connected together and grounding lead sheath through the instrument, readings of 3 W_1 and 3 I_1 can be taken at voltages, E, and curves plotted. In the next set of readings high voltage is applied to conductor A and conductors B and C are connected to the lead sheath. These readings correspond to W_2 and I_2.

Then

$\cos\theta_1 = \frac{W_1}{E\, I_1}$ = power factor from conductor to lead sheath.

$$\dot{I}_1 = I_1 \cos\theta - j\,(I_1 \sin\theta)$$

$$r_1 = \frac{E^2}{W_1} \text{ and } c_1 = \frac{I_1 \sin \theta_1}{2\,\pi f E}$$

$$\cos \theta_2 = \frac{W_2}{E\, I_2} = \text{power factor due to } Y_1\ Y_4 \text{ and } Y_6$$

$$\underset{\cdot}{I_3} \doteq \frac{\underset{\cdot}{I_2} - \underset{\cdot}{I_1}}{2} \text{ and } W_3 = \frac{W_2 - W_1}{2}$$

$$I_3 = 1/2 \sqrt{(I_2 \cos \theta_2 - I_1 \cos \theta_1)^2 + (I_2 \sin \theta_2 - I_1 \sin \theta_1)^2}$$

$$\cos \theta_3 = \frac{W_3}{E\, I_3} = \text{power factor between conductors.}$$

$$r_4 = \frac{E^2}{W_3} \text{ and } c_4 = \frac{I_3 \sin \theta_3}{2\,\pi f E}$$

Three-Phase Calculations

The equivalent dielectric circuit in Fig. 11 gives two separate circuits, one Δ (between conductors), and the other Y (between conductors and lead sheath).

E_ϕ = line voltage (variable).

$e_\phi = \dfrac{E_\phi}{\sqrt{3}}$ = voltage to lead sheath.

$W_{3\phi}$ = total three-phase loss (dielectric) per foot

I_ϕ = current per conductor per foot

$\cos \theta_\phi$ = three-phase power factor

$W_{3\phi} = 3\ (W_3' + W_1')$ where;

W_3' = W_3 read from curves at voltage E_ϕ and,

W_1' = W_1 read from curves at voltage e_ϕ

$\underset{\cdot}{I_\phi} = \underset{\cdot}{I_3'} + \underset{\cdot}{I_1'}$ where;

$\underset{\cdot}{I_3'} = \underset{\cdot}{I_3} \sqrt{3}$ at voltage E_ϕ and,

$\underset{\cdot}{I_1'} = I_1$ at voltage e_ϕ

$$\cos \theta_\phi = \frac{W_{3\phi}}{\sqrt{3}\, E_\phi\, I_\phi}$$

also;

$$c_\phi = \frac{I_\phi \sin\theta_\phi}{2\,\pi f E_\phi} = \text{capacity, farads per conductor per foot.}$$

DISCUSSION ON "PROBLEMS OF OPERATION AND MAINTENANCE OF UNDERGROUND CABLES" (HARPER). "THE FUNDAMENTALS OF SUCCESSFUL HIGH-TENSION CABLE JOINTS" (ROPER). "THE INFLUENCE OF DIELECTRIC LOSSES ON THE RATING OF HIGH-TENSION UNDERGROUND CABLES" (BANG AND LOUIS) AND "INSULATION CHARACTERISTICS OF HIGH-VOLTAGE CABLES" (CLARK AND SHANKLIN), NEW YORK, JUNE 27, 1917.

W. I. Middleton: There should be the heartiest cooperation between the cable users and the manufacturers. A great deal of unnecessary trouble could be avoided if the manufacturer had a more definite knowledge of the conditions under which the cables are to operate.

The failure of the joints due to the compound used is very interesting. Many compounds, as the author says, are good insulators in themselves, but are not good when in combination with other compounds equally as good.

There must be more than a mechanical mixture otherwise there is liable to be a separating out of the different ingredients and stresses are set up within the joint or within the cable itself if the dielectric constants of the cable compounds are vastly different from the insulating material itself.

We have emphasized for a number of years the necessity of having the dielectric constant of the cable compound similar to that of the insulating material with which it is used.

Mr. Roper's paper has covered all of the important features of cable splicing, but I am convinced that the personal element enters into the making of a good joint probably more than any other one thing. Some of the most successful cable splicers are making what might be called a very unscientific joint without any of the new ideas that have been advocated recently, depending almost entirely upon the quality of the material used and the quality of the workmanship, observing, of course, all of the fundamentals mentioned by Mr. Roper. Joints of this kind on 25,000-volt cables are being made by two men in from four to five hours.

Undoubtedly the cable joint of the future will be a mechanical joint, eliminating to a great extent the personal element that now enters into the hand-wound type. There is no joint on the market today that can compete with the hand-wound, hand-filled joint when well made. As Mr. Roper says, some of the more expensive joints may be considerably stronger, but their high cost and the length of time taken to make them is a drawback to their being universally adopted.

As to the cheaper methods of making joints, some of them are open to severe criticism from the fact that they have given trouble and some have not been tried out sufficiently long to have gained the confidence of the splicers.

There is one other feature which has not been taken into consideration in regard to joints, and that is the electrostatic capacity of the joint itself. Those who have experimented with joint-making in connection with cables have found that a cable will stand a very high voltage test and the joint itself will stand a very high voltage test. In the combination of the two, the joint will break down. I think there is a chance for investigation along the line of the dielectric constant or the electrostatic capacity of the joint itself—there is more stress on the joint in combination with the cable than there would be if the joint were tested alone.

Referring to the paper by Messrs. Clark and Shanklin, when the insulation of a cable is subjected to an electrical pressure the material is electrostatically strained by molecular displacement, which may end in rupture if the pressure prove sufficient. If the pressure be removed before breakdown occurs, it will in most cases gradually recover its normal unstrained condition. The ability of a cable dielectric to withstand this strain or to rapidly recover from such strain is important when used on an a-c. circuit. Whatever may be the real mechanism of this phenomenon, the result is that with rapidly alternating pressures molecular vibrations are set up in the dielectric, heating it and causing loss of energy similar in manner to that which takes place in a rapid magnetization and demagnetization of a piece of iron. Therefore, the less action in the dielectric the less heat. Associated with this loss due to molecular vibration is the resistance of the dielectric. The losses due to hysteresis and conduction in the dielectric must be supplied by the charging current and pressure, and these losses increase the power factor of the cable to the necessary extent by bringing the pressure and charging current more nearly into phase.

The specific resistance and permittivity of the different insulating materials used in making high-tension cables are vastly different, some materials have a very high specific resistance and a high capacity, while others have a low resistance and low capacity. The degree and manner in which these different materials are affected by temperature as regards resistance, capacity and electric stress is also vastly different. A careful study of all of these characteristics should be made before they are used in the construction of high-tension cables.

Paper insulation has a smaller permittivity constant than cambric or rubber and a lower resistance. The temperature coefficient of the permittivity is small, while that of the resistance is very large. This is shown in Tables II and III, and cables having this large temperature coefficient must of necessity have a larger power factor at higher temperatures.

When we take into consideration the high temperature to which the experiment was carried, I think the power factor results are about what we might expect. The results obtained with the different cable compounds are very interesting, and worthy of further investigation; so much depends upon the mechanical makeup of the cable that a great many tests should be made before we arrive at any decision.

The results we have obtained in the factory do not agree with those given by the authors in some instances. For instance, the change in specific gravity with temperature of rosin-oil compound shows about 4.4 per cent. Mineral-oil compound shows 4.3 per cent. The rosin-oil compound shows no loss of dielectric strength with increase of temperatures over the working range of the cables, that is up to 150 deg. Beyond that point there is a falling off:

Temperature		Breakdown
83 deg. fahr.	28 deg. cent.	30,500 volts.
100 deg. fahr.	38 deg. cent.	30,750 volts.
125 deg. fahr.	52 deg. cent.	30,000 volts.
150 deg. fahr	66 deg. cent.	29,500 volts.
175 deg. fahr.	80 deg. cent.	26,000 volts.

A common base for mineral-oil compounds shows a marked decrease of dielectric strength with increase of temperature:

Temperature		Breakdown
80 deg. fahr.	27 deg. cent.	24,500 volts.
100 deg. fahr.	38 deg. cent.	15,000 volts.
125 deg. fahr.	52 deg. cent.	15,000 volts.
150 deg. fahr.	66 deg. cent.	11,750 volts.
175 deg. fahr.	80 deg. cent.	9,100 volts.

In our experience in experimenting with the rosin-oil and the mineral-oil base compounds, we have found the rosin-oil compound to be the best compound. Samples of paper cables impregnated with rosin-oil compounds show a marked increase in dielectric strength with increase of temperature over a considerable part of the working range of paper cables.

Temperature		Breakdown
32 deg. fahr.	0 deg. cent.	61,200 volts.
42 deg. fahr.	6 deg. cent.	64,300 volts.
60 deg. fahr.	16 deg. cent.	63,025 volts.
78 deg. fahr.	26 deg. cent.	67,130 volts.
98 deg. fahr.	37 deg. cent.	64,400 volts.
120 deg. fahr.	49 deg. cent.	72,600 volts.
140 deg. fahr.	60 deg. cent.	69,460 volts.

Paper cables impregnated with rosin-oil compound show a marked increase of alternating-current capacity with increase of temperature which would tend to relieve the dielectric strains in a cable, as the cable temperature increased; which is quite important.

Temperature		Capacity
33 deg. fahr.	4 deg. cent.	0.0410 microfarad
41 deg. fahr.	5 deg. cent.	0.0420 microfarad
50 deg. fahr.	10 deg. cent.	0.0460 microfarad
60 deg. fahr.	15 deg. cent.	0.0463 microfarad
70 deg. fahr.	21 deg. cent.	0.0500 microfarad
80 deg. fahr.	27 deg. cent.	0.0525 microfarad
90 deg. fahr.	32 deg. cent.	0.0506 microfarad
100 deg. fahr.	38 deg. cent.	0.0595 microfarad
110 deg. fahr.	43.5 deg. cent.	0.0700 microfarad
120 deg. fahr.	49 deg. cent.	0.100 microfarad
130 deg. fahr.	54 deg. cent.	0.053 microfarad

C. L. Dawes: There are several points in the paper of Messrs. Bang and Louis which I believe should receive comment. At the present time the price of copper, which is twice as great as it is in normal times, and the prevailing high prices of all labor and materials have increased the cost of cables enormously. Therefore, to conserve the national supply of copper, lead, etc., and to keep down excessive investments during the present period of high prices it is desirable to force cables as well as other apparatus to the limits of their carrying capacity. The data given in this paper would, I believe, lead engineers to rate their cables 50 per cent and less of their safe heating limit.

Instead of constructing a duct representative of actual operating conditions the authors have surrounded their duct lines with heat insulating materials in a manner analogous to that employed in lagging electric furnaces. For instance the line is surrounded on three sides with 5 in. (12.7 cm.) of dry sand (apparently) and on the bottom the sand is 3¾ in. (9.5 cm.) thick. Dry sand has a thermal resistivity about ten times that of concrete or masonry and from three to four times that of moist soil. In practise perfectly dry sand or ashes are seldom if ever encountered. Outside this sand the duct line is still further heat insulated with wooden boards which have a heat resistivity practically equal to that of the dry sand. Furthermore most of the heat generated within the duct line must pass out through these boards into the air. It is obvious that the heat dissipating ability of these boards is but a small fraction of that of a similar surface against moist earth. Therefore it is not surprising that with a total difference of temperature between the copper and the room of 69 deg. cent. that only 3 deg. cent. of this difference existed between the copper and the lead and only 4 deg. cent. between the lead and the duct.

There is another phase of practical operating conditions which the authors have entirely overlooked. In most central stations, the peak load exists only for perhaps a half hour, and owing to the heat storing capacity of the cable and its surroundings it is almost always possible to force a cable considerably in excess of its normal rating for such short periods. Some manufacturers rate a 0000 cable similar to that used by the authors, at 239 amperes under ordinary conditions of operation.

In their experiments the authors filled 7 out of 9 ducts with heavily loaded cables or they would have been heavily loaded if carried to their normal ratings. In practise it is usually possible to intersperse such cables with cables having a much lower loss as for example arc cables, primary and secondary mains, etc., keeping the cables having a high loss in the outside ducts.

In my opinion the measurement of potential is open to some question. The potential transformers in this test were operating near or in excess of their current rating and at 58 per cent of their voltage rating. Therefore, a change of loss, permittivity or power factor in the cable would mean a very appreciable change in the high-tension voltage, and also some slight change in the iron losses. No correction for these factors is given by the authors. The method of supplying the two losses from separate sources, however I believe is rational.

The making of cable joints has always caused operating companies considerable trouble. I believe that some of the difficulty in obtaining good cable joints is due to the fact that the ordinary cable splicer is a practical man and perhaps a very good workman but does not appreciate the many seemingly negligible factors which have a very important bearing upon the dielectric properties of insulating materials. Among these factors is the exclusion of air and moisture mentioned in the paper. The elimination of all sharp points and edges in the metal of the joint cannot be over-emphasized.

Philip Torchio: I want to discuss especially the papers of Messrs. Bang and Louis, and Clark and Shanklin. About five years ago Mr. Rayner presented a paper at the Institution of Electrical Engineers of England and covered a good deal of the field covered by the two papers here presented. Numerous other investigations have also been made. These analytical papers are very valuable, but we must not fall into the error of generalizing too much. In this respect I call attention to points like the following: In the paper by Messrs. Bang and Louis, near the end, there is a statement comparing specimens *A*, *B*, *C*, with *D*, and the conclusion is—"Or a rating more than three times greater for specimen *D* than for *A*". While scientifically correct, that statement may mislead people if they are not in a position to closely analyze it. For instance, it would be useless to make a comparison with a 0000 cable *A*, which carries 38 amperes, while another cable, of the same size, would carry 127 amperes. We know that if a three-phase 0000 cable can not carry more than 38 amperes, that there is something the matter, not with the cable, but with something else, and what we have to do is to remedy the conditions outside. Scientifically, the statement is perfectly correct, but is subject to misconception.

The same authors also refer to the Institute Rules determining the copper temperature of cables compared with insulating fibrous materials used for other purposes, like armature coils, and hence a conclusion and comparison in the curve is made that, if we operate a cable at 105 deg., as allowed in armatures for such type of insulation, we can carry power to practically unlimited amount.

Three years ago when I was a member of the Committee on Standards and we were working out these problems, I put this question to a gentleman that I think all of us recognize as perhaps one of the highest authorities in cable making, who was then Chairman of the Sub-Committee on Cable Ratings, Mr. Fisher. "We have just adopted 105 deg. for copper temperatures for coils, and now we go into the next room and we limit it to 65 or 70 deg. for cables. Why should there be such a difference?"

Well, Mr. Fisher said what Messrs. Clark and Shanklin have said, and what any practical man would say, he mentioned that these cables are impregnated, but are in a green condition, not entirely cured. They are bent, coiled up on reels, and roughly handled in installations. The cables also are installed under different levels and ground conditions, and inequalities of heat dissipating properties of soil, etc. Now, all of these conditions may cause large distortions or disturbed conditions of the structure of the cable insulation. I have seen drawn out cables that were originally round, but when they came out were triangular in shape; we never know the insulation value under such conditions of use. It is useless to talk about an increase of dielectric stresses and dielectric loss, etc., when we have to meet these practical conditions of operation.

We have to recognize these conditions; and must not be led astray, because if we substitute some other insulating oil in the high-tension cables, the old troubles are in a great measure still there.

In the same way I would call attention to the opening sentence in the paper of Clark and Shanklin, where they say: "The experience of operating companies here in America over a period of years shows that practically all failures on high-voltage underground-cable systems can be attributed to heating." It may be that it just happens that my personal experience on high-tension cable is just contrary to that. Our cable troubles due to heating are exceptions. Just recently we had two burn-outs caused by localized heating due to external sources like a boiler installed, under a sidewalk alongside a trunk line, or escaping steam from steam mains.

In our 16,000-volt cables we have had practically no failures. In the case of our 24,000-volt cables, which were a new type we compelled the manufacturers to make, we had difficulty at the factory in making the cable withstand the voltage test, and twenty lengths were made, and discarded. Some manufacturing changes were made which gave better results but the principal difficulty due to a special form of wrapping was not entirely eliminated except at the last. The first portion of cable lengths gave us trouble, as seven or eight failures occurred; with the latter portion of cable lengths there were no failures.

Therefore, I do not consider it exactly a fair statement to say that cable failures are due to heating. I think there are also mechanical reasons and structural reasons which have to be looked up.

The measurements of dielectric losses are very interesting. To my knowledge the first reference of the importance of dielectric losses in cable specifications and the first measurements of these losses on cable installations were made sixteen years ago by me and reported at the Convention of the Association of Edison Illuminating Companies in 1902. The tests were made on a 10,935-foot cable feeder paper insulated and a 24,756-foot cable feeder, rubber insulated. As to the tests applied to cables manufactured in or before 1901, it may be of interest to have the results from those cables compared with the results obtained by Messrs. Clark and Shanklin for modern cables. The method of measurement was by means of a Rowland dynamometer. I quote from the report as follows:

"In explanation of the tests it may be briefly stated that the triplex cable was considered as constituting six independent condensers, of which three are made up of each conductor against a lead sheath, and three of each conductor against the other two conductors. The capacity current and the watt losses in dielectric hysteresis of one of these condensers was first determined for a given frequency, voltage and temperature, and from this data the power factor of the capacity current was obtained. For the other condensers the capacity current was measured under the same conditions of frequency, voltage and temperature, and the dielectric losses are calculated, assuming for the capacity current the same power factor as obtained before. The total dielectric losses for the whole cable were calculated by adding the working voltages of the different condensers under normal operating conditions. The results were as follows:

Dielectric Losses in Triplex Cables Operating at 600 Volts—25 Cycles.

	Paper cable	Rubber cable (with a section of paper)
Length, feet	10,936	
	Rubber	24,736
	Paper	1,774
	Total	26,510
Copper, cir. mils	250,000	250,000
Insulation	10–32 in.	10–32 in.
Temperature (approx.)	80 deg. fahr.	80 deg. fahr.
Current to ground, amperes	0.59	2.7
Cycles	25	25
Volts impressed	6,300	6,360
Watts measured	175	2,424
Apparent watts	3,720	17,200
Power factor	0.0471	0.141
Charging current, working conditions, amperes	0.047	2.16
Charging voltage	6,400	6,400
Voltage between legs	6,400	6,400
Voltage to earth	3,700	3,700
Current to earth	0.341	1.56
Current to other legs	0.129	0.60
True watts to earth	59.3	813.8
True watts to other legs	44.9	625.2
Total watts lost from 1 leg	104.2	1,439
Total watts lost from 3 legs	312.6	4,317
Total watts lost per foot	0.02859	
Loss in 1774 feet paper cable		51
Loss in 24,756 feet rubber cable		4,266
Total watts lost per foot, at a temp. of 80 deg. fahr.	0.02859	0.1723

The above results can be used as a basis for investigating what would be the losses under different conditions of frequency, voltage or temperature, taking into consideration the fact that the dielectric losses are approximately proportional to the frequency, to the square of the voltage and to a certain function of the temperature, not yet determined. The temperature, however, increases considerably the dielectric losses, and from results obtained from tests on insulating materials carried out by Mr. C. E. Skinner of the Westinghouse Electric & Manufacturing Company it seems probable that the breakdown of insulation is usually started at one point by the development of heat at a higher rate than can be dissipated, the increase of temperature causing the dielectric losses to grow larger, thereby making the conditions worse and worse, until the insulation is charred and finally breakdown results.

Another factor affecting the dielectric losses is the shape of the electromotive-force curve, the sine curve giving the minimum dielectric losses, which is explained by the presence of higher-frequency harmonics in the curve differing from the sine shape.

The data here presented are too incomplete to attempt to arrive at absolute conclusions, but it may assist in gaining an idea of the great importance of the dielectric losses in cables:

(a) In applying high-voltage tests and the duration of the same.

(b) In specifying insulating resistance of cables as the heating due to dielectric losses alone, affect greatly the insulation resistance of the cable.

(c) In the selection of rubber or paper-insulated cables for very high voltages and high frequencies, independently of considerations of cost.

(d) In the adoption of pressure wires in high-tension cables, as the capacity current of the pressure wires is often larger than one-half the capacity current of the main conductors.

(e) In the importance of uniformity of rotation of prime movers and good design of generators to avoid distorted curves of electromotive forces."

During the last few years I have made several tests, and have not arrived yet at a clear understanding of the law governing the proportionality of losses with frequency. In that paper written fifteen years ago I said that the dielectric losses were proportional to the frequency. Subsequent tests which I have made show that they are not absolutely proportional to the frequency. Looking over some curves we have made on a variety of insulating materials, we have found that approximately the losses vary less than the square root of the frequency, and I would like to ask Messrs. Clark and Shanklin if they can give the law, if there is a law, covering the valuation of dielectric losses with frequency.

I want just to emphasize one point about the importance of these losses that I should, perhaps, have stated before. I do not believe it is practicable to operate a cable above the temperature of steam in air. If you operate a cable at 105 deg. in the duct, that duct will get dry and grow hotter and hotter. So there is no use in presenting papers dealing with temperatures above 100 deg., as such papers are misleading. Let us confine our study below the 100 deg. mark so as not to deal with the impracticable.

If we limit the study to a practical limit of say 80 deg. for 6600 volts, 25 cycles, the losses would be almost negligible, for any kind of insulating compound. With a cable at 13,200 volts, at 60 cycles, the difference for different compounds would be more, so that for same copper temperature the current capacity would have to be reduced ten or twenty per cent with a higher dielectric loss compound. Hence the dielectric losses in cables assume only importance with voltages above say 10,000 volts and higher frequencies.

I want also to briefly make a comment in reference to varnished-cambric and rubber-insulated cables. I made some tests in 1910, to obtain the resistance and dielectric strength variation with temperature for varnished-cambric and paper cables, which showed that the insulation resistance fell off much more rapidly for the varnished cambric than for the paper, and also that the varnished-cambric, three-phase 250,000-cir. mils, 10/32-insulation cable breaks down on the average at 20 deg. at 97,000 volts; at 100 deg. at 38,000 volts, and at 150 deg.; at 32,000 volts. This is inconsistent with the data given in the paper; however, I think the answer is given by a side comment of Mr. Shanklin's when he stated that the linseed-oil compounds have different characteristics from the mineral-oil compounds, and I imagine that in his case the varnished cambric is of the type using the mineral compounds, but the varnished-cambric cables of 1910, were very bad.

C. W. Davis: The company with which the writer is connected has had its engineers at work upon these problems for many years, and while it has pursued dielectric loss and heating investigations along very similar lines and has achieved results which in many ways confirm those outlined in these papers, there is enough difference between its results and those presented in these papers to lend an added interest to the latter.

The graphical method of determining the critical temperature has the great advantage of simplifying the problem. We have made use of this method as outlined by Prof. Miles Walker in his discussion of Rayner's paper before the I. E. E. of Great Britain in 1912. Prof. Walker explains in that discussion how the critical temperature can be obtained by drawing a line tangent to the dielectric loss curve parallel to the heat dissipation curve.

Mr. H. W. Fisher, R. W. Atkinson and others have pointed out how dissipation curves of a duct system can be obtained and it has been explained that this information is a pre-requisite in the determination of resulting cable temperatures, and in particular the critical temperature of high-voltage cables. Until the cable user, however, is able to come to the manufacturer's assistance and make such careful and such extensive measurements of the conditions of his duct system as to give the manufacturer an accurate record of what can be fairly taken as average conditions and as the worst conditions to be met with in any given locality, it is obvious that the manufacturer is forced to base his judgment on what may be more or less insufficient data.

There can be no doubt about the desirability of making use of dielectrics of low power factor in the construction of high-voltage cables. It was in recognition of this fact that the engineers with whom I have been associated, namely, H. W. Fisher and W. A. Conner, were led to embark upon a course of experimentation upon mineral-base compounds some time in 1908, I believe. And their much earlier experience with cable made entirely of mineral waxes was in part responsible no doubt for their selection of a mineral-base compound for this new purpose. By 1910 a very considerable amount of such mineral-base-compound cable had been manufactured and installed under their direction for use at 10,000 volts and over, and there are several hundred thousand feet of such mineral-base-compound cable in satisfactory use to-day.

A number of difficulties were met with in the early days of the manufacture of cable made with these mineral-base compounds. Most of these difficulties have been overcome and the others in the course of time and under longer observation have proven less serious in practise than had been imagined. It has only been within the recent past, however, and after a number of years' experience with such cables, under actual working conditions in widely separated parts of the country, that the conclusion was reached that cables using mineral-base compound can be made (if we are satisfied with reasonably low rather than the lowest attainable dielectric losses), which are fairly comparable in their freedom from failure from all causes, with cables made of vegetable-base compounds.

It would be interesting to know on what basis of reasoning Messrs. Shanklin and Clark are led to the conclusion that practically all failures on underground-cable systems can be attributed to heating. Records relating to millions of feet of cable, with which I am familiar, show that for cable of the particular make I have in mind only three instances of burn-out can with any degree of assurance be attributed to over-heating of the kind that is apparently referred to, although the records referred to date back to 1902 and relate to cable operating at voltages from 10 kv. to 25 kv. three-phase, made of both vegetable-base compound and mineral-base compound. These three cases represent the smallest class of failures encountered with these cables. Where such an enormous quantity of cable is involved and where the conditions of service run the full gamut between the best and worst from practically all the large cities in the country, it seems only a fair conclusion that the cable of vegetable-base compound (which included much the greater portion of that here spoken of) is either of substantially lower dielectric loss than the vegetable-base cable referred to by Messrs. Shanklin and Clark and Messrs. Louis and Bang or that the duct systems of the country only rarely include any such extreme conditions as they allude to. Numerous tests on high-voltage cable of vegetable-base compound with which I am most familiar have failed to show anything like as high results on the average as those referred to by Messrs. Shanklin and Clark or Messrs. Louis and Bang, either for new or old cable. I am inclined to believe, therefore, that the cable of vegetable-base compound represented by their data is exceptional in charac-

ter, either because of the original treatment or becau
been injured by repeated overloads. This, coupled wi
system of exceptionally poor thermal properties, seems
most probable cause of the trouble referred to; and in
of the excellent results that have been obtained and
being obtained with vegetable-base cables of various
ture in voltages from 10 kv. to 25 kv., it would seem
unjustifiable to assume that the data presented in the
give anything like merited recognition to the claims
made cable using vegetable-base compounds.

It is interesting to note that one of the largest and n
cessful makers of high-voltage cable in Europe has used f
years and is using today, we believe, a vegetable-base co
which, in a specimen we tested at voltage up to 14,0
three years ago and again a few months ago, had a pow
of 4 per cent at 150 deg. fahr. and 30 per cent. at 200 de
this cable having been taken from a piece that has been o
successfully in a very hot duct system since 1910. It is n
infrequent for us to find samples of new cable using ve
base compounds that show power factors of 7.5 per cen
deg. fahr. and 30 per cent at 200 deg. fahr.

The change of resistivity and power factor with vo
low temperatures has been noted by us in connection wit
mens which have been bent when cold, and also with sp
made under practical working conditions which have n
bent while cold. While this does not necessarily mean t
behavior of the specimen may not be due to the same
suggested by the authors in connection with their co
samples, it is nevertheless suggestive of the possibility th
causes may be involved.

We have, on several occasions during the last four
years, noted the peculiar appearance of high-voltage thr
ductor paper cable which had started on the road to dest
through the combined effect of dielectric losses and coppe
in over-heated cables. The appearance of such cabl
responds exactly with the description given Messrs. S
and Clark; and tests made on new cable and on cable take
to such a burn-out show changes of the same character in
by this paper.

In passing it is worth noting that Klein in 1913 in a p
the E. T. Z. pointed out the very great change in power
with voltage at low temperatures on certain single-con
paper-insulated cables of various copper dimensions and
thicknesses of insulation. The curves given by Mr.
showing power factor, are almost direct counterparts
curves given in Mr. Clark's paper for specific resistivi
kilovolts.

So far as we are aware the first data published showi
cyclic change of resistance with temperature are giv
Tedeschi in 1912 in the *Archiv Fur Elektrotechnik* where

cycles at various voltages up to 5000 volts were carried out on a number of insulating materials. It is of further interest to note that Pungs, in his article on insulating materials in the same periodical for 1912, calls attention to the fact that the insulation resistance of oils and compounds, particularly at relatively high temperatures, is very closely comparable with the alternating-current resistance as determined by dielectric loss measurements.

H. R. Woodrow: We were very glad in looking over Mr. Roper's paper to see that many of the points that he has brought out are in accord with our experiences along the same line. In general the conclusions that we have deduced from a large number of tests and experiments which were made for the purpose of finding a type of joint which would be successful for use on a 25,000-volt cable are in accord with those given by Mr. Roper. The type of joint adopted was completely described in the *Electrical World* of April 15, 1916.

The need of improving the methods of making such cable splices was recognized several years ago and extensive studies and tests with different kinds of joint-filling compounds were conducted in connection with different types of carefully made joints; but all these joints failed to give uniform and satisfactory results. Paraffine compounds were found unsatisfactory because on cooling they contract, leaving pockets. Heavy-gum compounds having high melting points did not flow into the crevices, and moreover under short-circuit stress cracks were formed which did not heal and finally caused breakdown. Later experiments were conducted with liquid fillers (fluid at low temperatures) such as rosin or mineral oils used in the insulation of cables. The chief difficulty experienced at first with liquid fillers was their tendency to run into the cable and leave voids in the joint. After considerable experimenting, however, this difficulty was completely overcome by the design of joint described in the *Electrical World* of April 15, 1916. This joint is filled with oil and has the hand-wrapped insulation, but over the outside insulating belt is a grounding gauge over which is a covering of cotton wick which holds the oil in the joint. Tests have shown that the oil may all be drained out of the sleeve without affecting the saturation of the joint.

Under laboratory test these joints have withstood, without failure, 50,000 volts, three phase, for over 340 hours, when the test was discontinued. Out of more than thirty experimental splices of all types, the oil-filled ones were the only ones which stood indefinitely the 50,000-volt, three-phase test. Some of these oil-filled splices were tested with equally satisfactory results, even when drained of all the oil in the annular space between the joint and lead sleeve.

Over 290 cable splices have been in service nearly three years on three 25,000-volt cables, supplying the New York, New Haven & Hartford Railroad. This is a single-phase load with a very heavy fluctuation and there has not been a single failure in these joints. Prior to the development of these joints, even the best made joints of the older type failed as often as once a month on less than 16,000 volts.

We have made up the accompanying curves of Fig. 1, showing our comparative experience during the last few years with the oil-filled and solid-filled joints.

About a year ago we began applying this type of joint on our 15,000-volt lines, and up to the present time not a single failure has occurred, which shows a marked improvement over the solid-filled joint.

Mr. Harper has brought out some points which we have also found in our tests. With these joints filled under vacuum, the insulating compound, which is an oil, penetrates into the cable for several feet, which obviates the difficulty due to a partial draining of the cable forming air pockets.

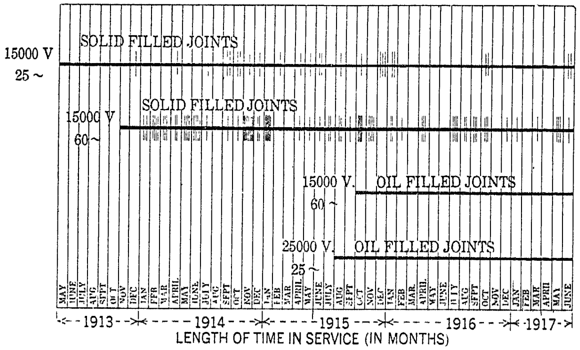

FIG 1

This oil-filled type of joint requires practically the same amount of time to make as any hand-wrapped joint but there is an additional time required if the vacuum is applied. The vacuum filling has an advantage on the extra high-voltage cable, although we found the other method, when it was impregnated without using the vacuum, gave good results on lower voltages. The vacuum operation will require an hour to an hour and a half per joint.

John B. Whitehead: The dissipation of heat from a conduit line, takes place by heat conduction; radiation and convection being absent. Under these circumstances the temperature that is attained in the conduit line depends on the temperature gradient of the surrounding medium.

In these papers it is assumed, that we have a definite temperature gradient around each conduit line; if that is the case, and it undoubtedly is the case in the single line of Bang & Louis, it accounts for the straight line relation between the losses in the cables and the resulting temperature. This in turn indicates that a given number of ducts should be laid so as to present the maximum surface of contact with the soil. To lay them so as to require the least amount of excavation will often result in conditions lowering the safe capacity of the cables, for example Mr. Harper shows that the capacity of the duct line is only limited by the outside surface, the effect of the inside, the interior conductors, not having much bearing on the total capacity.

The paper by Messrs. Bang and Louis, it seems to me, is an important step toward the possibility of rating cables definitely. The method suggested by them, however requires that the dissipating curve of the duct line should be known. For this there is needed a knowledge of the properties of the soil and the losses that are taking place in all of the ducts, except that particular one in which the cable in question, is laid. In addition the dielectric loss curve of the cable itself, must be known. It should be possible to standardize several forms of cross section of duct line, and to test their dissipation curves in various conditions of soil. With the dielectric loss curve of a cable known it would then be possible to state the safe load it could carry in several standard conditions. While we are still far from being able to do this, the paper has shown clearly the principles involved.

Passing to the paper by Messrs. Clark and Shanklin, and particularly to the suggestion that ionization takes place within the cable, and is shown by the rapid fall in the resistivity curves at lower temperatures. The evidence that there is ionization here, as presented by the authors, is two-fold; first, that on opening a cable there is evidence of bubbles of gas, open spaces apparently, and the additional curve which has been taken on a cable which has been prepared with tape, and in which there is loss due to the ionization of air. It seems to me there are several reasons why there cannot be any ionization, at least gaseous ionization, within the body of a normal cable.

First ionization of gas is invariably accompanied by sparking and carbonization. In my opinion, if you get either of these effects started in the cable, the cable will not stand up very long, but the result will be immediately progressive and the cable will break down. Second how can there be any gas in the cable? The author's suggestion that the distortion and bending of the cable leaves open spaces, and that these spaces are filled by volatilized gases is not clear to me:—What is going to volatilize the gases at the low temperatures existing? Third, there is no evidence of ionization at the higher temperatures. Ionization occurs very much more readily at high temperatures than low. I think if it started at low temperatures, it would start much more easily at high temperatures.

It is possible to account for all the measurements on the cable at low temperatures by distortion of the cross-section of the cable, bringing the conductors themselves nearer to the sheath at certain points? In this case there would be an increase of flux density, and consequently a local increase of temperature, resulting in higher losses, and consequently a reduction of the average resistance of the entire insulation. I ask, as bearing on those points, how the authors maintain the temperature of the cable uniform, and how they know it is uniform? I should like to ask also for some further description of the spaces which they say have been filled by ionized gas.

D. W. Roper: One point which was raised in connection with my paper was by Mr. Middleton, where some reference was made to the electrostatic capacity of the joint being different from the cable, that might be the cause of trouble. We have had several cases of trouble of that nature, some years ago, in which the difference in the electrostatic capacity of the cable may have had some influence on the failures. The joints were, we will say, fundamentally bad in one particular, but it was a very interesting case of joint trouble. One joint in a long transmission line, sixteen miles long, had failed about one mile and a half from the generating station, and as we found later, the joint had exploded, as it were, the lead sheath being split longitudinally, and a large portion of the compound blown out. The conductors were left practically intact in their curved form necessary for the application of the insulation. There were also some other troubles at the station occurring coincidentally, so that it was not possible to make a test with the usual test set. The cable was therefore tested by closing the switch at the further end of the line. This, of course, gave us an open-air arc between those conductors which had been exposed by the explosion of the joint. Later, when the repairs to the cable and in the station were completed, we again tried out the cable and found it stood the regular test, but on being placed in service it remained in service only a few hours before another similar breakdown occurred. This was repeated several times, and these failures were all near the original joint trouble. Before we got the cable back in service permanently, we remade all of the joints within about three-quarters of a mile of the original burnout, after which the trouble ceased. Apparently the subsequent breakdowns, following the initial failure, were due to some high-frequency and high-potential disturbances that traveled along the cable and affected only the joints. Possibly the electrostatic capacity of the joints had some bearing on the trouble, because the troubles were confined to a comparatively small distance from the original failure.

There are a few points I would like to bring up in connection with the other papers which have been presented. The authors have mentioned hot spots in connection with the cables. Do these hot spots occur in cables that are thoroughly impregnated, and if so, is it feasible to devise a specification or test to tell when the cables are properly impregnated? In two of the papers, mention is made of limiting the amount of current or power that should be transmitted through any one conduit, and in one paper a definite limit is suggested. It would be interesting to know what is the voltage and power factor of the load being carried on these cables. It would be more interesting if figures would be given of the total watts lost per foot of cable, and the loss per foot of conduit, and the number of ducts.

One of the papers mentions the submerging of the cables in water, and another paper tells of the beneficial effects of sprinkling the duct. Mr. Harper, I believe, mentioned the water for cooling. It is not clear, however, whether it was intended to simply submerge the cables so as to make them all of uniform temperature, and perhaps increase the conductivity of the surrounding soil, or whether it was intended to have a current or flow of water pass through the conduit, so as to give practically the conditions found in the water-cooled transformer. The submerging of cables is a good device, under some circumstances, but it has its disadvantages. We had a case of that kind which was adopted involuntarily, owing to a series of rains that brought the ground water level a foot or more above the level of the conduit, and then a cable broke down in this submerged area. We had to pump out a mile or more of conduit, before we could get at the cable, because as we lowered the water level the water ran in from the adjacent conduit lengths for some distance on each side, and it was hardly feasible to get down and plug up the ducts without the use of a diver. That is a point which should be taken into consideration when planning to submerge cables.

A. F. Bang: Replying to Mr. Dawes' remarks about the low rating indicated in our curves, I want to underscore again that the ratings which we have given refer only to particular cases and only to *constant* load conditions. If the load varies you can safely carry a materially greater peak load than indicated by the average load in our curves. If the load factor is small our deductions do therefore not apply. This problem of variable load conditions is very important but is not covered by our paper.

I was interested in what Mr. Torchio said about the reason for limiting the temperature on high-tension cables to values considerable lower than on low-tension cables and what is ordinarily considered safe for electrical machinery. I still believe though that the essential limit is the critical point as defined in our paper and that we will be able to safely go to much higher cable temperatures than specified in the Institute Rules if we are only sure we will not *reach this point.*

We have made up certain curves, for instance, to determine the critical temperature for a duct line with a varying number of cables installed and found this temperature to vary very much, say in the case of two cables it might be 90 deg. cent. and for thirteen cables 65 deg. cent., though the cables in both cases had the same dielectric characteristics.

C. A. Adams: I would ask if any precautions were taken to guard against the errors common to low power factor measurements by wattmeters where instrument transformers were employed?

A. F. Bang: Our measurements were made by measuring on the low side the input into potential transformers which were connected on the high side with the cable. The power factor on the low side of the potential transformers would never be below 50 per cent and any error in the watt meter indications, due to low power factor would, I take it, not be appreciable before the power factor went below 10 per cent.

The only source of error of importance that I can see is the fact that our method was a differential method, *i. e.*, to get at the cable losses we had to subtract the iron and copper losses in the potential transformers from the total indications. If now these losses were three or four times as great as the cable losses, as was the case at low temperatures, a 1 per cent error in the readings might naturally cause a much larger error in the final result; but at low temperature these losses are anyhow small and unimportant and at higher temperatures the above mentioned error would be negligible, since the cable losses here, especially in the poorer samples, would be many times greater than the transformer losses.

One strong point in favor of our measurements is the fact that they were made with actual three-phase current and potential applied, so that there can not be any doubt but that the cables were actually subjected to the same stresses as take place in ordinary operation.

Wallace S. Clark: Referring first to Mr. Harper's paper, Mr. Roper neglected to state at one time the Chicago Edison Company did actually and deliberately operate a lot of cables submerged in a tunnel under the river. They were low-tension cables, it is true, but the idea was there. They got to a point where they could not carry the current. It seems in operating lead-covered cables actually under water, there is one bit of data we must have before we can suggest it, and that is the rate of corrosion of the lead sheath with water flowing through the conduit, and that water carrying more or less air. Corrosion of iron in water, assuming the water to be fresh, is largely a matter of the amount of air and rate of circulation of the water.

Referring to Mr. Roper's paper, there is one point in joint operation that has not been mentioned, and that is, in actual operation the joint begins to heat on the center line of the joint and gradually heats towards the outside, and when it cools it reverses the process and the result is, if your compound is too hard, you have a hard shell of compound on the outside of the joint and compound in a softer condition in the center. You cannot use compound without some shrinking, and the result is you are apt to have shrinkage in the middle of the joint where you do not want it. I would like to say on this subject—I do not know whether I am officially right, but I will call it the Torchio joint—I will say that this is really the first clear improvement of a joint I have seen that I was willing to indorse.

Referring to the paper by Messrs. Bang and Louis, the agreement is rather better with our paper than would be indicated by the actual figures. They do not state in the paper the number of cycles, at least, I did not notice it, but I did notice on one of their drawings they had 25 cycles. Our measurements were made at 60 cycles, and therefore we would expect the measurements to run higher than theirs, but taking that into consideration, the agreement is not bad.

In regard to the temperature at which cables can be operated in connection with the rules formulated by the Sub-Committee of the Standards Committee, of which Mr. Fisher was chairman. I will say that I fathered it as much as any one else, and I will take the responsibility for it. I did not know then what I do today, and one of the reasons that these experiments were undertaken was to substantiate whether the principle of the rule is right or not, and I believe it is right. Whether the actual numerical values are right or not is another question, but assuming we had taken 110 deg. cent. for fibrous insulation, which Dr. Steinmetz and Mr. Lamme proposed, and had put it into the rules, and all the gentlemen who are operating underground cables had followed that suggestion, I do not think we would have received thanks from anybody. I think everybody would have had trouble. You cannot do it, that is all, for the average cable. Some day the manufacturers will make better cable, and we will be able to push the operating temperature up, but you must remember that a hot spot on a cable is a bad proposition.

There is one other point—if you buy a motor and you are going to run the windings at 100 deg. cent., and if that motor runs ten years without burning out an armature, you would not criticise the manufacturer very severely, provided it operated all the time at full load. But how many of you gentlemen are prepared to renew cables every ten years? You must have a cable depreciation around one per cent, and if it is much more than that, you are absolutely against it from a financial standpoint. I believe that principle of the rating is correct.

To answer Mr. Torchio's questions in part, until you get up to an over stressed cable, the dielectric loss appears to be pretty closely proportional to the square of the voltage.

With regard to the number of cycles, we have as yet only made tests at 60 cycles, we have been about a year and a half at this, and have spent a good deal of money on these tests.

With regard to the accuracy of our tests, you will find described in the paper that we took a 20-foot length of cable and tested it, and then cut it in two, in the middle, and duplicated the results on the halves. That test was made at my suggestion, and it seems to me it is a pretty clear statement as to the accuracy of the methods of testing used by Mr. Shanklin, who really did all the work.

Then a word on Mr. Whitehead's comment. We have given all of the results we obtained. Of course, they are open to various interpretations. There is absolutely a free field for that, and we have tried to leave that field free. We did find when we bent a cable a good many times and tested it that it showed a sharper rise in the curve than cable which had not been maltreated, but whether that is due to ionization or something else, I do not know. That is merely a theory.

I want to defend that rule on another point. Mr. Torchio spoke of tests on a 0000 cable at 260 amperes. With the 105 deg. rise which the Bang and Louis paper spoke of, I think the current is about 100 amperes. How in the world could a Standards Committee have put out a rule for the carrying capacity of cables that would equalize such data?

One other thing in regard to Mr. Torchio's figures—while I have not had a chance to compare them, his cable had about 10/32 or 11/32 in. thickness of insulation, and worked at 6600 volts, so that the stress per 32d was comparatively low, and naturally you get very low power factor losses there. If he had pushed up his voltage stress per 32d as it is pushed up in the 25,000-volt cable the dielectric losses would, of course, be much more important.

Delafield Du Bois: I want to suggest a possible explanation of the peculiar curves between specific resistance and voltage, as brought forth in the Clark and Shanklin paper, and discussed in part by Mr. Torchio. We know that in oil, moisture can exist in two forms, either in condensed form and globules, or in an absorbed state, exactly the same as moisture exists in air, either in the form of drops or globules, or as absorbed in the air. In the same way, whether the moisture is in the form of globules or absorbed moisture depends largely on the temperature. It would seem that the curves at 25 deg., showing a great falling off of resistance with increase in voltage, were due to the presence of this moisture in the insulation or oil in the form of globules, and as the oil becomes heated, we would expect that phenomenon to disappear, as the water was absorbed by the oil. I think the phenomena can be explained in that way without resorting to ionization, or at least ionization may have some effect, but the fact that the moisture is absorbed by the oil as the cable becomes hot is largely responsible for the shape of the curve shown in this paper.

Edward B. Meyer: Referring particularly to Mr. Roper's paper on high-tension joints, I believe that, as already stated by one of the previous speakers, the most important factor entering into the making of a satisfactory joint is the education of the splicer. In the system of the Public Service Electric Company of New Jersey we have quite a number of 13,200-volt underground cables installed by our own men. Some four or five years ago we found a number of the new cables installed failed under the initial high-potential breakdown test. Analysis of the failures developed the fact that most of the failures occurred in the splices made by one or two of the six splicers who had worked on that particular installation. We then conducted a campaign of education among our men, and after carefully working up a set of specifications we found that our joint-making methods were considerably improved and that the failures on high-tension joints were reduced. It seems that the importance of paying attention to the minute details in the making of the joint does not receive proper consideration, so that it is necessary to properly educate the men before you can expect to obtain good results.

Now, then, in reference to the heating of cables, we have also experienced some trouble from that source. Last year on account of the late delivery of cables, it was necessary to operate some of our tie feeders at almost continuous maximum load, thus resulting in a very high load factor. After two or three days of continuous operation, the cables failed. The cables were found to fail at loads very much under the normal rating of the cable. In the case of the 350,000-cir. mils three-conductor cables, our normal rating is 300 amperes, whereas the tie feeders failed in some cases with a load under 200 amperes. Radial feeders in the same conduit line and in ducts adjacent to the cables which failed carried 300 amperes and their load factor was considerably less, with the result that they did not fail. We made a change in our system by operating some of the smaller generator units in such a way that the load factor of the cables was reduced, resulting in less interruptions to our service, owing to the fact that the cable temperatures were maintained within reasonable limits. With the high load factors the sheath temperatures in some cases went as high as 150 deg. fahr. resulting in failure. Recently we again discovered some trouble on the same line, and investigation showed that a steam main crossed our duct line, and since this condition has been taken care of the cable failures have ceased.

John L. Harper: Mr. Roper has asked a question regarding the voltage and power factor in the cable system to which my paper referred. The working voltage was 12,000 volts, and the particular line that I had reference to, where we tried out this method, had about 85 per cent power factor.

Mr. Roper's remarks in regard to the submerging of the cables leads me to believe that most engineers have felt about the submerging of the cable much as one feels toward the method of taking a bath by falling off a dock rather than having a shower bath, or preferably by the prepared tub method. We have all had bad experiences with cable ducts being submerged and then trying to get the water out—we have had to pump out miles of it, and a considerable part of our time has been spent in getting our conduit systems properly drained so that they would be dry.

In our particular system at Niagara Falls all of our manholes are drained to the sewers, and in preparing for submerging the cables we have arranged stoppers in the drains from the manhole to the sewer, so that by pulling out a few of these stoppers the whole system can be drained quickly. The conduit would not dry out quickly enough to make a short interruption in the watering effective in heating the cable. We also found spots where the conduit will not hold water, for instance, where it has been built through rock, and the concrete surrounding the ducts is so porous, that the water seeps out through the rock and down through the sewers. Under those circumstances, we have arranged a spray beside the cable at the end of the duct, and found in taking up the center of a 500-ft. line that the conduits were quite damp in there, and that the humid atmosphere carried away the heat even if the water did not go clear through.

In regard to Mr. Clark's remarks on the deterioration, from air and water of the lead sheath surrounding the cables, in our city they require all permanent service water pipes put under pavement to be made of lead, so that I believe we will not find any immediate difficulty from the lead sheathing oxidizing or deteriorating.

I would be very glad to hear further discussion as to the probable life of underground cables. Engineers who have had to get out plans for financing the construction of conduit lines, or the writing off of underground cables, determining amortization factors for the conduits and cable systems, etc., never have hesitated to place the time required for replacing an ordinary cable at fifteen years, although I have personally had cables in operation longer than that time, and they seem to be in good condition yet.

One of the later speakers made some comments in regard to the time required for making joints. In our experience at Niagara Falls, we find that it is better not to hurry the cable splicers too much, as the perspiration is liable to run off of their foreheads and get into the joint, and it sometimes makes a lot of new work necessary.

Also about the educating of the cable splicers. There is a peculiar adaptation of the principle of education, applicable only to Niagara Falls. Of course, Niagara Falls is the center of the universe as to electrochemical work and nearly all electrochemical processes have some refuse. The easiest way to get rid of this refuse is to dump it into the sewer. Our manholes draining to the sewer have at certain times become saturated with a compound known as chlor-benzol, one of the heavy gases from munition works; and a cable splicer going into a manhole gets his lungs full of this gas and reaches the condition that one would expect to be in when he had ten cocktails. If there is any education that we can give a cable splicer in how to avoid the effects which will persist from chlor-benzol, we will be glad to find it out.

Comfort A. Adams: In Mr. Roper's paper, referring to cable-joint filling compound he says: "It should have sufficient dielectric strength for the purpose, which strength should not be materially reduced at the maximum operating temperature of the cable." I wish to contribute a little information as to the behavior of some of these filling compounds at low temperatures.

A couple of years ago when investigating the behavior of a filling compound made up of rosin oil and vaseline used in the joints of an English cable supplied to one of our large public service corporations, I found that failures occurred more often at low temperatures than at high temperatures. I took a sample of this compound and tested it in the laboratory. At temperatures within the range of the observations recorded in the papers under discussion, the compound behaved satisfactorily; that is, it had a high dielectric strength and gave no trouble. A thickness of this compound which withstood 40,000 to 45,000 volts over a wide range of temperatures above 15 deg. cent. being quite as good at high temperatures as at the lower ones, when subjected to such temperatures as exist in the earth in the winter time, in the vicinity of the freezing point, broke down after being subjected to 14,000 volts for 45 minutes.

Reference is made in the paper by Clark and Shanklin to the change of constitution, the stability, as it were, of the compound. In the case cited above there appears to be a distinct change of constitution of the compound, and not so much a chemical as a molecular or mechanical change. For example, in a cable joint filled with this compound, we found that after it had been subjected to high electrical stresses for a considerable period at low temperatures, the compound, instead of having a buttery or wax-like structure, had a very distinctly granular structure, which was preliminary to failure. Thus there are cases in which the low temperatures are most to be feared.

Referring to the paper of Messrs. Bang and Louis, I wish to add a little to my question in regard to the method of measuring the dielectric losses and to support what Mr. Robinson said in regard to the probable errors. I feel quite confident that these errors are of sufficient magnitude to affect the results materially.

Referring to the subject of ionization and internal corona, there are a few experimental facts which have come to my attention which may be of interest. First, in connection with rubber-covered stranded cables. Where the rubber is pressed in between the strands it does not always penetrate much below the outer circle of the conductors, thus leaving little V-shaped air sections. Where the rubber bends away from the copper strands there is sometimes a sufficient voltage gradient to cause corona. The effect upon the rubber compound is to produce what appears to be cuts transverse to the axis of the wire. Sometimes the cuts are close together and sometimes far apart. Sometimes they are located in groups more or less widely separated; but in every case they occur along the line where the rubber bends away from the copper, *i.e.*, where there is a high gradient in air. The cuts are not ordinarily visible to the naked eye until the rubber sheath is bent backwards so as to open the cuts The depth of the cuts is a function of the voltage and of the time, *e.g.* a cable which failed under a quick rise test at about 65,000 volts failed in 45 minutes at 10,000 volts. Samples cut off from the test piece in the long time test at various stages showed a progressive deepening of the cuts, even to a depth equal to three-fourths or more of the wall thickness, before failure occurred.

The same wall of insulation, made of the same material but squeezed further into the V's between the strands withstood a long time test of 25,000 volts without corona cutting, because the point at which the rubber bent away from the copper was so far down in the V as to reduce the gradient in the air below the corona point.

In many cases where the between-strand penetration was on the whole good, corona cutting and failures were almost always traceable to locally defective penetration, or to air spaces due to the buckling of the wall away from the copper. In other words failures occur in nearly every case as a result of internal corona in air spaces when the maximum gradient in the rubber compound itself is considerably below its dielectric strength.

Although this corona cutting in the rubber compound appears to the naked eye as clean knife-like cuts, examination with a microscope shows the walls of the cuts to be burned or fused, the appearance resembling that of furnace slag on a small scale.

With paper-covered cables there are also occasional signs of internal corona although its effect is different owing to the different nature of the dielectric. The evidence is also more rare since the whole cable is ordinarily pretty thoroughly impregnated under vacuum, so that in the perfect cable there are no air spaces. However, the examination of a paper cable after failure frequently reveals dry insulation, *i.e.* imperfect impregnation in the region of the fault.

It is practically impossible to make up a cable joint without some such air spaces. This is undoubtedly the reason for the relatively large number of failures in joints in spite of the fact that the total thickness of insulation is greater there than in the cable itself.

What happens within a joint or cable during a high-voltage test is of great interest and of practical importance. Some light was thrown on this subject by some rather elaborate tests on 25,000-volt cable joints made for one of our large public service electric companies a year or two ago. During these tests we listened carefully to the sounds within the joint either by placing the ear directly on the joint casing, or through the medium of a stethoscope.

When the voltage reached about 15,000 there commenced a gentle "sizzling" sound as of a light brush discharge. With increase of voltage, this sound gradually increased without changing its general character. When the voltage reached a certain point (from 35,000 to 70,000 depending upon the joint) a new sound, which we called "spitting," developed. This spitting was intermittent and at irregular and sometimes considerable intervals. It sounds very much like a breakdown of the insulation with a quick reestablishment thereof. In fact in joints where the chief insulation consists of a semi-liquid filler compound, these spits are actual failures which heal themselves when the alternating short-circuit current passes through its zero value, as shown by oscillograms. But in the case of paper wrapped joints, impregnated, the spits involved only a partial failure. The inner layers of paper were punctured, the discharge then spread along the joint between adjacent layers of paper and was apparently carried as a very high-frequency displacement current through the outer layers of paper over a comparatively large area, as there was no evidence whatever of a puncture of the outer layers. After the spit the puncture of the inner layers was apparently healed by the melted impregnating compound.

These spits were such as to be very noticeable to a trained observer, but not to the ordinary tester stationed at the switchboard some distance from the test sample, and not at all in the case of the test of a cable laid in the ground unless some special device is employed for this purpose.

Even when no spits occur it is altogether probable that internal corona or other unobtrusive discharges will in time gradually weaken the insulation to the breakdown point whenever there are air spaces or voids subjected to sufficiently high stresses.

The moral of all this is that our present methods of testing high-voltage apparatus in some cases partially destroys the insulation without actual failure of the ordinarily detectable sort.

In some cases our test specifications are at fault and in some cases the test methods are defective.

D. W. Roper: It is very interesting to note that Messrs. Clark and Shanklin have determined from their experiments that they can distinguish between a good cable and a poor cable by some measurements on a short length of the cable. It is perhaps somewhat discouraging when you go into the detail to find out that they tell the difference by reading the power factor of the charging current when you apply high-pressure testing current to the cable, and then to learn in another portion of the paper that there is only one instrument in the country that can make these measurements properly. We hope they will continue their work until they devise some method of discriminating between good and bad cables that is available to some of the other members of the profession.

The question was raised regarding the life of underground cables. In Chicago there was installed about twenty years ago some 00 three-conductor paper-insulated cable on 9000 volts, and operated on that pressure up to about three or four years ago, at which time its use was discontinued when we abandoned the Harrison Street station. During the past winter this cable was pulled out of the conduit, installed at another location and again put in service on 12,000 volts. It has been in service for a number of months without any trouble. That may give some clue as to the life of high-tension cable.

Some question has arisen regarding the accuracy of the measurements given in the paper by Messrs. Bang and Louis. It may be of interest to know that the results which they give were confirmed by some tests which we made two years ago in an entirely different way. The tests were made by heating up a length of cable with inverted current transformers, so that we could at the same time apply normal working pressure to the cable by means of potential transformers which were also working inverted. The conductor temperature was measured by means of thermopiles soldered directly on to the conductors and brought out through small holes in the lead, which were protected by means of a little oil cup, a homemade affair, to prevent breakdown at that point. Although the results we obtained are not exactly the same as submitted by Messrs. Bang and Louis, the results are very similar, and show the marked difference in the temperature of the conductor due to the different kinds of insulating materials.

Claude N. Rakestraw: In the fifth paragraph of Mr. Harper's paper, he says, "Assuming cables to be made of good material and with good workmanship and design, I venture to propose the following as the fundamental causes of failure, viz: Joint troubles, mechanical injury to lead sheaths, and overheating." It seems to me that he has eliminated from consideration one of the most important causes of failure, and that is the electrolytic failure. Trouble with electrolysis is one thing which the operating engineer always has to fight in the large cities, and is one of the most difficult troubles to overcome of those which he encounters. Perhaps Mr. Harper intends to include electrolytic troubles under the term "mechanical injury to lead sheaths". In any event it would seem that in regions where electrolytic injury of the lead sheath is present, or apt to be present, the submerging of cables in water is apt to accentuate the trouble, and not only to aid in the destruction of the sheath, but once an opening has been made in the sheath to admit the water that will complete the destruction of the cable. That is one point which the paper has not taken into consideration.

Mr. Roper says: "It is probable that no two operating companies use the same methods and materials in making up their 20,000-volt cable joints." That is undoubtedly true, and I would make the suggestion that the Committee on Transmission and Distribution might well arrange some kind of a symposium asking the various operating men in the various cities of the country to give a brief description of the kinds of joints they make, the materials they use, the failures they encounter, etc. If such a suggestion is well thought of, such a symposium could be arranged, and I think it would provide a mass of data which would be of great assistance to all of the operating engineers.

H. E. Weightman (by letter): I do not believe it is advisable to use water in the ducts to keep the cables cool as has been suggested, on account of the possible effect of electrolytic corrosion due to a lowering of the specific earth resistance on the application of water.

Further, it would tend to increase the life hazard in testing or repairing in cable manholes with any of the cables live and it is not always possible to kill all circuits in a manhole when repairing one.

It seems to me that larger ducts, say six inches (15 cm.) diameter and ventilated manholes would help solve the problem. If duct runs are too long for good ventilation why not run small vertical risers to the surface at intervals to help air circulation. These risers could be located at the summit between manholes.

P. H. Chase (communicated after adjournment): The papers by Messrs. Bang and Louis and by Messrs. Clark and Shanklin do not state whether the conductors were round or sector shape. I believe that some valuable information might be obtained from comparative tests on cables of round and sector-type conductor. These would show the difference in dielectric losses between the two shapes of conductor, which is to be expected from theoretical considerations, for it is evident that with a sector type of conductor, there is a very material concentration of stress at the points of relatively sharp curvature. This concentration cannot exist to as great an extent with round conductors. Consequently there must be increased dielectric losses and higher temperatures at these points. The cable therefore is comparatively weak at these points, a factor of greater importance, the higher the voltage.

Though exact data are at present lacking, it has appeared from the experience of one operating company that the failures on the cables having sector conductors have developed more as short circuits between conductors rather than as faults to ground, which seems to point toward insulation weakness between conductors. I mention this point in order, if possible, to bring out any experience of other operating companies along the same lines, which indicates a relative increase in failures of cables with sector conductors, compared with cables having round conductors. Of course, some weight must be given to the fact that cables with sector conductors have a greater heat loss per foot with a less than proportionate increase in outside surface, and also that duct lines in the past few years have tended to become more and more congested with a resulting increase in the operating temperature of the cables; but on the other hand, it is to be expected that the insulation on the earlier cables had deteriorated and therefore would be more liable to failure.

In the paper by Messrs. Clark and Shanklin, reference is made to a type of 3-conductor cable described by Mr. Höchstadter, which has a thin sheath over each conductor, and the belt insulation is eliminated. I would like to know if any members have had any experience with this type of construction, particularly on higher voltage work.

R. W. Atkinson (communicated after adjournment): The study and thorough knowledge of the problems of dielectric losses in high-tension cables are of two fold importance. Most obvious is the direct effect of these losses upon temperatures and upon current rating. A far greater amount of data are required to throw light on the problems which concern the life of a cable and the permanency of its original properties.

A very brief description of a method of measurement which has been used very successfully by the writer during the past several years is in order. The instrument itself is a reflecting dynamometer. The fixed coils of the instrument are connected in series with a non-inductive resistance shunted across the test specimen. The moving coils, two coils arranged astatically, are connected in series with the specimen under test. An arrangement of shunts makes it possible to adjust the instrument to take care of any specimen from the smallest to one taking a charging current of several amperes. By an arrangement of switches, the same instrument is used as an ammeter to measure the charging current of the specimen. There are certain small residual errors in phase angle of the different parts of the circuit; these have been measured and calculated and compensation has been made for them. Of late, the apparatus has been used with special voltmeter coils so as to avoid the necessity of high-resistance multipliers for the shunt coils of the wattmeter. The phase-angle errors which would otherwise exist due to reactance of the voltmeter coils have been compensated by comparison between measurements by this method and by the older one.

The same apparatus has been used for measurements of energy loss with three-phase voltage applied to three-conductor cables. An arrangement of switches makes it possible to connect the single-phase wattmeter successively from one phase to another, the instrument and switches being at all times near ground potential.

The writer has been very much interested in calculation of three-phase losses from single-phase measurements, as outlined in the paper by Messrs. Clark and Shanklin. In some cases, the agreement between three-phase power factor as calculated from single-phase measurements by the methods outlined by them, and the actual measured power factors, is very satisfactory. In other cases, the disagreement is very considerable, in some cases being such that the single-phase results themselves show the method of calculation to be not applicable. This latter condition occurred where the voltages on specimens with insulation thickness less than that of the three-conductor cable on which the three-phase calculations were made by the authors, were carried as high or higher than the 25 kilovolts used by the authors. In some instances, the quantity W_3' became negative. Of course this extreme result can be explained in a measure by consideration of the fact of the different magnitudes of the stresses in portions of the cable as mentioned by the authors. The point is raised, however, that it is not allowable in general to calculate three-phase losses as suggested in the paper, without full knowledge of similar cable actually tested under three-phase conditions.

The writer has made a great many comparisons between the three-phase power factor and the power factor of a different single-phase test. This single-phase test consists in applying voltage to two of the conductors of a three-conductor cable, the third conductor being connected to the sheath and to the grounded middle point of the transformer, the conductor under test being connected to the two extreme terminals. For convenience, and by analogy with telephone cable testing, we have called this single-phase test a "mutual" test. There is always some little difference between the three-phase power factor and the mutual power factor, but we have found the two fairly well comparable under a wide variety of conditions. The simpler single-phase measurements will, however, continue to be extremely useful on account of their very much greater convenience, for comparison of specimens actually closely similar and for general comparison of even quite different specimens.

Extremely valuable data in the study of dielectric losses are the variation of these losses with frequency. The writer has made measurements of dielectric losses of a good many specimens on both 25 cycles and 60 cycles. The loss at 25 cycles is always somewhat less than the loss at 60 cycles and, in some cases, is lower in almost direct ratio with the frequency. In other cases, there is little difference between the losses at the two frequencies. In general the losses at one frequency cannot be stated in terms of the losses at the other and it is hard to make any specific general statements regarding the relation between the losses at the two frequencies. The tendency is for the losses at the two frequencies to be relatively closer together at high temperatures than at low temperatures. Where there is a considerable increase of power factor with voltage increase, the tendency is for a greater difference between the losses at the two frequencies to occur at the higher voltages.

In order to show how extremely satisfactory dielectric loss characteristics may be combined with other obviously unsatisfactory general characteristics, the writer had prepared a sample of 3-conductor cable consisting of 3 conductors No. 0000 B. & S.G. insulated with 8/32 in. (6.3 mm.) dry paper around each conductor, the whole being laid up with filler and cabled and then covered with a "belt" insulation of 2/32 in. (0.794 mm.). of paper. This cable was made up in all particulars in the same way as an ordinary cable except that though it was thoroughly dried, it was not saturated with insulating material of any description but was lead covered immediately after removing from the drying oven. Dielectric-loss tests were made upon this cable. It was found that the losses at all temperatures were extremely low and particularly was it noticeable that the effect of increase in temperature was extremely small. Even at a temperature in excess of 100 deg. cent., the loss at 13,000 volts 60 cycles 3 phase was less than 0.1 watt per foot. Other very interesting data were obtained, but this serves for illustration. It is obvious that such a cable would be actually used only in some condition where dielectric losses are of more importance than any condition ever likely to exist in practise. Moreover, it is obvious that the fact that this cable is capable of withstanding certain extreme conditions which would not be withstood by other types of cables, is no evidence of its superior qualities for practical purposes.

There seems to have been some consideration of the use of high-voltage cables at excessive temperatures. There are a very great many more considerations involved in the use of high-tension cables at excessive temperatures than the mere question of dielectric loss of the cable at time of manufacture or installation. One comparatively obvious consideration will be discussed. Starting with the assumption that the cable, as manufactured, is thoroughly filled with compound, and is installed in such a way that no compound is lost, nor has opportunity to escape in service, the question arises as to what will happen when the cable is heated and the compound expands. The answer is that the pressure of the expanding compound will permanently stretch the lead sheath. When the cable cools obviously its condition is different than before, and will continue to change as the compound gradually drains toward the lower portions of the length of cable, particularly in those lengths which are installed on a hillside. Means have been proposed for providing for this expansion and contraction of the cable compound. C. W. Davis has suggested the use of expansion chambers at the joints, so designed that when the cable cools the original compound is forced back into the cable. As the writer understands, in the test by Messrs. Clark and Shanklin, their test specimens were subjected to the various heat cycles with both ends submerged under oil. If any of the insulating material is expelled by the expansion due to heating, its place will be taken by the influx of the oil under which the ends were submerged, when the cable cools.

We have tested a piece of cable and found it to show no greater effect from heat cycles than the cables tested by Messrs. Clark and Shanklin when tested with the ends submerged in oil. We have then taken the same cable and have subjected it to a heat cycle with a maximum temperature of 130 deg. cent. The ends of the cable were, however, treated differently; they were sealed by pouring around them in a paper cone, a high melting point resinous material, completely covering the conductors. However, during the heating of the cable, a considerable amount of the insulating compound—a mineral oil by the way—was forced from the ends due to its expansion. We give below a short table showing the effect of the removal of the compound, upon the single-phase power factor, these tests being made at room temperature.

Kilovolts	5	10	15	20	25	30
Power factor before heating	0.8	0.8	1.2	2.25	3.8	6.3
" " after heating..	0.8	0.8	1.3	9.5	19.4	22.0

The problem of dielectric losses is a broad and important one. It must be considered with due regard to its place as one of the many problems of dielectric characteristics.

H. W. Fisher (communicated after adjournment): There is so great a variety of vegetable and mineral oils and compounds that at the present time it would be impossible for any one to state definitely whether all mineral or all vegetable compounds would be the most satisfactory for high-voltage cable purposes. It is the opinion of the writer that the best results may be obtained by the proper combination of one or more kinds of vegetable and mineral oils.

Low dielectric loss is not the only consideration. Mr. Atkinson has shown in his discussion that a cable made with perfectly dry paper has a lower dielectric loss than almost any other type of cable and yet no one would want to use such a cable for high-voltage transmission. In our experimental department we have made cables, with certain mineral compounds, which had remarkably low dielectric loss and yet the cable would not have been suitable for practical operation because the compound was so fluid that it would be certain to flow from high to low points after such a cable was put in service.

The lead of a cable is unavoidably stretched during the process of installation and hence there are places into which compound can gradually flow if said compound does not have sufficient viscosity at high temperature. The viscosity of cable compounds therefore is something that must not be overlooked.

The real test of a cable is the length of its life under ordinary working conditions. Therefore any conservative manufacturer is slow to adopt a new compound until he has reasonable evidence that cable saturated with said compound will have a long life under the usual conditions occurring in practise.

At one time the writer helped to develop a compound which had remarkably low dielectric losses at high temperatures. Cables saturated with this compound showed equally low dielectric losses, but when samples of the cable were subjected to 13,000 to 15,000 volts, under temperatures occurring in practise, the cables finally broke down and an examination revealed the fact that owing to the fluidity of the compound at high temperatures, there were comparative dry spots at different points of the cable where the break downs occurred.

This indicates the desirability of using compounds which are not too fluid at high temperatures.

Developments made on the suggestions of the writer were instrumental in producing a compound which had very remarkable viscosity properties, the viscosity changing very little from ordinary temperatures up to temperatures of about 100 deg. cent. Here was a compound which, so far as viscosity was concerned, seemed to be ideal but when it was subjected to other tests, it was found that its power factor was very high at high temperatures and that it had unstable characteristics which absolutely prohibited its use as a saturating medium for cables. The writer has referred to this remarkable compound in order to illustrate how difficult it is to secure material which has idealistic characteristics in all respects.

In the above discussion, the writer does not wish to be understood as going on record that fluid compounds may not be used with success in the construction of cables, but it is his opinion that very fluid compounds cannot be used in paper-insulated cables as they are made, installed and operated at the present time.

A. F. Hovey (communicated after adjournment): Regardless of what method of joint making is adopted, Mr. Roper's suggestions are to the point and if they are carefully carried out good joints should result. The cable company with which I am connected has been making use of a paper-tube joint since 1903 and where this joint has been made by our own workmen we have had exceptionally good results with it, on all voltage up to and including 25,000 volts. In fact at 25,000 volts, in so far as our records extending over several years permit us to say, the evidence seems to clearly support the belief that we are having no more trouble than we would expect on 10,000 or 12,000-volt joints, and we have had exceptionally little trouble at either voltage. While in general, the method of making tube joints for 25,000 volts is similar to that for 12,000 volts, greater care is exercised throughout for joints at the higher voltages and we do not permit our men to make such joints until they have become skilled while making experimental joints. We have found that properly made joints using paper tubes will easily stand operating conditions equal to those, that the very highest class cable will stand. Tests for breakdown and for dielectric loss at various temperatures indicate that joints made with such materials as we have selected for high-voltage work will not deteriorate under operating conditions in the way indicated by Mr. Harper, in his paper on "Problems in Operation and Maintenance of Underground Cables."

Obviously, the temperature of a joint is so much lower than the maximum temperature (roughly mid-way between manholes) to which the connecting cable is subjected that nothing like the same requirement exists for low dielectric loss in the jointing compound and jointing material that exists in the materials used in the body of the cable itself Besides, the separation of the conductors is greater in the joints than in the cable and this again, reducing the stress, tends to also reduce the loss at any given voltage and temperature. It is only for working voltages above 15 or 16 kv. that there would seem to be any great need for a filling compound which, when mixed with the cable compound, would not result in a material having markedly greater dielectric loss at high temperature or lower puncture strength than the cable compound with which it is used and even this need must be felt only under the most exceptional conditions, as for instance near long vertical runs in hot stations, vaults or manholes. The compound that I have made use of during the last couple of years for the very high-voltage cables has a dielectric loss at high temperature even when intermixed either with mineral or vegetable compounds, which is quite low and probably far lower than there is any necessity for, because my experience is directly contrary to Mr. Harper's, and even where we have made use of jointing compounds of relatively high dielectric loss and lower puncture strength at higher temperatures than the 25,000-volt compound I have just referred to, we have found but one case in 15 years where we can, with any good reason, attribute joint or cable failures to an inter-mixture of jointing and cable compounds.

Our present method of making high-voltage joints involves the use of a vertical chimney at one end of the lead sleeve. The lead sleeve is so arranged that the end with the chimney shall be low and the end for the out flow of compound shall be at the high end of the lead sleeve. The compound is then poured into the lead sleeve through the chimney. The inclination of the sleeve and the current of hot compound which we force through the sleeve at high temperatures, but at temperatures that will not injure or deteriorate the tubes or cable insulation and to the extent of two or three gallons before we allow the filling compound to remain in the sleeve, entirely washes out moisture and air. So far as we know this is the best method of accomplishing this particular result, at least with tube joints. The idea is due to one of our engineers, and I believe a patent application has been made covering the method in question.

Mr. C. W. Davis was the first person in this country, so far as I am aware, to propose the use of vacuum filling for high-voltage joints. The idea occurred to him about 1905 and a patent application was filed at the time. Later, however, it was abandoned. We have made a very considerable number of tests on joints using vacuum, both with hard filling and soft filling compounds, but have never been able to achieve results which were as good as were achieved regularly with our standard methods, unless we went to such abnormal care as to increase the cost of the joint to a prohibitive expense. Our present day joints, using paper tubes, for 25,000 volts are still made without vacuum.

In the early days of high-voltage cable practise, the company with which I am connected made joints using very soft oily filling compounds. These compounds have many advantages, but they have a very great disadvantage where the cable is laid on an incline, or where one part of the section of cable is at a higher level than the other. We found back in 1903 and 1904 that with oil filling the oil would, in some cases, practically empty itself out of some joints and run for long distances through the cable, into joints at a lower level. While we make it a practise to-day to use oil filling at times under exceptional conditions, we prefer, as a general thing, to use a rather harder compound.

H. C. Louis (communicated after adjournment): Answering some criticism regarding our methods of measurement, it is admitted that there may be some question as to the absolute accuracy of these for measuring very small losses, but they are sufficiently accurate for measuring higher losses. At low temperatures the dielectric losses are very low, but when they are low their relative effect is small, so consequently it is of no great importance for our purposes to measure these with refined accuracy. However, at higher temperatures the dielectric losses being higher, they are of great importance, consequently it is of importance that the measurements here be accurate. It is at these higher points that our method is accurate, and is of more than sufficient accuracy to show up marked differences, where such exist, between different cables.

Presented at the 322nd Meeting of the American Institute of Electrical Engineers, New York, June 27, 1917.

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THE INSULATOR SITUATION

BY W. D. PEASLEE

Abstract of Paper

This paper gives a résumé of the insulator situation as it exists at the present time and shows the gradual increase in requirements that have been held essential to a successful insulator.

A statement of the apparent causes of the very rapid deterioration of insulators, even when stored and subject to no electric stress is given, and the conditions necessary to the production of an insulator that will reduce this deterioration cost are discussed.

Microphotographs showing the structure of the porcelain from several insulators are given. Interesting flows and defects common to porcelain insulators are shown. Three means are given for improving the insulator situation, one of which is the use of fused quartz as an insulator material, the laboratory feasibility of which has been shown. Investigations as to the commercial manufacture of quartz insulators are under way.

THE INSULATOR situation has for the last few years engaged the earnest thought of all engineers directly or indirectly responsible for the operation and maintenance of high-voltage transmission lines.

Not long ago the following general postulates were laid down by some operating engineers, despite the warnings of some research men, as the ultimate requirements of insulators for reliable service, and much was written in the technical press regarding efficiency as applied to line insulators with respect to these features.

1. The insulator must not flash-over at operating voltage even when dirty and wet.
2. The insulator should flash-over on excess voltage at normal frequency and not puncture.
3. Sufficient surface leakage distance should be provided to prevent excessive leakage current. (To prevent burning of wood poles and pins.)
4. It should be mechanically strong enough for the load imposed upon it.

Insulators by the carload have been made that meet fully those requirements, as far as normal frequency voltages are concerned, that is, those that do not fail under factory test do. These new insulators, designed according to the best then existing engineering knowledge, were placed on our transmission lines and the insulator manufacturers began the study of making more and cheaper insulators while the engineers operating the lines began to get interested in other things, when the insulator situation again became of prime moment.

It was found that after about two or three years in service these insulators were failing at a rate of from 10 per cent to 40 per cent per year, apparently without logical regard to location or severity of service, climatic or electrical. Many very valuable and brilliant methods for detecting insulators about to fail were developed and the best that was hoped for was a chance to take them off when convenient before they went out at an inopportune time.

About this time a rather startling fact was discovered by several investigators almost simultaneously. If a certain lot of insulators, having passed successfully factory routine tests, are placed in a warehouse for a couple of years, a goodly per cent of them will fail at the end of that time on tests less severe than they passed to escape from the factory junk pile. In other words, they deteriorate at an alarming rate even doing nothing and apparently just "because". This experience the author can testify to with some regret, and it has been corroborated often enough, so that it is now a well recognized fact among workers in this field.

Not satisfied with the generally accepted theories in explanation of these failures the author began in 1914 an extended research into this field which finally led to the use of the microscope in an endeavor to ascertain, if possible, the mechanism of the failure of porcelain insulators. As a result of this microscopic work in connection with some experiments on transients, he laid down seven requirements in corroboration and extension of the four previously mentioned.

By the time this paper* appeared the work had been further advanced by means of an excellent high-powered petrographical microscope and the results lead to a decided doubt as to the ability of the porcelain insulator to meet the situation at all, even under the best conditions of manufacture.

From a study of the insulator troubles of several systems for the past three years, and from a microscopical analysis (using polarized light and crossed nicols) of several hundred insulators whose records were known, the following points regarding the deterioration of the porcelain insulator seems to be pretty well established.

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*W. D. Peaslee, A. I. E. E. TRANS., Vol. XXXV, 1916, p. 1187.

1. Perfectly good insulators that will pass all sorts of tests right after manufacture deteriorate at an alarming rate even when not in use. I have personally known of insulators stored in a warehouse, all of which have passed test on an oscillator, and also 60-cycle flash-over. Two years later 14 per cent of these insulators failed under 60-cycle flash-over. That is, they punctured at a voltage under the flash-over voltage.

2. It is very apparent that something takes place, inside the cap in a suspension unit, and between the shells in a pin-type unit, that causes this deterioration when not in service, though it is not impossible that the electric tests have some part in starting or causing some of this deterioration.

3. There seems also to be grounds for suspecting that in some manner certain insulators deteriorate at a rather rapid rate due to dielectric stresses combining with other stresses in such a manner as to materially hasten the failure of the insulator.

The results of the investigations seem to show that four factors are of predominant importance in this connection.

1. Mechanical stresses due to temperature changes necessarily resulting from the different coefficients of cubical expansion of the metal, cement, and porcelain; mechanical stresses due to the expansion of moisture in the cement on freezing and the changes in volume of cement with different moisture contents; and mechanical stresses due to improper firing and cooling of the porcelain.

2. Dielectric flux concentrations due to improper design of insulator shapes and defects in the porcelain. This can be remedied, but its general effect* is as has been shown, to permit the stressing of the porcelain to very high values by means of transients, which like the poor are always with us.

3. Dielectric flux concentration due to the fact that the porcelain is inevitably made of two or more materials of widely different physical properties, one isotropic and the others anisotropic in character, as strikingly shown by the microphotographs. It is well known that the dielectric constant or specific inductive capacity of an anisotropic medium is not the same in all directions but varies according to the angle between the principal optic

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*W. D. Peaslee, A. I. E. E. TRANS., Vol. XXXV, 1916, p. 1187.

axis and the direction of the electric force. With the random alignment of the anisotropic crystals in a porcelain body, relatively large, minutely local flux concentrations result. While partially fluid in the process of firing, most insulator shapes develop flow lines wherein the crystals, anisotropic in nature, align themselves in such a manner that the permittivity of the dielectric along these flow lines is considerably greater than in other portions, resulting in definite flux concentrations along these flow lines.

4. Mechanical stresses due to the differences in coefficients of expansion of the constituents of the porcelain acting to establish a separation surface between the crystals.

Considering the cement, the volume changes of Portland cement under change of moisture content are well known, and examination of the cement in two hundred and eighteen insulators whose history was well known has shown the following facts:

All but fifteen of the insulators that had failed electrically (207) had cement whose absorption, after two hours baking at 120 deg. cent. and four hours boiling in water, was over 12 per cent and in some failing quickly the absorption ran up to 46 per cent. Of those which had failed mechanically or were still good (11) only one had cement whose absorption ran over 6 per cent.

While these data are no more than a surface indication they are offered as an indication of a possible source of deterioration. It can readily be seen that absorptive of such amounts of water the cement in such insulators must expand and contract viciously under temperature and humidity changes. Investigations are at present under way using various water-proofing compounds and different mixtures of cement to find the mixture that with a permissible strength sacrifice will give the lowest absorption. The preliminary results are encouraging and it is hoped that some valuable information will be available soon from these tests.

The porcelain from which insulators are made consists in the main of partially dissolved crystals of silica (SiO_2) in a ground mass of feldspar glass and crystals of sillimanite of a size depending upon the heat treatment of the porcelain. It has been found by some investigators that it is entirely possible and easy to determine within ± 25 deg. cent. the temperature at which a porcelain has been fired by the examination of a thin section under the microscope.*

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*Technologic Paper No. 80 Bureau of Standards, Constitution and Microstructure of Porcelain.

Referring to Fig. 1 if the needles are anisotropic substances whose maximum permittivity occurs along the principal axis the resulting field distortion will be roughly as shown. Now combine a conglomerate mass of such needles scattered through a medium of different average permittivity still, and sprinkle through this a quantity of relatively large irregular anisotropic crystals of still different average permittivity and it will readily be seen that there are infinite possibilities for minute flux concentrations, and *whenever for a finite length of time the dielectric flux exceeds a certain value in a dielectric, the destruction of the dielectric is certain.* Also, failure of the dielectric heats it, and the negative temperature resistance coefficient of porcelain renders a progressive breakdown extremely easy when the initial failure occurs.

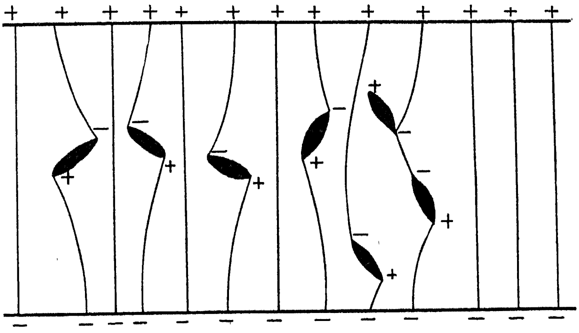


FIG. 1

Microscopic examination of a great many samples has shown that the flow lines mentioned are mechanically weak and have a great tendency to develop minute cracks, apparently during cooling. These cracks are often lined or filled with crystals of sillimanite (Al_2SiO_3) which are anisotropic and more or less in alignment. The combination of such cracks and the above mentioned flux concentrations along flow lines is a source of further weakness in the porcelain insulator. This point is brought out very clearly in several of the illustrations.

If the four factors last mentioned are the predominant ones in causing insulator depreciation, we can of course lay out the requirements of an insulator that will avoid them.

1. A cement must be developed that will be elastic enough to take up the necessary changes due to expansion and contraction of the insulator parts under temperature changes. Condensite

and other compounds have been tried in this but so far as the writer knows, no positive results have been obtained that are as yet of commercial value.

2. Improve the design of the insulator to avoid flux concentrations and the placing of dielectric of different permittivities in series and improve the workmanship to avoid defects.

3. Make the insulator from an amorphorous, non crystalline substance, containing only one constituent, or only those of the same electrical and temperature characteristics.

Certain of the compounds that have been experimented on give promise of some success as a cement of the character described, and fused quartz except for inherent difficulties in working is ideal as an insulator material.

The temperature coefficient of expansion of fused quartz is about 0.000,000,59 or about one-fifth that of porcelain. Its dielectric strength is very high and its mechanical strength and elasticity are all that could be asked. It withstands indefinitely the action of high-frequency voltages such as those from the Poulsen arc and is to the action of the elements the most resistant material known.

Due to its low temperature coefficient of expansion an incandescent piece of fused quartz can be dropped into a pail of water without effect, except an apparent improvement in elasticity. This is a valuable property when it comes to running it into chilled molds.

The drawback is that it has a tendency to evaporate before flowing enough to mold and on fusing to explode and form a milky substance full of tiny air holes and useless as an insulator and is also chemically very active at extremely high temperatures. Means of overcoming this have been found and until the microscope slides of the product were examined it was thought a perfect product had been secured, as the tiny carbon particles and bubbles so marked under the microscope in Fig. 10 and Fig. 11 are not visible to the unaided eye, merely giving the quartz a blue color. Means have been found to overcome even these defects and on a laboratory scale the manufacture of fused quartz as an insulator material is assured. The problem now engaging our attention is the production of this material in commercial sizes at a cost that will not be prohibitive.

CONCLUSION

A study of the growing mass of data on insulator deterioration, forces upon our attention the following insulator situation.

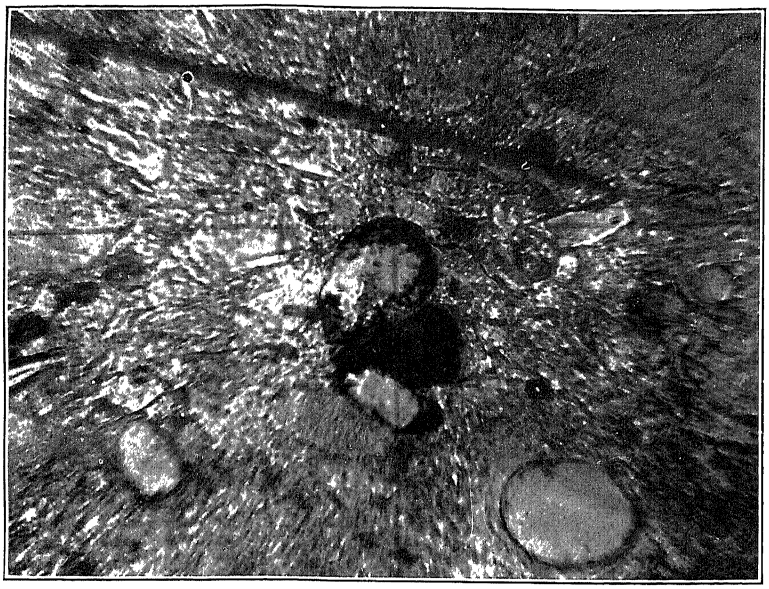


FIG. 2—300 DIAMETERS—TAKEN FROM AN INSULATOR MANUFACTURED IN 1912

This type has showed marked deterioration and has been removed from service by one company and replaced by another type—Note the shrink crack full of sillimanite crystals and the large partially resorbed quartz crystals showing in high relief—The darkness of the ground mass is caused by minute distributed sillimanite crystals—The darker bodies are aggregates of sillimanite with some kaolin.

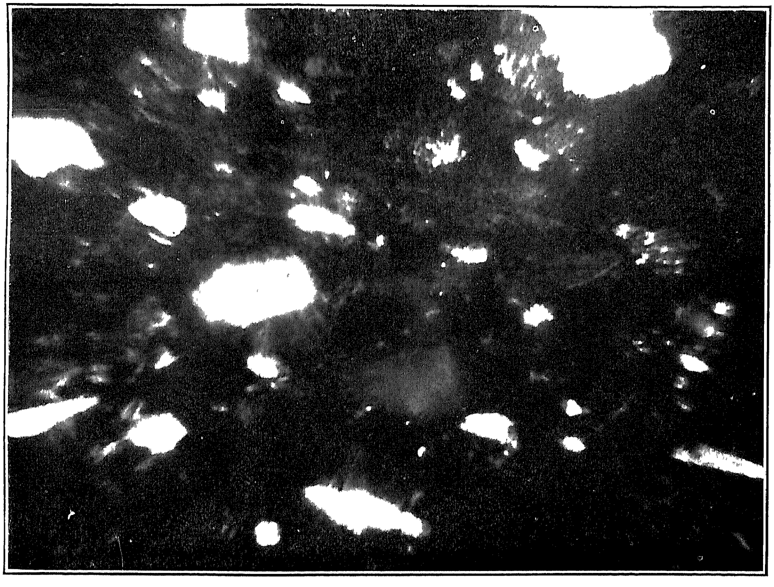


FIG. 3—300 DIAMETERS—SAME AS FIG. 2 [PEASLEE]

Taken through crossed nicols showing anisotropic nature of quartz crystals (bright white masses) and of sillimanite (appearing as a light fog).

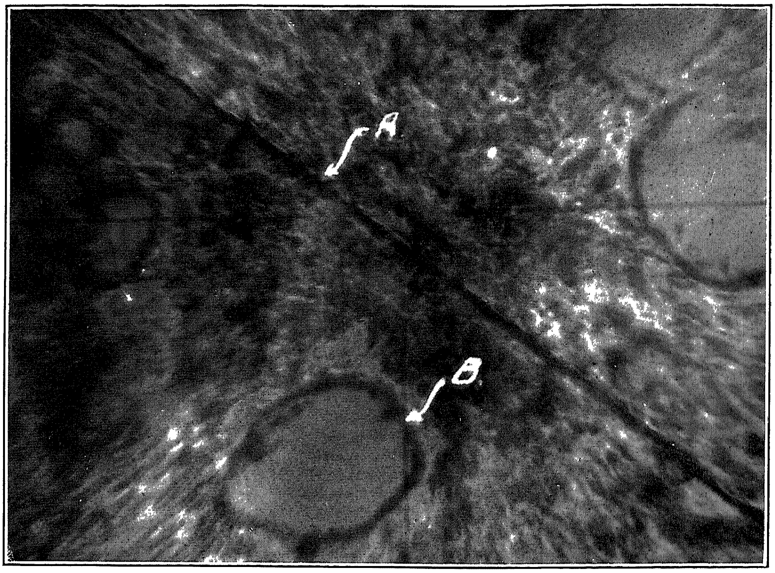
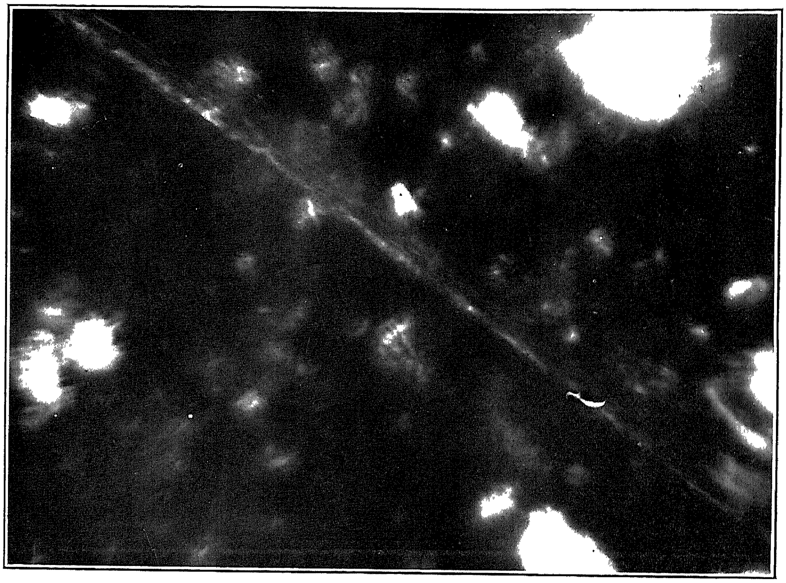


FIG. 4—300 DIAMETERS—SECTION FROM A WELL KNOWN MAKE OF HIGH-VOLTAGE PIN-TYPE INSULATOR

Failed under test before being put into service—A is crack lined with silliminate crystals—B is a large partially resorbed quartz crystal—The dark masses are sillimanite particles scattered through the glassy ground mass.



[PEASLEE]

FIG. 5—300 DIAMETERS—SAME AS FIG. 4 EXCEPT UNDER CROSSED NICOLS

The dark portion of this and other slides taken under crossed nicols is the isotropic feldspar ground mass.

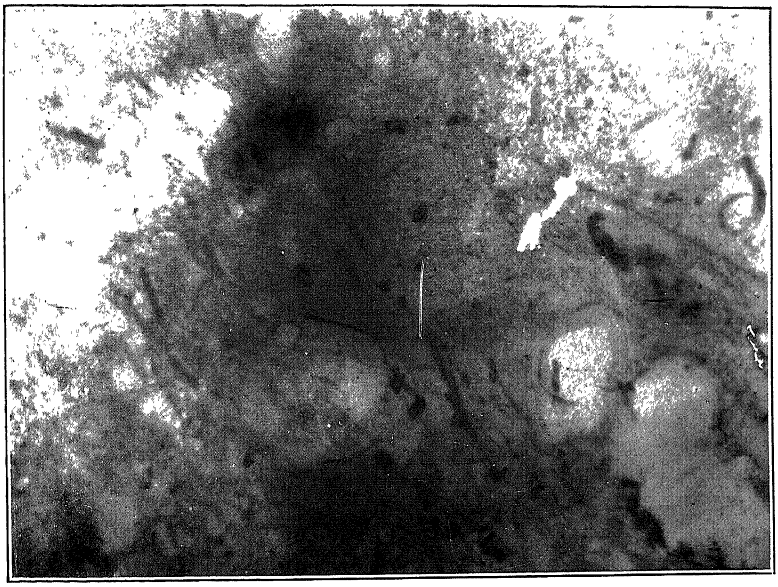
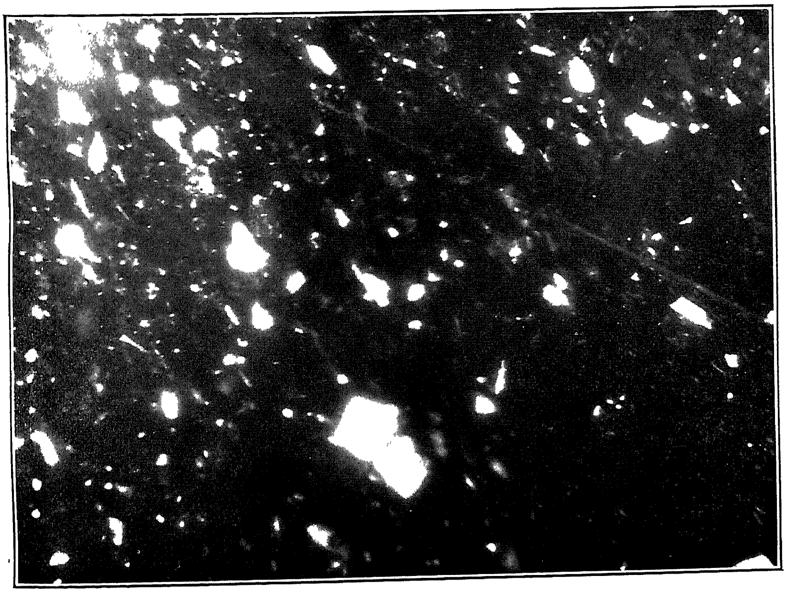


Fig. 6—300 Diameters—Slide From an Insulator That After Passing Factory Test Was Stored in a Warehouse Two Years

Failed under flash-over voltage when taken out—The crack A is apparently due to expansion and contraction and is not a cooling crack as no crystals have formed in it—The cement in this insulator was very porous and showed the effects of expansion and contraction—Note the flow structure crossing the picture between the large quartz crystals and the alignment of sillimanite along this flow.



[PEASLEE]

Fig. 7—300 Diameters—Same as Fig. 6 Under Crossed Nicols

Note the great amount of fine quartz crystals and their concentration due apparently to flow in the mass before or during firing.

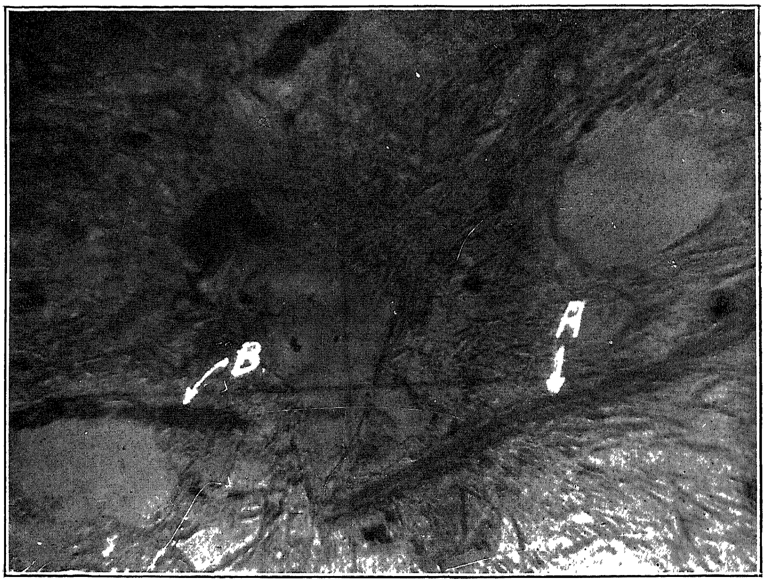
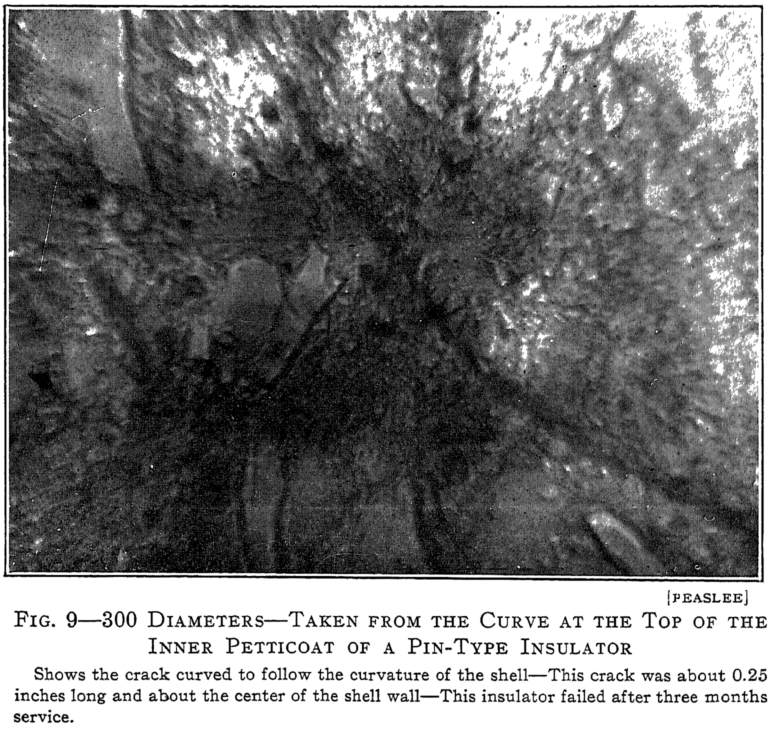


FIG. 8—300 DIAMETERS—TAKEN FROM A SUSPENSION UNIT REMOVED FROM THE LINE FOR LOW MEGGER READING AFTER ELEVEN MONTHS SERVICE

The insulator punctured below flash-over, 60 cycles—Note the very large quartz crystals partially resorbed and the large sillimanite crystal B and the short crack A lined with fine sillimanite crystals—This crack is only 0.01 inches long but is a potential weakness and this insulator was liberally sprinkled with them.



[PEASLEE]

FIG. 9—300 DIAMETERS—TAKEN FROM THE CURVE AT THE TOP OF THE INNER PETTICOAT OF A PIN-TYPE INSULATOR

Shows the crack curved to follow the curvature of the shell—This crack was about 0.25 inches long and about the center of the shell wall—This insulator failed after three months service.

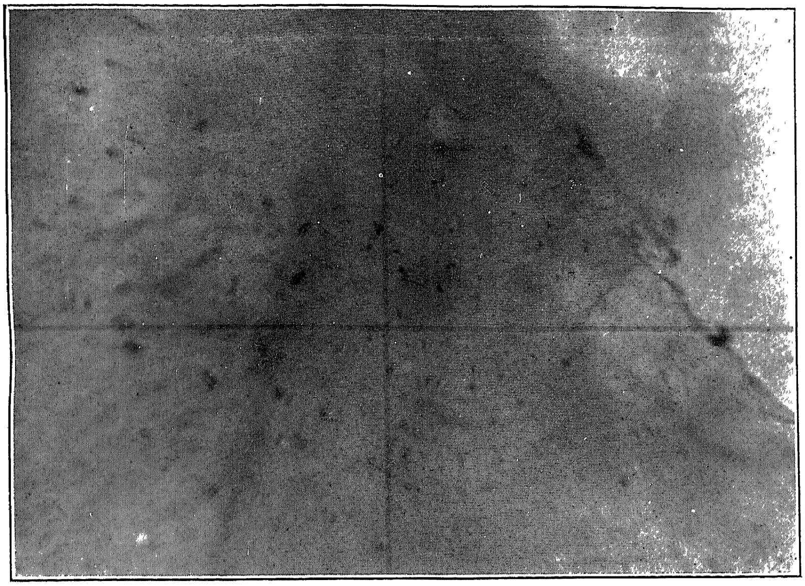
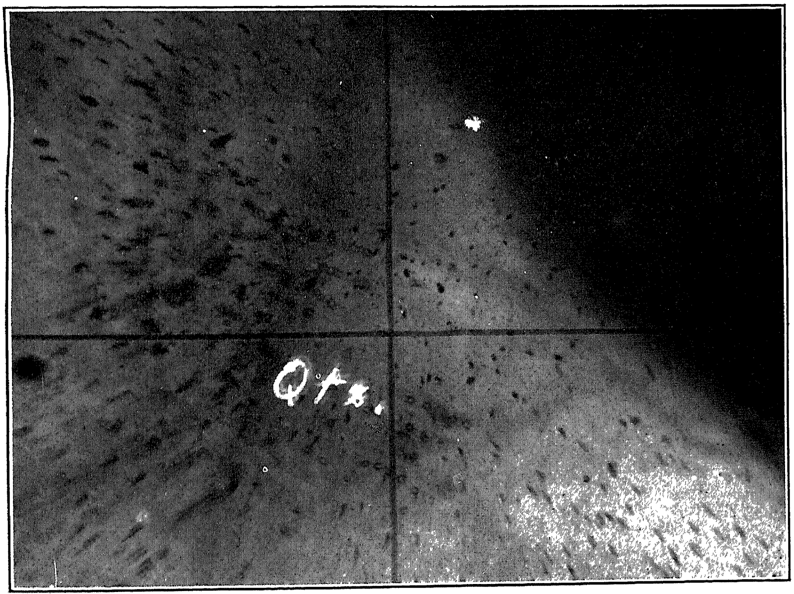


FIG. 10—300 DIAMETERS—FUSED QUARTZ MANUFACTURED IN SEARCH FOR A SUITABLE INSULATOR MATERIAL



[PEASLEE]

FIG. 11—300 DIAMETERS—SAME AS FIG. 10 EXCEPT UNDER CROSSED NICOLS

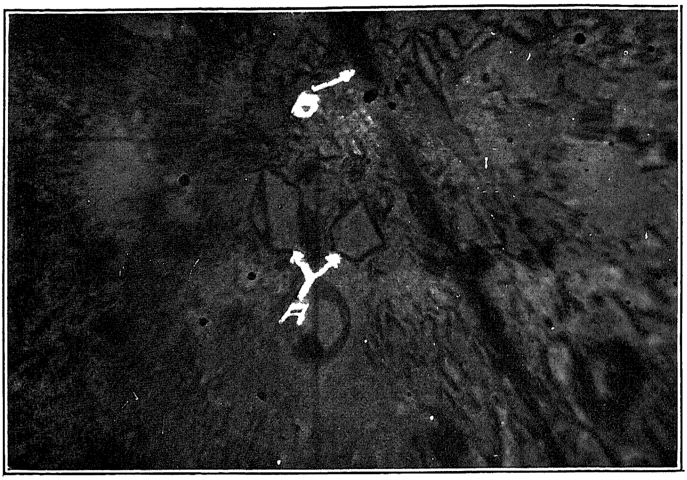
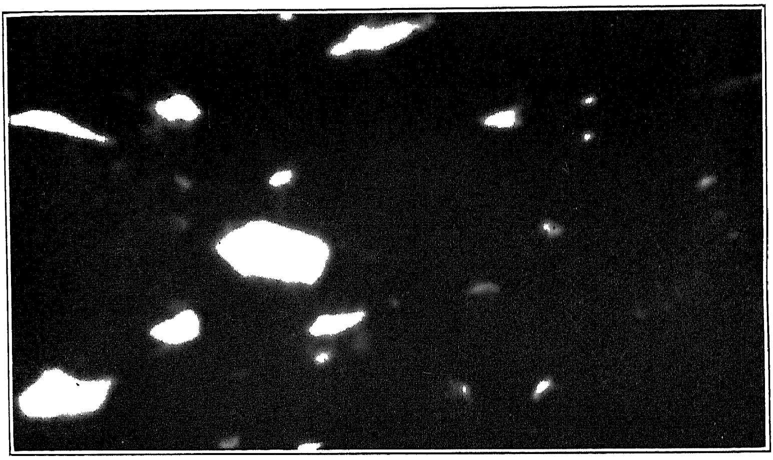


FIG. 12—300 DIAMETERS—CRACK IN A SUSPENSION UNIT THAT WAS REMOVED FROM THE LINE ON ACCOUNT OF LOW MEGGER READING



[PEASLEE

FIG. 13—300 DIAMETERS—CROSSED NICOL VIEW OF SECTION FROM AN INSULATOR OF FOREIGN MANUFACTURE

Shows very clearly the isotropic ground mass (black) the anisotropic quartz (white) and the foggy masses of anisotropic sillimanite.

1. The porcelain insulator as commercially available today is not a success, it is deteriorating too rapidly both in and out of use and the gathered evidence points to certain inherent features of manufacture and composition that are responsible in large part for this unreasonable deterioration.

2. The design features may be overcome by a correct appreciation of the mechanism of breakdown and the application of correct principles of dielectric flux distribution.

3. It has been found possible to make and manufacture fused quartz on a laboratory scale and indications are that it can be done on a commercial scale for a permissible price, and it is hoped that the development of this and other studies now being made by many engineers may lead to a lessening of the enormous cost of the present insulator depreciation encountered by the operating companies.

The investigations herein mentioned have been made in the high-voltage laboratory of the Oregon Agricultural College and recognition is hereby expressed of the aid of Mr. A. Strieff, Assistant Instructor, and Mr. J. A. Hooper, a member of this year's senior class, both of whom rendered invaluable assistance in the many tests necessary to such an investigation.

Presented at the 332nd Meeting of the American Institute of Electrical Engineers, New York, June 27, 1917.

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EXPANSION EFFECTS AS A CAUSE OF DETERIORATION IN SUSPENSION TYPE INSULATORS

BY J. A. BRUNDIGE

Abstract of Paper

In seeking causes for the rapid deterioration which has been encountered in suspension type insulators, two leading hypotheses, viz., porosity and mechanical cracking through expansion effects, have been advanced by different groups of investigators. These are briefly outlined in the paper, after which the author presents data in support of the latter. Some of the operating problems attendant upon insulator deterioration are also discussed.

Introduction

THE CONTINUED and seriously high rate of deterioration occurring in all parts of the country in high-voltage suspension insulators of the cap and stud type, has caused investigators in their efforts towards the design of a durable insulator, to make a more careful study of the problem than heretofore, for the purpose of trying to fix the underlying causes of failure. Had the problem been a simple one, it would doubtless have been solved before now, and in view of the number of factors involved, especially from the manufacturing standpoint, it is not only natural, but in fact a favorable sign of active and thorough investigation, that somewhat different views may at first be held as to the predominating causes of insulator deterioration by those engaged in making such studies.

As a result a number of engineers, together with a majority of those concerned in insulator manufacture, while recognizing that porosity undoubtedly has been responsible for a considerable percentage of failures, hold that mechanical cracking of the porcelain, brought about by expansion effects (possibly assisted by heavy cable loads) is the principal cause for the trouble experienced. This hypothesis is equivalent to saying that unless the more recent designs and methods of assembling have sufficiently reduced the mechanical stresses set up inside the cap, troubles with this type of insulator may be expected to continue, regardless of the degree of perfection and freedom from porosity attained in the manufacture of the porcelain shells.

On the other hand, there are prominent investigators who have quite definitely taken the stand that practically all insulator deterioration is ultimately attributable to porosity, which either makes the porcelain weak electrically after moisture has been absorbed, or causes cracking through the capillary absorption and crystallization of salts taken up from the adjacent cement used in assembling, or causes a combination of both. The porosity hypothesis is in part based upon the results of experiments made with apparatus designed to measure extremely high resistances. With such apparatus, it has been shown that all porcelain is more or less conducting and that quite wide resistance differences exist between various specimens. A considerable variation in the resistance of certain specimens when subjected to treatment for the purpose of altering their moisture content has been also noted. These results, which are not surprising when considered in the light of the extreme sensitiveness of the instruments used and of the well known difficulty of producing well vitrified porcelain, have led some of these investigators to believe that all porcelain is inherently porous to such an extent as to make it unsuitable for manufacturing satisfactorily durable insulators.

The engineers of the company with which the author is associated in investigating the problem from an operating standpoint so far as somewhat limited facilities have permitted, and while having no desire to minimize the importance of porosity effects, have arrived at the belief that expansion effects are causing the major part of insulator deterioration, and the following notes and comments are given in support of this view. It is appreciated that many of the facts mentioned have only circumstantial value, but when considered together it will be evident that this belief is not without foundation.

GENERAL

Statement of Conditions Inside of Cap. Before referring to observed results, it is instructive to review the conditions existing inside the cemented metal cap of a suspension insulator.

It is well known that Portland cement after setting does not attain its ultimate hardness for a considerable period—usually months—but when this state is finally reached, it will withstand and transmit fairly large compressive stresses. Under these conditions, the porcelain inside an insulator cap is directly subjected to varying forces which it is poorly adapted to withstand. The temperature expansion coefficient of porcelain being only approximately one-half that of steel, it is perceivable that from this source alone stresses amply sufficient to cause damage must occur as the result of seasonal temperature changes. Expansion in the steel stud on hot days is doubtless responsible for much of the trouble.

But another and even more serious form of expansion takes place inside insulator caps where Portland cement has been used for assembling, the existence of which has not been generally recognized. This is the expansion and contraction of the cement itself that the influence of varying moisture conditions. Mr. A. H. White* in a series of experiments extending for a period of nearly five years, on both old and new samples of concrete and cement obtained from various sources, has shown that from a dry to a completely moistened state, neat cement briquettes will expand as much as 0.15 per cent, which is equivalent to a temperature expansion change in cement of nearly 270 deg. fahr. (150 deg. cent.). In addition, it was shown that the effect was gradually progressive, the samples exhibiting a slight increase in length with each cyclic change. Other investigators have also noted this behavior of Portland cement, the generally accepted explanation of which is that it exhibits reversible collodial characteristics to a limited extent, analogous to the softening and swelling up of glue in the presence of water. The progressive expansion taking place with the cyclic changes is attributed to the incomplete hydration of some of the particles of cement, which are initially sealed up and protected, but as the expansion and contraction goes on, more of these are exposed to moisture so that additional setting takes place, accompanied by a certain amount of permanent expansion. This expansion was clearly indicated in the experiments mentioned by the numerous hair cracks which generally developed throughout the mass of the sample briquettes.

It is believed by the author that the much greater deterioration which has been experienced with insulators hanging in the tension or horizontal position as compared with that occurring in those suspended vertically, may be accounted for by the fact that the cement in the former is much more directly exposed to the elements, thus affording greater opportunity for the taking up of water and the reverse, whereas the cement in units in the upright position is not subjected to such marked extremes; also because of the greater relative exposure of the horizontally placed units to the hot sun, resulting in their undergoing more abrupt temperature changes. The great difference in the rate of deterioration observed in the two positions cannot be wholly accounted for by the fact that the tension insulators usually support heavier mechanical loads. While it is believed that excessive loads accelerate deterioration, several instances have been encountered where tension insulators carrying loads less than those on adjacent suspension insulators have shown percentage failures from 2 to 3 times as great and equalling the failures observed in tension insulators generally elsewhere on the same systems.

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*Proceedings Am. Society for Testing Materials, Vol. XIV, Part II, 1914, p. 204.

DATA AND OBSERVATIONS

The following notes relate to data from megger and oscillator tests made during the last three years on transmission systems located in widely separated parts of the country embracing an aggregate of over six-hundred-thousand suspension insulator units. It may be added that in drawing conclusions from the megger tests, care has been taken to select only readings made under favorable conditions and by testers experienced in handling the instrument, who were cognizant of its limitations and of the precautions to be observed in its use.

Electrical Stresses Not a Necessary Factor in Deterioration. Owing to the greater electrical stresses to which the end units of insulator strings are theoretically subjected, it might be expected that these would fail in comparatively greater numbers if such electrical stresses were a material factor. With one exception, the data on which have not been fully confirmed, no evidence has come to the notice of the author that the end units do fail at a greater rate. The following record from oscillator tests made on strain insulators units from a 60-kv. line in 1916 may be taken as typical. Potential was applied at 100 kv. for 5 seconds:

Unit No.	Location of defective units in string (No. 1 unit next to conductor)			Total defective
	Wire A	Wire B	Wire C	
1	40	35	34	109
2	40	33	39	112
3	37	37	39	113
4	42	42	30	114
5	40	38	30	108

Another phenomenon of much more conclusive nature showing that deterioration takes place independently of electrical stresses is the loss often observed in insulators where they have been left for some time in the original crates. In one instance reported from the Pacific coast, it was found that the percentage deterioration in a lot of several hundred units left over from line construction amounted to practically the same as that occurring in the ones in service. The crates containing the excess insulators were of the usual open type and had been stored in a yard for a period of approximately four years. Tests were made by means of the megger and the loss totaled nearly 5 per cent.

Insulator Failures are Not Proportional to Degree of Firing in Kiln. Referring to a lot of 877 units subjected to oscillator tests in 1915, and which represented the entire commercial range of firing, it was found that failures did not occur any more frequently in underfired shells than among the others. In fact, the overfired shells were the ones which showed a tendency to fail in excess of the average. This would indicate that the failures observed were influenced more by mechanical strength of the porcelain than by the degree of porosity. The degree of firing was only roughly estimated by classification of the glaze colors. but the results shown in the following table taken in conjunction with like data from other sources are believed to be sufficiently definite to justify the above conclusion.

	Total number	Bad	Per cent bad
Black Green	176	37	21.0
Green black	30	7	23.4
Greenish brown	12	4	33.3
Black	191	48	25.1
Dark brown (normal)	210	40	19.1
Brown	231	35	15.2
Light brown	27	6	22.2
	877	177	20.2per cent

In the above, dark brown is considered to represent normal firing, the brown and light brown underfired, while the black, greenish brown, green black and black green were overfired, the last exhibiting blebs.

A more striking instance is the failure on a certain system of a large number of foreign insulators, the porcelain of which was exceptionally well fired and uniform in character. Although excellent service was afforded for the first three years, trouble afterwards became very serious and it is reported that practically all of these insulators have now been removed from the lines, after approximately six years operation. In this case, it has been fairly well established that the deterioration was due to cracking which was accentuated by a very unfavorable design of the head, including long embedded studs. Other insulators of American manufacture of a similar design have likewise failed on the same system in quite large numbers. It is interesting to note that the insulators of this design were prone to develop operating troubles on particularly hot days.

Rate of Deterioration. In testing with the megger it has been found that the great majority of the units read zero or nearly so when first discovered. The few units which did not read zero at first have, so far as observed, since changed in resistance but slightly and such of them as the author has examined show unmistakable evidences of porosity.

The rapidity with which deterioration takes place in insulator units is in many cases quite remarkable. On a certain high-voltage line, it is reported that insulators sometimes fall in resistance from infinity reading on a 2000-megohm megger to apparent zero during the course of a few days. An example is cited of an anchor tower carrying 120 units which had been gone over with the megger for removal of defective insulators. Four days later, the testing crew took occasion as a matter of interest, to remegger the insulators on the tower and were quite surprised to find that five more units had failed in the meantime. It is not stated whether any rains had occurred during the interval, but the weather had been quite hot.

That insulators suddenly fall in resistance from infinity reading on a megger to practically zero is believed to be indicative of cracks rather than of porosity. The absorption of moisture in a piece of porous ware is known to be slow process and there are no grounds for suspecting that at a certain critical point of saturation the resistance drops so quickly as has been found.

Porosity in Small Amounts not Believed to be Excessively Harmful. The theory which has been advanced that conducting areas are formed by moisture within the body is undoubtedly valid for those cases where the porosity is marked, but the author believes that the performance of the numerous pin-type insulators which have been in use for comparatively long periods and also of the Hewlett type units, some of which have been giving good service for approximately 10 years, shows quite conclusively that reasonably small amounts of porosity are not necessarily injurious. All of these older insulators are unquestionably of greater porosity than many of the suspension type manufactured comparatively recently and which have failed to a serious extent. It is true that pin-type insulators with cemented shells have failed in large numbers after considerable periods in service, but the cause can usually be traced to cracking from purely mechanical sources. An illustration is found on a 60-kv. southern line where the insulators, which were of the 4-part pin type, operated perfectly for ten years, after which over 25 per cent failed during one summer. Without exception these insulators were found to have cracks in the upper shell generally following the side tie-wire groove and sometimes extending radially out to the edge of the petticoat.

It has been determined by Prof. Creighton that where a porous insulator has become saturated with moisture, it is easily punctured by application of potential at ordinary frequencies. This fact led to the suggestion by others that insulators only slightly porous, after having been in service for several years so as to absorb moisture, might be discovered and weeded out before reaching a dangerous condition by subjecting them to a 60-cycle test at near flash-over voltage in order to line up the particles of moisture to form a conducting path, similarly to the lining up of particles of foreign matter between the electrodes in an oil testing set. Experiments were undertaken by a transmission company to apply this hypothesis, but the results obtained were negative in character so far as concerned substantiating the presence of porous ware. Such breakdowns as did occur could nearly always be traced to manufacturing defects in the porcelain.

Indications of Cracking. While the author has not had opportunity for the careful examination of any large number of faulty insulator units by the removal of the metal caps, such work has been done has afforded fairly consistent results. In order to remove the caps without disturbing the porcelain, a helical cut of suitable pitch is taken with a hack saw, beginning at the lower edge, which enables the metal at the side of the cap to be removed in the form of a strip, thus freeing the upper portion.

With the caps taken off it is still by no means an easy matter to locate the existing cracks unless they have become opened up somewhat or are discolored through age. It has been found, however, that in some instances the cracks may be identified by carefully examining the surface of the cement which has been in contact with the porcelain. The cement may show a fine crack corresponding to the crack in the porcelain, or only darkened lines which for lack of a better term have been called "weathering lines". Such lines, marked by arrows, are shown in Fig. 1. In this view, the metal stud has been tilted slightly away from the camera to separate the weathering line from the edge of the porcelain, thus bringing it into better view. The line on the cement in the cap section has been produced from the same crack, as will be seen by comparing the two.

While the cause for the weathering lines is not definitely known, it is probable that they are the result of ozone liberated by slight electrical discharges through the crack while the unit was in service. Such discharges undoubtedly occur and their presence is borne out by a slight alteration often observed in the appearance of the surface of the fracture in the porcelain which becomes glossed and faintly iridescent when held to the light at the proper angle. This faint play of colors is not found on surfaces of pieces broken after removal from service.

OPERATING PROBLEMS

Deterioration in insulators is proceeding at such a rate that it is debatable whether all suspension insulators of the cap and stud type now in service will not have to be replaced within a comparatively few years. Obviously this constitutes a very serious situation for the operating engineer and it is imperative that insulators approaching the end of their usefulness be detected and removed from the line before conditions develop threatening the continuity of service.

The question arises as to whether anything will be gained by subjecting all insulators in service to a periodic test at high voltage for weeding out those pieces undergoing incipient deterioration. It is apparent that if a test could be devised which would insure safe operation for a period of three or more years without the necessity for attention to the insulators in the meantime, much of the cost of insulator patrol and maintenance of the lines would be obviated. It is believed that a high-voltage test might prove useful should it be established that porosity is a serious factor. On the other hand, it is not likely that much

PLATE XXVI.
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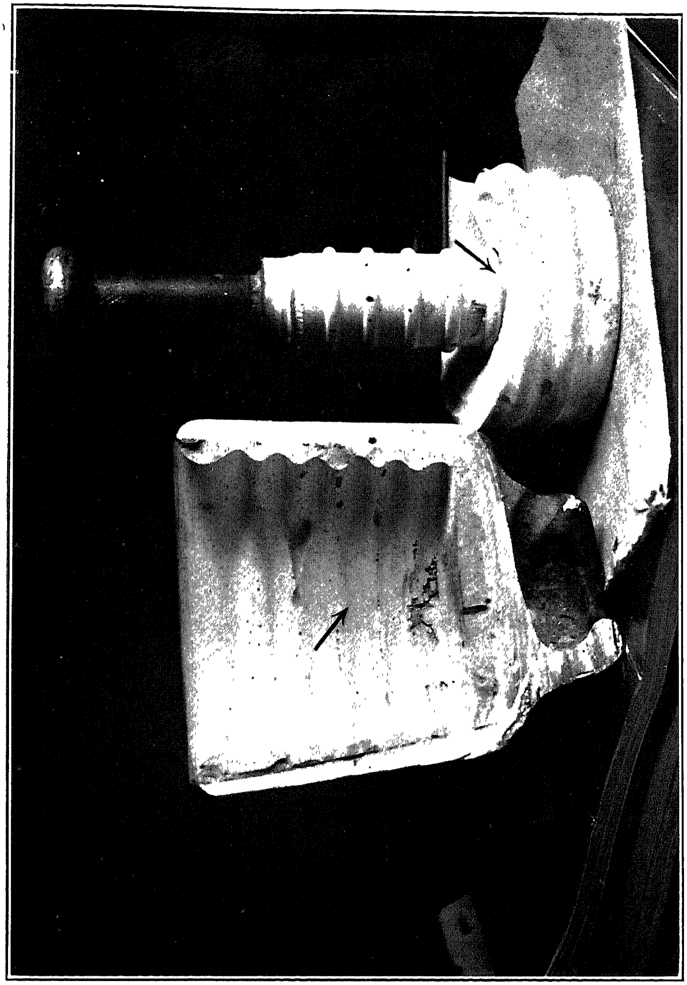

Fig. 1

[Brundige]

benefit would be derived from the testing of units in this way which are liable to deteriorate through cracking. It is known that each application of high potential will indefinitely continue to break down additional units and many of these would doubtless be of service for a considerable time before necessity arose in the ordinary course of events for their replacement. Moreover, failure by cracking proceeds at such a rapid rate that many of the insulators might make a good showing on test and fail very soon afterwards.

Conclusion

Summarized briefly, the reasons for believing that expansion effects are the principal cause for deterioration in suspension-type insulators are as follows:

1. The conditions existing inside the metal caps are sufficient to explain, through temperature expansion alone, much of the trouble experienced. Expansion occurring in the cement because of moisture absorption is of an even more serious nature.

2. No evidence of importance is at hand showing that electrical stresses have caused the trouble under discussion.

3. Except for bad cases of underfiring which are generally easily detected, losses of insulators in service do not bear any apparent relation to the degree of firing.

4. So far as observed, deterioration takes place quickly, such as would be expected during the formation of cracks. On the other hand, slightly porous ware absorbs moisture only very slowly, and were porosity the predominating cause for failure, large numbers of units would undoubtedly have been discovered in the intermediate resistance stages.

5. Cracks of more or less long standing have been identified in a majority of cases where it has been possible to carry on post mortem examinations on defective insulator units.

6. The relatively good showing made over long periods by pin type insulators having cemented parts and by Hewlett units, none of which represent as high development of the porcelain manufacturer's art as some of the later suspension insulators, would indicate that porosity is not the factor of most immediate concern.

It is to be borne in mind, however, that after losses from expansion effects have been more or less completely eliminated, porosity in porcelain may remain as a problem of considerable importance and that the work done in this direction by investigators

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It is to be borne in mind, however, that after losses from expansion effects have been more or less completely eliminated, porosity in porcelain may remain as a problem of considerable importance and that the work done in this direction by investigators will become increasingly valuable, unless perhaps a better material for insulators will then be available. Progress in overcoming expansion will doubtless include, as one of the steps, the elimination of Portland cement from the designs and this incidentally will be of aid in simplifying the problems arising from porosity.

Presented at the 332nd Meeting of the American Institute of Electrical Engineers, New York, June 27, 1917.

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PRESENT PRACTISE IN THE DESIGN AND MANUFACTURE OF HIGH-TENSION INSULATORS

BY A. O. AUSTIN

Abstract of Paper

As considerable time or a severe condition is necessary to show up serious insulator defects, the favorable line conditions on the earlier lines permitted the use of inferior material and designs. The apparently satisfactory operation of inferior insulators together with the large production necessary did much to retard improvement in the insulators, for improvement was practically impossible unless same could be accomplished without materially increasing the cost.

The desire to increase production and improve the material necessitated radical changes in the manufacture and equipment, so that the well equipped plant today is far different from the ordinary pottery which it resembled a few years ago.

The rapid development in the transmission field has materially changed conditions, and rendered much apparatus obsolete, the early insulator being no exception.

As causes of losses have become evident, means have been found to eliminate the serious effects of same. The recognition of the increased value of reliability together with the study of operating conditions has materially changed the insulator situation, so that the material going to the scrap pile today is more suitable for line work than the best product a few years ago. The loss from porosity has been reduced to a negligible quantity by improved firing methods and a closer selection.

To prevent the serious cracking loss noticeable on old insulators, has been the most difficult problem. To prevent trouble on old lines, it may be necessary to give the insulators a temperature, as well as an electrical test.

Trouble from this source on modern insulators is prevented by careful attention to the temperature gradient, increased mechanical strength to resist internal stresses, and a lowering of the internal stresses by means of an elastic joint.

The performance of the modern insulator is very gratifying, and its performance must not be judged by insulators which are really obsolete.

THERE IS often an impression that it is possible to eliminate all insulator troubles by the use of some new design, test or manufacturing method. In order to eliminate trouble, however, it is absolutely necessary that all depreciation be eliminated. The very nature of the dielectric, the large number, necessitating low cost, and the many hazards to which the insulators are subjected in operation gives us no right to expect that we can hope to equip a line with insulators which have absolutely no depreciation.

Although the depreciation for some one particular cause may be made negligible in the insulator, it is highly important that this desirable result is not obtained at the expense of a greater hazard in some other direction. The best line practise will be that which reduces the principal hazards as far as possible, but takes into account the hazards produced by depreciation, so that a system may be planned and operated accordingly.

From the operating standpoint the insulator problem is one of maintenance, for the state of the art is such that the hazards can be reduced to a negligible quantity where the fundamental factors governing reliability are given due consideration and the line is maintained in a good state of repair.

The length of time required to show up the effects of depreciation and the rapid development of the art together with the low cost of the insulator is largely responsible for the greatest losses.

Material improvements have been made in the design and manufacture so that the performance of the modern insulator cannot be judged by the performance of insulators made even a few years ago.

The production of a satisfactory insulator is no small problem, if the cost is to be comparatively low. Designing necessitates a very thorough knowledge of manufacturing conditions or their possibilities, for it must be remembered that an insulator part usually goes through from 20 to 30 operations and passes through the hands of over 20 operators. To produce the best results, it has been necessary to eliminate heavy labor wherever possible, so improved machinery has been adopted and the modern insulator plant has departed very much in appearance from the pottery and the methods which it followed only a few years ago.

Fig. 1 shows one of the mold conveyors where the ware is cured before taking it out of the molds. Equipment of this kind saves much of the heavy labor, permits of easy inspection and tends to produce uniformity in the insulator and at the same time results in a considerable saving both as to labor and losses in manufacture.

Similar work has been carried out wherever possible in the manufacture, Fig. 2 showing the ware as it is ready for drying. With equipment of this kind, five or six hundred insulators are subjected to practically uniform drying conditions on all sides and at the same time cracking and the handling cost is cut to a small fraction of what it was formerly.

The inspection of ware is very rigid at all stages in the manufacture and a large portion of the ware rejected today is as good, if not better, than the first quality material of several years ago.

It was only a few years ago, that it was difficult to obtain a test of 40 or 50 kv. on a single part without having losses of from two to forty per cent. The state of the art is such at this time that it is possible to test single parts at much higher voltages than large four-part insulators of a few years ago.

Fig. 3 shows some large insulator parts on test, under test conditions equivalent to 140 kv. The average loss even under severe conditions of this kind is seldom over one per cent. This has only been possible by careful systematic work. Many of the older lines are equipped with three- or four-part insulators which would have had an assembly loss of 40 or 50 per cent, if they had been given the above test which can be easily carried by a single part today.

In Fig. 4 is shown one of the assembly cars used for suspension insulators. A car of this kind handles 700 insulators and permits of their being cured under the best possible conditions and with a minimum amount of handling. The increased cost, owing to a very much higher standard, has necessitated improved handling methods at every point in order to keep down the cost which would otherwise be considered prohibitive.

Much has been written about the principal electrical characteristics of the insulator, so the discussion of design and manufacture will be largely confined to the two most important elements producing depreciation, cracking and absorption.

CRACKING

The cracking of insulators is by far the most serious cause of depreciation on most lines, and has been an important factor in the design of pin-type insulators for some years. In order that the method adopted to reduce this loss may be better understood, it is necessary to consider conditions as found on the line.

For the past seven or eight years, much cracking has been noticeable on pin-type insulators, some lines being entirely reinsulated while on others, it was necessary to locate the faulty material by visual inspection or by ringing out with a stick. To avoid this loss, pin-type insulators were replaced by suspension type in at least one instance.

The more recent cracking of suspension insulators has attracted far more attention than a similar loss in the pin type. This is largely due to the fact that they could be easily located on the line and to the higher standard of operation on most of the lines where they were used.

A curve which is characteristic of the depreciation for many insulators is shown by *A* Fig. 5. It will be noted that this curve rises rather rapidly for a short time due to absorption and then rises very slowly for five or six years operation when it jumps very rapidly due to cracking.

In order to see the effects of this depreciation, it is necessary to study the effect upon operation.

Curve *B* shows the operating hazard or probable interruptions for the depreciation shown in curve *A*. It is assumed that a four-part insulator will fail when three parts become bad. Curve B, rises very rapidly after six or seven years as the hazard increases in direct proportion as the cube of the depreciation. This curve shows why cracking is often very serious before its effect is noticeable on the operation.

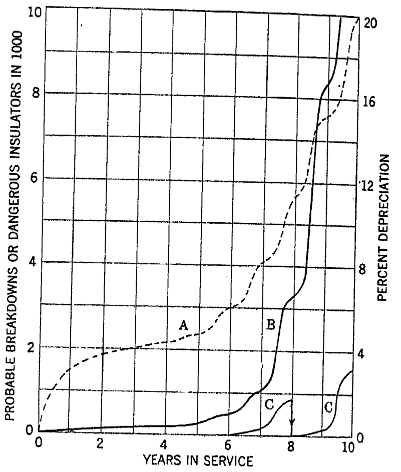

Fig. 5

Curve *C* shows the operating hazard where all defective material is removed every four years. While the removal of faulty material greatly reduces the hazard, it is seen that the cracking during the tenth year would cause 14 probable breakdowns in a lot of 10,000, although there might have been only one or two the year before.

The insulators which crack are apparently affected in no way up to the instant of cracking, hence, it is impossible to anticipate their failure by any practical electrical test. Porous material on the other hand, can be detected and removed before it becomes valueless.

Owing to the small margin of safety and the nature of dielectrics used for insulators, it is not reasonable to assume that depreciation can be entirely eliminated. A low depreciation is

PLATE XXVII.
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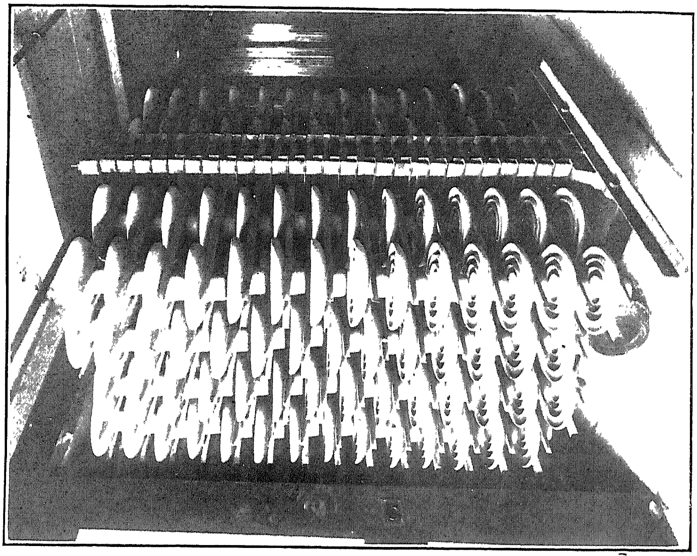

FIG. 2

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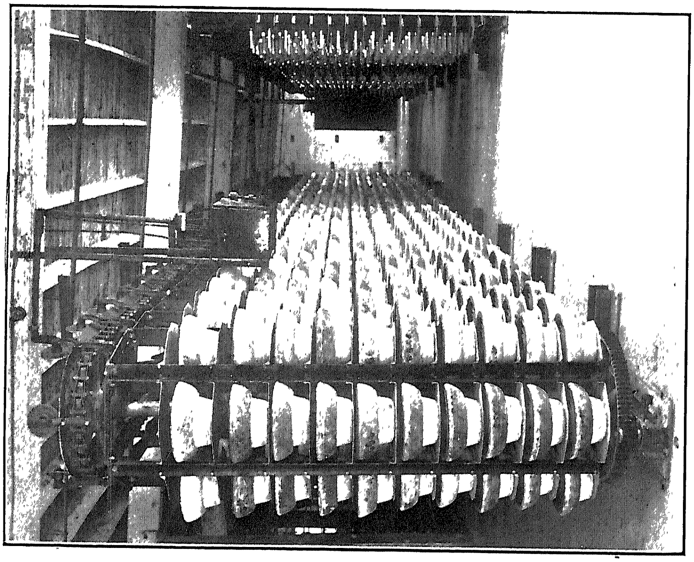

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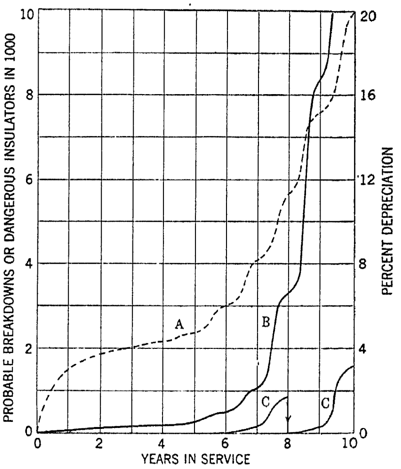

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A. [illegible]
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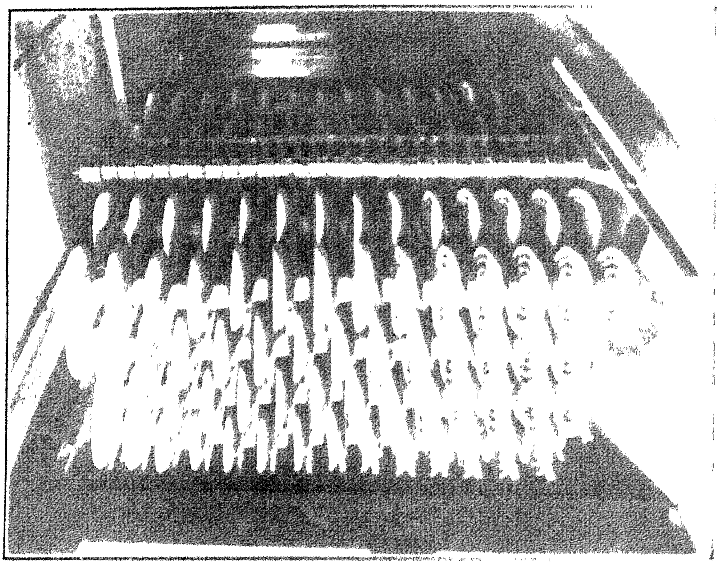
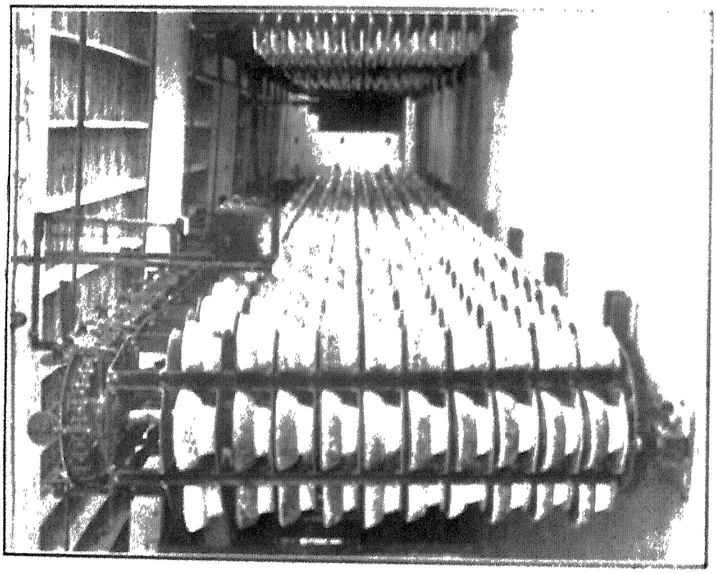




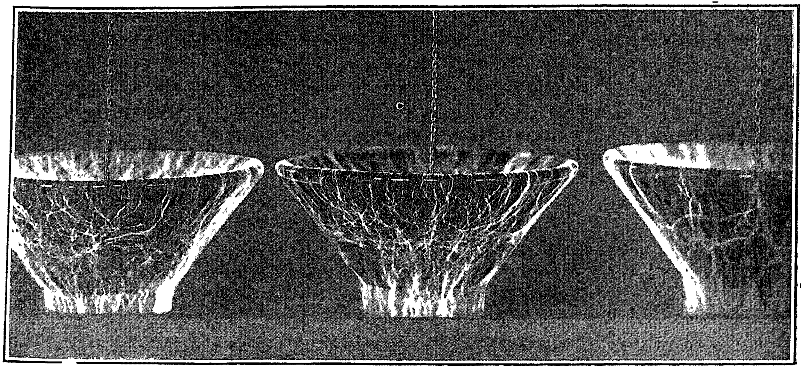


FIG. 3

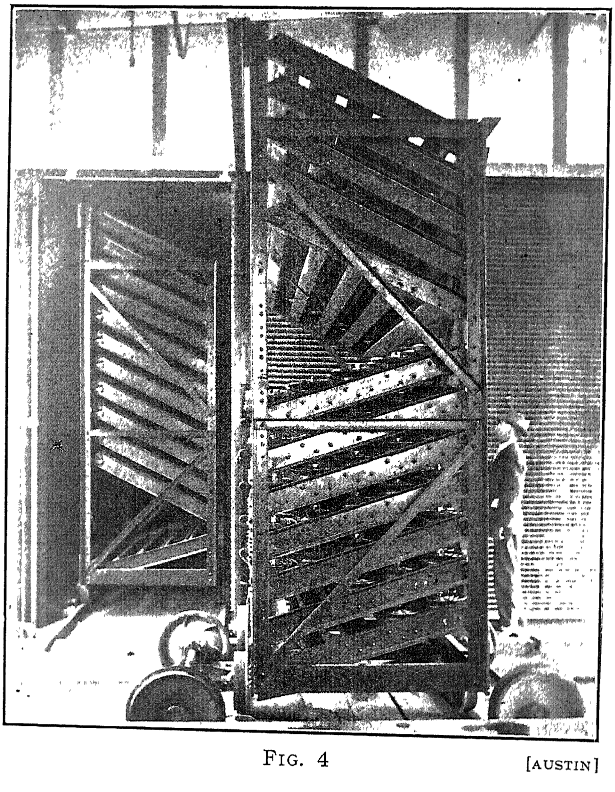


FIG. 4

[AUSTIN]

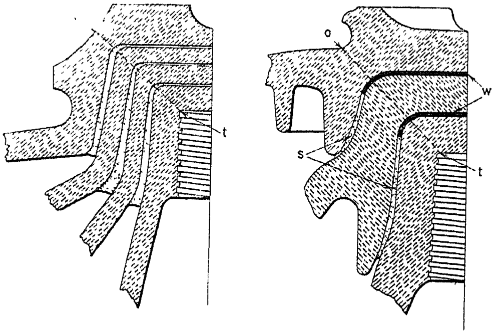
well worth while, however, as it permits the use of insulators of fewer parts with their smaller maintenance.

Examination of many insulators which have cracked on the line indicates the following:

1. That the largest and apparently the strongest insulators crack soonest.
2. That the size, shape and number of the cemented joints effect the cracking.

Old insulators when heated up slowly will often crack before they reach 150 deg. fahr. The parts which fail are the tops or shells outside of large cement spaces.

When old insulators are heated up and then have their tops sprayed with water similar to a thunder storm, a very much larger per cent of cracking takes place. This, however, is usually confined to the head of the insulator.



A FIG. 6 B

Insulators like that shown in *A*, Fig. 6 may have a very high loss on a single heating and cooling cycle. Designs similar to *B*, however, stand very much more severe treatment without loss.

There is a very noticeable cracking on the heads of insulators similar to *A* after five or six years weathering. If insulators are subject to the elements, it appears to make no difference whether they are in service or not.

When old insulators which are heated and sprayed with water, crack, the failure is announced by a sharp report, indicating a force of considerable magnitude. Insulators which have stood a heating and cooling cycle of higher range are little effected by a number of cycles of lower range.

These together with the absence of any electrical indication of weakness go to show that the cracking is due to a high stress rather than fatigue.

It has been pointed out that the stress may increase due to a slight expansion of the cement with time. This stress in itself may not be serious, but combined with others may cause failure. The cement adheres more firmly to the porcelain with time so that any adjustment by slipping may become less. It is also possible that the modulus of elasticity of the cement may increase with time. The accumulation of dirt may cause a greater leakage of current, heating up the inner parts, or the insulators may get hotter in the sun. The poorer heat conductivity of the cement is still another factor.

While it is true that porcelain or glass will fail at a lower ultimate if the stress is applied for a long time the stress required to cause failure is very high.

Under operating conditions an insulator may reach 150 or 160 deg. fahr. in the sun. Under these conditions insulator *A*, Fig. 6 will have a temperature gradient along the line *O—t* like that shown in *A* Fig. 7. If rain falls on the upper surface, the temperature may be represented by *A′* after a few minutes. Under similar conditions, the temperature gradient for insulator *B* is shown by *B* and *B′*.

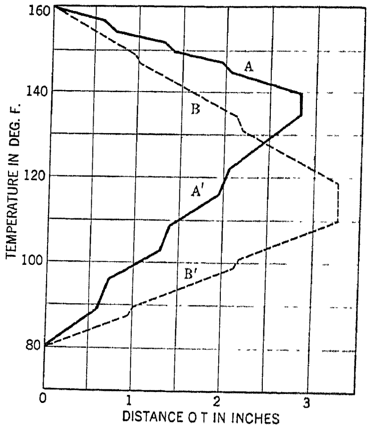


FIG. 7

It will be noted that the inner parts of the older type of insulator get very much hotter than those in *B*. It is further noticeable that the radiation of heat from the inner parts is less in *A* than in *B*, so that when the heads of the insulators are cooled to the same temperature, the temperature gradient will be much steeper in *A* than in *B*, so that the stress set up between the porcelain parts regardless of the cement will be very much higher in the older type insulator.

The cement has a higher linear coefficient of expansion than the porcelain and will produce a very high stress if of large cross section. The practise then of reducing the size and area of the cement joint may reduce the stress set up by the cement to easily less than half that for the older insulators.

The advantage in fewer cement joints and small heads on the insulators has been carried out extensively in the modern two-piece insulators for voltages up to 50 kv.

Much effort has been expended to incorporate the following in the modern pin-type insulator:

1. A few strong parts.
2. Small heads with corresponding cement sections and areas.
3. Minimum amount of nesting permissible with mechanical reliability.
4. Elasticity in the joint.

A few strong parts keep down the renewals when the line is cleared up for the loss will increase nearly as the product of the per cent loss on a part and the number of parts. This is important and will show up sooner or later on the line.

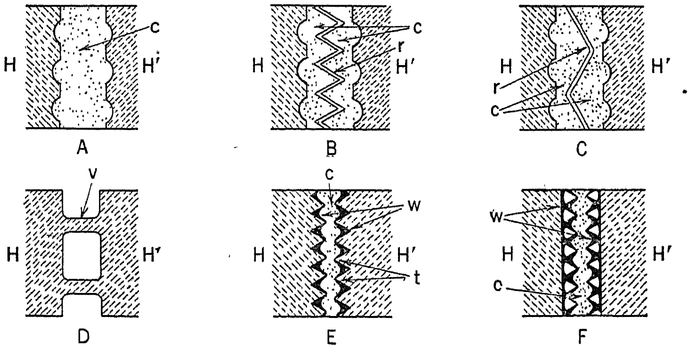


FIG. 8

The axial pressure tending to force the top out of the head in *A* is entirely eliminated in *B* by coating the ends of the shells by a wax *w*, or the use of a cushion.

The grip surfaces *S* are sanded and as the grip is of very high efficiency, a minimum surface may be used for mechanical reliability. Since this surface grips in all directions equally well, there is a tendency for the insulator to hold together even when badly damaged mechanically. This is particularly important for an insulator where the parts are not deeply nested.

ELASTICITY OF THE CEMENT JOINT

In Fig. 8 are shown several types of joints which are of particular interest in considering cracking. In *A* is shown the ordinary cement joint with scored surfaces held together by cement *C*.

In *B* and *C* is shown an elastic or yielding insert *r* placed in the joint. In *B* the possible movement would be greatest across the joint and in *C* lengthwise of the joint. This then is one method for regulating relative movement in the joint either axially or radially.

Although a joint as shown in *B* or *C* will relieve any internal stress, it is not good for handling heavy loads and the danger of the insulator falling apart and dropping the line is increased, so it is not used except in special cases.

The insulator joint will be satisfactory in so far as it fulfills the following:

1. Minimum area for a given grip.
2. Absence of slipping.
3. Elastic yield or ability to distribute a heavy load.

The coated sanded surface fulfills all these requirements to a marked degree, as will be seen by a consideration of *D*, *E* and *F*.

By reference to *D*, it is seen that any force between *H* and *H'* must be exerted through the small struts *V*. If the force is one of compression, the stress in the main members *H* and *H'* will be very small compared to that in the struts *V*, being in inverse proportion to their cross sectional areas. If there were a number of these struts, their area could be so proportioned that they would compress or crush and limit the stress at any point in *H* or *H'*. This method of connecting the different members would prevent looseness of the parts and would give a good stress distribution in practically any direction for the strain would occur in the small struts *V*.

In *E* is shown a practical method of accomplishing the stress distributing feature of *D*. The main insulating members *H* and *H'* are provided with ridges or projections *t*. If the joint is entirely filled with cement, conditions will be no better than in *A* so the surfaces are coated with paraffine or other material *W* which accumulates in the bottom of the grooves leaving the points only very thinly covered. The bearing of the cement is then confined to the tops of the projecting ridges or points which act in the same way as the struts in *D*. Under these conditions, it is seen that the stress per unit area at the points *t* and the adjacent cement will be many times that in *H* or *H'* so that the joint will give and relieve the main parts from heavy strain, produced by unequal expansion between the parts or cement.

A low coefficient of elasticity can be obtained in *E* by controlling the size and depth of the ridges or points and the coating used to regulate the effective area of contact between them and the cement.

While *E* fulfills the general requirements, the grooves tend to concentrate the stress and the failure of a point may start a crack in the body of the piece. It also has the further objection in having a small give in the direction of the joint.

The sanded surface *F* provides the projections similar to *E*, but the space filled by the wax is much greater. This joint not only limits the stress much better in all directions but is easily made and is free from the objectionable feature of scoring.

Ordinary surfaces cannot be covered with wax or paint to relieve the stress unless the load is very light, as the bearing may be concentrated in one or two points, causing a low ultimate or cracking.

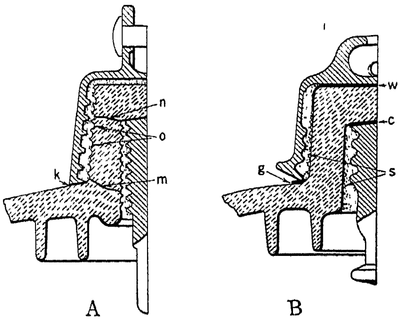


FIG. 9

By proper control of the sanding, it is possible to eliminate hard spots or undue concentration of stress in many insulators, giving a higher ultimate in the suspension insulators without the danger of cracking.

CRACKING OF SUSPENSION INSULATORS

The cracking of insulators was covered to a large extent in a paper on Insulator Depreciation, December 1914. This discussion outlined very briefly, the causes of cracking and methods of reducing same. In order to insure the mechanical reliability of the insulator, it is advisable that the cap bear on the flange to prevent the escape of cement while setting. This contact area is shown at *K* in Fig. 9. Cutting the cap with a slot just above this point or the putting in of new cement at *K* showed that the insulator could be heated much higher without starting a crack at *m*. This proved that the stress was due to the elongation of the cap and expansion of the cement.

A higher assembly temperature would tend to eliminate cracks *o*, but would do little good at *m*, if the cement at *k* expanded, so the first improvement was to reduce the bearing area at *K* using a cap like that shown in insulator *B*. Insulators cemented in this way stood many cycles between freezing and boiling without showing damage. In practise, however, it was found that cement which filled in below the cap would not loosen and drop out; furthermore, the cap came in contact not on a 45-degree line as intended, but out on the flange in some cases.

To insure positive relief at point *K*, rubber gaskets were first used at the lower edge of the caps, or they were dipped or painted with wax or asphaltum but various types of paper felt gaskets were finally used. Some of these gaskets were left in and some removed, the present practise being to remove the gasket.

The present practise is shown in *B*, Fig. 9. The head of the insulator is covered with a wax or paraffine *w*, a disk *C* is used at the end of the pin and a gasket at *G*, the latter being removed after the cement has hardened.

All grip surfaces are sanded and a moderate assembly temperature used so that dangerous shearing stresses will not be set up in cold weather. With machined or highly refined caps, it is possible to go to very high assembly temperatures. Without these refinements, however, a high assembly temperature is not advisable.

The use of the elastic sanded surface for limiting the stress together with the assembly shown in *B*, so materially improves conditions that the danger of cracking is certainly very much reduced or practically eliminated at least for a very much longer period.

The higher the mechanical ultimate in the insulator, the more difficult it becomes to design or manufacture so as to keep down the internal stress.

Although the sanded surface permits of the design of very much higher ultimates in the cemented type of strain insulators, it is probably best practise for very heavy loads to use insulators which place all the stress on a wood or fiber core similar to those shown in Fig. 10. Insulators of this type are in operation on the highest voltages and on loads up to 35,000 pounds and can be designed for practically any mechanical ultimate without making the conditions any more severe as to the dielectric.

While a close selection is made as to vitrification, it might be well to note that the heating of the insulator will melt the paraffine or wax on the head of the insulator which will impregnate the cement and porcelain, should it be porous.

Further refinements will be made from time to time, but it would seem that the present standard is such that they will have to be made without materially increasing the cost.

The coated sanded surface still offers refinements at minimum cost and it would seem that improvements in the near future will be along this line, particularly if it seems necessary to provide elasticity in the pin for better stress distribution.

ABSORPTION

The loss by absorption or porosity may vary widely on different systems and for insulators manufactured at different times. Since most of the porous material can be detected by the trained eye, the question naturally arises as to the conditions which permitted its installation. It is even more important to know how this can be guarded against in the future.

The routine tests made on insulators up until 1903 were made with dry electrodes and were often made on the nested parts at very low voltages as compared to present practise. Under these conditions very porous material would pass tests successfully and it is certain that some lots of insulators included fifty per cent of material which would be classed as porous with present standards.

Although tests were made from 1903 on, with water inside and outside the insulator parts, most of the insulator parts were tested inside of two minutes after applying the water and then packed and shipped to be assembled in the field later and installed without further test. Under these conditions some of the poorest material was eliminated but large percentages of insulator parts were frequently installed which furnished but little insulation by the time they were in place.

Much material was installed as late as 1905 which showed a fifty per cent loss on second test after soaking 24 hours. Most of this material was used on wood construction and on the Pacific coast where conditions were favorable; otherwise, very serious trouble would have resulted.

By the end of 1906 tests had not only been raised, but insulators were assembled in the factory and were beginning to be given a final test. This did much in some instances to eliminate porous material, but as assembled tests were seldom over 70 per cent of the flashover of the insulator, many pieces of porous material could pass the assembled test which would have been detected on a higher test voltage.

Many different designs were made in the next few years and in some instances desirable forming properties were developed in the clay to facilitate manufacture. These desirable properties were developed at the expense of vitrification with the result that some large lines were equipped with material which, although standing a high unassembled part test, was very weak dielectrically after installation on the line.

While there was a general improvement in the vitrification up to 1910, operating conditions were becoming more severe, which had the tendency to offset this improvement.

Thicker material was used on parts to give greater dielectric strength, and as it takes a considerably longer time for absorption to take place with increasing thickness, the greater weeding out of the higher test voltages was largely offset.

Much of the ware put out in 1909 and 1910 showed a loss of less than two per cent by absorption when examined several years later, although material was considered as vitrified over approximately a four-cone range, 80 deg. cent.

To obtain properly vitrified material which will not fail by absorption it is necessary,

1. That the body composition be such that it will vitrify to the extent that the pores will be sealed against absorption.
2. That all material be fired to the proper temperature and time for the body composition used.

To obtain the first condition many checks are necessary in order that uniformity be maintained. The general composition may be entirely satisfactory and spoiled in the manipulation due to a streak of off mixture from settling. This streak or a path left by the burning out of lint may absorb water and render the insulator worthless later. Material improvements have been made in the compounding and working of the clay in the last few years, so that some of the worst hazards cause but little concern at the present time. Better equipment had to be designed and placed in operation to eliminate the most serious cases of trouble, while additional checks were used to cut down others.

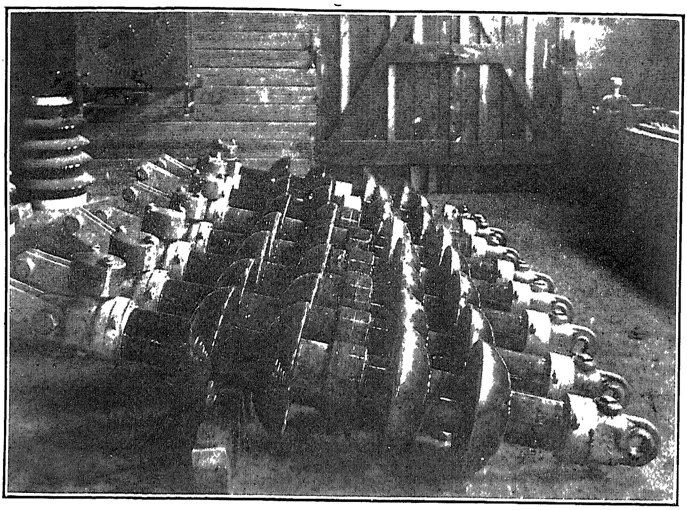


FIG. 10

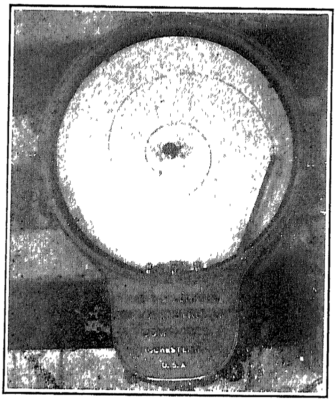


FIG. 11 [AUSTIN]

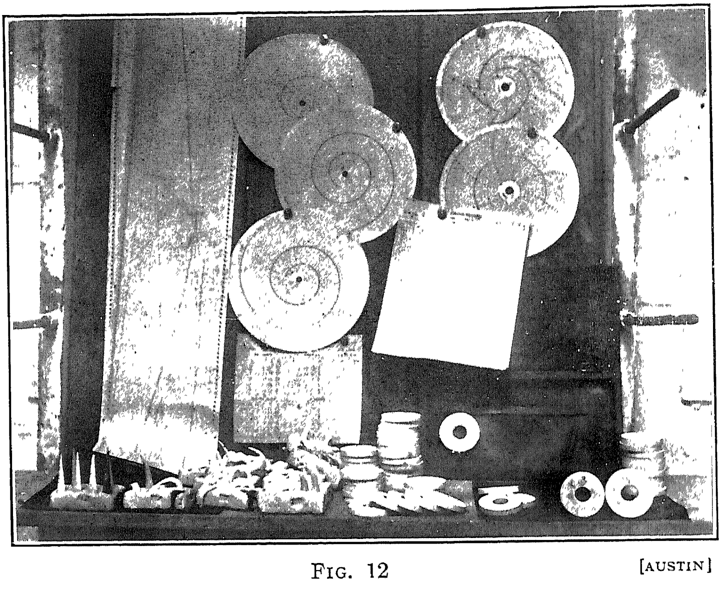


FIG. 12

[AUSTIN]

Uniform viscosity in the slip and time of pumping the filter presses were prime factors in eliminating streaks of off mixture.

FIRING OR BURNING OF THE WARE

The important factors in obtaining satisfactorily vitrified ware are:

a. A means of determining the firing history of each individual piece.

Where a glaze is used for the insulator which has a wide range of color for a slight difference in temperature, the selection as to temperature or firing history is fairly easy for the trained eye. Glazes such as slate, white, yellow, caramel or Albany slip glazes where the calcium content is too high, change little over a wide firing range and make selection as to firing difficult or practically impossible.

b. A body composition that is satisfactorily vitrified over as wide a temperature range as possible.

In guarding against porous material, the use of a body composition of such range of vitrification that proper selection can be safely and economically made inside this range is an important factor.

c. Close control of the firing of the ware.

Some body compositions have such a very short range of vitrification that a selection extending not over a range of two cones may include both over and under fired material.

Only a few years ago, the use of a glaze which had too uniform color over a wide temperature range together with a body composition of short range of vitrification, while apparently very satisfactory at the time was largely responsible for a considerable number of insulators becoming bad through absorption.

It is always possible to obtain good ware by selection where factors *a* and *b* are favorable although factor *c* is anything but good. Unless the firing control is good, it is not possible to make a close selection of ware without a heavy rejection which would materially affect the cost.

Much work was required to improve firing conditions so that the selection could be reduced from a four-cone range, 80 deg. cent., to practically a two-cone range, 40 deg. cent. Kilns had to be reconstructed so as to have less difference between the hottest and coolest places, and a method of feeding the fuel used which was largely independent of the personal element.

Recording mercury thermometers were used to indicate temperatures up to 1000 deg. fahr. These thermometers do not control the firing, but serve as a check of the method and give data for correction of same.

Reference to the chart in Fig. 11 shows that the rise in temperature is under perfect control, which is very desirable so as to eliminate firing checks in heavy ware. In addition to the use of mercury recording thermometers up to 800 or 1000 deg. fahr., a recording electric pyrometer is used to indicate the rate of temperature change in firing and cooling.

For indicating the finishing temperature, pyrometric cones are used and also disks or rings whose shrinkage can be accurately measured.

Fig. 12 shows a set of records and trials now used for a single firing. Until two years ago, the Seger cones and the experience of the kiln burner were the only guides in firing. Highly skilled firemen, even where gas was used for fuel were unable to produce a satisfactory result for the narrower limits until a system was adopted that was largely independent of the personal factor. Consequently the control was not nearly as good as at present.

The fact that much ware put out some years ago selected over twice the firing range of that at present had an exceedingly small percentage of porous material, would indicate that there is little cause for concern with present material, if factors *a* and *b* are at all favorable and a careful selection made.

The present practise of using much thicker porcelain tends to minimize the danger due to absorption, for it is possible that a drying out action may start before moisture has extended entirely through the part. The time of complete penetration is controlled by many factors, but may be regarded as increasing as some power of the relative thickness when comparing two pieces of the same vitrification.

Unless conditions are favorable, it is seen that resistance tests which require that moisture extend entirely through the piece to detect porosity may require entirely too much time unless made on thin pieces. Before tests could be made on thick pieces, they would be on the line.

It is the general factory practise to depend largely upon dielectric tests made before and after soaking or assembly. These tests indicate the point where even slight absorption takes place and the selection for vitrification is such as to keep away from the danger point.

TESTS

While a thorough discussion of tests is beyond the scope of this paper, those in general use deserve some attention.

With the thickness of material used in the present insulator, tests just below flashover are hardly severe enough. If, however, the regulation of the circuit is poor and the voltage is raised so that a violent flashover is produced, a very effective weeding out of poor material is obtained.

The test in Fig. 3 is of this nature, and is undoubtedly the best all around test method in use. A few insulators or a great many can be tested with certainty, which is no small advantage where thousands of insulators have to be tested daily.

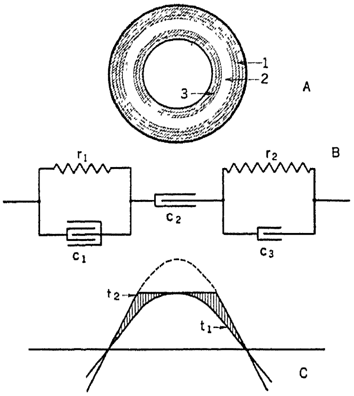


FIG. 13

A brief analysis of conditions in Fig. 13 will show why this test is so efficient in detecting material which is porous.

In A, the section of an insulator is shown which has absorbed moisture so that there are two wet zones 1 and 3 adjacent to the cement surfaces with a dry zone 2 between.

The porous zones may be regarded as condensers and resistances in multiple, which are in series with a condenser represented by the dry zone. The arrangement is shown diagramatically in B.

The impressed voltage under different conditions is shown in C.

t is the voltage on the series of zones at 60 cycles, the maximum voltage being just under flashover.

If the voltage is raised until the insulator flashes the effective voltage on the series will be represented by t_2.

The resistances r_1 and r_2 may be very high so that the porous zones will carry an appreciable portion of the total voltage on the piece. Owing to the phase displacement of the voltage over the different zones, the dry zone will be subjected to a large percentage of the total voltage.

It is evident that where t_2 represents the impressed voltage, conditions will be much more severe on the dry zone for the leakage over the resistance will be greater. That this is necessarily true is seen, for if the frequency was zero, zones 1 and 3 would, of course, carry no voltage, the entire voltage being concentrated on zone 2.

This shows that the lower the frequency or the flatter the wave, the greater the stress on the dry zone. In t_2 the rise in voltage is more rapid than in t_1 which tends to place a higher stress on zones 1 and 3 than in t_1, but owing to the greater length of time the maximum voltage is applied the dry zone will also receive more stress than in t_1.

It is evident that increasing the frequency to 200,000 cycles will give little time for the leakage of charging current through r_1 and r_2, so that a porous piece will act as though it were a perfectly good piece and the chance of elimination on test by failure of the dry zone is very much less.

In t_2 the insulator flashes and is subject to several surges during each alternation. This affects the concentration of stress on the dry zone C_2, but little, while subjecting the wet zones C_1 and C_3 to their proportion of the voltage, which they would carry if they were dry.

From this, it follows that a test made at normal frequency so that the insulator flashes heavily without picking up a power arc tends to place maximum stress on all zones.

The fact that the conditions of normal frequency as well as high frequency are present at the same time undoubtedly accounts for the effectiveness of this method of test in eliminating material which tends to become faulty under line conditions.

CONCLUSIONS

It is hoped that the above brief outline of insulator practise will indicate the possible differences between an old and a modern insulator, for it would appear that much concern is felt over what are chiefly the defects of old insulators.

The insulator is subject to many hazards and as the design is a compromise at best, it is not surprising that certain properties should be developed at the expense of others. It is gratifying, however, to note that the chief troubles have been along lines not looked for.

The rapid development of the art has rendered much transmission apparatus obsolete, and the insulator is by no means an exception. When in addition to this, we take into consideration that little is generally known even at the present time as to the conditions under which the insulator has to operate and the necessary properties in same to withstand these conditions, it is surprising that the insulator has reached its present state of perfection.

It has been a number of years since the operating voltage has been limited by the insulator, and the performance of many lines put in during recent years shows that the modern insulator has more than justified expectations even in the face of higher standards of operation and increasing severity of line conditions due to the growth of the system or mechanics of the line.

The fact that the design is a compromise calls for the most careful thought in the design and manufacture, and the successful line will be constructed with insulators which have been improved along lines which are shown to be necessary by time and experience rather than along radical lines that may incorporate elements which will cause serious trouble in unlooked for quarters after a few years operation.

It is much easier to locate the cause of trouble than it is to effect a cure, for the insulator works on exceedingly narrow margins, and it is difficult to find a means of strengthening the insulator for one set of conditions without materially affecting another.

It is well to note that the above paper deals chiefly with the means which have been employed to eliminate some of the worst defects without sacrificing the properties in other respects proven to be good.

The abnormally low depreciation and successful performance of well insulated lines under the severest conditions show that the modern insulator is very successful compared to the earlier product, and if properly made, the standard is such that further improvements are not warranted if they greatly increase the cost.

For old lines, it is well to consider the increasing hazard with time due to an increasing rate of depreciation. To maintain a given standard of performance, it is then necessary to either split up the system into smaller sections as the line becomes older or to go over the line at decreasing intervals of time to remove the faulty material. It is possible that considerable benefit can be obtained on some systems by putting the old insulators through a temperature cycle, as well as the usual tests, to eliminate faulty material.

The fact that some of the weakest points in the insulators were greatly improved by design or manufacture without materially affecting the cost, did much to insure progress for a considerable period when troubles were only occasionally noticeable. The present situation shows that these improvements give every promise of being not only very valuable, but necessary for a long life and are worth many times the effort required to produce same.

Discussion on "The Insulator Situation" (Peaslee), "Expansion Effects as a Cause of Deterioration in Suspension Type Insulators" (Brundige), and "Present Practise in the Design and Manufacture of High-Tension Insulators" (Austin), New York, June 27, 1917.

Harris J. Ryan (Abstracted by J. C. Clark): The rate of failure of the cap-and-stud type of suspension insulators by cracking in the porcelain caps is due largely to two temperature factors, namely:

1. *The range of temperature* through which the insulators are carried by heat from the sun or atmosphere.
2. *The total number of temperature cycles* that the insulators have been put through.

To understand this better, consider what happened to a particular batch of insulators that were received from the factory in 1912. This batch numbered 166. It was divided into two lots; those of the one lot numbered 74 and were labeled A; the others numbered 92 and were labeled 1-13 A. They are listed in the first two columns of Table I. On delivery, those of the A-lot were stored four years in a place where they had undergone only slight changes in temperature from day to day and from season to season; those of the 1-13 A-lot were placed in service four years on a transmission line where the sun and atmosphere put them through daily cycles of temperature changes and further changes in temperature from season to season. After the four years, *i.e.*, in June 1916, all were sent to the laboratory for durability studies. High-duty megger tests prior to December first, 1916, determined no failures among the A's and 8 failures, or 8.7 per cent among the 1-13 A's. Tests by 60-cycle spark-over voltage (much as specified in Mr. Austin's present paper) applied for 10 seconds or less, Nov. 29 and Dec. 5, 1916, caused no failures among the A's and 7 failures, or 7.6 per cent among the 1-13 A's. None of the 74 A's and of the surviving 77 1-13 A's appeared to be porous to an appreciable extent. They seemed to be sound in all essential respects.

On the conclusion of these preliminary tests, both lots were placed on the ground in the laboratory yard, caps down, disks horizontal and left to "weather." Effects, if any, caused by changes in the cement due to absorption and evaporation of moisture were maintained active by the winter rains. In their absence water was poured into the stud cavities from which it evaporated, twice each week. The minimum winter and maximum summer temperatures to which they have thus been subjected were about 0.0 and 50 deg. cent., respectively. The daily temperatures ranged through a few degrees minimum to about 35 deg. cent., maximum. When this yard treatment had extended through 192 days for the A, and 196 for the 1-13 A-lots they were retested by megger and 60-cycle spark-over voltage. Again none of the A-lot failed while 2 failed by megger

SUSPENSION TYPE INSULATOR TESTS

TABLE I—YARD TREATMENT

Series	A			Service 1-13 A			L. A.			B		
Figure		3			3			4			4	
Maker		A			A			B			B	
Date of mfgr.		1912			1912			1912			1916	
No. tested		74			92			98			58	
No. treated in yard		74			77			79			57	
Yard, days		192			196			224			189	
Dates: Month	6	12	6	6	11	6	11	3	6	6	12	6
Dates: Day	21	5	14	21	29	12	9	3	20	14	5	11
Dates: Year	16	16	17	16	16	17	16	17	17	16	16	17
		‡			‡		‡				‡	
Failed 20-kilovolt megger	0	0	0	5	3	2	2	0		0	0	0
					**			**	*			
Failed S. O., cap		0	0		7	6	17	4	17		1	0
Failed S. O., disk		0	0		0	2	0	0			0	0
Failed prior to yard treatment		0			15			19			1	
Failed after yard treatment		0			10			21			0	
Failed after yard treatment %		0.0			13.0			26.6			0.0	
Failed total, No.		0			25			40			1	
Failed total, %		0.0			27.2			40.8			1.75	

Series	Service B. L.			Service C			Service G			H		
Figure		4			3			9				
Maker		B			A			C			C	
Date of mfgr.		1913			1913			1908			1916	
No. tested		92			82			59			15	
No. treated in yard		84			69			57			15	
Yard, days		186			168			201			201	
Dates: Month	6	12	6	7	12	6	7	11	6	7	11	6
Dates: Day	14	11	14	7	29	14	19	25	13	8	25	13
Dates: Year	16	16	17	16	16	17	16	16	17	16	16	17
		‡			‡			‡			‡	
Failed 20-kilovolt megger	2	3	0	0	1	0	0	0	0	0	0	0
			*		†	*						
Failed S. O., cap		3	16		12	5		2	0		0	0
Failed S. O., disk		0	2		0	0		0	0		0	0
Failed prior to yard treatment		8			13			2			0	
Failed after yard treatment		18			5			0			0	
Failed after yard treatment %		21.5			7.2			0.0			0 0	
Failed total, No.		26			18			2			0	
Failed total, %		28.3			22.0			3 4			0.0	

Series	J			Q₁			Q₂			Service S		Totals
Figure		5			7			8		3		
Maker		A			D			D		A		
Date of mfgr.		1916			1916			1916		1912		
					short cap			long cap				
No. tested		92			47			47		102		858
No. treated in yard		92			47			44		91		786
Yard, days		200			235			235		129		
Dates: Month	8	11	6	10	3	6	10	3	6	2	6	
Dates: Day	16	22	9	28	3	19	28	3	19	10	18	
Dates: Year	16	16	17	16	17	17	16	17	17	17	17	
		‡		‡			‡			‡		
Failed 20-kilovolt megger	0	0	0	0	1	0	0	0	0	1	2	22
											**	
Failed S. O., cap		0	2	0	0	2	2	3	5	10	22	136
Failed S. O., disk		0	0	0	0	1	1	0	1	0	0	7
Failed prior to yard treatment		0			0			3		11		72
Failed after yard treatment		2			4			9		24		93
Failed after yard treatment %		2.2			8 5			20.4		26.4		
Failed total, No.		2			4			12		35		165
Failed total, %		2.2			8 5			25.6		34.3		19.2

* 2 in each lot failed from 30 to 70 kv.—required for flashover 85 kv.

** 1 in each lot failed from 50 to 70 kv.—required for flashover 85 kv.

† 6 in each lot failed from 50 to 65 kv.—required for flashover 85 kv.

‡ Date upon which yard treatments began.

and 6 through the caps and 2 through the disks failed under spark-over voltage, making a total of 10 failures, or 13 per cent, for the 1-13 A-lots, due to the yard treatment. The total failures as a result of all tests was 0.0 per cent for the A's and 27.2 per cent for the 1-13 A's, and of these 25 per cent failed in a manner to indicate that the ultimate cause was cracking in their porcelain caps.

Remembering that the two lots were alike originally, and that their yard treatments were simultaneous and identical, the immunity from loss for the A-lot and the heavy loss for the 1-13 A-lot present a difference in durability that must be due to differences in their treatments during the four years that occurred after they were delivered by the factory and before they were received at the laboratory. Those of the A-lot were stored in the dry at a nearly even temperature. The 1-13 A-lot were in service on the line subjected to electrical and mechanical loading stresses, to temperature changes produced by the sun and atmosphere and to moisture more or less salt laden. Megger tests showed no evidences of failure due to moisture absorption by the porcelains. Mr. Peek has called our attention to Mr. Nicholson's having found[10] by increasing the mechanical loads applied to suspension insulators from 45 to 80 per cent of the ultimate that a corresponding increase occurred in the percentages which punctured from 5 to 80 per cent. He found that all punctured on the application of flash-over voltage as the ultimate mechanical load was approached. No transmission engineer has brought forward definite results showing that the ordinary mechanical-loading stresses cause these insulators to fail. Much evidence exists which indicates that failures are rarely caused by such loading stresses. An engineer in charge of one of our largest 100-kilovolt transmissions said recently that, strange as it may seem, those insulators which were left on the lines without mechanical loads have shown a greater percentage of failures. There are numerous reliable results which show conclusively that few sound insulators of the type and form here considered are injured electrically in service. When thus injured, most, if not all, are easily detected by inspection and laboratory tests. For example, it is well understood that the voltage duty carried by the insulator next to the line is highest, that it diminishes as the position-distance of the insulator unit from the line conductor increases to about one-half of the highest value for the insulator unit that is adjacent to the tower. Contrary to this arrangement of voltage duties, Mr. John A. Koontz found that the insulator units located next to the 100-kilovolt line conductors in suspension strings fail at the lowest and those next to the towers fail at the highest rates.

It is to be remembered herein that all of the units in a suspension string project into the shadows cast by those above them except the one next to the tower which is exposed to the full heating effect of the sun, that the one next to the line conductor has a slight advantage over the rest through cooling by any upward air currents that may occur. Mr. Koontz's results were obtained through megger tests of 39,000 insulators largely of the A, B and G, Table I, class, on a 100-kilovolt line. The tests were made in 1914 with a 1000-volt, 2000-megohm-range megger. The actual results are given below:

TABLE II

	Number	Per cent
Total number tested, on lines, in power houses, substations and switching stations	39,000	
Total number found defective	1,397	
Total percentage found defective		3.56
Numbers and positions in strings:		
No. 1 (At top of suspensions, or nearest tower in dead ends)	314	25
No. 2	257	20
No. 3	228	18
No. 4	295	23
No. 5 (next to line conductor)	175	14
		100

In 1915 Mr. Woodbridge reported[1] that "between September 1, 1912 and November 15, 1914, there were 22 cases of insulator failures on the lines (of the Sierra and San Francisco Power Co., Stanislaus River Power House to San Francisco, 134 miles) from all causes other than mechanical and known lightning". The lines were equipped with 5-unit, Locke Cat. No. 273, two-part porcelain insulators, operating voltage, 104-kilovolts. The two-part porcelains of these insulators are conical and their thermal characteristics must differ considerably from those of the disc-units listed in Table I. Even so, the results are a help in eliminating the electrical and mechanical loading factors and in isolating the failures due to cyclic daily and seasonal temperature changes. Two (2) only of the 22 failures occurred during the seven months of late fall, winter and early spring when the lower and more uniform temperatures prevail; the remaining 20 failures occurred during the five months, May to September, wherein prevail the higher and more widely fluctuating temperatures. Fourteen failures occurred in a 10-mile, fog-pervaded stretch on the west shore of San Francisco Bay wherein the temperatures are made to fluctuate over wide limits during the summer months by a succession of calm, clear atmospheres alternating with cool fog-forming ocean breezes that come in through the Golden Gate. Twenty miles below this section the lines extend west across San Francisco Bay through a gap in the Coast Range, thence across the narrow Livermore Valley, through the *Midway Hills*, on across the wide and evenly warm San Joaquin Valley and the foot-hills of the Sierras to the power house. Five of the remaining eight failures occurred among the Midway Hills where the temperatures must fluctuate considerably, being the region of temperature conflicts due to topography, direct heat from the sun, cool breezes from the ocean and hot reverse winds from the great valley. Mr. Woodbridge says "only one failure occurred in the foot-hills of the Sierras, in which section the line reaches its highest elevation, approximately 3000 feet and where thunder storms are most frequent".

These results of Messrs. Woodbridge and Koontz eliminate the working electrical and mechanical stresses as the cause of numerous sporadic failures and they strongly indicate such cause to be due to cyclic temperature changes, because the failures were most among those insulators that were continually being subjected to the widest temperature changes. The conclusion that the failure of suspension insulators by cracking is not due to electrical and mechanical duty is further strongly supported by the rapid failures of the LA lot to be taken up later and which had never been in service. There remain, therefore, the difference in treatment as to cyclic changes in temperature and to moisture content of the cement. It would seem, if further growth or swelling of the cement caused the cracking in the porcelain caps of the 1-13 A-lot, that the abundant moisture-absorption cycles which all were put through in the yard treatment would have caused at least a few to fail among the A-lot. Such conclusion obtains further strong support through the fact that all insulators in this class fail most rapidly during warm dry summer weather, especially in dead-end strings facing the sun and in country of light rainfall and hot summers. The engineer in charge of insulator tests on the lines from which the S-lot were removed has written: "during the extreme hot weather which we had on June 14, 15 and 16 (1917) the insulators on the northeast side of the mountains, just beyond the point where we commenced testing, began to fail on both circuits of the line causing a complete interruption to our service over the tower line. Those which failed on the fifteenth were repaired on the afternoon and evening of that day. However, the next day was a little warmer and more failures occurred which caused the line to go down a second time. The same performance was repeated on the afternoon of the seventeenth when the temperature in the neighborhood of the insulators on the tower probably reached 125 or 135 deg. fahr. It was observed that the majority of the strings of the insulators which failed were on the dead-end towers and were turned up with the bottom of the insulators toward the sun during the hottest part of the day. After further investigation of the conditions prevailing at the time the particular insulators failed, I found this was not true in all cases."

Thus it follows that the only factor which remains to account for the difference in the failures by cracking during the yard treatments of the A, 0.0 per cent, and 1-13 A-lots, 13 per cent, is the difference in cyclic temperature changes that they were put through prior to such yard treatment. The indication is, therefore, that when the insulators which are liable to fail by cracking have been put through 1000 ± cycles of temperature changes ranging to 10 ± deg. cent., they will fail by cracking at an accelerating high rate. We may call this result *thermal fatigue*, since many temperature variations in the insulators have produced changes of some sort in their makeup, so that they will fail rapidly when the temperature changes are continued. The results in Table I were obtained through durability studies that were begun at Stanford for the California Power Companies one year ago.[2,3,4,9]. This table is virtually an extension of Table I of the paper by Clark that appeared in the TRANS., 1916, p. 1453.[3] In Clark's table and on the photographs, therewith, the insulators and their manufacturers were distinguished by corresponding letters. With the addition of the LA and S-lots, the insulators and results that are listed herewith in Table I are the same as those listed in Clark's table and are distinguished by the same letters. The results as a whole may now be considered.

Origin and Age of Insulators. The same manufacturer furnished the A, S and 1-13 A-lots in 1912, C lots in 1913 and the J-lots in 1916. They are identical in make-up except that in the J's the cement for attaching the metal caps and studs was applied for a portion of the length, only, of the porcelain caps obviously for the purpose of reducing thermal expansion stresses. Another manufacturer furnished the LA, BL and B-lots in 1912, 1913 and 1916 respectively. These lots are identical except for the hardware of the LA's which had been altered so as to shorten the pitch-length and to provide arcing beads at the base of the metal caps. The G and H-lots were furnished by another maker in 1908 and 1916 respectively and are essentially the same in form and construction. Another manufacturer furnished the Q_1 and Q_2 lots in 1916. They were identical except that the caps in those of the former were two-thirds of the length of the latter.

Service, Location and Extent. The insulators in the 1-13 A, BL and G-lots were placed in service shortly after their receipt from their manufacturers, on 100-kilovolt lines, mainly at low altitudes in central California with moderate temperature changes. They were in service *four*, *three* and *six and one-half years* respectively, prior to their arrival at the laboratory. Those of the C-lot were in service *three years* on a 100-kilovolt line mainly at low altitude in southern California with less moderate temperature changes; and of the S-lot, *three years*, on a 60-kilovolt, high-altitude line in southeastern California with extreme temperature changes.

Storage, Location and Temperature Changes. Those of the A-lot were stored at a temperature "that varied but little" in a boiler house in the bay region of central California for four years; *i.e.*, from the time of their receipt from the manufacturer to a year ago when they were sent to the laboratory. Those of the

LA-lot were left four years in crates and corded in the open in southern California where there is a succession of hot rainless summers and where they were subjected to severe daily and seasonal temperature variations. The BL's, J's, H's and Q's were not stored having been delivered at the laboratory a year ago direct from their makers. The G's were in service six and one-half years, in storage with moderate temperature variations one and a half years, and then delivered to the laboratory a year ago.

Results in Regard to Porosity. No insulators that appeared to be porous were given this yard treatment. Although alternately wet and dry during most of two hundred days in the yard none altered its insulation resistance in a manner to indicate porosity of its porcelain. Furthermore, in advance of such yard tests, few in these lots had failed because of porosity. It must be remembered that they were selected by their contributors and sent to the laboratory for durability studies on the general understanding that they were sound and fully fit for service.

Results as to Failures Caused by Cement. Although the cement, particularly around the studs, was subjected to many cycles of water absorption and evaporation, no results so far have been obtained to indicate that the swelling of the cement and its thermal expansion are important factors in bringing about the failure of these insulators. If any had failed from either or both of these causes, some of them would have been found among the A, B or G-lots which differed from the rest only by not having been fatigued with temperature changes.

Results in Relation to Failures Due to Mechanical Loading. None of the insulators under our observation failed in a manner that indicated the initial cause to be ordinary or moderate mechanical loading. Next to the highest percentage of failures was encountered in a lot that had carried no mechanical loads after leaving the factory. On the other hand almost the lowest percentage of failures occurred in the batch that had carried a 100-kilovolt line in actual service for 6.5 years. As already stated Mr. Nicholson found four years ago that high mechanical loads will cause many suspension insulators to puncture on the application of flash-over voltage. This in the light of the facts now presented, is a result to be expected. If cracks in the porcelain caps are deepened by temperature changes and the mechanical stresses they induce it would seem that heavy mechanical loads should produce much the same result. It is evident that Mr. Nicholson's studies should be continued so as to obtain the combined effects of temperature changes, mechanical loads and applications of flash-over voltage. It may well be that high mechanical loads and numerous and wide temperature changes will cause many of the suspension insulators now in common use to fail rapidly.

Results as to Failure by Cracking. All failures under tests that followed the yard treatment were observed to be due to cracking.

Of the 92 that failed in a total of 786 after yard treatment, the high-duty megger found 5 (porosity or non-porosity thereof not yet fully determined) and the application of spark-over voltage for ten seconds or less caused 6 to fail through local cracks in the disks, and the remaining 82 to fail through cracks in their caps. (When an insulator failed under the spark-over test immediately after it was found to possess a normally high insulation resistance the result was assumed to be due to cracking.) Thus of the 93 that failed after yard treatment, 88 are known to have failed through cracking, and it is likely that further study will reveal that most if not all of the remaining 5 failed through the same cause.

That this large amount of failure through cracking was due to a form of "fatigue" brought on by a large number (1000±) of cycles of temperature ranges of 10± deg. cent. is determined specifically as stated in the beginning by the results obtained from the A and 1-13 A-lots. While the A's lost none, the 1-13 A's lost 13 per cent after a yard treatment of 196 days due as we have seen to the fact that the former had endured only 200 ± temperature cycles while the latter had gone through 1700 ± cycles. Other factors than cyclic temperature changes, which might be assumed to account for such results, have been eliminated.

The B's and BL's have duplicated the results obtained from the A's and 1-13 A's except that the B's are a year and the BL's four years old. Both came to the laboratory a year ago, the B's direct from the factory and the BL's from a power line after three years of service. They differed during yard treatments of 189 and 186 days respectively only in the age of their porcelains and cements. The results from the A's and 1-13 A's have demonstrated that difference in age, as such, can not be much of a factor where there is so little difference in design as that found herewith. If the difference in the ages of the cements is neglected as a factor then it is seen that the comparison of the results by yard treatment of the B's and BL's forces the same conclusion; viz., that they are of a type and make of insulator that failed rapidly by cracking produced by temperature changes after they have been put through 1000 ± cycles of temperature changes, the failures having been 0.0 per cent for the B's and 21.5 per cent for the BL's.

The results from the LA's insist upon the same conclusion. Twelve thousand of them were stored in such a manner that they underwent in southern California 1500 ± less moderate temperature cycles than those produced by the sun at low altitudes in central California. At the close of such storage they were found to have lost by ordinary megger, 5 per cent and by one-minute, 50-cycle, flash-over voltage test, 16 per cent or a total of 21 per cent. Out of a batch of those that survived these tests and were sent to the laboratory last fall 98 were tested. Two were found to have failed by high-duty megger and 17 failed on the application of 10-second spark-over voltage, making a total of 19 insulators or 19.4 per cent. These laboratory tests were made early in November, 1916, while the megger and one-minute flash-over tests at the place of storage had been made during the preceding July, an interval of 100 ± days. The numerous failures through initial laboratory tests were manifestly due to the remaining severe summer temperature cycles over and above the preceding 1500 ± cycles that this lot of insulators had been through. After yard treatment from November 9, 1916 to March 3, 1917, affording 114 moderate temperature cycles due to that many mild winter days, the spark-over test caused 5 per cent to fail. From March 3 to June 20, 110 more days of severer temperature cycles, thermal fatigue progressed so that the spark-over tests at the close of this period caused 21.5 per cent more to fail. Thus it is seen that the less moderate spring and early summer temperature cycles caused the insulators in this LA lot to deteriorate at a rate that was more than four times the corresponding rate for the more moderate temperature cycles of the preceding winter.

Of the C's, after 3 years of 100-kilovolt line service under the less moderate temperature cycles in the lower altitudes, one failed on megger test and 12 by 60-cycle spark-over voltage, a total of 16 per cent, immediately in advance of the yard treatment. After 168 days in the yard 5 insulators, or 7.2 per cent of the number treated in the yard, failed on spark-over test. Of the S's after three years in service on 60-kilovolt lines at higher altitudes and severe temperature cycles, one by megger and 10 by spark-over, total 11 per cent, failed immediately in advance of the yard treatment. After 129 days of such treatment 2 failed by megger and 22 by flash-over, a total of 26.4 per cent of those treated.

It is especially to be remembered that the A's, 1-13 A's, C's and S's are identical in make and nearly so in age, that the A's from even-temperature storage lost none, and the rest from service with its temperature changes lost heavily under the yard treatments, and that other factors that might be assumed to account for these results have been shown to be almost wholly absent in the circumstances.

The foregoing insulators, though of two different makes, are evidently in much the same class as to cracking. All were found to have a certain liability to fail through cracking after encountering 1000 ± temperature cycles. The remaining Q —Q_2, J and G—H lots are in differing classes of their own in respect to thermal fatigue or cracking. The Q's had not been in service, came to the laboratory direct from the factory and promptly developed thermal fatigue and failure by cracking. They came in two varieties distinguished by the length of their caps, the Q_1's having two-thirds the length of the Q_2's In advance of the yard treatment of 235 days none of the Q_1's failed by megger or spark-over and thereafter 4 insulators, or 8.5 per cent, failed by spark-over. Correspondingly for the Q_2's, three failed in advance and 9 insulators, or 20.4 per cent, failed thereafter. These results are of value showing that the short caps in surrendering some ultimate mechanical strength have gained considerably in durability during the first year following their manufacture.

The J's have the same manufacturer as the A's, 1-13 A's, C's and S's. Outwardly the porcelains appear to be much the same in all. The J's differ from the rest, however, in some departures in design and assembly that have evidently been made with a view to lessening the failures due to thermal-expansion strains. They arrived at the laboratory a year ago, encountered no failures in advance of a yard treatment of 200 days and thereafter two of them, or 2.2 per cent, failed on spark-over test. It is of particular interest to note here that these insulators have not as yet made as good a durability record as have the older A's from the same manufacturer.

The G-lot were purchased 9 years ago. They were in service on a 100-kilovolt line for 6.5 years, and then stored for 1.5 years before their arrival at the laboratory a year ago. They are among the earliest insulators employed for 100-kilovolt transmission on the Pacific coast and are of the so-called flat disk type. Microscopic sections were made of porcelains taken from each kind of insulator mentioned in these tables. Such sections have been examined in the field of the polarizing microscope under the guidance of Dr. A. F. Rogers, Professor of Mineralogy and knowledge gained through the available technical papers.[5,6]. Studies are being made also of the mechanical designs of these insulators. The porcelain in the G-lot of insulators differs somewhat from the porcelains in the other lots, listed in Table I. Its grains of flint are finer and more uniform in size and distribution. The temperature of burning appears higher, though we are not sure of this as yet. It stands lowest in abrasion and almost the highest in hardness as determined by tests made in the materials laboratory by Mr. W. L. Parker.[7] Although designed about ten years ago the caps and hardware of these insulators were so formed that the mechanical stresses produced by temperature changes may have been less than in the general run of later designs. To date, although by far the oldest, these insulators are showing the best all-around durability characteristics of those that have been received at the laboratory. Exception hereto is made of course for the latest makes wherein there has not been time enough as yet to develop the facts as to durability. So far none have been lost among this G-make of insulators after a yard treatment of 201 days. The insulators in the H-lot are the same in design and make-up except that they were received from the factory one year ago.

In studying these results as a whole in relation to the design and make-up of the insulators from which they were obtained it is not apparent that faults in design are the only cause of thermal fatigue or cracking; it is apparent to a limited extent that the qualities of the porcelains, cements and methods of assembly are also causes for the extent of this large class of insulator failures. No studies have been made as yet in the reduction of ultimate mechanical strength that occurs when the maker seeks to lessen the mechanical stresses due to temperature changes. This is a matter of importance as Mr. Austin demonstrates.

It is a matter of speculation as to whether the cracks in the porcelain caps or disks were started and deepened by temperature changes or existed in the porcelains in advance (through faults occurring in manufacture) and were subsequently deepened by temperature changes. There are evidences to support either assumption and perhaps both may be correct. If the former assumption only is correct the remedy for cracking is to be looked for in design; if the latter only is correct the remedy is to be found by improving the porcelain; and if both are correct the remedy will require the adoption of both expedients.

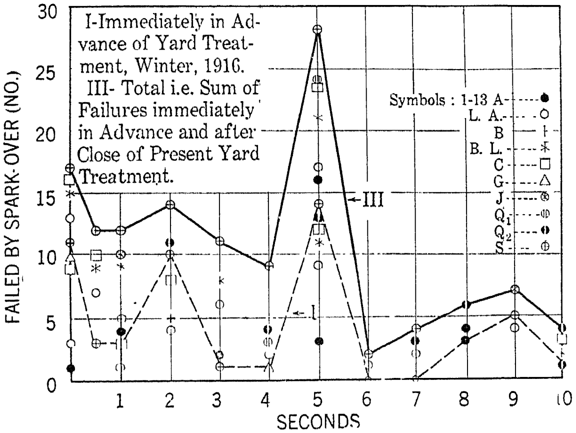


FIG. 1—SUSPENSION TYPE INSULATOR TESTS
Failures in Relation to Duration of Spark-over Test

The times in seconds of application of 60-cycle spark-over voltage in relation to the numbers of insulators of the several lots that failed in advance of and after the close of the yard treatment are plotted in Fig. 1. The corresponding integral failures are plotted in Fig. 2 showing all of the failures that occurred under the spark-over test applied for any interval of time of 10 seconds or less.

The conclusion herewith was stated at the beginning, and in effect thus: The rate of failure of suspension insulators of the cap-and-stud type, by cracking, is due largely to the number and range of natural temperature cycles that they have been put through.

The third paragraph of the Brundige paper constitutes virtually a discussion of the recent papers on insulators by Woodbridge, Clark and Ryan.[2,3,4]. Exception is taken to the statement of the author that we "have quite definitely taken the stand that *practically all insulator deterioration* is ultimately attributable to porosity". The stand as actually taken was quite different as anyone may see in reading the conclusions stated in those papers,[4] viz: "Recognition of porosity as a contributing cause of suspension insulator failures has not lessened the importance of design features that reduce cracking through differential thermal expansions and failure through electrical overstresses and the heat of heavy flash-overs." A year ago when our present laboratory studies were undertaken it was known through the earlier Brundige paper[8] and the experience of transmission engineers generally that the great majority of suspension insulators which failed were cracked in the caps. Abundant evidences were also at hand which showed that the qualities of porcelain and cement are factors that bring about failure by cracking or otherwise as well as the thermal-mechanical effects.

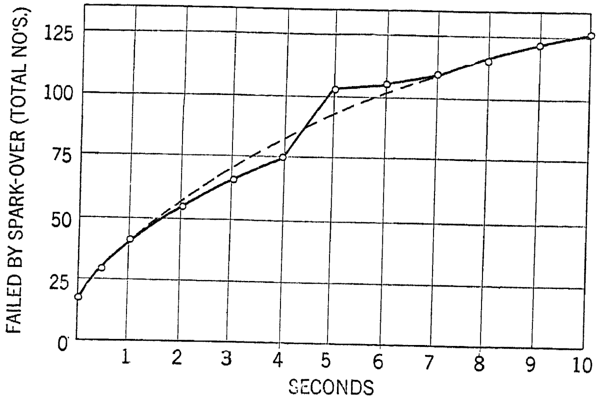


FIG. 2—SUSPENSION TYPE INSULATOR TESTS
Aggregate Failures in Relation to Duration of Spark-over Test

To segregate these factors, studies were first made of the qualities of the porcelain used in the construction of a variety of suspension insulators in fair numbers with all means then available. Spark-over tests were not employed because of the need of knowing as much as possible about the qualities of the porcelains before injuring many of them by the application of high voltage. This order of studies was adopted also to develop the meaning of results by megger test. The papers were prepared in October and November, 1916, and covered the initial laboratory studies just specified. We were wholly unprepared to estimate the relative proportions of insulators that fail from one cause or another and we made no effort to do so. Later on we have undertaken to determine effects due to thermal-mechanical actions subject to control by design and setting-up.

The present papers have their origin in the transmission practise, the research laboratory and the factory. They are of high value. Strangely enough the practise paper lowers and the factory paper raises the estimate of the importance of the quality of porcelain for high-voltage insulators. This is the result of an oscillation that has occurred in the process of each coming to the point of view of the other and going a little beyond. Let there be no mistake,—porcelain to be used for the cap-and-stud type of suspension insulators, must be non-porous, tough and strong as indicated in the factory paper. Likewise durability improvements have surely been made in these insulators when the mechanical stresses in the porcelain produced by temperature changes have been materially lessened without losing ground seriously on other accounts.

In testimony of the value of Mr. Austin's glaze-color method for scaling the degree of burning applied to each individual insulator piece, the following results of tests are given: Four unmounted suspension insulators were received from the factory that illustrated clearly the way the method works out in practise. They were labeled A, B, C and D in the order of their degree of burning and color strength, A and C being the darkest and lightest in color and burned to the highest and lowest temperatures, respectively. They were tested for absorption or porosity by noting the rise in temperature produced by a 5-minute application through the caps of 10 kilovolts at 325,000 cycles, undamped, using mercury electrodes without corona formation. These radio-frequency tests were made before and after brief periods of immersion of the outer surfaces of the caps in potable water. A few high-duty-megger measurements were made to note any relation that might exist between degree of burning and insulation resistance. Except where the porosity is very great no changes in insulation resistance would be produced by a few hours of water immersion:

TABLE III.

Shade	On arrival		After 11.5 hrs. water immersion, rise in temp. deg's C.	After total of 16.5 hrs. water immersion	
	Rise in temp. deg. C.	Mega-meg-ohms 20 C.		Mega-megohms 20 C.	Rise temp. deg. C.
A, dark	4.6	0.70	4.5	0.60	4.5
B, medium dark	4.9	0.70	5.2		
C, medium light	5.0	0.67	5.1		
D, light	13.6	0.63	26.1	0.57	39.0

J. Cameron Clark: It appears that Capt. Peaslee is rather optimistic about the possibilities of the quartz insulator. Its physical characteristics appear very fine, for example, it can be heated very hot and plunged into cold water without cracking. However, I find well-informed ceramists rather uncertain about the permanence of fused silica as a material for insulators. There are certain peculiar inversions in the structure of the material which are not generally well understood, but which the ceramists understand, and which take place rapidly at very high temperatures. On the other hand, it is thought possible that the material may change *slowly* and become brittle at lower temperatures than even 100 deg. cent. It seems to me we ought to have the benefit of the experience of users of quartz in other arts as to the permanence of the material.

In the introduction to Mr. Brundige's paper, he states that some people are attributing practically all the insulator trouble to porosity. Both Mr. Ryan and myself wish emphatically to disclaim taking that position. In fact, our experiments were, from the point of time at least, largely concerned with *attempts to get water into insulators.* Those experiments show that most porcelain is decidedly not porous at all, *i.e.*, it has no apparent porosity, or porosity due to open-pore systems. Yet, of course, this does not minimize the disastrous effect that a small percentage of porous units can have. The conclusion that perhaps most porcelain has no appreciable percentage of open pore systems is simply based on the fact that after months of soaking it has not been found possible to change the insulation resistance of the porcelain. Again, in the cases of some insulators which looked suspiciously low in insulation resistance, no change was found, even after drying them several hours at 150 deg. cent.

In the paragraph entitled "Insulator Failures are Not Proportional to Degree of Firing in Kiln," Mr. Brundige concludes, since he could detect slightly more failures among over-fired units than among under-fired ones, that porosity was of lesser importance among causes of failure. I would very much prefer to see a 60-cycle test to determine which of these units were porous, rather than the kind of test that was used. Perhaps some of that material had absorbed moisture to quite a large extent, which would very readily have been shown up, upon the high-voltage megger test, and still the insulator would easily have passed the test which was applied.

As to the rate of deterioration, Mr. Brundige draws the conclusion that it must be cracking that has used up the insulators, because of the fact that some of them, after having passed the 2000-megohm-megger test have gone down within a few days, showing that there was no appreciable insulation resistance left. I think it is important for us to remember that a 2000-megohm megger is not by any means adequate to detect all of the varying degrees of porosity in insulators. Instead of a 2000-megohm megger, it should have been a two-million-megohm megger, one having one thousand times the range of the one mentioned by Mr. Brundige. The resistance of a good porcelain unit is of the order of one million megohms, or as we called it in our papers in January, one megamegohm. Dry, sound units range roughly from a half million megohms to two million megohms, depending on the design, but if they are without cracks, or absorbed moisture they will be of the general order of one million megohms.

Obviously, if you apply a 2000-megohm megger to detect units which have varying degrees of porosity, you will pass, as good, many of those which are practically gone, catching only those which are really not worth talking about at all. When we appreciate the fact that a 2000-megohm unit is down to a point where it has only 0.2 of 1 per cent left of its original resistance, I think it is not at all surprising that it may go in a few more days. It has probably been going down for a year or two, and by the time when we catch it with a 2000-megohm megger it really does not have anything left. It is entirely possible that if Mr. Brundige had dried out these units that were detected by a 2000-megohm megger, he could have raised them so that they would have passed any ordinary 60-cycle arc-over test or oscillator test. Regarding porosity in small amounts, it should be remembered, I think, that there are two different kinds of porosity, either or both of which may exist in a given porcelain. Porosity may exist in very small sealed up voids, in which case it may be in evidence only by reduction of the density of the porcelain; or, it may appear in open, communicating pore-systems. I believe that we may have perfectly good units wherein the porosity by density is six, seven, or eight per cent, but the porosity in such units exists entirely in very small sealed up voids. Obviously, an appreciable percentage of the other porosity means disaster.

I do not believe we are at all warranted in drawing any conclusions regarding porosity from the examples of Hewlett units cited by Mr. Brundige. In the case of the Hewlett unit, we have a very remarkable insulator. It is unfortunate that we do not still have it,—that it is not being largely manufactured as a high-voltage insulator. In the Hewlett unit there is no cement to cause trouble. We have a unit that is glazed all over, and due to that fact, it may have a fair amount of total porosity, and still be a satisfactory unit in not taking up water. It is not at all comparable with the cap-and-stud-type insulator.

In regard to the older units purchased nine or ten years ago, to which Prof. Ryan refers in his paper,—we had approximately 100,—we were never able to detect any porosity through communicating pore systems. Many of them were soaked long periods and many of them dried out in attempts to change their insulation resistance, but they had uniformly a resistance of nearly 2,000,000 megohms at 20 deg. cent.

Philip Torchio: Are you referring to the Hewlett insulator?

J. Cameron Clark: No, these are the old flat-disk cap-and-stud type. I think that on account of the record of certain of these older units, it is not safe to make the statement that all of these older insulators are unquestionably of greater porosity than many of the suspension type units manufactured recently. Whether it was an accident that they were made so good is a question, but it seems that the makers of that same unit cannot duplicate the performance today.

Conclusion No. 4 in Mr. Brundige's paper, states that if it were proper to charge up the trouble to porosity we would find units having porcelain in all stages of resistance. As a matter of fact, we *do* find that very thing when we megger them with a proper megger having a range of zero to five million megohms. The results of such tests are reported in my paper, "Experiments on Porcelain Suspension Insulator Units," A. I. E. E., TRANS., 1916, p. 1453.

Referring to Mr. Austin's equivalent insulator circuit, (Fig. 13), his discussion states that "increasing the frequency to 200,000 cycles will give little time for the leakage of charging current through r_1 and r_2, so that a porous piece will act as though it were a perfectly good piece." I suspect that Mr. Austin's reasoning here has been led astray by his knowledge that *damped* high-frequency voltage, as applied by the oscillator, will not detect water-logged porcelain. However, the application of sustained 200,000-cycle voltage will certainly result in passing relatively great values of charging current through r_1 and r_2 to the middle dry zone. This fact has provided Professor Ryan with a method of detecting porous porcelain after short periods of immersion in water. Professor Ryan's paper describes this method briefly and presents a table of results secured by its application to several insulators purposely fired at different temperatures.

E. E. F. Creighton: These three papers add to our knowledge of the problems of porcelain insulators. Mr. Brundige points out the difference in opinion by investigators regarding the deterioration, whether by porosity or cracking. Prof. Ryan has shown without question that the prevailing trouble on the Pacific coast was due to porosity. Mr. P. M. Downing sent me a dozen 60,000-volt pin-type insulators from the lines of the Pacific Gas & Electric Company, and the test indicated absorption of moisture, not cracks. Mr. Austin suggests indirectly why the insulators in California happen to be porous. He states that porosity can be detected by an expert by a visual inspection and also, in a different place, that "this material was used . . . . on the Pacific coast where conditions were favorable, otherwise very serious trouble would have resulted." At the time this installation was made, there were no engineering requirements on absorption. (As far as that is concerned, the engineering specifications today on absorption are worthless.)

The records that have come to my attention of insulator failures in the East show a large percentage of defects by porosity, although I am of the opinion that cracking has somewhat predominated. However, it should be understood that in many cases where insulators were porous, the caps were removed by boring overlapping holes along the iron, and as a result of this damaging method the insulators have been actually cracked in the process of boring. I know of many cases where this method of removing the caps was used, and I discovered by my own experience that insulators I had known to be porous by measurement before removing the caps were cracked after the caps were removed. Since adopting the method of sawing off the caps with a hack-saw, it has been possible to find a larger percentage of the insulators porous and without cracks. Mr. Bang of the Pennsylvania Water & Power Co., who kindly furnished me the last thirty, picked them out of 200 by high megger reading, and I find that the majority of them are porous. It is not always that engineers can have contrary convictions and still both be correct.

To any one familiar with the narrow range of about 40 deg. on 1350 deg. cent. for firing, and the difficulties in maintaining uniform temperature throughout the kiln, it is not surprising to see batches of insulators come through from time to time either slightly underfired and porous or slightly overfired and brittle.

Mr. Brundige has reviewed the logical factors which caused the breaking of the porcelain, namely, differential expansion between iron and porcelain, expansion of Portland cement by continued hydration, and to these may be added the expansion of the magnesia content in the Portland cement which hydrates very slowly. There are other factors under investigation.

Mr. Brundige speaks of small degrees of porosity which may not be excessively harmful. It will be interesting to determine just what degree can be considered harmless. It is recognized among brick-makers that there is a minimum degree of absorption which withstands freezing without cracking the brick. Perhaps there is a minimum degree of water absorption by insulators which will permit freezing without opening up the pores still further.

There is a certain insulation resistance which is necessary. The capacity reactance ($1 \div c\,a$) of a perfect suspension insulator is about 75 megohms. Therefore, if the resistance is several hundred megohms and does not decrease with further age, it may be used in a string of insulators without changing the distribution of potential of the units. However, an insulator measuring a few hundred megohms will be punctured by the 60-cycle potential if other units in the string should fail in sufficient numbers to apply high voltage. Until more knowledge is gained of the effect of freezing on slightly porous porcelain which has absorbed moisture, it seems safer practise to remove even these insulators of comparatively high insulation resistance from the line, say 1000 megohms. Prof. Ryan finds good insulators measure a resistance of the order of a million megohms. One thousand megohms, as indicated by a megger, indicates a comparatively poor resistance. The highest resistance I have found in highly vitrified 10-in. suspension insulators is three and one-half million megohms.

It would be interesting if Mr. Austin would state further how "most of the porous material can be detected by the trained eye." These observation tests, if I understand correctly, are on both the color of the glaze and the surface of the white porcelain where it is exposed. I have found even experts frequently at fault in detecting porosity by the color of the glaze, and in my opinion the observer is led astray as frequently as he guesses correctly. Some colors of glazes are particularly sensitive to temperature and, other things being equal, the change in color will be an indication of the temperature. However, the change in color may be due to a change from one oxide to another, and a little difference in the air supply at the fire will cause more or less oxidation of the glaze and a consequent change in color long before the vitrifying stage of the porcelain is reached. I investigated coloring from another angle. I used an insulator, picked by an expert by its color as underfired and porous, but which by test showed perfect vitrification and absolutely no absorption. I placed this insulator over the dome of the kiln where the temperatures fall much below the vitrifying temperature of the porcelain. This insulator was darkened to the color of the highly vitrified porcelain of the same make. I did not analyze the glaze. The darker color might have been due to absorption of carbon. There was a very interesting paper presented at the last annual meeting of the American Ceramic Society on the subject of color of glaze. The same glaze mixture gave distinctly different colors due to a difference in the atmosphere of the kiln. My feeling is that the detection of porous insulators by this method is too unreliable to use. I should be glad to have this statement challenged.

Mr. Austin makes some interesting observations regarding tests of insulators, and I agree with him in most of his conclusions. While it may be that the 60-cycle test using flash-over and poor regulation is desirable from the standpoint of ease of testing a large number of insulators at once, and no doubt is better than the test using good regulation and no flash-over, still it is not as effective in weeding out poor material as the oscillator test. While an engineer may question the limit of severity which he wishes to impose in his insulator, I think there can be no question about the relative severity of these two tests. In one big porcelain factory, where they make insulators strictly for their own use, the oscillator test is used exclusively because it was found to pick out more defective porcelains. The expense of testing is no greater than with 60-cycles, formerly used. Since the porcelains were only a small part of expensive apparatus any extra loss was undertaken to get the better porcelain. In the early use of the oscillator, several years ago, the losses were much greater than in the previous 60-cycle testing, but due to improvements and care in the manufacturing processes, the losses were soon made as favorable as before.

In the cross-section diagram of porcelain in Fig. 3 of Mr. Austin's paper, if an insulator is so porous as to absorb moisture under simple immersion, as indicated by the shaded portions, the porcelain, when dry, will puncture by the oscillator test. Such an insulator would be palpably defective. But assuming that moisture had penetrated very slightly porous porcelain, as indicated in this figure, then the high-frequency potential would stress the dry layer to a very high value, and would probably damage it. This is due to the fact that the layer of moisture next to the surface has a dielectric constant of about 80, and therefore the equivalent thickness of porcelain is comparatively small.

When, however, the moisture extends entirely through the porcelain, so as to give a fairly uniform potential gradient, the resistance enters as a factor in preventing puncture by the high-frequency discharges. The *modus operandi* of puncturing a wet insulator consists, in the initial stage, of boiling out the moisture. If there is not sufficient energy in the discharge to do this in a brief time, naturally the insulator will not puncture, and the oscillator test, which is so effective on dry porcelain, becomes ineffective. However, if a combination of 60-cycles and high frequency is used—in other words, if a higher 60-cycle voltage is used in getting the oscillations rather than going through an oscillation transformer, sufficient energy will be taken from the circuit to boil out the moisture, and the comparatively enormous power of the high-frequency discharge will be able to do its effective work.

As to the possibility of damaging good porcelain by the high-frequency test, I have examined many dozens of punctured insulators, and I have yet to find a puncture which was not due to a visible defect, either a crack or porosity. On the other hand, potentials as high as 100,000 volts at 200,000 cycles have been applied by the hour to insulators which did not puncture, and although all possible care was taken subsequently in their examination, no defect, such as blebs, laminations, or porosity could be detected. In brief, the experience indicates that good porcelain is not damaged by a reasonable test of high frequency, but many smaller degrees of defectiveness can be developed by the oscillator test than by the 60-cycle test.

Mr. Peaslee, says that there are "grounds for suspecting that in some manner certain insulators deteriorate at a very rapid rate, due to dielectric stresses combined with other stresses in such a manner as to materially hasten the failure of the insulator." While the mechanical causes of failure have been thoroughly discussed by a number of engineers during the past few years, knowledge of the dielectric flux concentration is comparatively little known. It is of fundamental interest, and it may be a fruitful field of investigation. My experience has led me to conclude that trouble arises, in the usual designs of insulators, from defects rather than unequal stresses on the quartz, clay and feldspar as they exist separately or in different combinations. In the underfired condition, where the three ingredients are only slightly united, the weakness of the air ducts is so great that the ingredients of the porcelain are not stressed to the limit before the air begins to conduct and allows puncture to take place. Where the porcelain is well vitrified and not defective, these unequal stresses treated by Mr. Peaslee should appear.

Careful distinction must be made between deterioration of insulators and deterioration of porcelain. We can all agree that the insulators deteriorate, but it is a mooted question whether porcelain deteriorates, except by opening of the pores due to freezing of moisture therein and by potential gradient beyond the electric strength.

After the war opened in Europe the English ceramists found it impossible to obtain the French flint which they had been accustomed to use in their ceramic work. Instead, they were forced to use ground quartz, and as a result the factory losses increased to an alarming proportion. There appeared cases which indicated that the porcelain had actually undergone disintegration after it had been taken from the kilns. This subject was thoroughly treated by Mellor and presented in the TRANSACTIONS of the English Ceramic Society. While quartz and flint are chemically the same, $Si\,O_2$, they have different characteristics when heated, due to a difference in molecular structure of the material. Flint tends to lower its specific gravity quickly in one or two firings, whereas quartz requires a dozen or more firings to reach a lower stable value of specific gravity.

The use of ground quartz in the ceramic factories in America is very common. At one time experiments indicated possible deterioration of our porcelain with age at atmospheric temperatures. Further investigation, however, showed that the deterioration was not due to any change in the structure of the quartz. Furthermore, Hewlett insulators made some ten years ago were carefully examined, and they withstood a higher dielectric test than they were given initially ten years before. In fact, their puncture voltage was as good as the porcelains today. This porcelain was made by Mr. E. L. Barringer. It was well vitrified. It evidently shows that porcelains can be made which will not deteriorate at atmospheric temperature in any reasonable time, if ever.

On the whole, the three authors show a pessimism regarding the insulator situation which I do not share. I believe it is only a question of a comparatively short time until insulators will be available which are without defects and without any material deterioration; furthermore, that this desirable condition will not be confined to one type of insulator.

E. M. Hewlett: The increase in voltage from 60,000 to 100,000 called for a change in insulator design from the pin type to the unit-suspension type. When looking into the various possible designs for this high-voltage insulator, we did consider the cemented type. Our experience with cemented insulators covered 10 or 12 years before this time, for both indoor and outdoor service, and we had suffered a great deal of trouble with this composite type of insulator made up of porcelain cemented to iron and pins. We recognized the strain on the porcelain due to the expansion of different materials with different coefficients of expansion, and the difficulty of analyzing and satisfactorily designing an insulator to overcome these conditions. Considering the problem of maintaining (as nearly as possible) an uninterrupted service on a long line supported by a great number of insulators, an insulator with a composite structure made up of porcelain, cement, and iron, with its unknown internal strains, did not seem attractive for such conditions. To avoid these strains and simplify the problem, we considered that an insulator designed so as to bring the porcelain under compression strain, without additional expansion strain, would give the most satisfactory results. This we accomplished in the Hewlett-insulator design. While this construction, of course, presented some design and manufacturing difficulties to be overcome, we felt that it was better to work out the problem along this line than to try to reconcile and correct all the difficulties in the composite, cemented type of insulator. We were able to work out the problem satisfactorily in the Hewlett insulator as you see it, and I am glad, in view of the statements brought out here this afternoon, that we decided to follow this course.

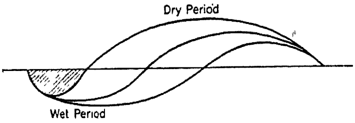


FIG. 3

I wish to call your attention to the relation of wet and dry periods to the absorption of moisture and consequent deterioration of the porcelain insulating units. From Fig. 3 you will notice an assumed relation of wet and dry periods in which one-fifth of the time it is raining and four-fifths of the time it is dry. This is of course not an average all through the year, but may be an average during some seasons; at any rate it will answer the purpose of illustration. Now let us compare two insulators which are both non-porous, or both quite porous. One insulator is so built that it begins to dry immediately on entering the dry period; the other has cement in its structure, which absorbs moisture after the expiration of the wet period. This practically means for the second insulator an extension of the wet period from one-fifth to two-fifths or three-fifths of the total cycle. Assuming that it takes as long for the insulator to dry as it does for it to absorb moisture; if the wet period is prolonged, so that it is greater than one-half of the total period, there is never a time in which the insulator is losing as much moisture as it gains, consequently there is a building up of moisture and a gradual saturation of the porcelain. The uncemented insulators, which do not have a sponge to hold the water to be absorbed, are not handicapped by this building up of moisture and saturation as is the cemented type.

There has been a great deal of discussion about the cracking of the cemented insulators, and the cause has been attributed to absorption of moisture, change in temperature, expansion of metal parts, expansion of cement, corrosion of metal parts, and consequent expansion. Some of the small cracks, which are started by this expansion, are enlarged by the moisture freezing in them as frost cracks rocks, and the small initial crack may finally grow to a deep enough crack, so that the insulator is weakened and only a relatively low potential is required to break down the unit. These troubles seem to be common in the cemented type of insulator, but in the single porcelain unit where the fittings only impose compression strain on the porcelain and where the porcelain is wholly glazed, such troubles are reduced to a minimum.

The link suspension units, as originally made, when fired in a kiln were supported at the center and a bald spot of unglazed porcelain was left. In the present design, the insulators when being fired are supported on the same sort of a block, but in a sand cushion. This makes it possible to glaze the insulator all over, and while some of the sand sticks to the glaze, the glaze is intact underneath the sand and thoroughly covers the porcelain. Thus with the porcelain entirely glazed, and with no absorptive material on the surface to hold the moisture past the wet period into the dry period, we have an insulator which has a natural tendency to dry, rather than one which has natural tendency to absorb moisture.

I believe that future development eventually will be carried on along the line of an insulator, such as the compression or link type, which is not subject to absorption or expansion, rather than the cemented type.

The reason that the Hewlett insulator has not been manufactured to a great extent, is to my mind the fact that it costs a little more than the cemented type. Furthermore the link insulator has been designed with a liberal safety factor for mechanical strains, but to go beyond this safety factor would interfere with the electrical effectiveness of the insulator. The cemented type was made to withstand greater strains. The feature of higher strain factor seems to appeal to many people and they would rather take the cheaper insulator with the extra pounds of permissible strain than the more expensive one with sufficient strength, and better insulation.

If specifications for the purchase of insulators would be formulated with proper reference to the mechanical and physical limitations of the materials, and the operating requirements of the system, I think the whole situation would be different by now, insulators would be built as they should be and there would not be so many installed, which have to be taken out again.

Philip Torchio: Mr. Hewlett, when you enlarge the part where the clamp goes, and there is a collection of water, have you found any damage caused by the freezing of the water?

E. M. Hewlett: The hole in the insulator for the clamp is quite short, and when water freezes in this hole the ice can expand into the open at both ends, so that there is very little strain on the wall of the insulator itself. We have subjected insulators in test to the influence of freezing water, and were unable to break them, neither have we heard of any troubles from this source in service.

Percy H. Thomas: What we have been looking for in the last two or three years, the thing we almost have, but not quite secured, is really authoritative data as to whether it is cracking or porous porcelain that causes the trouble with high-tension insulators. I am inclined to take pretty seriously the evidence that Mr. Brundige has brought out. I believe his opportunities for observation are very broad. They are not, perhaps, as good in the laboratory as some other men may have, but as regards practical conditions, he has a wide range to judge from. The practical conclusion, it seems to me hinges on the fact that electrically the insulator, particularly under normal practise, does not get heavy strains—a 100,000-volt line has something like 60,000 volts to ground, and if there are six disks the normal strain is only 10,000 volts per disk, while the insulator may have a puncture voltage of 150,000, or something of that kind. So that even if one insulator does have a check in it, and it does show on the megger, this does not mean there is going to be a breakdown, and it is not necessary to get panicky over this defective insulator. The megger test shows a percentage of total insulators which almost never runs more than 10 or 15 having low resistance.

If, however, the cracking indicates a mechanical deterioration, a physical weakening of the insulator, so that it will not stand the same pulling stress, that is a very serious matter, because a single insulator disk failing mechanically drops the string. That suggests a question I would like to ask Mr. Brundige, or any one else who has the data, namely whether he has any definite results of pulling tests on insulators which have indicated a certain percentage of cracks or show on the megger a given degree of electrical weakness.

There is another conclusion that perhaps we may draw from the fact that there is danger that disks will check and crack, especially if this checking does not mean a serious mechanical deterioration. Assuming we cannot get perfect porcelain, or perfect insulators why is it not better to use two-piece suspension units—they are not apparently over popular, but they are used in a number of plants. I think that very serious consideration is warranted as to whether the added expense is not worth while. We have two shells in one unit, and can have almost as many units as if we had a mere single shell in each, and it requires simultaneously the puncturing of two, to get the same effect as puncturing a single piece unit. I imagine if it were possible to prove mathematically the actual gain that exists in the two-piece insulator, that purchasing agents would not hesitate to pay the extra price.

A fused silica or quartz insulator will certainly have some advantages, but from such experience as I have had with fused silica, I am afraid it is likely to be brittle. Time and trial must offer the proof, and it will not do to draw conclusions now. In the case of the manufacturers who are making strong, thick sections, possibly the brittleness will not be serious. The danger of brittleness should not prevent a trial of the quartz insulator.

There is one other consideration I should like to suggest, and that is this—following up the idea that if there is no mechanical weakening of the insulator from a slight check, such as lessens its electrical power, and if we can use the porcelain in such a way that it does its mechanical work, without being subjected to any severe electrical stresses, we then reduce the severity of the requirements of the porcelain very much. With a practical way to use the suspension insulators with the porcelain in tension, then the electrical stress can be very largely taken off the porcelain as a puncture stress, and I think any of our present porcelain would be able to stand the electrical conditions without failure.

John B. Taylor: My remarks are limited to Mr. Peaslee's paper, and more especially to the photomicrograph plates, because it is not at all clear to me that these plates, though interesting, have along with them enough data to justify drawing any conclusions from them. Just so that the record will be clear, I assume all of these plates are polished, thin, transparent sections, such as the petrologist uses in his work. A quartz being one of those classes of crystals which have different properties and directions, which show up differently in polarized light, so that one of these plates with the white patches on it gives no indication of the particles of quartz in between, which by the law of chance, are differently orientated, depending on the different cut of the slate, and the different plane of polarization of the light, which was not brought out in the plate. It is true that quartz, like the grain of wood or the crystal structure in iron, or other materials, has different properties, mechanical, optical, electrical, or magnetic, perhaps, in its different directions, but that is no reason why we should take it for granted that this may be an important factor in insulator design. I think it is possible to look up the right values for the specific inductive capacity of quartz for the different optic axes, and that should be added before we are justified in drawing any conclusion from the theories advanced.

I think also that these photomicrographs are apt to be deceiving to people who are not working with them. The magnifications are high, 300 diameters in all cases. If we take the average size of these white patches of quartz and roughly estimate that size, perhaps, as one centimeter square, it means that in a cubic centimeter there are some three millions of these particles. There has got to be quite a substantial change in these in the one direction, and on top of that a very substantial difference in specific inductive capacity and breakdown capacity to lend this theory any force.

C. Francis Harding: A good deal has been said about the cracking and thermal effects in suspension type insulators, but little connecting such facts with the so-called puncture test. These puncture tests have been taken with perhaps a grain of reluctance by a good many insulator manufacturers and utilities, but they do show something, I feel, in regard to the actual strength of the porcelain under electrical stress.

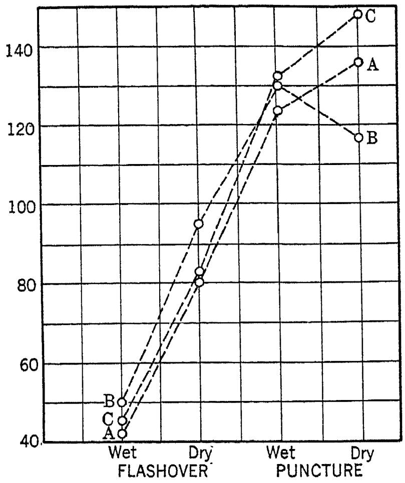


FIG. 4—TESTS OF SUSPENSION INSULATORS. SINGLE UNITS

Recently I had occasion to make some acceptance tests on a number of manufacturers' insulators, and the usual puncture test on each unit of the suspension insulator was made by placing the units under oil, and applying the voltage between the two metal parts of the unit. After these tests had been made on a group of the samples, it was thought desirable to make some porosity tests by soaking the insulators for twenty-four hours to determine the absorption. After that had been done we were interested in placing these units which had been thus soaked on the puncture test, and some rather surprising results were obtained. Fig. 4 will show that effect. These insulators were supposed to be used with three units in each string, on a 33,000-volt line, and there were some four or five manufacturers' insulators tested. The tests at the left of the figure are of little interest, being the usual flash-over test, but the two sets of ordinates at the right entitled "Puncture, dry," and "Puncture, wet," I wish to call attention to, the puncture voltages being indicated in kilovolts on the ordinate scale. Each of the curves represents insulators of similar rating submitted by different manufacturers. Two manufacturers' insulators tested higher on the dry puncture test and one group, surprising as it may seem, had a higher puncture test after the soaking for twenty-four hours in water than before.

Now, that may be due—and I wish these tests taken for just what they are worth—to the fact that a comparatively small number of insulators of each manufacturer were tested as compared with some other tests which have been presented by Prof. Ryan. On the other hand, the tests may have some value when it is stated that group, *B*, which tests higher after immersion than before, not only represents the average of a number of insulators, but every one of the individual insulators which was tested after soaking for twenty-four hours in water had a higher voltage puncture than the average of a corresponding number of the same make, tested without soaking in the water.

Just what effect that may have in practise I do not know, unless it may be this: The only explanation I can offer for this result is the fact that it is quite possible that the stress is more evenly distributed after the surface is wet than before, and so the insulator may require a higher puncture voltage as a result of the condenser action, assuming the wet surfaces of the insulators as plates of a condenser. It may, however, have some bearing on this discussion of the failure of insulators due to temperature changes on the line in dry and wet weather. For instance, during the dry weather, when the higher temperatures obtain, you would have possibly, if this theory is correct, a concentrated stress at some one point on the insulator, depending on its design. In the wet weather, you would have not only lower temperatures, but a simultaneous, more even distribution of stress throughout the dielectric. These combined effects might give an additional thermal strain and cause breakdown.

J. T. Barron: I notice in Capt. Peaslee's paper that he states that he left a number of insulators in the storeroom for two years and had 14 per cent of failures on 60-cycle flash-over. I think if his paper is based on insulators of such characteristics as these, which are rather exceptional, that possibly the main deductions which he makes are not true of the average insulator.

I think that a lot of the data in Mr. Brundige's paper also are questionable; because in the past we have received a large number of insulators which proved exceptionally poor and then later received a large number of the same type of insulators manufactured under better conditions which have been exceptionally good.

I notice Mr. Austin states that the production of a specified insulator is no small problem if the cost is to be comparatively low. In the discussion, the cost of the insulator has also been taken into consideration. I think that is wrong. Eighteen months ago we were paying about five cents a pound for our porcelain in the pin type insulator, and today we are paying ten cents a pound. Everyone is buying what they believe to be the best insulator, and the engineer, rather than the purchasing agent is making the selection. I think Mr. Austin is trying to cover the insulator manufacturers when he talks about the low cost, being a determining feature.

Mr. Austin states that the size, type and number of cement joints affect the cracking, and also that the largest and apparently the strongest insulators crack the soonest. In Fig. 6 he gives us two pictures showing approximately 60,000-volt insulators, the old one being a four-piece and the new one a three-piece. The three-piece unit, which I believe is a new type put forward by the insulator companies in the last year or two, is apparently the stronger insulator, and yet it is stated that the strongest insulator will crack the soonest. I would like to have him clear up that point.

The proposition of cracking, I believe, is generally confined to the head of the insulator. Several of the manufacturers insist that the two-piece insulator is better than the three-piece. If the head cracks with a three-piece insulator, there is quite a good chance of there being no failure of the entire unit, which I believe is different with the two-piece insulator.

Prof. Creighton referred to the possibility of 60-cycle tests, with poor regulation at flash-over, being a good test, but not necessarily taking the place of high frequency. I have tested some twenty-five thousand insulators with a normal line rating of 45,000 volts, which had been tested at the factory with 60 cycles, very poor regulation, and these insulators under high frequency have given us a failure of 0.4 of one per cent, or about four in one thousand. I think that this shows that this kind of a test is quite beneficial in weeding out practically all of the defects as all other types of insulators which we have tested at high frequency and which had only the normal 60-cycle test at the factory have shown more than one per cent failure.

Philip Torchio: It has been stated already that the impression conveyed by these papers is calculated, in a way, to weaken the confidence in the reliability of service of extra high-voltage transmission lines, using series of suspension insulators. I know that none of the authors intended to convey such an impression, I would ask, however, that some of the operators, or possibly Mr. Brundige in closing, would cover the point that the papers refer to, deterioration of individual units, and not to failures of series of insulators in service emphasizing instead the degree of reliability and the very small number of interruptions—or possibly entire lack of interruptions—to the service due to the failures of the insulators on extra high-tension lines.

J. A. Brundige: At the beginning of Prof. Clark's remarks I was somewhat disturbed to think I had wrongly interpreted Prof. Ryan's paper to mean that he considered the majority of insulator troubles attributable to porosity; but as I listened to Prof. Clark's further remarks, I began to feel better, for I believe it is evident that he, at least, still has a strong leaning to the porosity hypothesis.

Mr. Thomas has asked the question whether we have found on testing, any mechanical weakening of the insulator units after having been shown faulty by means of the megger. Several units have been tested in this manner, and nothing has been found which will indicate any mechanical weakening from such causes, nor does it seem probable that there should be a weakening of this character. Reports have been received from other operating companies also, which seem to bear this out.

Regarding a statement made by Prof. Creighton that the general tone of the papers this afternoon is rather pessimistic, the author must admit that he is not over-confident with respect to the future performance of many, if not all of the cap-and-stud-type suspension insulators now in use. Another speaker in commenting along the same lines has suggested that as my paper deals with the performance of insulators manufactured several years ago, the data therein should not be used to estimate the possible performance of later suspension designs. While it is true that the paper is mostly based on a study of insulators of the older types, it seemed very necessary to make such a study, for as you have seen today, it is very evident that there is no general agreement even yet on the underlying causes of insulator failures. Without definite information in this direction we are rather in the dark regarding the possible behavior of the newer types. There are doubtless not many transmission engineers, who, in view of their experience with past insulators, would accept the newer types on faith and leave them on the line for several years without making periodic inspections and tests.

This will answer Mr. Torchio's question whether it is not true that there are now relatively few line interruptions due to insulator deterioration. While there are comparatively few actual interruptions from this cause, it is generally due to the fact that the operating companies follow the condition of their suspension insulators quite constantly and test them out by the megger or other methods at least once a year in order to weed out the defective units.

A. O. Austin: I quite agree with Mr. Torchio that if we look only at the troubles of the insulator, the situation appears rather pessimistic and tends to affect the development and financing of the largest power-transmission projects. The insulator, however, has passed beyond the experimental stage, the line problem being largely one of operation and maintenance.

There is much evidence to show that the insulator outlook is very bright at the present time. The increasing severity of operating conditions has been successfully met and even the problem of handling lightning has been quite easily and successfully solved. The largest size insulators, as well as the smallest, can be made puncture-proof and in addition, the larger sizes do not spill during even the most severe lightning storms.

It was not many years ago, that it was thought impossible to handle the lightning problem successfully. It was reasoned, however, that the stress which could be placed upon the insulator was limited and not nearly as severe as generally supposed. Stress can only be placed on the insulator through the conductor, which in turn is surrounded by air having very definite limitations as we all know. It awaited only for the production of good insulators to prove the point.

When an exceedingly severe stress is thrown on the conductor, the air breaks down and limits the stress, so that an insulator having a flash-over around 400 kv. seldom, if ever, spills on the line. Starting with this assumption and using common sense in design of the insulator so as to obtain factors of safety for true operating conditions, resulted in an immediate improvement in the insulator performance.

A short-spaced insulator, although having a comparatively small factor of safety at best, has given a phenomenal performance. Line after line has been thrown in without a single failure, while a few years ago, a line would have to go through a weeding out process before it would attain even a reasonable degree of reliability.

Lines having old insulators or ones assembled without some of the later refinements may have much faulty material which must be removed from time to time to give good operation. This is a problem of operation and much trouble would be saved if the weeding out of poor material was given more attention. It is not necessary that all of the poor material be removed to place the line in good condition, for the removal of only a part of the faulty material may reduce the probable trouble to a negligible quantity. This is readily seen by looking at the curves on operating hazards, for various percentages of depreciation shown in the paper* presented before the Institute in 1914.

The tendency in modern insulator design is to give careful consideration to the factors governing maintenance and first cost, for it has been proven by many lines that there is little cause for insulator trouble for at least several years even with only a fair design, if well made.

That maintenance has been considered very carefully in the design and manufacture of insulators is seen by a comparison of the modern insulator with the best of a few years ago. In insulators of the same rating, it is noticeable that the modern insulator has a fewer number of parts; the three-part taking the place of the earlier four-part; the two-part taking the place of the three-part, etc.

As it is clearly recognized that the probability of trouble from matching up of faulty members is increased with the reduction of the number of parts and furthermore the manufacture of insulators with fewer and better parts is much more difficult, it is seen that modern practise is directed at maintenance rather than electrical design.

The very satisfactory operation of many lines which have been installed in the last few years shows that the modern insulator with fewer number of parts is not deficient in any way so that we may take advantage of their lower maintenance or depreciation.

Experience has also shown that insulators having a large number of parts have a higher percentage loss based on the total number of shells, so that the loss will increase at an even greater rate than the number of parts in an insulator, other conditions being the same.

Much of the present insulator trouble can be prevented if the line is gone over with a megger or other suitable apparatus and the faulty material eliminated. It is very difficult to get the operating man to take the necessary time to weed out the faulty material, and it usually happens that a line is in very serious difficulty before any attention is given to this important matter.

Information which was very misleading has tended to discredit the advantages gained in going over a line with a megger or other suitable apparatus to weed out faulty material, with the result that a great many systems, which could have saved themselves considerable trouble and expense, have put off the work of going over the line until they were up against an impossible operating condition.

The following is a good example of how dangerous it is to draw hasty conclusions. Several years ago, a line which had experienced some trouble was gone over with a megger, which indicated a loss of approximately 2.5 per cent. This in itself looked very good. The line, however, was equipped with two-piece suspension insulators, so that the 2.5 per cent depreciation indicated, included insulators only which had both parts bad in the same unit. As a part loss of 15 per cent or 30 bad pieces in every 100 of the two piece units will give only 2.5 per cent insulators having both parts bad, it will be seen that the 2.5 per cent indicated by the test was far from the true state of affairs. In this case, there would be 25 per cent of the insulators with one part bad; 2.5 per cent with two parts bad, or a total of 27.5 per cent, although the method used showed only 2.5 per cent. The removal of 2.5 per cent would still leave 25 per cent of the insulators which had one part defective, so it is readily seen that going over a line under these conditions might not result in any material improvement, and should not be used as a basis, for another line where over 90 per cent of the faulty material could have been removed with ease.

It is seen that the two parts in an insulator practically multiplies the depreciation. This will show up sooner or later, and has been the chief reason for rendering the two-piece suspension insulator obsolete.

To minimize the defects of depreciation, it is advisable to have an insulator composed of a large number of comparatively small detachable parts, which is easily carried out in the suspension insulator. In the case of the pin type insulator this is not practicable except in special cases. We can, however, minimize the effect of depreciation by reducing the number of parts as well as increasing their factors of safety to meet the internal stresses set up.

It is gratifying to know that insulator troubles have not come along the lines expected. A large part of the cracking is due to an attempt to insure mechanical reliability. This was only natural, for in a large insulator, the standard of mechanical reliability to the electrical to give the same degree of operation must be from one to three thousand times higher. This follows from the fact that there may be a large percentage of electrical depreciation before trouble results from matching up of the faulty members, where-as an infinitesimal mechanical failure will produce an impossible operating condition, owing to the dropping of the line.

This consideration is an exceedingly important one and eliminates the consideration of many insulators for important work, although they may have decidedly valuable properties from the electrical standpoint.

The importance of good material, is, of course, very well recognized; the methods used to determine same, however, must be practicable.

It is not possible to inspect every piece as to the exact dividing line between properly and improperly vitrified material. By using glazes which indicate the firing range, it is possible, however, to be on the safe side and set out any material which may be doubtful. If good manufacturing conditions prevail, this does not necessarily produce a great hardship. If certain glazes are used; which give no firing information, it is not possible for an inspector to judge the ware properly. Slate glazes, white glazes, chromium enamels, or brown glazes, which have too high a calcium content are not very sensitive to temperatures, so it is not possible to judge the firing of the ware properly.

Only a limited quantity of ware of this kind can be fired with certainty, as it must be placed in zones in the kilns which are known to be reliable. This point has been overlooked in the manufacture or selection of insulators to gain appearance. Where precautions were taken, one of the large lines showed a loss after several years operation of only 1.5 per cent which is exceedingly low. If this same material had followed the general practise and been fired promiscuously throughout the kiln, the losses would have been 10 or 15 per cent due to porosity, as the slate glaze made it practically impossible to detect the low-fired material.

When I first went into the insulator business some years ago, none of the manufacturers were correcting for moisture, which varied over a range of 25 per cent in some of the materials. Under these conditions, it is not surprising that the results have been very erratic in many cases.

Mr. Barron has raised a question as to the comparison of the pin insulators shown in the paper. In the case shown the conditions are probably less favorable to the large three-piece insulator than generally prevails as this insulator really has a much higher rating than the four-part insulator of earlier design to which it is compared. The small cement section and the method of assembly provides against any internal stresses, although the size of the insulator is very much increased. The large insulator is one which has been manufactured for some few years, and up to the present time, I know of no single failure. This is not unusual for modern insulators, as there are many in the same class.

We must, however, take care of the mechanical stresses, the larger and heavier insulators requiring much more care in their refinements. Unfortunately, the work that is most worth while in the assembly of an insulator does not show in the built up insulator and some of the largest and apparently strongest insulators may prove to be the poorest for operating conditions, unless proper provision is made for internal stresses.

We need not look upon the insulator question at all pessimistically simply because some of the older insulators or poorly made types are causing trouble, for we can find any number of insulators now that are producing exceedingly satisfactory results. The cost has not been so great and the depreciation has not been so large even under the worst conditions, but what the rapidly developing art can easily stand the expense.

The modern insulator probably has a lower depreciation rate than any piece of apparatus connected with the system.

We have not hesitated to scrap electrical machinery of all classes for new types with improved efficiencies, and we must do the same thing with the older insulators simply as a matter of good business.

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THE ENGINEER'S DESTINY

PRESIDENT'S ADDRESS

BY H. W. BUCK

THERE has been much discussion concerning the position of the engineer in modern times, but conditions are changing so rapidly and points of view are undergoing such a fundamental evolution that it is well from time to time to review the relations of the engineer to his surroundings and to secure if possible the proper orientation.

The change and improvement in the engineer's position in the world in recent years has been so rapid as to surprise even those who were the optimists in the under-dog days of the engineering profession. In the middle of the last century, when the engineering and technical schools began to be formed in this country by men of far-seeing vision, the classical scholars looked on askance and took pains to differentiate these upstart institutions from their own traditional schools of learning and to ostracize those who pursued the new courses by classifying the professions as "learned" and technical.

Times fortunately have changed. The engineering profession is coming into its own. To-day the engineer is being swept along with an irresistible force, by a tide which he himself has created, and it is well therefore for the engineer to take his eyes off his work occasionally and to observe his constantly changing surroundings. A flood of scientific and technical accomplishment has swept over the face of the earth, revolutionizing life, commerce and international destinies. Even the turmoil in which the world now finds itself can probably in the last analysis be traced to the over acceleration of world affairs resulting from the work of the scientist and engineer.

During all this development period of the engineering profession—during the past century the engineer has worked his way along alone and in silence, so to speak, seeking his reward rather in the joy of accomplishment and in the realization of his dreams than in worldly recognition and accumulation. The very inherent greatness of the pioneers who have laid the foundations upon which we now build prevented them in a way from acquiring a more worldly position in affairs. This tradition, however, is not a virtue beyond a certain point, and the engineer by nature is too willing to give way to others. The time has come when he should take a more worldly position in the world which he himself has created.

In our general relations to intellectual development we may consider that we are just emerging from a classical period where tradition, custom, prejudice, ignorance and dogmatic religion were the controlling forces. Movements which took place in world affairs were largely political, following the paths best suited to the advantage of the ruling classes. There was little real progress, because there was no development of scientific knowledge and its application in engineering. Scientific truth held no standing. The worship of tradition caused a powerful reaction against any scientific discovery which might necessitate a readjustment of established habits of thought and life.

For centuries before the dawn of the scientific and engineering era great changes took place throughout the world, but little real progress occurred. Races rose and fell, always falling back to the starting point, for there can be no upward trend in racial development without the solid basis of scientific knowledge to grow upon. China made great progress and developed its early civilization under scientific activity but during recent centuries it has lived under the worship of classical tradition and has become inert.

A constant change in point of view, which is so largely brought about through developments in scientific knowledge, seems to be necessary for progress in civilization. Our civilization to-day differs from that of a century ago in proportion to the scientific and engineering evolution which has taken place during the period through its reactions on life in all of its phases. Such discoveries in science as the law of gravitation, the evolution in species, the laws of electro-magnetic induction, etc., have probably had a more profound effect upon the development of the human race than any other acts in history.

The engineering profession has passed through the preliminary stages of its growth and has reached a position where the engineer should work and act not only with proper attention to his work itself but with full consciousness of the important relation of his work to human affairs in general. Among the early pioneers in engineering were many notable instances of men of great breadth of view. Men like Watt, Fulton, Whitney, McCormick, Erickson and others. Specialization had not at that time begun to work its narrowing influences. Of recent years, however, under the stress of commercial development and economic conditions, increasing specialization has taken place and the engineer has become obliged to compass his mind with an ever narrowing horizon. This specialization produces extraordinary proficiency in particular fields, but has the objectionable effect of narrowing the character and outlook of the man and of reducing his value as a citizen. We must take care lest commercial considerations and the modern mania for efficiency in the narrow sense does not force our engineers to lose sight of the world around them in their concentrated attention to the part rather than to the whole. This excessive specialization is a danger which threatens the future standing of the engineer.

It is interesting to recall in this connection the results of a recent canvass made by a joint Committee on Education on the qualities which, in the opinion of some five thousand leading men, engineers and others, best fitted a man for a successful career as an engineer. As a result of this vote only 13 points out of 100 were assigned to purely technical knowledge as an essential, the other 87 points being allotted to broader qualifications such as judgment, character, human understanding, etc. This is merely a quantitative statement of the many general demands now being made of the engineer and it illustrates how the work of the engineer can be broadened out. It is an encouraging symptom.

A most significant movement of recent times in the engineering world has been the development of cooperative action among engineers of all classes, and this tendency will, I believe, serve to offset the evils of specialization. It is the growing recognition of the fact that all branches of engineering are interdependent. We electrical engineers, I believe, are well aware how much we need the assistance of other branches of engineering for the successful fulfillment of our purpose.

This cooperative movement has quite recently been given tangible expression in the formation of the Engineering Council, an act, I believe, of far-reaching consequence. Under this organization as a beginning the Civil, Mechanical, Mining and Electrical Societies together with the United Engineering Society are tied together for cooperative action through a joint body of twenty-four representatives. This body will meet at frequent intervals and will deliberate on matters of general interest to engineers. It is an encouraging beginning toward universal cooperation among engineers in all branches of work.

In this Engineering Council we have for the first time an engineering body representing some thirty thousand engineers of sufficient scope and standing to create an engineering public opinion. Its influence is likely to be far-reaching in building up the prestige of engineers in both technical and civic affairs.

A further development which has reached full recognition only in recent times is the mutual appreciation which has grown up between the engineer and the worker in pure science. The engineer looks to the scientist to provide him with raw materials of knowledge with which to work out his applications, and the scientist must look to the engineer to make his discoveries so fruitful that the full effectiveness of his work on the frontier of research can be sustained. Both are working together in order to unfold nature in the most effective way for the benefit of man.

We electrical engineers, I think, feel a particularly close bond with the pure scientist in that recent developments in physical science have disclosed an intimate relationship between electrical phenomena and the nature of energy and matter.

All of the important movements which are taking place at the present time, which center around the engineer and his work mean, I believe, that the engineer is soon going to leave his position of isolation in independent fields of work and realize that he owes an obligation to the community broader than his daily engineering work, and will contribute to the general welfare his talents and experience. It matters not whether the problems before him are political, sociological, industrial or technical, I believe that the engineering type of mind, if the proper breadth of view has been acquired, is best fitted to undertake them.

It is not necessary, perhaps, in important administrative positions to have civil, electrical or mechanical engineers as such, but we do need men in those positions who have had training of the type which engineering gives, with the mental balance, the power of analysis which such a training develops, the resourcefulness and the faculty of recognizing and properly apportioning the various elements in a problem. There is a quality of mental honesty which engineering experience highly develops which is sorely needed in public life. The scientific and engineering professions should rise up and furnish such men from their ranks for the best welfare of the country.

The classicist contends that a world dominated by scientists and engineers would be cold, materialistic and atheistic and lacking in those qualities of art and sentiment and the imaginative outlook which every civilization so highly prizes. To this doctrine and its injustice to the engineer I want to take emphatic exception. The world today may be inclined toward materialism but it is not dominated by the engineer, far from it, but by other classes. The engineering mind on the other hand is characterized by a highly developed creative imagination and possesses to a high degree exactly those qualities of mind and temperament best suited to combat materialism. There have been many instances in history of great artists who have been great engineers and vice versa, and I believe that the two temperaments lie in close relationship. Furthermore, scientists and engineers as a class, have a strongly developed spirit of international understanding and sympathy which may serve as an important safeguard against excessive nationalism and aggression.

And so, gentlemen, I believe that we can confidently look forward to a new era for the proper fulfillment of the destinies of the engineer. Out of this world chaos we now see men of engineering and scientific training rising to positions of commanding prominence on all sides. It is simply the working of the inevitable law of the survival of the fittest.

In this great movement not only must the individual engineer play his part, but the great engineering societies must realize the power of influence which they are developing in an ever increasing degree in the community and the obligations which devolve upon them.

And so I hope that the American Institute of Electrical Engineers as it passes along from one administration to another will acquire an increasing realization of its duty, not only in furthering the growth of science and engineering, but in furthering the influence of the engineer in the affairs of the country and of the world.

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ADDRESS BY PRESIDENT-ELECT E. W. RICE, JR.

IT GIVES me great pleasure to meet you here tonight and to be thus introduced as your President-Elect. I thank you, and through you, all the members of our great Institution, for the honor which you have conferred upon me in selecting me to be, for a time, your official leader and chief servant. I hope I may have the strength, the ability and the opportunity to render such service as to justify, in some measure, your confidence. I fully appreciate that, with such a great honor, is coupled an equally great responsibility and I value the position as an opportunity to be of service to you, and through you, to our Country.

The standard set by the long line of illustrious and industrious men who have already filled the position of President, will be constantly before my eyes and will, you may be sure, stimulate me to do my best.

It is a great satisfaction to feel that, by a wise custom of our Institution, President Buck and Ex-President Carty will remain on our Board of Directors throughout the term of my administration. I shall lean heavily upon them for their experienced counsel and assistance.

No body of men can get together at the present time without soon discussing the subject of the war, which is uppermost in everyone's mind.

The war is the one dominating factor in the world life and thrusts itself before our thoughts whether we wish it or not. We are in the war at last and will remain in it to the end. Whether it shall be a bitter end or a bright end will depend largely upon ourselves, as it is now our war.

It has been stated many times that modern war was largely a question of mechanics and engineering, a statement with which we must all agree. It is self evident that engineering must, therefore, take a leading and dominant position in the war work. Now the electrical engineer stands for about the latest thing in engineering development; his activities embrace practically all other fields of engineering, being, so as to speak, the last word in engineering. The electrical engineer must, therefore, realize that this is his war in a very personal and particular sense.

War calls for supreme sacrifices and the deepest devotion, but it also demands something more difficult to give, and that is *work*. War may be said to be the personification of work, not only individual work, but especially organized and disciplined work,—disagreeable, dirty, heartbreaking, backbreaking, nerve-racking work, but always work. No nation of loafers ever won a war. Other things being at all equal, that nation or people who are willing to work the hardest will surely win the victory. Now I wish to point out that the enemy we are fighting is recognized as the most industrious organization in the world. Our enemy has prepared for war for fifty years and has been working with ever-increasing energy ever since the war started three years ago. We made no adequate preparation during all this time and therefore started with a fearful handicap of lost time and lost opportunities. We must not delude ourselves that our enemy is exhausted, but remember that he has the advantage of a flying start. We must accelerate at an incredible rate if we are to get our war-motor going fast enough, soon enough to catch up.

Our enemy boasts that we have started too late. We must, by the hardest work directed with scientific skill and accuracy, organize and effectively utilize all our power of work to make his prophecy an idle boast.

The country is trembling with eager anxiety to help. Men and women are offering their services and their money. All eyes are turned toward Washington and to many everything seems confusion, and as a result, we are full of criticism. Now I think it is clear that nothing is to be gained by destructive and captious criticism. We must discipline ourselves with patience, and if we take a broad view, we must admit that progress is being made. We must remember that a democracy of a hundred million people, whose thoughts and habits have been entirely those of peace, cannot change to the methods of war in a day, or a month, or even a year.

War is a business and must be handled as a highly organized, centralized, autocratic enterprise. We must, no matter how repugnant it may be to our habits and thoughts, temporarily adopt such methods of our enemy as are known to be efficient and successful, because the penalty of failure is death. War is so repugnant to our ideals that it takes time to realize the necessity for and make the collosal changes demanded in every direction. We must, therefore, as I have stated, avoid captious criticism and confine ourselves to constructive crticism, and that sparingly and sympathetically administered.

There is one idea which we must abandon. The great majority of our people, who have no acquaintance with science or engineering, is prone to imagine that this war will be settled quickly by some wonderful new invention, as if by an act of legerdemain; but you engineers realize that such a thing is practically impossible. It is so hopeless that it is cruel to permit any such idea to take hold of the American public. Neither is it possible for the war to be settled by the act of some hero or superman. It can only be settled by the united efforts of thousands of men, each contributing his bit. Team play in our civil army at home is as essential as in our fighting army abroad.

I venture to suggest that we cannot all occupy desks at Washington, and it is well for us, and for the country, that we cannot. We can, however, put ourselves and our business in such condition as to meet whatever demand is made upon us. Only relatively few can be useful in the direct service of the army and navy, but there is plenty of honorable work and useful work for us to do. The most effective work for most of us will be in the shops and offices at home, and everyone who does his work loyally and well, is as much a factor in our organized war as the man at the front.

Now, properly understood, the fact that no single great invention is likely to be made which will win the war, is no cause for discouragement. It does not mean that there will be no improvement, no new inventions, no new methods devised and put into effect. It simply means that we must not wait for the miracle which will never appear, but get to work and energetically take advantage of all present knowledge. We must survey the field, get at all the facts, carefully determine our plans and then proceed to put them into practical execution.

Take for example the matter of shipping. This perhaps presents the greatest immediate problem of the war, frightfully complicated as it is by the submarine. I feel sure that it can be successfully solved, if we are content to solve it by the simple, common-sense methods used by engineers and successful business men in the ordinary course of business. The problem must first be carefully investigated, all available data quickly obtained and checked, and all new conditions considered, after which a broad-gaged well considered plan, or plans, can be formulated, criticised and then put into effect.

Of course it is elementary to say that we must provide shipping in enormous quantities to replace that destroyed and to provide for increased demands. It is evident that time is the essence of the problem. We must, therefore, build the greatest tonnage in the shortest time. The ships must be manned and navigated to their destination and the most efficient methods provided for docking, unloading and loading.

With the situation such that the race is between ship building and ship destruction, with the destruction many laps ahead, it is vitally important that ships should be loaded and unloaded with the utmost expedition. We have recently heard of an instance where a large ship, after running the gauntlet of a voyage to England, was forced to visit several different ports, and waste one month's time, before starting the return voyage. This loss of time is equal to the loss of a complete voyage. The net tonnage delivered per month is the only thing that counts, therefore ship-tons saved are worth more than ship-tons built. Quick methods of loading and unloading at specially devised terminals, here and in Europe, should be developed and put into operation. The methods are known. It simply remains for us to organize and apply them.

We must see to it that the kind of ships, in respect to size, material and speeds, are such that the greatest tonnage may be moved across the seas in the shortest time. In the time element must of course be considered the time required to build such tonnage. If an investigation should indicate that cargo ships can be built which will successfully withstand one or more torpedo attacks, and which can also be provided with speed and armament sufficient to give them a good chance of fighting off and getting away from a submarine, they should be built no matter whether such ships cost more, or are less adapted for use after the war, or take a little longer time to construct than those of the ordinary type.

It is entirely within the range of possibility that such ships may prove to be the only ones which will be able to navigate the seas with any decent chance of surviving. It would seem clear that, unless the submarine is swept from the seas, it is hopeless to build a large tonnage of slow moving, relatively small and inadequately defended ships, as the net tonnage which could be delivered by such a fleet of ships will be too insignificant to be of any material value. We would have bet on the wrong horse and lost; therefore, I hope that we will have the foresight to build as large a number as possible of big, comparatively torpedo-proof cargo ships, as soon as possible.

We should also, at the same time, consider whether it is worth our while to continue building large dreadnoughts, battle-cruisers, and the like, which cannot possibly be finished for years to come. Our ship building facilities are limited, and if the facilities now devoted to the construction of dreadnoughts could be immediately diverted to the construction of large fairly indestructible, high-speed cargo ships, which can be built in half the time, we would be taking a great step towards solving the problem.

So much for what might be termed the "defensive method" of attacking the problem. Along with this defensive plan, we should put into execution every practical offensive plan of attacking the submarine, such as methods of detection when submerged, methods of attack by means of destroyers, mines, aeroplanes and special artillery. All such methods should be, and probably are being developed, and while no one of them will prove to be the panacea by itself, collectively they will be of the greatest value in reducing the menace. However, I think it is well to emphasize the fact that the only safe and sane plan of action is to assume that we can only win by pushing the development of all practical looking methods of attack and defense, at the same time, and to the limit of our ability.

Now I am well aware that there is nothing theatrical or startling, or novel, in the above suggested solution. For this reason it is not likely to appeal to the great non-technical public, but there is no doubt in my own mind that it represents the scientific and common-sense method, and that if followed with patience, persistence, vigor and diligence, it will prove successful, and if successful, the war cannot be lost. All the other problems of the war—the aeroplane, army, navy, food, manufacturing, farming, transportation, etc.—can be successfully solved by the same scientific, but simple and common-sense methods.

It is a great satisfaction to notice that this country has at last awakened to the importance of developing that great American invention—the aeroplane, and of manufacturing it on a great scale. We should do everything to help accelerate this work. If we can get aeroplanes of the right kind to Europe, soon enough and in sufficient quantities, experts tell us that it will do more to win the war than a large army.

We must also not neglect the development of the submarine, because if we fail to find a way to drive the submarine from the seas in short order, and fail to make relatively unsinkable and uncatchable ships, we may have to rely on big freight submarines, properly convoyed by fighting submarines, if necessary, in order to get food, material and soldiers to Europe.

We must not forget that, after all, all these things must be done by men collectively and that, therefore, it is essential for us to think and act collectively, and with reasonable unanimity. We must co-operate and not nullify our power by quarrels among ourselves. This means that we must be willing to give consideration to the views of others, be ready to make reasonable compromises and be constantly actuated by a spirit of conciliation. We must make every effort to get men of great experience, industry and sound common-sense in positions of trust and influence. We can then hope to have the helpful suggestions offered by other men of experience and wisdom given intelligent and proper consideration. We must give our chosen leaders reasonable time to make and carry into execution their large plans. We must get behind our leaders and loyally support them, and if after a long and fair trial, we find that we have made a mistake in our selection, we should then promptly replace such leaders by those more competent who will surely be found. This is the only way in which a democracy can work and form an effective and efficient organization.

I think I have said enough to indicate that there is plenty of work ahead for engineers at home, as well as abroad; in civil life, as well as camp life. Engineers have a great opportunity in this war and a heavy responsibility. You have special knowledge, experience and a forward looking point of view which the country needs, and it is your duty to see to it that you are given the opportunity to make effective use of your talents, in the service of the Nation, and if you are not given that chance, you must persistently demand it until you get it, and then I feel certain that the victory will be on our side, our civilization will be saved, and the world will be made a safe place for all decent people who will then be able to turn again to the satisfaction and joy of a useful and peaceful existence.

Annual Reports of Technical Committees presented at 332nd meeting of the American Institute of Electrical Engineers, New York, June 27, 1917

ANNUAL REPORT OF COMMITTEE ON TRANSMISSION AND DISTRIBUTION

To the Board of Directors,

The Committee on Transmission and Distribution submits the following report for the year of 1916-17:

In order to find a starting point for the work of this committee a letter was addressed to each of its twenty-three members asking for definite suggestions as to the most urgent problems concerning transmission and distribution. Replies were received from thirteen individuals containing twenty-four suggestions. Twelve of these suggestions related to high-tension insulators, seven to underground cables, one to lightning protection, one to relays, one to towers, one to reactances, and one to suppression of transient voltages. It appeared from the result of this inquiry that troubles with high-tension insulators and underground cables were uppermost in the minds of those who replied, most of whom were either operating men or men in close touch with the operation of transmission and distribution systems.

After conference with a few of the members of the committee, it was decided to confine the work of this year to the subjects of high-tension insulators and underground cables. It was further decided that on account of the urgency of the problems involved, particularly regarding high-tension insulators, that the subjects be treated so far as possible from the practical standpoint, theoretical discussions being eliminated except so far as they indicated definite practical results.

HIGH-TENSION INSULATORS

During the last few years the troubles with high-tension insulators have become cumulative. During the summer seasons of 1915 and 1916 the failure of insulators on some long distance transmission lines resulted so disastrously in the way of interruptions that it was evident something had to be done unless we were content to relegate long distance service to the category of intermittent or second class power. Particularly on the Pacific coast these troubles had become acute. Some of our best engineers were already working on the problem. A number of the far western companies had clubbed together and employed the best talent they could find to carry on the investigation. Professor Ryan of Leland Stanford, Jr. University is now busy carrying out experiments on a very large scale to determine the effects of porosity in porcelain. All who are acquainted with Professor Ryan and his work know that his results when published will carry authority on this point. It was a great disappointment to us that the paper promised by Professor Ryan on the results of his investigation, to be presented at this meeting, had to be deferred a few months on account of extraordinary weather conditions, making it impossible to complete the data.

Professor Peaslee of the Oregon Agriculture College has attacked the problem from the chemical and microscopical point of view while others have been investigating the effects of expansion and contraction of insulators on dielectric strength. The manufacturers of insulators have also been busy. This is shown in Mr. Austin's paper presented at this meeting. The manufacture of high-tension insulators from fused quartz is recommended by Mr. Peaslee. It is hoped that some one properly equipped will pursue this line of investigation.

What has been accomplished? The last word has not been said on any of these lines of investigation and will not be said for a long time. So far as porcelain insulators are concerned we know (1) that porosity must be reduced to the lowest practical limit to prevent absorption, (2) that joints must be so designed and made that the insulators will not crack from expansion and contraction effects, (3) That insulators must have ample margin of mechanical strength.

UNDERGROUND CABLES

It is only within the last few years that users of underground cables have begun to realize the importance of disposing of heat due to copper and other losses. Conduit lines were designed 15 or more years ago with the idea that there would be from three to five watts lost per foot of cable. Even engineers seemed to have a sort of blind faith that these watts would get away somewhere, somehow. No one ever stopped to consider what would happen in case these watts did not get away. However, we have found out in the school of experience. The lesson has been costly, but it has been learned. It is scarcely exaggeration to say that as much skill is required to design a conduit line as to design a dynamo.

The problem is to dispose of heat. This may be done in two ways: (1) by minimizing the amount produced, (2) by quickly conducting away the heat that is produced. The first is largely the problem of the cable manufacturer. We have two papers this year which show great advance in our knowledge of dielectric losses. The use of certain insulating materials in saturating paper insulation greatly reduces these losses. This is well brought out in the papers by Messrs. Bang and Louis, and by Messrs. Clark & Shanklin. The reduction in losses is not a small amount requiring laboratory tests to find. Cable ratings under some conditions may be more than doubled. These investigations are highly profitable and should be continued. The second point relating to conducting the heat away from cables after it has been generated has been brought out by Mr. Harper. No operating engineer can read this paper without a deep and sympathetic feeling of comradeship with the author. Most of the underground conduits now in use were built without any regard to their capacity to dissipate heat and thereby immense sums of money have been thrown away.

Mr. Roper's paper on high-tension cable joints is the very highest authority on this subject and is worth careful study. Probably there are more breakdowns at the present time in joints than in all the rest of a cable system. Practically all of these breakdowns are avoidable by carefully following the recommendations laid down by Mr. Roper.

SUGGESTIONS FOR THE FUTURE

For the future activities of this committee, I suggest that it pursue the high-tension insulator problem. The work has been started, but there is much to be done. Investigations so far indicate pretty clearly where the weaknesses lie and this is but one step from the solution of the problem. We suggest that further investigations be made on the subject of dielectric loss in cables to the end that the laws governing it may be definitely known. It will then be possible for the manufacturer to give an even quality of cable with minimum dielectric loss.

We further suggest that the use of 20,000- or 30,000-volt cables be investigated. Some 20,000-volt cable is now in use in this vicinity, but we greatly desire to hear all about it.

L. E. IMLAY, *Chairman.*

ANNUAL REPORT OF TRACTION AND TRANSPORTATION COMMITTEE

To the Board of Directors,

The Traction and Transportation Committee submits the following report for the year 1916-17:

At the beginning of this year, the scope of the Traction and Transportation Committee, which superseded the Railway Committee in accordance with the recommendations of the Special Committee on Re-organization of Technical Committees, was considerably enlarged, and in order properly to cover the field, more members were added. The main committee was divided into five sub-committees, as follows:

Urban and Interurban Railways: H. H. Adams, Chairman, L. P. Crecelius, W. J. Harvie, E. D. Priest, C. Renshaw.

Trunk Lines and Heavy Traction: N. W. Storer, Acting Chairman, A. H. Babcock, George Gibbs, Hugh Hazelton, E. B. Katte, John Murphy, W. B. Potter, Wm. McClellan.

Electrification Data: E. B. Katte, Chairman, R. Beeuwkes, J. V. B. Duer, C. H. Quinn.

Interference of Electric Railway Circuits With Telephone and Telegraph Lines: C. F. Scott, Chairman, A. H. Babcock, J. B. Taylor, H. S. Warren.

Effects of Grounded Railway Circuits on Pipes and Underground Structures: E. J. Blair, Chairman, J. V. B. Duer, John Murphy, A. S. Richey, H. S. Warren.

Through the Sub-Committee on Urban and Interurban Railways, it is hoped to bring the Institute into closer touch with the great body of electric railway men who now derive most of their support and assistance from the A. E. R. A. One paper on "Gear Losses" has been secured by this committee, and others on "Railway Motor Ventilation" are in preparation.

One paper bearing specially on the question of "Heavy Traction" has been presented during the year. This was the paper by Mr. R. E. Hellmund, on the subject of "Regenerative Braking of Electric Vehicles," which was presented at the Pittsburgh meeting of the Institute in January. The paper itself is a very complete treatise on the subject covering the problems and the various solutions involved in regenerative braking with different systems of electrification. It forms an excellent basis for discussion.

The paper and the discussion that followed showed plainly the great progress that has been made in the practical application of regenerative braking on heavy traction lines in this country. The advantages of the constant-speed type of locomotive with the induction motor in use on the Norfolk and Western Railway were clearly set forth; also the advantages of the variable-speed type of locomotive in use on the Chicago, Milwaukee and St. Paul and the Lake Erie and Northern locomotives where direct current is used, were fully brought out. The important fact to be derived from the discussion is that regenerative braking is an accomplished fact and must always be considered in any electrification scheme where heavy-grade work is involved. It has been demonstrated beyond question that it is a much more reliable and safe method of holding trains on descending grades than the use of air brakes alone since, in any case, where regenerative braking is used, the air brake is always available to stop the trains in emergencies.

While regenerative braking has not been applied to trains of multiple-unit cars, or to single cars, in this country, on a commercial scale, the hope and belief were expressed that successful application of regenerative braking on both would be only a question of a short time. The application in such work will be due, not to the safety and reliability of operation, as in the heavy grade work, so much as to the economies which can be effected in power consumption. It is conservatively estimated that not less than 20 per cent of the power now consumed in the ordinary multiple-unit trains in city and suburban service, can be saved by the application of regenerative braking. This will involve increased capacity in the driving motors and a small amount of additional control equipment, but the net saving will far outweigh the additional cost of equipment and maintenance.

The effort of the Sub-Committee on Electrification Data to collect information from existing railways, met with the usual obstacles. It was decided that any costs which might be figured on the electrifications now in operation would be of very little value to the Institute, since the conditions under which the installations were made varied so widely, and unless all of the facts could be known the figures would be very misleading.

The Sub-Committee on Interference of Electric Railway Circuits with Telephone and Telegraph Lines did a considerable amount of work, but it was not deemed advisable to make the results public at this time.

The Sub-Committee on the Effects of Grounded Railway Circuits took up the question with the National Committee on Electrolysis, with the special object of seeing that the effects of the high-voltage direct-current lines where the drop in the return circuit necessarily reaches comparatively high figures, was thoroughly considered.

For the coming year, it is recommended that the committee continue the work which has been undertaken this year. While any papers which would raise the question of system of electrification are distinctly undesirable, it is recommended that a series of short papers by representative steam railway men be secured, which would explain in considerable detail the problem which must be met by the electric railway engineers in superseding the steam locomotive on the heavy transportation lines. This should include not only the problem as it confronts us today, but what the future has in store for us as far as these railway men can predict. There is a constant tendency towards the use of much heavier and longer trains, larger locomotives and higher speeds, both in freight and passenger service. We need further information as to what the limits are likely to be; how much power must be concentrated in one train; how necessary it is to operate the trains under close headway during emergency conditions; how much it is worth to the railways to be able to do this; what are the physical limitations that will be involved, etc., etc.

The present committee is larger than can be used satisfactorily and it should be reduced to one-half or two-thirds the present number, and composed of men who are able and willing to devote their time to the consideration of problems of electric railroads.

N. W. STORER, *Chairman.*

ANNUAL REPORT OF COMMITTEE ON ELECTROPHYSICS

To the Board of Directors:

The Committee on Electrophysics submits the following report for the year 1916-17:

Very rapid advances are being made in physics; entirely new fields are being opened up, new conceptions formed. In order that engineering may advance as rapidly as it should, it is necessary for the engineer to keep familiar with and to apply the discoveries of the scientist and the physicist. Much of the work of the physicist is published in various scientific and philosophical magazines seldom read by engineers. It is difficult for the engineer to get the perspective view which he needs and should have. Realizing this, the Electrophysics Committee were able to secure Dr. R. A. Millikan, President of the American Physical Society, to deliver a lecture on "Modern Physics" at the mid-winter convention in New York, on the evening of February 15th. The enthusiasm with which this lecture was received by an overflowing audience is without precedent. Dr. Millikan's lecture will be published in the TRANSACTIONS. We believe at least one such lecture should be delivered annually.

Cooperation and mutual understanding between physicists and engineers are of the utmost importance. The Technical committee of the Physical Society bears a somewhat similar relation to its society with respect to engineering as our committee does to our society with respect to physics. We have had the most helpful cooperation from this committee and much of the success of our Electrophysics session at the mid-winter convention on February 15th was due to their active assistance. The members of the Physical Society were invited to our meeting and took part in our discussions as well as attending Dr. Millikan's lecture. Their technical committee also arranged for meetings of the Physical Society in New York immediately following our meeting, which many of our members attended. We believe that joint or parallel meetings are of great value. It would be desirable, if practicable, to have interlocking members on the committees. For instance the chairman of one committee an ex-officio member of the other committee. In addition, to Dr. Millikan's lecture the following important scientific papers were read:

"Corona and Rectification in Hydrogen", by J. W. Davis and C. S. Breese.

"The Electric Strength of Air-VII", by J. B. Whitehead and W. S. Brown.

"Oscillating-Current Circuits by the Method of Generalized Angular Velocities", by V. Bush.

Many important physical papers of great value and many scientific discoveries made by engineers, which naturally first appear in engineering journals, are lost to physicists because physicists do not read these journals and because when they are reviewed and reported they are recorded under the heading of "Engineering" rather than under the heading of "Physics". This is a great economic loss, and attempts should be made to remedy this condition.

We believe efforts should be made to continue and to further cooperation between engineers and physicists; it is to their mutual advantage.

F. W. PEEK, JR., *Chairman.*

ANNUAL REPORT OF COMMITTEE ON LIGHTING AND ILLUMINATION

To the Board of Directors,

The Committee on Lighting and Illumination submits the following report for the year 1916-17:

The Lighting and Illumination Committee sought, early in the year, to find some specific activity upon which it might legitimately concentrate its efforts with profit to the members of the Institute and with some measure of good to the art of lighting. This ambition was not realized, as the committee was unsuccessful in finding such an activity that was not already under consideration or that ought not more logically be undertaken by the Illuminating Engineering Society, which is specially organized for the investigation of lighting problems in general.

The Lighting and Illumination Committee considers that it may render a valuable service to the Institute by securing papers on illumination topics that are of peculiar interest to the members of the Institute or are of such general interest as to warrant their presentation in abstract form before this body. The committee has several suggestions for papers of this character to hand over to next year's committee as it was not feasible to procure them in time for presentation this year.

The committee also considers that it may render some service to the Institute in incorporating in this annual report a very brief statement of recent progress in the science and art of illumination, referring to a few of the most salient features of that progress, and not in any sense attempting to classify categorically the recent developments or even to suggest a true perspective. For such a presentation the members of the Institute are referred to the Report of the Committee on Progress of the Illuminating Engineering Society.

PROGRESS IN ELECTRIC ILLUMINATION

The marked progress in electric lighting in the past few years is founded largely on the reduced cost of luminous flux. The higher lamp efficiencies which have been realized in the newer electric lamps, together with the gradual decline in cost of power, have combined to cheapen luminous flux to such an extent that the average customer can with advantage sacrifice cost to a moderate degree in order to enhance to a greater degree the satisfactoriness of his installation. On this thesis may be explained, at least in part, the tendency toward higher illumination intensities, both indoors and outdoors, the more general use of indirect or partially indirect lighting systems, and the demand for so-called "filtered flux," as in the lamp which supplies luminous flux approximating daylight in quality, or as in the yellow lamp or the various kinds of tinted glassware.

It is not intended to leave the impression that the lower cost of luminous flux from electric lamps is uniquely responsible for this marked development in artificial illumination practise. Along with this growing economy must be considered the important progress manifest in the increasing appreciation of the value of the newer ideas of illumination and the equally important progress in developing the means by which these more highly developed ideas may be realized in practise.

A few particular cases of progress may be mentioned. One of the more important of these is perhaps the approximate realization of the quality of daylight in artificial illumination, secured through the development of a special bluish bulb, which, at some sacrifice of the lamp efficiency, modifies by selective absorption the quality of luminous flux emitted by the gas-filled tungsten lamp, so that the transmitted flux has a spectral distribution of the same general character as sunlight. The practical possibilities of such a device are immediately evident. One of the most important applications is to be found in the lighting of museums of art,—an application which has recently been made with marked success.

"Flood lighting" constitutes another particular case of recent progress worthy of special mention. The application in the lighting of the Statue of Liberty is the most prominent illustration of this type of illumination, but it is being used in the most varied ways, many of which have marked practical value. In connection with flood lighting may be mentioned the recent activity regarding projection lamps in general, and the present prominence of the question of automobile headlights and other searchlights.

One other example of progress is to be found in the considerable increase in the use of ornamental "White Way" lighting for main business streets, and ornamental boulevard and residential street lighting. This, of course, has resulted in a general revising of the standard of illumination throughout streets of lesser importance, but joining more or less closely the "White Way" districts. Along with the better lighting of business streets there is a gradual tendency to improve residence street lighting and to make better use of the light flux available on such streets by more rational subdivision of units and by utilization of light flux heretofore wasted for the lack of proper directing devices.

In closing this brief report reference should be made to the growing acceptance of the idea of legalizing lighting codes. Two years ago a Committee on Lighting Legislation of the Illuminating Engineering Society presented to the society a report containing a code of factory lighting. During the past year this code has been adopted and become a law in the states of Pennsylvania and New Jersey.

Automobile headlight glare which has been the subject of considerable legislation has in general been treated in a very unscientific manner until very recently. Now, however, there is decided evidence of an awakening among technical men on this subject and a better understanding of the problem to the end that satisfactory solution of the problem appears to be in sight.

E. P. HYDE, *Chairman.*

ANNUAL REPORT OF COMMITTEE ON ECONOMICS OF THE ELECTRIC SERVICE

To the Board of Directors,

The Committee on Economics of the Electric Service submits the following report for the year of 1916-17:

During the past year this committee has carried on most of its work by correspondence, but there have been a number of the members who were conveniently situated with reference to New York City. The activity of the committee is best indicated by the fact that it had charge of the program for the Institute meeting on November 10th, 1916, the subject of the papers at that meeting being, "Effect of Recent Court Decisions on the Work of Inventory and Appraisal," by Philander Betts, Fellow; "Continuous Inventories, Their Preparation and Use," by Harry E. Carver, Member; "Growth and Depreciation," by Julian Loebenstein, Associate.

On April 14th, invitation was received from the Valuation Terminology Committee of the National Electric Light Association to send three representatives to the joint session of that committee with representatives of other societies, to be held on May 4th. As the result of the invitation, Mr. Clifton W. Wilder, Mr. Wm. B. Jackson and Dr. Philander Betts were designated to attend the same. Of these, Mr. Jackson was the only one who was able to attend, but Mr. Cohn was present, representing Mr. Wilder.

At the committee meeting referred to there were in attendance members of the National Electric Light Association, American Institute of Electrical Engineers, American Society of Mechanical Engineers, American Gas Institute, American Electric Railway Association, American District Steam Heating Association and the Railroad Presidents' Conference Committee.

After considerable discussion as to the form of organization, the meeting should adopt, Mr. J. N. Shannahan was elected Chairman, and after a lively discussion, it was decided not to make any recommendations as to the formation of a joint committee at this time.

The definitions of the terms which the N. E. L. A. committee have prepared, were discussed and it was decided that each person who was appointed to represent an association at the meeting should receive or be sent a copy of the definitions of the N. E. L. A. and of the A. E. R. A., and that each should be asked to send to Mr. Shannahan criticisms of the definitions: these to be manifolded and sent to each of those authorized to meet with the committee, so that there will be working material as a basis for discussion when the enlarged committee is again called together. The committee then adjourned to re-assemble at the call of the chairman.

The American Society of Civil Engineers has recently discussed similar definitions in connection with the discussions of appraisals. These definitions differ so greatly from the definitions accepted by economists, financiers and engineers, generally, that the importance of the work of this joint committee is at once apparent.

It is recommended that the committee be continued, with general instructions to continue its conference with representatives of the other societies.

PHILANDER BETTS, *Chairman.*

ANNUAL REPORT OF COMMITTEE ON PROTECTIVE DEVICES

To the Board of Directors,

The Committee on Protective Devices submits the following report for the year 1916-17:

During the year two meetings have been held under the auspices of this committee; one at Boston on the rating of high-tension oil switches, and one at Chicago on protective relays for transmission systems. At the meeting in Boston it developed that the engineers were not all in accord as to how or whether oil switches should be rated in terms of the current which they could safely interrupt. A sufficient number of engineers, however, were in favor of such a method of rating to indicate that such a rating would be very desirable if possible. It is also to be noted that the edition of the National Electrical Safety Code, issued as Circular No. 54 by the Bureau of Standards, and dated November 15th, 1916, has a rule requiring that oil switches "be marked with the current which they can safely interrupt." It is recommended that this question be referred to the Standards Committee.

During the discussion on the relay papers it was brought out that there were several types of relays which were known by different names. One type of relay, which is more correctly called an excess-current relay, is generally called an overload relay, although it is not the best practise to use such relays on the generating or transmission system as a precaution against overheating due to slight overloads. They rather operate in case of large excess currents or short circuits, and there is no doubt but that their name has misled many engineers to use them improperly to the impairment of the service rendered by their system.

Another type of relay is variously called a reverse-current relay, reverse-power relay, reverse-energy relay, and unidirectional relay. This type of relay is intended to operate only with excess currents in one direction.

A third type of relay, used for connecting two transmission lines operating as one line with divided conductors of different constants, is variously known as biased relay, percentage differential relay, and ratio balance relay.

During the meeting in Chicago it developed that because of a lack of a conventional name, considerable space in the papers and time in the meeting was consumed in defining the terms used in the discussion. While it is entirely probable that the details of these various types of relays will be altered and improved from time to time, it is quite evident that the types of relays will endure, and it is suggested that the nomenclature of these several types of relays be referred to the Standards Committee.

The discussion on the relay papers in Chicago indicated that a number of the companies in the larger cities were rapidly changing their practise in the operation of transmission lines due to the development in relays. Heretofore most of these companies, in order to obtain localization of the trouble upon the breaking down of a transmission line, have operated their lines radially, that is, singly and not in parallel at the substation ends. This plan of operation confined the service interruption following a breakdown, to the load being carried on this transmission line. With the advent of the later types of relays, which have been developed by several manufacturing companies, a number of the central station companies are now operating their transmission lines in parallel in a number of combinations, as they find by experience that the relays are reliable for this service. This method of operating transmission lines in parallel results in a considerable economy in transmission line copper due to the diversity in the load on the various transmission lines. With the present high prices for copper, any development in relays which will permit greater economy in transmission line copper should be of interest to a large part of the Institute membership.

It also appeared from the discussion that some companies were using a simple, low priced relay and securing practically the same service as other companies with a higher priced, more complicated relay, and under practically the same conditions. It also appears that some companies are experimenting with types of relays under certain conditions, which have been tested by other companies under practically the same conditions and abandoned because unsatisfactory.

In view of this situation, which has been known to the committee for a number of months, but which was accentuated by the discussion at the Chicago meeting, the committee has undertaken to prepare a questionaire regarding relays, but owing to the circumstances during the past few months they have been unable to put this questionaire in form to be sent out to the various power transmission companies. It was the hope that the replies to this questionaire would enable the committee to make some definite recommendations based upon the experience of the various companies. It is recommended that the work on this questionaire be continued by the Committee on Protective Devices during the coming year, and that the information gathered by the questionaire after being tabulated and summarized, be presented by the committee in form for discussion at one of the technical sessions of the Institute.

D. W. Roper, *Chairman.*

ANNUAL REPORT OF COMMITTEE ON INDUSTRIAL AND DOMESTIC POWER

To the Board of Directors,

The Committee on Industrial and Domestic Power submits the following report for the year of 1916-17:

At the October 1916 meeting of the Board of Directors the committee name was changed from "Committee on Electricity for Industrial and Domestic Use" to "Industrial and Domestic Power Committee." A rather large committee was selected, as the chairman desired to consider a new field of activity and wanted to get as wide an expression of opinion as possible.

Three meetings of the committee were held; the first in Philadelphia in October, the second at Pittsburgh in January and the third at Chicago in March. In addition a sectional meeting was called in Cleveland in December.

Of the twenty-two members on the committee 16 have attended one or more of the meetings and of the other six the chairman has personally seen and talked over the committee work with all but one.

The committee has been represented at every meeting of the Meetings and Papers Committee.

The presentation of papers under the auspices of the committee was confined to two sessions at the midwinter convention in February on the general subject of "Control of Industrial Motors." The committee responded well to the request for a list of papers available for presentation before local sections, and with the cooperation of other committees compiled a list of about 110 such papers.

In 1914 the committee started an investigation of standard sizes for carbon brushes. Up to that time no effort had been made along this line with the result that nearly 5,000 variations of sizes and special work were in use. The present chairman has actively followed this line and in 1915 interested the Electric Power Club. After a great deal of work on their part a meeting was called in Cleveland between representatives of the Electric Power Club and the Carbon Section of the Associated Manufacturers of Electrical Supplies, with the chairman of this Committee as the chairman of the meeting. Subsequent meetings were held and the following standards have been adopted by the Electric Power Club as general engineering recommendations effective July 1st, 1917. It is recommended that the A. I. E. E. Standards Committee consider these and give their approval if nothing is found incompatible with engineering practise and development.

STANDARD SIZES FOR CARBON BRUSHES

1. Length of Round and Rectangular Brushes

Up to and including 1¼ inches	increase by steps of ⅛ inch
Over 1¼ and to 3 inches	increase by steps of ¼ inch
Over 3 inches	increase by steps of ½ inch

Preference to be given steps of ¼ inch below 1 inch length and ½ inch above 1 inch length.

2. Width and Diameters

Up to and including ¼ inch	increase by steps of 1/16 inch
Over ¼ inch to 2½ inches	increase by steps of ⅛ inch
Over 2½ inches	increase by steps of ¼ inch

It is recommended that ¼-inch steps be used wherever possible. Diameters of round brushes shall vary by steps of 1/16 inch.

3. Thickness

Up to and including ¾ inch	increase by steps of 1/16 inch
Over ¾ inch	increase by steps of ⅛ inch

It is recommended that ⅛-inch steps be used wherever possible above ½-inch thickness.

4. Limits

Rectangular and Square Brushes

Length	plus or minus 1/32 inch
Width	exact size down to 1/64 inch undersize
Thickness—Plain brushes	plus 0.001 inch minus 0.003 inch
Plated brushes	plus 0.001 inch minus 0.004 inch

For square brushes, thickness limits to apply to both the width and thickness.

Round Brushes

Diameter:—

Up to and including ¼ inch	exact size to 0.006 inch undersize
5/16 and ⅜ inch	exact size to 0.008 inch undersize
7/16 inch and above	exact size to 0.010 inch undersize
Length	plus or minus 1/32 inch

Box gages to be used for thickness and diameter.

5. Length of Shunts

It was decided that no standard lengths could be used, but it was agreed to define the length of a shunt as the distance from the top of the brush to the center of the slot or hole in the terminal.

6. Holes or Slots in Shunt Terminals

It was decided that exact sizes are not essential, but that the following shall be adopted as a maximum for the screws specified, and the minimum shall allow sufficient clearance to permit the screw entering the slot or hole without binding.

No. 8 and No. 10 screws	7/32-inch hole or slot
No. 12 and No. 14 screws	¼-inch hole or slot
5/16-inch screw	11/32-inch hole or slot
⅜-inch screw	13/32-inch hole or slot

7. Bevels

Bevels on carbon brushes shall vary by steps of five degrees and shall be accurate to within one degree above or below.

The length of a beveled brush shall be the distance from the end to the toe of the bevel, if beveled on one end only, or the distance from toe to toe measured parallel to the face of the brush when both ends are beveled. In other words the length will be that of the square-ended brush from which the beveled brush was made.

8. Plated Brushes

Dimensions, limits, etc., to be the same as for plain brushes, except thickness limits, for which see Paragraph 4.

The greater part of the year's work has not been finished but the committee hopes to have the papers ready for presentation next fall. It undertook to begin a study of motor and control application to various industries. It believes there is a best type direct-current motor, a best type alternating-current motor and a best type controller for every important machine in every industry. The committee has undertaken this year to begin a study of three fields; cement industry, passenger elevators and metal working industry. For this purpose three sub-chairmen were appointed, one in charge of each division. As stated above, the work is not completed but, if the committee is held largely intact for another year, it is believed we can make complete recommendations for the cement industry and passenger elevators. The metal working industry is too extensive to cover in such a short time and the sub-committee in charge of this division is making a study of some of the most important machine tools. The results are to be presented in convenient table form.

Care will be taken to keep recommendations of such a general nature that they will not discriminate between manufacturers. In every case where recommendations are made they will include both alternating and direct-current motors without favoring one over the other.

The committee enlisted the services of local Industrial and Domestic Power Committees and responses were received from Seattle, Milwaukee, and Cleveland with assurances that any work outlined by the committee would be carried out to the best of the ability of the local committee members. The Industrial and Domestic Power Committee recommends that a careful study be made to determine to what extent local and national technical committees can cooperate in their work to mutual advantage. It has been suggested that perhaps the technical committees depend too much on specialists and not enough on the vast fund of practical experience of the membership at large.

E. H. MARTINDALE, *Chairman.*

ANNUAL REPORT OF COMMITTEE ON TELEGRAPHY AND TELEPHONY

To the Board of Directors,

The Committee on Telegraphy and Telephony submits the following report for the year 1916-17:

Cooperation in Avoiding Interferences

Cooperation between the representatives of lighting and power interests and the operators of telephone and telegraph plants, directed toward avoiding hazards and interference, has progressed, and the engineers of these different utilities are becoming better acquainted with the problems involved leading toward the friendly settlement of differences. Specific cases of interference are being met by the application of principles, some of which have been established as the result of experience and others which are in more or less advanced stages of development.

Telephony

Applications of the developments which made transcontinental telephony possible have been extended so that now the cities of over 50,000 population in the United States, as well as the territory adjacent to them, have been placed in telephonic communication with each other. These improvements which have increased the range of telephone transmission many-fold have been made with only slight changes in the lines and equipment and with no change whatever in the subscriber's station apparatus.

Beyond the advantage to the public of such a universal system of communication, its special value under war conditions, in problems of national defense, is of transcendent importance. This was demonstrated during the past year by a mobilization of systems of communication made at the request of the Secretary of the Navy. This demonstration was conducted in cooperation with naval officers under the command of the Chief of Naval Operations. Continuously during three days the Navy Department at Washington and the Navy Yards and Naval Stations in the Continental United States carried on all their communication with each other and with the naval force in that territory by telephone and telegraph over the wires of the Bell System. During this period instantaneous telegraph and telephone communication between any points involved in the mobilization was possible.

Included in this demonstration was a test of wireless telephony. The battleship New Hampshire was connected with the system by wireless telephony and was able to receive and send communications while the vessel was at Hampton Roads and even as far out to sea as the southern drill grounds. Conversation between the Mare Island Navy Yard on the Pacific and the battleship New Hampshire while at sea was possible. Captain Bennett, in command of the Mare Island Navy Yard, conversed with Captain Chandler on the New Hampshire which was at that time in a storm on the Atlantic. This conversation was transmitted over the transcontinental telephone wire circuit from Mare Island, California, to the radio station at Arlington, Va., and there transferred automatically to the wireless from Arlington to the ship at sea, the return conversation taking the opposite course.

The mobilization was intended primarily to test the efficiency of the wire system in time of war. The Secretary of the Navy and the Admiral in command at Washington talked to the commanding officers of the naval stations on the Pacific coast, on the Gulf and Atlantic coasts and on the Great Lakes. The results of this mobilization were most satisfactory and demonstrated that the plant and organization involved were in complete readiness to respond in case of national emergency.

Telegraphy

In the subject of telegraphy, beyond the increasing use of automatic devices and methods, there have been no new developments completed concerning which reports can be made at this time.

Radio-Telegraphy

Along the lines of commercial application of radio-telegraphy there has been an extension in the long distance service by the Marconi Company opening up communication with Japan via Honolulu. There are also high-powered U. S. navy stations located in California, Hawaïi, the Canal Zone and the Philippines. The two German stations located at Sayville and Tuckerton have done a large business in transmitting messages to and from Germany directly.

The tendency in radio-telegraphy in its use on ships has been towards standardization in the form of the sets. This is particularly true in the spark type of sets. There has been an increasing use of sustained waves on ship stations. The general service rendered by the ship stations has continued to be of large and increasing value.

The use of sustained waves is increasing. The power required for this method is generated by arcs or by high-frequency alternators either directly or through frequency transformers.

F. L. RHODES, *Chairman.*

ANNUAL REPORT OF THE MARINE COMMITTEE

To the Board of Directors,

The Marine Committee submits the following report for the year 1916-17:

The shipbuilding industry has been submerged with work both of the merchant variety as well as the naval type of vessel. Heavy demands have been made upon the time and talents of the electrical engineer. New and extensive applications of the electric motor have been made. Many of the merchant designs have departed from the conventional direct-current lighting plant to a complete equipment of alternating-current lighting and motor service including engine room as well as deck auxiliaries. These plants have followed commercial land practise using 250-volt, 60-cycle, 3-phase alternating current. Two or more merchant vessels now under construction will be electrically propelled. The navy preparedness program included over one hundred and fifty vessels and the capital ships are to be electrically propelled.

PRESENT ACTIVITIES

The rules for marine installations compiled last year and adopted by the National Board of Fire Underwriters were accepted this year without change by the American Bureau of Shipping. These rules were simultaneously submitted to Lloyd's Register of British and Foreign Shipping and their representatives in this country suggested that our committee confer with the British Institution of Electrical Engineers. We are at present in correspondence with the rules committee of the British Institution and are awaiting ways and means of accomplishing the final acceptance of Lloyd's Register. This acceptance will complete the work of standardization of marine installations. A sub-committee was appointed at the beginning of the year to consider tentative rules for the installation of wires and wireways specifically for electric propulsion but this sub-committee in a partial report indicates that the subject cannot be well handled while the designs proposed are still in an unsettled state. One technical paper has been in course of preparation since last August and three other papers were requested. The stringency of the times has not permitted the writers to effect an accomplishment. The confidential restrictions placed on naval affairs forbids the publication of technical data at this time and this committee is of the opinion that general or suggestive papers would not be of value to the TRANSACTIONS of the Institute. The applications of electricity to the merchant service are in the nature of innovations and their value remains for determination after their performance has been carefully observed in practise.

FUTURE SUGGESTIONS

The unfinished negotiations with Lloyd's Register of British and Foreign Shipping should be brought as soon as possible to a favorable conclusion. The preparation of tentative rules covering the installation of electric propelling machinery and correlated matters should be entered upon as soon as opportunity permits, so as to avoid restrictions or complications that may take place due to lack of rules or the transfer by interpretation of old rules to a new and not fully understood application. Continuity in committee work is essential to progress and the chairman of this committee has consistently laid before the secretary of the committee all matters connected with the actions both external and internal of the committee so that full information is available to the succeeding committee. It is believed that sub-committees which have not been able to complete their work should be continued until discharged. In view of the conditions briefly alluded to it seems almost impracticable at present to suggest any methods for the production of technical papers. It may be expected, however, that in a few months the conditions may change, sufficient experience gained, and results obtained, to allow of the collection of much valuable information for the TRANSACTIONS.

H. A. HORNOR, *Chairman.*

ANNUAL REPORT OF COMMITTEE ON THE USE OF ELECTRICITY IN MINES

To the Board of Directors,

The Committee on the Use of Electricity in Mines submits the following report for the year of 1916-17:

PRINCIPAL DEVELOPMENTS IN THE USE OF ELECTRICITY IN MINES

The more notable advances in the use of electricity in mines during the past year have been in the directions enumerated below:

The development of storage-battery locomotives and combination storage-battery and trolley locomotives.

The development of trolley locomotives in sizes up to 35 tons.

The development of the so-called "arc-master" for use with mine locomotive controllers.

The increased use of power purchased from central stations.

The application of electric motors to the ventilating fans of coal mines.

General improvement in the design and construction of mine electrical equipment, and increased use of interpoles and improvement in the methods of impregnating motor windings to make them moisture resisting.

The increased use of 2200 volts and higher voltages for underground transmission and for direct application to stationary motors used in mines.

The development of explosion-proof motors for coal mines.

The development and widespread adoption of self-contained portable electric lamps for miners. (This is believed to be one of the most effective safety measures that has been taken in some time).

The development of a practical electrically operated reciprocating drill.

The increased use of electrically operated coal cutting equipments.

Increased interest in safe electrical equipment and measures for safeguarding the use of electricity in mines.

ACTIVITIES OF THE MINES COMMITTEE

The Mines Committee was reorganized rather late in 1916 and for that reason has not accomplished as much as the members could have desired. The committee has recommended to the Institute that active steps be taken to increase the membership of the Institute among electrical engineers engaged in mining work. They have also recommended that a vocational classification be made of the membership of the Institute, to the end that all technical committees can readily determine what engineers are interested in the particular work of the committee.

The committee has also recommended that meeting at which mining papers are presented should be the meeting held at Pittsburgh, or one of the sessions of the Mid-Winter Convention, in the event of the Mid-Winter Convention's being held simultaneously with that of the American Institute of Mining Engineers. The committee points out that under no other circumstances can we be assured of an audience sufficiently interested in mining subjects to warrant the committee in asking the more prominent mining electrical engineers to present papers.

The principal work of the committee has been done in connection with suggested rules for the use of electricity in bituminous coal mines. These rules were prepared by two members of the committee in collaboration with a majority of the rest of the committee and other mining men and electrical engineers. The rules were prepared under the supervision of the United States Bureau of Mines, who stands sponsor for them. These rules were formally submitted to the Institute by the Mines Committee, with a request that they be approved and adopted by the Institute. The rules were referred first to a joint committee of the Standards Committee and the Code Committee, who found that there was nothing in the rules incompatible with the Institute's Standardization rules. The Mines rules were then referred to a joint sub-committee composed of members of the Mines Committee and Standards Committee, who reported favorably upon them to a joint session of the Standards Committee and the Mines Committee, and the joint session recommended to the Board of Directors that the Institute approve these rules in the interests of safety, good engineering practise, and standardization. The Board of Directors then referred the rules once more to the Code Committee, which at the date of writing this report still has the rules under consideration.

SUGGESTED FUTURE ACTIVITIES

It is suggested that the aim of the Mines Committee in the future be to get nearer to the electrical mining men, to urge the Institute to do things to interest such men and attract them to membership in the Institute.

An effort should be made to recruit the Institute ranks with as many of such men as possible and have mining sessions at which the best electrical men in the industry present papers. The time for such a move was never more opportune. It is suggested that if the Institute will interest itself in standards for mine safety and other mine electrical standards, such an attitude will do more than anything else to interest a class of members that the Institute needs very much.

H. H. CLARK, *Chairman.*

ANNUAL REPORT OF IRON AND STEEL COMMITTEE

To the Board of Directors,

The Committee on the Iron and Steel Industry submits the following report for the year of 1916-17:

The Iron and Steel Committee is arranging to cooperate, if possible, with the Association of Iron and Steel Electical Engineers for providing papers for their annual convention in September. Details of this cooperation have not yet been worked out.

From a study of the relations between the A. I. E. E. and the A. of I. & S. E. E., it is evident that it is desirable to formulate some kind of policy for the A. I. E. E. to follow.

The A. of I. & S. E. E. came into existence about ten years ago, as at that time it was thought by the mill operators that the activities of the A. I. E. E. did not cover the field in a way helpful to them. It was, therefore, decided to form an independent association that would deal with the problems of steel mill operators in a proper manner. The only persons eligible to full membership are electrical superintendents of steel mills or other plants allied with the steel industry. Members of manufacturing companies are only eligible for associate membership and take no part in voting.

It is evident from a knowledge of the constitution of the two societies that there is a natural division between their activities. The A. of I. & S. E. E. is primarily interested in the collection and exchange of data relating to operations. Such data are of interest only to persons interested in the steel industry. The A. I. E. E. is interested in the development of the electrical apparatus generally and questions of design of apparatus and special functions are of interest to many of its members. The information that may be obtained from papers and discussions is of interest to the whole industry whenever it contains an element of more than local application. It would, therefore, appear that as far as papers are concerned it would be natural for those covering operating features or special methods of installation, operating costs, etc., would be within the province of the A. of I. & S. E. E. Papers dealing with the design of the apparatus or with the relation of apparatus used in steel mills to apparatus used in other industries, would naturally appear to be within the province of the A. I. E. E.

There are, of course, many points at which there may be a conflict. For instance, a paper dealing with the interconnection of a number of independent generating stations in steel mills so as to utilize their combined capacity to better advantage, would be a natural subject for discussion before the A. of I. & S. E. E. On the other hand, such a paper would also be of interest to the A. I. E. E. as it relates to a problem which is an every day matter of power generation and distribution. There are bound to be a number of such papers read before both societies. On the other hand, a paper dealing with the design of a rolling mill motor would naturally be the subject for the A. I. E. E. as a rolling mill motor is only a special form of one class of electrical machinery. However, the paper dealing with method of installing such a motor and the peculiar conditions under which it might operate would be proper subject for the A. of I. & S. E. E., as it would only be of interest to the members of this association and not the membership of the A. I. E. E. generally.

In this connection, it must be borne in mind that the majority of full members of the A. of I. & S. E. E. are members of the A. I. E. E. and it is also a fact that practically every one in the A. I. E. E. interested in the steel industry are members or associates of the A. of I. & S. E. E.

This matter has been gone into at some length as it would appear advisable for the A. I. E. E. to adopt a fixed policy regarding acceptance of papers to be read before it on subjects dealing with the iron and steel industry.

An important activity of the A. I. E. E. is being duplicated to some extent by the A. of I. & S. E. E., *i.e.* Standardization. It does not appear that it is advisable for the A. of I. & S. E. E. to attempt a standardization of apparatus in competition with the A. I. E. E. but it would appear desirable for the A. of I. & S. E. E. to adopt the standards of the A. I. E. E. and to have a sub-committee of the A. I. E. E. Standard Committee to take care of any special characteristics that may be found necessary for the successful operation of apparatus in the steel industry. In this way a useless duplication of the work can be avoided and also the natural confusion that will arise regarding requirements if there are several standards. The manufacturing companies are certain to adhere to the A. I. E. E. standards and there would be continual difficulty regarding exceptions to any other set of standards.

It would appear that there is a field of activity in the steel industry that should be probably covered by the A. I. E. E., but at the same time an effort should be made to avoid the presentation of papers which interest only a small section of the Institute membership and which would be more effective if read before an association all of whose members would be interested in the subject. It is, however, desirable that an effort be made by the A. of I. & S. E. E. to avoid unnecessary attempts at standardization if the same result can be obtained by cooperation with the A. I. E. E.

It would seem desirable that a small committee of the A. I. E. E. should be formed to confer with a similar committee of the A. of I. & S. E. E. to arrive at an understanding regarding the activities of each society in connection with the iron and steel industry. In this way a definite settlement can be arrived at and a policy fixed for the future. The A. I. E. E. committee should preferably consist of members not associated with the iron and steel industry. All members of the A. I. E. E. connected with this industry are necessarily more or less associated with the A. of I. & S. E. E. and it is rather difficult for them to differentiate between the claims of the two societies.

WILFRED SYKES, *Chairman.*

ANNUAL REPORT OF THE EDUCATIONAL COMMITTEE

To the Board of Directors,

The Educational Committee submits the following report for the year of 1916-17:

The Educational Committee has devoted its attention to the subject of devising a scheme to assist and encourage the younger members of the Institute in developing themselves by undertaking the study of research and practical problems which it is hoped will be collected for them by the Educational Committee in the future from the older members of the Institute and presented in the form of competitions. In doing this your committee has been carrying forward and enlarging the policy of the Educational Committee of the preceding years.

It is the sense of your Committee that the Educational Committee should endeavor to assist the younger members of the profession to improve their training in engineering by carrying on volunteer graduate work of a practical character under the advice of the older members who are in the active practise of the profession.

There is apt to be a gap in the intellectual training of an engineer from the time he leaves his technical school, while he is engaged in the important but ruder type of work of the shop or field, till he has "found himself" and made a sufficient reputation to be taken into the office and put upon the higher problems of engineering. During the time of this gap numerous young men feel a want, a longing to do something which will improve themselves and give them an opportunity of demonstrating their capacity to the older members of the profession. At present only the exceptional ones have sufficient originality and progressiveness to think out problems for themselves and tackle them. The majority of the good men devote their spare hours to reading and the remainder just drift along.

While reading and study will develop one's knowledge it will not develop originality, inventiveness and resourcefulness. Only the application of knowledge to specific problems will do that.

The young men in this category are particularly the younger members of the Institute forming the substantial part of the membership of the "Branches," but also the enrolled students and students in technical schools should be considered. The Institute should consider that these men cannot attend the general meetings very frequently and that the majority of the papers presented before these general meetings are too specialized to appeal to them. Yet these men are the future backbone of the Institute and the profession. It is therefore the duty of the Institute to hold their interest and train them. This is a matter which should interest the Sections Committee also and should aid in strengthening its work and it also can aid in carrying on this work.

Thus your Committee urges a cooperation between the Sections Committee and the Educational Committee in the work of providing programs of papers and study for the younger members through the medium of the branches.

The committee has sketched out a scheme of action which it recommends should be developed and followed up by successive Educational Committees throughout a term of years.

The program should begin with a meeting of the Institute devoted to this subject for which papers would be invited from a few of the older men which should deal with the subject of what qualities and training after graduation are most valuable in the young engineering graduate. A few written discussions of the main papers should also be invited. At the same meeting there should be presented some letters from the younger members stating their point of view and the obstacles they encounter in the way of advancement.

The Educational Committee should collect suggestions for research and hypothetical problems of a practical and theoretical nature from older members and present them to the membership through the medium of the branches. Work on these problems should be organized in the form of competitions similar to those carried on successfully for a number of years by the Engineers Society of Western Pennsylvania. The committee acknowledges the helpful and valuable suggestions imparted to it on this subject by Dean Leete of Carnegie Tech. School. To carry out this scheme the Educational Committee should actively solicit suggestions from the older members and should send out from time to time a list or program and announce a time at which the solutions would be considered and graded. The Educational Committee should assume the responsibility of grading these reports but should appoint judges from engineers outside of its membership. Later the most praiseworthy solutions should be published and a meeting of each of the branches set for the discussion. If possible awards should be made for the best solutions although it is probable that the honor of having the paper published and discussed would be a sufficient incentive.

The Educational Committee from time to time should invite prominent members of the Institute to write papers on fundamentals or principles of the design and operation of electrical apparatus or systems which would make our proceedings of more interest to the younger members. As examples of papers of this character are two by Mr. B. G. Lamme on Commutation, 1911, and on Iron Losses in D-C. Machinery, 1916.

Your committee recommends that relations be established with the Educational Committees of the other three Founder Societies for an exchange of views and coordination of methods as we understand that the A. S. M. E. is endeavoring to carry out a similar policy to strengthen its hold on the younger members through the cooperation of the Educational and Sections Committees.

In general the plan outlined is intended to supply the missing link in the development of the engineer between the time of graduation and the time he really begins to accomplish something and becomes an identity in the engineering profession.

W. I. SLICHTER, *Chairman.*

ANNUAL REPORT OF THE POWER STATIONS COMMITTEE

To the Board of Directors,

At the first meeting held October 13, 1916, the scope and activity of the Committee were analyzed with a view of avoiding the present unnecessary duplication found in the numerous committees of the various sister societies. The opinion has been advanced that many of the overlapping committees would do better to limit their work to the subjects peculiarly proper to their own society, and omit the investigation of subjects falling more nearly in the work of other societies, utilizing the work of sister society committees to complete the field when necessary.

For instance, the design of turbines, stokers, bearings, and boilers is very properly the work of the A. S. M. E. It is a waste of time for the A. I. E. E. to go through all the work of the Boiler Code again: Therefore, the boiler code of the A. S. M. E. was offered to the standards committee with the recommendation that it be approved, since, presumably, the Mechanical Society should be the highest authority on a mechanical subject. Action has been deferred for the present. In the opinion of the committee the right move has been made on subjects common to several societies in the formation of the joint committees on the cost of electric power.

With the above in view, a resolution was passed (a) that the function of this Committee is to investigate and report on the engineering of power stations beginning at the prime mover and ending at the outgoing cables; (b) that the investigation of specific problems on the remainder of the power stations, and allied engineering affecting it, shall be formally referred to appropriate committees of sister societies for action. This Committee will utilize the resulting action of such sister societies in completing its own reports. This Committee believes in usefulness of joint committees in getting the full results on a subject covered to the best advantage by each society in its own special field as far as is practicable.

The subjects offered for investigation are given below.

(1) *Safety First Work in Regard to Arrangement and Installation of Apparatus*: This was referred to Mr. Torchio, who has

reported that the reference material on this subject was already very fully covered by the National Electric Safety Code and the Fire Insurance Companies.

(2) *Combined Hydraulic and Steam Plants*: This was thought to be a subject of special solution in most cases, but a committee consisting of Mr. Scattergood, Mr. Harisberger and Mr. Meredith was appointed to get up a paper on the economics and design of hydroelectric and steam developments.

(3) *Excitation*: This subject was taken up broadly, the original suggestion only covering the use of the combination electric-steam-drive unit now becoming popular. A committee consisting of Mr. Wallau, Mr. Wood and Mr. Harisberger and Mr. Kruesi, Mr. Smith later substituted for Mr. Kruesi, on account of the latter's ill health.

A preliminary report has been prepared, which will in all probability be submitted in final form by September.

(4) *Methods of Heat Analysis and Distribution for Steam Power Plants*: Mr. Pigott, Mr. Gorsuch and Mr. Wood form the subcommittee and were instructed to cooperate with the N. E. L. A. Prime Movers Committee. Upon attempting this, it was found that the N. E. L. A. Committee thought its work was too far along to make it advisable to modify their report, but it will be used in working up ours. A paper on the subject is to be ready by September.

(5) *Air Conditioning for Generators*: Dr. Moss, Messrs. Lincoln, Moultrop and Torchio were appointed to investigate the status of the air washer and filter for generators. Preliminary report, March 17th, shows that while the advantages of air conditioning are undoubted, the results obtained do not always justify the expense of the equipment, on account of inefficient working of the washer. The committee is not satisfied with the data at present on hand, and is collecting further information.

(6) *Rating of Switches and Breakers for Dusty or Dirty Atmosphere*: The question of cutting the rating of switches and breakers in cement and cereal mills, or other dusty atmospheres, was brought up by Mr. Drabelle. It was the opinion of the Committee that this subject should be referred to the Committee on Standards, which was done.

(7) *The Ford Power Plant*: On examination of Mr. Allison's paper it was thought desirable that some actual operating data from this plant should be given, rather than predictions, and the matter was therefore laid aside for the present.

(8) *Improvement of Waterwheel Test Code*: The opinion of the Committee was that the problem should only be attacked from the electrical end by way of known generator losses, etc., leaving the mechanical end to be handled by the A. S. M. E. and the N. E. L. A. in whose hands it more properly belongs. Mr. Lincoln, with Mr. Egbert, and Mr. Reist, were appointed to handle this matter. No report up to the present.

(9) *Design of Mimic Switchboard*: This subject was brought up by H. R. Parker of the Toronto hydroelectric system; he was referred to several companies using these boards, as it was thought committee work for immediate construction purposes would be too slow.

The chairman of the Committee, Mr. DeRemer, has been forced to relinquish this work for the present on account of being called to Washington on Government work. The writer was appointed to carry on the work and wishes to apologize for any shortcomings of this report, as it has been prepared on notice too short to allow of bringing all sub-committee's reports up to date.

R. J. S. PIGOTT, *Vice-Chairman.*

ANNUAL REPORT OF COMMITTEE ON ELECTROCHEMISTRY AND ELECTROMETALLURGY

To the Board of Directors,

The Committee on Electrochemistry and Electrometallurgy submits the following report for the year 1916-1917:

In the absence of Capt. T. H. Schoepf on military service the following brief memorandum on the work of the Electrochemistry and Electrometallurgy Committee, has been compiled from the files of the chairman, with the hope of giving the new committee and chairman the benefit of the suggestions and opinions of the retiring committee.

No sessions on electrochemical subjects were held during the year as none of the papers reviewed by the committee, were suitable for presentation except at joint meetings with the American Electrochemical Society, and such joint meetings could not be arranged because of conflicting dates.

It was Chairman Schoepf's plan to work up some good papers for the coming year and he made an endeavor to find out from the most prominent engineers in the electrochemical industries, what subjects should be considered by the committee and presented before the Institute or before a joint meeting with the American Electrochemical Society. The replies to the first request by the chairman and the subsequent opinions of the members of the committee, on suggested subjects, seemed to indicate a wide difference of opinion regarding the advisability of presenting some of the proposed papers.

The most popular suggestions were electric furnaces and the fixation of nitrogen, however some of the electrochemical specialists feel that papers on these subjects, which have been treated repeatedly by other societies, would result in duplication, unless some hitherto untouched phase of the subject is to be dealt with.

Other subjects which have been suggested and discussed by the committee are: electrical porcelain, electric cells and batteries, electrolytic copper, conductors of copper vs. iron vs. aluminum, coal vs. water power for electrochemical industries.

The last subject, seems from the correspondence, to be worthy of careful study and of great interest to both the steel and electrochemical industries, in view of the recent developments in large steam power units and the water power restrictions imposed by the Government.

It is the suggestion of the retiring chairman and some of the members of the committee that this timely subject be taken up by the new committee and presented before a joint meeting with the American Electrochemical Society.

Another suggestion which met with approval was to have Mr. Liljenroth present a comprehensive paper on nitrogen fixation including the Norwegian practise, if the proper approvals can be obtained and there is no military objection to discussing the processes in the coming year.

L. W. CHUBB, *Secretary Meetings and Papers Committee.*

Presented at the 332*nd meeting of the American Institute of Electrical Engineers, New York, June 28, 1917.*

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FORMS OF ELECTRIC POWER BEST SUITED FOR THE VARIOUS LOADS ENCOUNTERED IN THE OPERATION OF BITUMINOUS COAL MINES

BY R. L. KINGSLAND

ABSTRACT OF PAPER

The best form of power to be used in coal mines for drilling, cutting, hauling, pumping, lighting and ventilation, Synchronous converters versus motor-generator sets. Location of generating plant and substations.

THIS paper is intended to deal entirely with the practical application of electric power to bituminous coal mining.

There are two general forms of electric power available; alternating current and direct current. With alternating current we have two standard frequencies; *i.e.* 25 and 60 cycles, and with both alternating and direct current we have many standard voltages.

In the question of frequency the writer is much in favor of 60 cycles. This has become more and more the standard frequency for central stations. There used to be some advantages of 25 cycles for rotary-converter work, but this has been cut down until today the 60-cycle machine is considered the equal of the 25. This choice of 60 cycles can be further justified by noting that in selecting motors for various mine services to operate between the speed limits of 300 and 3600 rev. per min. we have the choice of 12 speeds in the case of 60 cycles and only 5 in the case of 25 cycles.

Different voltages are more easily obtained with alternating current than with direct current due to transformers. The rated voltage on a-c. machines may be misleading from a safety standpoint. This rated voltage is the effective voltage; whereas the peak, or maximum instantaneous voltage, is equal to almost one and one half times the average value. Thus a person coming in contact with an a-c. 300-volt line could receive 423 volts while with a d-c. line he could receive only 300 volts.

The question of transmission is one of the most important in connection with coal mining. If complete copper circuits are used, 3-phase 3-wire alternating current with unity power factor requires only three-fourths the copper that direct current of the same voltage does. In actual practise, however, in bituminous coal mining work under ground, the nature of the load is such that more copper is required for a 3-phase a-c. circuit than for a d-c. circuit of the same voltage. This is largely due to the lagging power factor caused by induction motor load. It is common practise in d-c. transmission underground, to use the rail for the return circuit. In the case of 3-phase alternating current the rail may be used in place of one of the wires. In this case the total feeder wire for alternating current unity power factor and direct current would be exactly the same for the same voltage drop. Actual conditions, however, in this case require even a larger excess of copper for the alternating current than where complete copper circuits are used.

What is known as low voltage is steadily increasing in favor for portable machines used under ground. This includes a maximum voltage of 300, with a 10 per cent divergency factor allowed. In other words the actual maximum voltage at any point on the lines cannot be in excess of 330 volts to come within what is termed low voltage. For stationary machines, medium or even high voltages can be used to advantage, and safely, if proper protection is afforded.

To summarize, the writer favors the use of direct current or 3-phase 60-cycle a-c. power at 330 volts or less for all portable machines under ground and considers that stationary machines should be treated individually.

When considering the separate classes of motor-driven machines it is well to keep in mind some of the principal characteristics of the most commonly used motors; *i.e.* the d-c. shunt and compound motors and the a-c. squirrel-cage induction motor. For our purposes these are all used as constant-speed motors. In the d-c. motors the speed varies almost directly with the voltage while with the a-c. the speed is practically independent of the voltage and depends on the frequency. The torque of the d-c. motors varies almost directly with the voltage, but in the a-c. induction motor the torque varies with the square of the voltage. As an example if we had one-half rated voltage on a d-c. shunt motor we would have one-half torque, while with one-half voltage on a squirrel-cage induction motor we would only have one-fourth the torque. This is a good point to keep in mind when considering a-c. motors for portable machines under ground.

For under ground distribution only two classes of copper conductors have been found practical—bare, and lead-covered steel-armored. Other forms of insulation can not be depended upon for any length of time, and are therefore not as safe as no insulation.

In order to cover all classes of load each will be taken up in turn. First we will consider the under ground loads, and then the surface loads.

The machine most used for cutting coal today is that known as the chain machine. The squirrel-cage induction motor is ideal for driving this machine, provided the proper voltage can be supplied to it. Shunt or compound-wound d-c. motors are more frequently used for reasons which will appear later.

Drilling machines used for making the shot holes are made in various types, some of them being driven by the same motors that run the cutting machines, and some by separate motors. A constant-speed motor is required, and high starting torque is not necessary.

Several types of coal loading machines have been developed, but none of them have come into general use as yet. Motors for these require the same characteristics as for coal cutting machines.

Combination coal cutting and loading machines are being experimented with, but they do not appear to be even as far advanced as the loading machines. Constant-speed high starting torque motors are also required for their operation.

One of the main factors in determining the form of electric power to use under ground in large mines is haulage. No practical means has been developed for heavy haulage by the use of alternating current and we are therefore limited to direct current. Haulage may be divided into three classes; trolley locomotive, storage-battery locomotive, and stationary hoist. For trolley locomotives d-c. series motors have proven by far the most practical. For battery locomotives the same is, of course, true. For stationary hoists much depends on the amount of load, the distance from power source by way of the mine workings, and the distance from the surface. More will be said of this later.

Pumps used in mines can be placed in two classes—those used for gathering the water to one or more central points, and those used to remove it from these central points to the surface. For the smaller pumps, or those coming under the first class, it is obvious that the same form of power should be used as is used for cutting, drilling and loading. The larger pumps will be treated with stationary hoists.

Lighting under ground can be most easily accomplished by the power that is used for other purposes where the lights are required. That is, it is not worth the extra installation and maintenance expense to string separate lighting lines through a mine. In the case of shaft mines or slope mines it is some times advisable to install separate circuits for the lighting where trips of coal are landed, and where the empty trips are made up. Tungsten lamps are the most economical lamp available for the size units required under ground. It has been the writer's experience, however, that these lamps do not give satisfactory services on voltages above 125 in sizes smaller than 200 watts. For use on 250 or 300-volt service I have found the carbon filament lamp to give the best results.

For concentrated loads of 100 horse power or over such as hoists, pumps, and underground substations I have found separate lead-covered steel-armored cables leading direct from the surface through special bore holes very satisfactory for supplying alternating current at voltages up to 11,000 star. Such cables are practically indestructable. For centrifugal or plunger pump installations constant-speed induction motors of the bore hole cable voltages can be used with perfect safety as there are no exposed contacts. Hoists can be treated in the same way and where necessary current-limiting devices can be applied so as to cut down the peak load. For under ground substations it is generally advisable to transform down to the required voltage in the substation room, in case synchronous converters are used. For motor-generator sets this is of course unnecessary.

The question of the relative merits of synchronous converters versus motor-generator sets, while not closely related to the question of selecting the best form of power, is of considerable importance to mine operators. The principal advantages attributed to the motor-generator set are higher power factor and closer voltage regulation. The advantages of the synchronous converters are lower first cost, small space occupied, and higher efficiency. The writer favors the synchronous converter because he has found that its advantages outweigh those of the motor-generator set in practise. I have operated a central plant having a mine load only, consisting of twenty five mines, where synchronous converters were used for supplying all d-c. power, and induction motors for fans, pumps, etc.; and where the power factor was ninety eight per cent lagging on peak loads, and never went below ninety per cent. I have found that the latest design of synchronous converters will carry momentary overloads of two hundred per cent and that motor-generator sets are not good for over one hundred per cent overload. The d-c. voltage at the substations was practically constant. When installing a substation inside a mine the smaller space necessary for the synchronous converter will be found to be of great advantage. It is easier to prevent squeezes from above and below with a small room than with a large one.

In locating a pumping station the possible location of a substation adjacent to it should not be lost sight of. This often means materially better voltage conditions for cutting and hauling coal without increased labor cost. The same man can operate the substation and the pump station.

In a great many bituminous coal mines ventilation is of first importance. For this reason we should have the most reliable ventilating equipment. As the squirrel-cage induction motor is the most rugged, as well as the simplest form of electric motor made, it lends itself especially well to this class of work. Through experience I have found that this type of motor gives entire satisfaction. In cases where it is advisable to reduce the volume of air delivered to a mine for certain periods, in order to save power or for some other reason, a two-speed motor of the same type can be used to advantage. Changing the speed of the fan from time to time as mine workings progress can be accomplished by changing the pulley sizes for a belted fan or the sprocket sizes for a chain driven, or geared fan. My own opinion favors belt drives as these relieve the motor from the cut off blows of the fan. In case a-c. power is not available for fan drives shunt motors should be used, but commutators prevent their giving the same reliable service as the induction motor.

In the preparation of coal one of the principal difficulties that motors encounter is fine dust, and plenty of it. The totally enclosed squirrel-cage, or where service requires it, the wound-rotor induction motor meets this condition most successfully. In either case, it is preferable to have a totally enclosed motor. Totally enclosed compound-wound motors should be used if alternating current is not available.

Shop drives and lighting can be classed together because both of these should depend on the primary source of power. They should not depend on the operation of any more machinery than is absolutely necessary. Lights will be required when everything else about a mining plant is closed down, and shop operations will also be required when all other equipment is at a stand still.

In conclusion I would say that for large mines the most practical power for the main supply would be three-phase sixty cycles at a voltage high enough to reach all necessary points with a loss of not over ten per cent of the power delivered. This allows the use of a-c. motors and lights on the surface and also wherever they can be used to advantage under ground. Synchronous converters delivering 300 volts direct current can be installed on the surface and under ground, as best suits the conditions; and by their use the amount of copper required inside a mine can be kept at a minimum.

If d-c. power is generated it is not economical and often not practical to install several small generating stations. The obtaining of fuel and water often present difficulties. Therefore the d-c. power plant is generally located near the tipple. This means that power can be fed only one way to the mine. With substations located so as to feed power to the load in two directions, only about one fourth the copper is necessary for the same length of line and the same voltage drop.

For small mines where locomotive haulage can not be used to advantage, alternating current may be used to advantage for mining machines and pumps. This is especially true if alternating current is available from a central station.

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A NEW ELECTRIC MINE HOIST AT BUTTE, MONTANA

BY R. S. SAGE

ABSTRACT OF PAPER

A description of equipment in connection with a recent Ilgner-Ward Leonard hoist installation at the Elm Orlu Mining Company, Butte, Montana. This installation is one of the two largest of this type in operation in this country.

Complete data on two sets of tests; the first conducted to analyze a typical day's run and the second to determine efficiency while hoisting under known conditions.

GENERAL

DURING the past year and a half, a period to be remembered for its tremendous industrial activity, extraordinary progress was made in the field of mine electrification; and in particular in the electrification of mine hoists of more than average capacity and importance, and for whose operation the highest type of electrical system was demanded. This progress extended to both the coal mining and the metal mining fields.

Illustrative of the extent of this progress, it may be said that the number of Ward Leonard and Ilgner-Ward Leonard hoist equipments which will have been put into operation in this country during the two years of 1916 and 1917 will equal the total number operating prior to 1916, and the aggregate horsepower capacity will be 50 per cent greater.

A recent Ilgner-Ward Leonard hoist installation of considerable interest is that at the mine of the Elm Orlu Mining Company at Butte, Montana. This equipment has been in regular productive operation since the first week of last February having been put into commission to replace an old steam hoist at that time. This type of hoist but recently made its appearance in the Butte district, and the present installations are but forerunners of many electrifications which will undoubtedly be made in the near future.

This equipment is of interest, first, because of its capacity, it being one of the two largest of this type in operation in this country, and, second, because of its unusual smoothness of operation, simplicity and ease of control, it being so noted throughout the Butte camp.

A series of tests under operating conditions were carried out, the results of which, together with a description of the equipment are given herein.

The hoisting equipment is installed in a very substantial brick building of one room, the motor-generator set with flywheel, the slip regulator and the switchboard occupying all of one end. Views of the hoist and motor and the power set are given in Figs. 1 and 2. The building is exceptionally well lighted, both by day and by night, and there is a particular absence of crowding and congestion. Two 20-ton* hand-operated traveling cranes provide adequate capacity for the handling of the heaviest parts.

DUTY

As is well known, for the driving of large high-speed shaft hoists, considerations of accuracy of control and safety of operation necessitate the use of equipments operating on the Ward-Leonard system. This system combined with a flywheel for load equalization, commonly known as the Ilgner-Ward-Leonard system, is necessary wherever the conditions of power supply require it. In this instance, as for the majority of equipments of this character, flywheel equalization was imperative, as the reservation charge is based on the maximum instantaneous demand. For this reason, flywheel capacity was supplied sufficient to completely equalize the maximum duty cycle met in balanced hoisting from the deepest level.

The shaft has been sunk to a depth of 1800 ft. (548.6 m.), and ore is being hoisted regularly from the 1100, 1200, 1300, 1400, 1500 and 1800-ft. (335.2 m. to 548.6 m.) levels. Ultimately, the ore will be taken from a seam 3500 ft. (1066.8 m.), below the surface.

The general characteristics of the hoist are as follows:

Inclination of shaft with horizontal........	90 deg.
Present maximum lift....................	1800 ft.
Ultimate " "	3500 ft.
Weight of skip and man cage.............	11,000 lb. (4989 kg.)
Weight of ore per trip...................	10,000 lb. (4535 kg.)
Weight of one rope (3500 ft.)............	12,500 lb. (5669 kg.)
Maximum speed of skip during hoisting....	2,500 ft. per min.
Diameter of cable.......................	1.5 in. (38.1 mm.)
Weight of cable, per foot................	3.55 lb. (1.58 kg.)

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* One short ton = 0.9 metric ton.

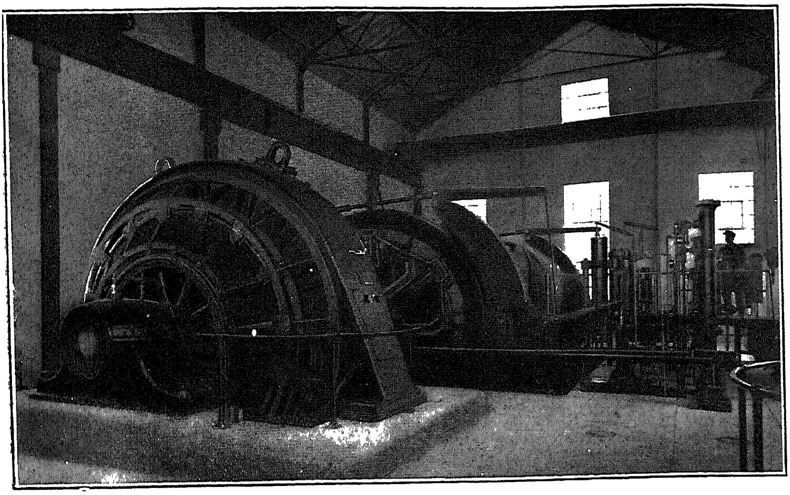

FIG. 1—VIEW OF HOIST MOTOR

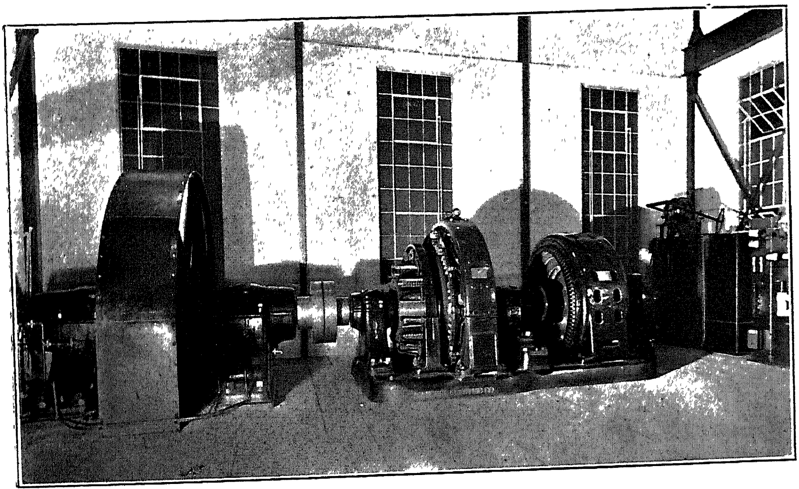

FIG. 2—VIEW OF FLYWHEEL MOTOR-GENERATOR SET [SAGE]

Drums—cylindrical—double-clutched.
Diameter of drums...................... 10 ft. (3 m.)
WR^2 of drums..........................3,600,000 lbs. ft.^{2}
Maximum hourly tonnage (2000 lbs.) 3500, ft. level............................. 155
Assumed mechanical efficiency............ 85 per cent.

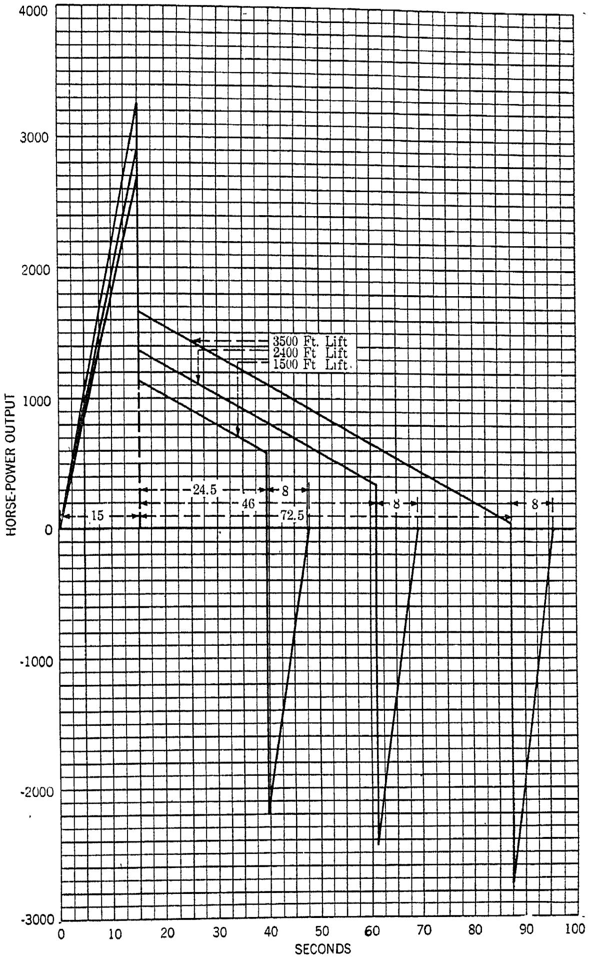

FIG. 3—CALCULATED HOIST DUTY CYCLES

The calculated cycles of duty for the various levels, upon which the capacity of the electrical equipment was based, are shown in Fig. 3. It is practically impossible to exactly predict the actual operating schedule in this class of service, but to insure continuity of service, the most important consideration in installations of this kind, the first requisite is that the driving machinery have a safe margin of capacity. A heating guarantee of 40 deg. cent. rise for all parts of the electrical equipment was therefore made, for continuous operation, when hoisting the specified hourly tonnage from any level. The efficiency curves for the various machines are given in Fig. 4.

The estimated limits of demand from the power supply for the various levels were as follows:

1500 ft. level..............	650 kw.
2400 " "	760 kw.
3500 " "	865 kw.

	100%	75%	50%	25%	Per cent. of maximum hourly tonnage
1500 ft.......	267	200	133	67	Hourly tonnage
	2.46	2.58	2.83	3 57	Kw-hr. per ton
2400 ft......	202	152	101	50.5	Hourly tonnage
	3.74	3 89	4.23	5.07	Kw-hr. per ton
3500 ft.......	155	116	77.5	38.7	Hourly tonnage
	5.54	5.75	6.17	7.45	Kw-hr. per ton

The estimated overall efficiencies corresponding to the maximum hourly tonnages are as follows:—

1500 ft. level—overall efficiency.......	46.2 per cent
2400 " " — " "	48.3 per cent
3500 " " — " "	47.7 per cent

The hoist is also used for practically all of the hoisting of an unproductive character, such as handling waste, men, timbers and supplies, and the hoisting of ore is done simultaneously with this sort of work.

The overall efficiency at which the ore is hoisted, that is, the ratio of the net work done in lifting a day's output of ore, to the kilowatt hours of electric energy used per day, is therefore much less than the figures given above. The mine is still in the development stage so that the unproductive work is a large proportion of the total required of the hoist.

At the present time the slip regulator is adjusted so as to limit the demand to 400 kw., as this is sufficient for the present depths, and the present manner of hoisting.

DESCRIPTION

The hoist consists of two steel-plate drums 10 ft., in diameter, grooved to hold 1900 ft. (579.1 m.) of 1.5-in. (38.1 mm.) steel cable per layer. The drums are mounted between three bearings, on a single shaft which has an extreme length of 40 ft. (12.1 m.). Each drum is provided with an axial-plate-type clutch and a parallel-motion post brake. The diameter of the brake tread is 14 ft. (4.3 m.). The post brakes are made of structural shapes and steel plate to form a box girder. The brakes are released and the clutches operated by means of oil through cylinders with floating-level control, served by an oil accumulator in connection with a motor-driven triplex oil pump. A two-horse-power motor supplies all the power required for the brake and clutch engines. The weights by which the brakes are set are in the form of cylinders moving vertically in guides formed by part of the brake engine. In addition to the regulation dial-type depth indicators, the drum flanges are extended to provide a surface for spotting marks.

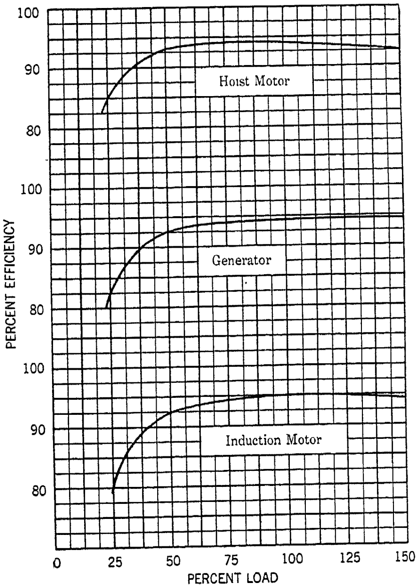

FIG. 4—EFFICIENCY CURVES

The bearings are all of the two-part type provided with gravity feed lubrication, in addition to oil rings. Sight-feed oil guages are provided for all the bearings.

To the drum shaft is coupled an 1800-h.p., 80-rev. per min., 525-volt, d-c. motor, front and side elevations of which are shown in Fig. 5. The motor is supplied with two bearings, and sole plates; the armature shaft is 14 in. (36 cm.) in diameter in bearing by 11 ft. 5 in. (3.4 m.) long, and has a forged half-coupling for connection to a corresponding half-coupling on the drum shaft. The armature is 9.5 ft (2.8 m.) in diameter, while the outside diameter of the magnet frame is 14 ft. 5 in. (4.3 m.). The motor has 16 main field poles, the coils being wound for 125-volt excitation, and has commutating poles to insure good commutation at the heavy overloads encountered in the service. The bearings in addition to being ring oiled, receive lubrication from the gravity feed oiling system. The extreme length of the hoist, with motor, is 51 ft. 9 in. (15.7 m.)

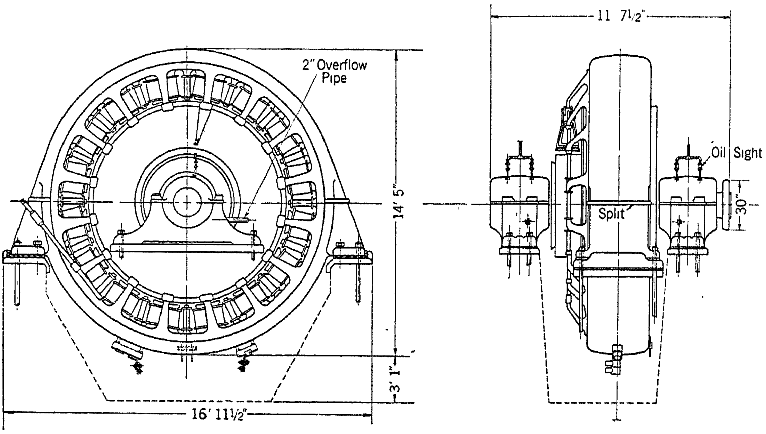

FIG. 5—OUTLINE OF HOIST MOTOR

The motor receives its power through a set, consisting of a generator driven by an induction motor, and a direct-coupled flywheel and exciter, shown in outline in Fig. 6.

The generator has a continuous capacity of 1300 kw., at 525 volts, and in order to successfully commutate the heavy currents at the low voltages, it is equipped with commutating poles, and, in addition, a compensating winding is placed in the pole faces of each of the 12 main poles. As in the case of the hoist motor, the main-field excitation is at 125 volts.

The induction motor is rated 1150 h.p., 3 phase, 60 cycles, 2200 volts, and has a synchronous speed of 514 rev. per min. It is of the wound-rotor type, with collectors for connection to the automatic slip regulator.

The flywheel is built up of thick rolled steel plates, 11 ft. 10 in. (3.6 m.) in diameter, so that the peripheral speed at synchronism is 19,100 feet (5.8 km.) per minute. The width of face is 20.5 in. (52 cm.) making the total weight 92,000 lbs. (41,730 kg.), exclusive of shaft. The plates are securely riveted together by steel rivets. The wheel is mounted between two bearings, and coupled to the generator by a Francke-type flexible coupling. A special grade of hard babbitt is used in the supporting bearings. Lubrication is afforded both by means of a low-pressure oil pump geared to the shaft supplemented by ring oiling. The geared pump begins to supply the bearings with oil practically

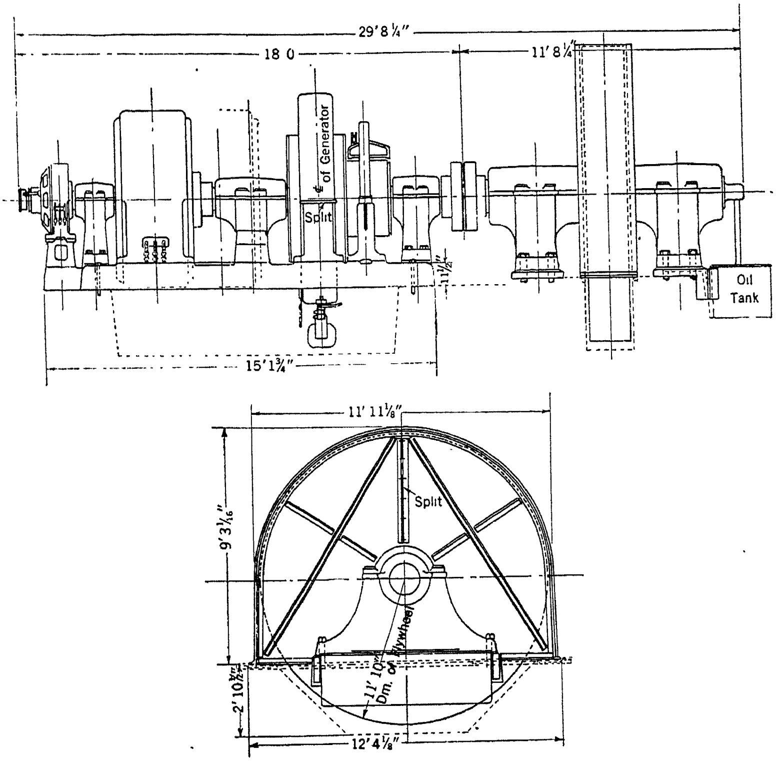

FIG. 6—OUTLINE OF MOTOR-GENERATOR SET

as soon as the wheel turns over. The oil reservoir is sunk below the floor line at the end of the flywheel bearing, and contains coils for the circulation of cooling water.

In order to reduce the demand on the power system, when starting the set, a small motor-driven, high-pressure oil pump is provided to supply oil pressure under the bearings, this pump being put into operation only when required for this purpose. With this starting pump in operation, the set and wheel started from rest with 80 per cent of full-load current from the power system.

A close fitting steel-plate cover is provided for the wheel above the floor line, the pit in the foundation virtually forming the lower portion of the enclosing casing.

The exciter armature is mounted on an extension of the induction motor shaft, the field frame being bolted to an extension of the base. The exciter has a continuous rating of 21 kw., at 125 volts.

A cast-iron base is provided under the induction motor and generator and sole plates under the flywheel bearings. The foundation under the various units is of re-inforced concrete extending down to the bed rock, insuring stability of alignment.

CONTROL

The hoist-motor speed and direction of rotation are controlled by varying the strength and reversing the polarity of the generator field. The field rheostat is provided with a large number of points giving a corresponding large number of speeds in both directions. Between the motor and generator there is inserted an overload circuit breaker and a single-pole line switch.

In the design and installation of the entire equipment, careful consideration was given to the matter of safety of operation, with the result that the effects due to carelessness on the part of the operators are obviated and protection to the men and apparatus is provided in almost every emergency.

The arrangement of the control levers is extremely simple, there being but one power lever, a forward movement corresponding to one direction of rotation and a backward movement, the reverse. This lever with the two brake levers, one on either side, completes the lever group. The clutches are operated by a throw, either in or out, of a crank through an arc of 180 deg.

A center-zero d-c. ammeter and voltmeter are mounted in the line of vision with the depth indicators.

A Welch safety device compels slowing down at the proper rate and provides protection against overwinding and overspeeding. As an additional protection, limit switches actuated by the skips themselves are installed in the guides. These switches, as well as the Welch device, cause the d-c. circuit breaker to open, in addition to setting the hoist brakes. In general, any emergency which will cause the d-c. circuit breaker to open will cause the brakes to be applied. The d-c. circuit breaker will open under any of the following emergencies:

(a) Extreme d-c. overload
(b) Overwind top or bottom
(c) Loss of exciter voltage
(d) Loss of motor-field excitation
(e) Overspeed of hoist
(f) Overspeed of motor-generator set.

The opening of the main line a-c. oil circuit breaker will not open the d-c. circuit breaker, nor set the hoist brakes, but the operator is free to continue hoisting as long as the stored energy in the flywheel will permit. The opening of the line switch is indicated in the usual manner by the ringing of a bell. When the d-c. circuit breaker has been opened under any of the above mentioned emergencies, and an application of the brakes made, the brakes cannot be released until the controller lever has first been returned to the central position.

After an overwind has occurred, it is necessary for the hoist operator to throw a small switch before power can again be applied to the motor and motion is possible only in the lowering direction.

The total energy in the flywheel at 94 per cent synchronous speed is 117,000 h.p-sec. of which approximately 50 per cent is available for operating the hoist in the event the set was disconnected from the power supply, the limitation being the speed at which the d-c. exciter is no longer able to hold up its voltage. If required, a complete trip with a fully loaded skip could be made from a depth of 1500 ft. (457.2 m.) on the energy of the flywheel, and a load of men could be hoisted from the deepest level.

Hoisting "out of balance" can be carried on without causing excessive speed reduction of the set nor necessitating a higher limit of demand from the power supply.

The liquid slip regulator used with this equipment is worthy of special mention, as being a decided departure in important details from the heretofore generally accepted design. In the latter, it was necessary to make a water-tight joint between the tile or porcelain vessels, and the cooling tank, and also between the tile and the electrodes, clamping rings or draw bolts being used for this purpose. In many instances, this construction in connection with temperature changes resulted in broken tiles, causing much annoyance.

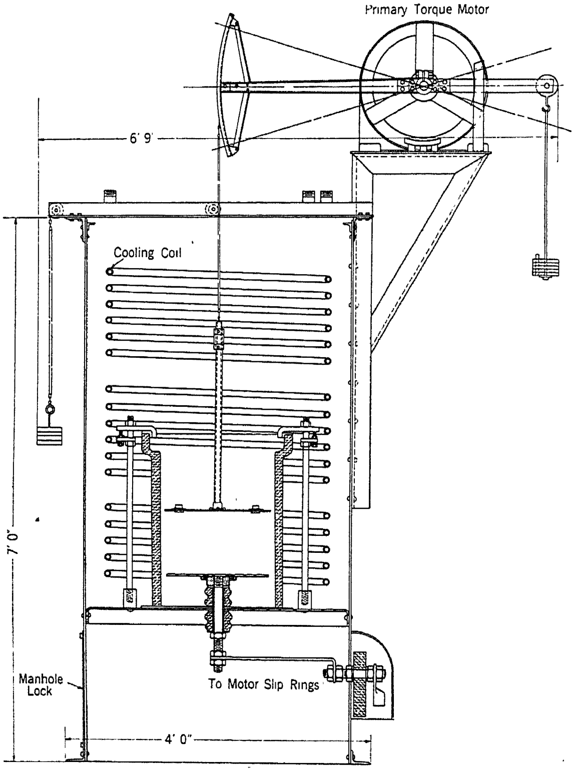

Fig. 7—Automatic Slip Regulator—Cross Section

In the present design, a sectional end view of which is shown in Fig. 7, the tiles are simply placed upon a rubber mat on the steel bottom of the tank, the only water-tight joint being that between the lower electrodes and the bottom of the tank. Only enough pressure is applied on the tile by the holding down bolts to hold it in place, the purpose of the tiles being merely to serve as barriers to prevent leakage between phases. Common sewer tiles are used for the purpose. The value of the input limit is adjusted in the usual manner by varying the number of the counterbalancing weights.

TESTS

The first set of tests was for the purpose of analyzing a typical day's run under ordinary conditions, as regards its relation to the electrical equipment.

In the second test, measurements were made of the electrical quantities for a few trips, to determine, if possible, the efficiency while hoisting under known conditions.

In conducting the tests, the following measuring instruments were used:

Curve-drawing a-c. wattmeter—30 in. feed per hour.
Graphic recording a-c. ammeter.
" " d-c. ammeter.
Indicating liquid column tachometer.
Graphic recording d-c. voltmeter.
Karlic tachograph.
Indicating a-c. wattmeter.
" a-c. voltmeter.
" a-c. ammeter.
Integrating polyphase watthour meter.

First Test. A log was kept of all the movements of the hoist from 7:00 a.m. to 7:00 p.m., recording the time, the nature of all trips, quantity of ore, etc. The only instruments in use were the a-c. recording watmeter—30 inches feed per hour, Karlic tachograph and integrating watthour meter.

As mentioned in the preceding paragraphs, there are no special designated periods of the day during which the hoist is used exclusively for the hoisting of ore, but this is carried on simultaneously with the unproductive work, as the conveying of timber, men and supplies. Moreover, the hoisting is of such an intermittent nature, that a straight line input to the motor-generator set is not even approximated, even at the low limit of 400 kw. Storage or bins are under construction, but were not in operation at the time of the tests. To illustrate the character of the load, a portion of a typical day's load chart is shown in Fig. 8. This curve is reproduced from one of the daily records from the 3 in. per hour curve-drawing wattmeter included with the switchboard equipment.

The results of this test are given below.

Duration of test	12 hours.
Actual time hoist in motion	3.5 hours.
Total number of trips made	280
Number of skips of ore hoisted	63
Percentage ore trips of total	22.5 per cent.
Total weight of ore hoisted	252.5 tons
Average weight of ore per skip	4.01 tons
Average lift of ore	1428 ft. (435.2 m.)
Net work done on ore-total	272 kw-hr.
" " " " " per trip	4.32 kw-hr.
" " " " " " ton	1.08 " "
Kw-hr. energy used during day	1500 kw-hr.
Total kw-hr. energy used, including non-productive work per ton ore hoisted	5.94 kw-hr.

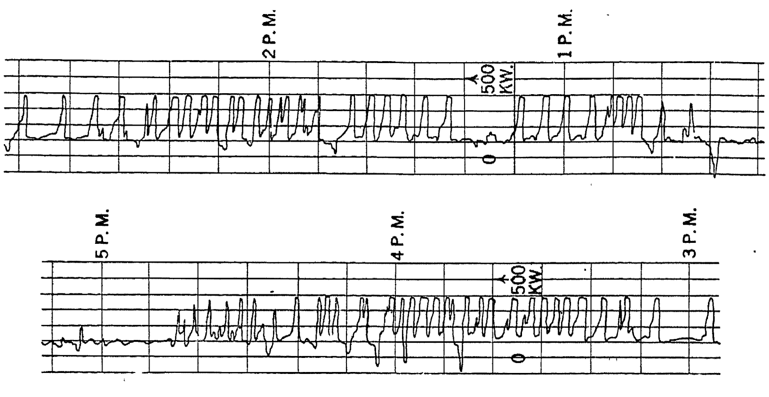

Fig. 8—Load Diagram of A-C. Input

Kw-hr. used while hoist at rest	680 kw-hr.
" " " " hoisting ore	582 " "
" " returned to power system	14.6 kw-hr.
Overall efficiency while hoisting ore	46.7 per cent.

The trips with ore were divided among the levels as follows:

Level	No. Trips	Actual Lift
1000 ft. (304.8 m.)	6	1068.5 ft. (325.6 m.)
1100 "	3	1170.0 "
1200 "	3	1271.2 "
1300 "	20	1372.5 "
1400 "	7	1473.5 "
1500 "	22	1574.5 "
1800 " (548.6 m.)	2	1915.2 (583.7 m.)

The weight of ore hoisted was obtained by observing the number of cars dumped into each skip and using an average weight per car obtained from a large number of cars.

The calculated net work done in lifting the ore is subject to some error, inasmuch as during about 25 per cent of the trips timber or men were lowered in counterbalance with the ore. On the other hand, men were hoisted with the ore during about 20 per cent of the trips.

During about 50 per cent of the trips with ore, at least one stop was made between the loading level and the dump, there being a maximum number of six such stops. The total kilowatt hours used was obtained from readings taken on the watthour meter at the beginning and end of the test.

The energy used during the ore trips was obtained from the record made by the 30 in. per hour curve-drawing wattmeter with the aid of a rolling planimeter. The energy returned to the system was obtained in the same manner.

The time during which the hoist stood at rest was taken from the tachograph record.

Second Test. In order to secure as rapid hoisting as possible, there was accumulated at the 1500-foot level, 30 cars of ore making possible 5 consecutive trips, as fast as the skips could be loaded.

Continuous records were made on the graphic meters of the d-c. voltage and current and of the a-c. current. A time marker marked every second on all of the records, simultaneously. The speed of the motor-generator set was read at frequent intervals and telegraphed to one of the graphic records at the instant of reading, a record of the consecutive readings being kept which was later transferred to the graphic record. The speed of the hoist motor was taken from the tachograph record and the 30 in. per hour recording wattmeter supplied a record of the power delivered to the motor-generator set.

Fig. 9 is a consolidation of all of the records and readings taken for the five consecutive trips.

By combining the two d-c. graphic records, the d-c. power curves were obtained.

The following conditions pertain to this test:

Hoisting done in balance	
Duration of test	565 seconds
Lift	1574.5 ft. (479.9 m.).
Number of skips hoisted	5
Weight of ore hoisted	20.3 tons
Average weight per skip	4.06 tons
Net work done—total	24.1 kw. hr.
Average time per trip	113 sec.

Actual running time total	310	“
“ “ “ av. per trip	62	“
Time at rest-total	255	“
“ “ “ average per trip	51	“
Average time for acceleration	22	“
Average time for running at full speed	15.5	“
Average time for retardation	24.5	“
Average rope speed	1525	ft. per min.
Max. “ “	2580	“ “ “
Average speed of M-G. set	486	r.p.m.
Hoist motor input (exclusive shunt field)	32.64	kw-hr.
“ “ output	28.2	“ “
Mechanical efficiency of hoist	83	per cent.
Energy input to M-G. set total	50.5	kw-hr.
Kw-hr. per ton	2.49	
Overall efficiency	47.7%	

The energy dissipated in the various parts of the equipment was found to be as given in the table below.

	Kw-hr.	Per cent. of net work
Losses in mechanical parts of hoist, sheaves, guides, etc.	4.1	17.0
Hoist motor losses	6.25	25.9
Generator losses	5.53	22.9
Induction motor losses	3.99	16.6
Flywheel losses	4.08	17.0
Slip regulator losses	2.12	8.8
Exciter losses	0.39	1.6
Total losses	26.46	109.8 per cent.
Net work done on ore	24.1	
Total energy consumed	50.56 kw-hr.	
Overall efficiency	47.66 per cent.	

The hoist motor input was measured directly, and the output was obtained by deducting the losses which were based on factory test results.

The power input to the motor-generator set when running light as measured by the 30 in. per hour curve-drawing wattmeter, and checked by the indicating wattmeter was 80 kw. at 512 rev. per min. For the average speed of the set during the test this was taken at 78 kw.

The segregation of the losses in the set was made from results of tests made in the factory. The running light input to the set includes:

Flywheel bearing friction and windage
Generator “ “ “ “
Ind. motor “ “ “ “
“ “ core loss and running light copper loss.
Hoist motor shunt field (with economy resistance in series)
Exciter loss with load of motor field.

The same total loss in the equipment is obtained by adding to the running-light loss of the set, the increase in losses while under load, these being readily calculated. On this basis, the rĕsult is as follows:

Motor shunt field (increase)	0.248	kw-hr.
Ind. motor copper loss (increase)	0.223	" "
Exciter loss (increase)	0.060	" "
Generator losses (excl. fric. & w'd'ge)	3.019	" "
Motor losses (excl. shunt field)	4.440	" "
Regulator losses	2.120	" "
Running light loss of set	12.250	" "
Friction in mech. parts of hoist	4.1	" "
Total	26.460	

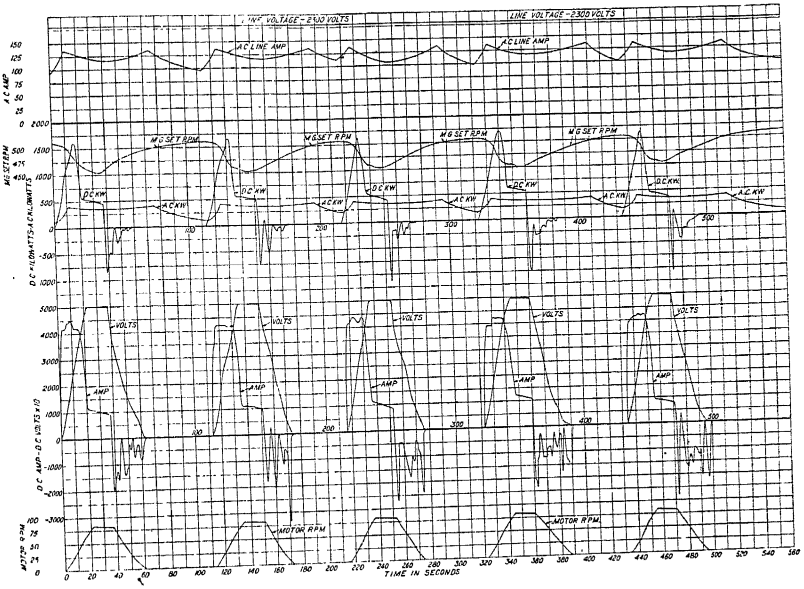

FIG. 9—CONSOLIDATED CURVES FROM TESTS

An item not included in the above segregation consisting of approximately 0.25 kw-hr., used in operating the accumulator oil pump and the brake solenoids is also included in the running light input.

The total energy accounted for is therefore 50.81 kw-hr., as against 50.5 shown by the wattmeter.

The energy output of the generator, as obtained from the d-c. power curves by the aid of a rolling planimeter was 32.64 kw-hr., and the input 38.17 kw-hr. The input to the generator was at the rate of 243 kw., so that during the trips, approximately 27.5 kw-hr., were required from the flywheel. The average speed reduction of the set during hoisting was 10.6 per cent on the basis of which, the energy given up during the five cycles was 96,500 kw-sec. or 26.8 kw-hr., which checks fairly well with the figure given above.

In general, the test results check very well with the estimated performance. The friction losses in the hoist, sheaves, rope and guides proved unexpectedly low, but the shaft guides are in very good repair, and the cage rides almost as smoothly as a passenger elevator. Also, the hoist drums run true and with no interference with the brake shoe blocks; the head sheaves and idlers are well lubricated and easy running.

The efficiency of hoisting exceeded that anticipated, even with the light loads, the load per skip being but 80 per cent of that considered normal. With heavier loads, as when hoisting high grade copper ore, the efficiency would be further increased. Hoisting was done at slightly less than 50 per cent of the maximum rate for this level and the power consumption was 2.49 kw-hr. per ton as compared with 2.83 kw-hr., expected.

The tendency should be for the efficiency to increase at the greater depths, the maximum probably being reached at the intermediate levels.

The time for retardation is considerably longer than that used in calculating the duty cycles, and is the result of the rather close setting of the Welch safety device. At present there is no particular reason for rapid hoisting, and the management being inclined to a policy of safety, require the slower rate of retardation. A reduction in the retardation period would further tend to a betterment of the efficiency of hoisting.

The standby losses are considered particularly low, for an equipment of this capacity.

It is believed that by decreasing the loading time so as to obtain a more uniform input to the power set, decreasing the retardation period and bringing the load up to normal, the hoisting efficiency of this outfit will reach 50 per cent at this level.

It is believed that the results of these tests give fairly accurate information as to the operating characteristics of the hoist at Elm Orlu, and indicate, in general, the results to be expected of the performance of other Ilgner equipments in similar service.

APPENDIX

Supplementary tests of the same nature as those referred to as Second Tests on page 667 were made when hoisting from the ore pockets subsequently installed at the 1500 ft. level.

During these tests twelve skips were hoisted from the bins or pockets, the time of loading averaging 27.3 seconds against 51 seconds during the previous test. The higher overall efficiency obtained, namely 50.2 per cent, is due to the decreased standby loss as a result of the decreased loading time.

It will be noted that the total lift was increased 43.5 ft. when hoisting from the ore pockets, thereby increasing slightly the work done per trip.

The conditions pertaining to these subsequent tests and the results observed therefrom are as follows:

Duration of test	1,081 sec.
Lift	1,618 ft.
Number of ships hoisted	12
Weight of ore hoisted	97,400 lbs.
Average weight per skip	4.06 tons
Net work done—total	59.38 kw-hr.
Average time per trip	90 sec.
Actual running time—total	754 sec.
Actual running time, av. per trip	62.8 sec.
Time at rest—total	327 sec.
Time at rest, av. per trip	27.3 sec.
Average time for acceleration	21.8 sec.
Average time running at full speed	17.0 sec.
Average time for retardation	24.1 sec.
Average rope speed	1,545 ft. per min.
Maximum rope speed	2,550 ft. per min.
Average speed of M-g set	481 rev. per min.
Hoist motor output	70.7 kw-hr.
Hoist motor input (exclusive shunt field)	81.5 kw-hr.
Mechanical efficiency of hoist	81 per cent
Energy input to M-g set—total	118 kw-hr.
Kw-hr. per ton hoisted	2.425
Overall efficiency	50.3 per cent

DISCUSSION ON "FORMS OF ELECTRIC POWER BEST SUITED FOR THE VARIOUS LOADS ENCOUNTERED IN THE OPERATION OF BITUMINOUS COAL MINES" (KINGSLAND) AND "A NEW ELECTRIC MINE HOIST AT BUTTE, MONTANA" (SAGE), NEW YORK, JUNE 28, 1917.

A. S. McAllister: Has the electricity-operated chain-cutter superseded the air punching machine in the cutting of coal?

R. L. Kingsland: It is hard to answer that question in a particular and comprehensive way. I think, to a large extent, the electric chain machine has superseded the air puncher, but at the same time, air punchers are being installed in new operations. It is my opinion however, that they are being installed in a very much smaller percentage of new operations than electrically-driven chain machines.

K. A. Pauly: I want to rather caution beginners, at least, from giving undue importance to the question of the cost of feeder copper. Mr. Kingsland discusses this question, but I think leaves it in such a way that the inexperienced engineer might unduly emphasize the importance of keeping down his main or feeder copper. It must be borne in mind that by the use of alternating-current apparatus whenever possible, we eliminate the stand-by losses of the converting apparatus, and, especially where the load is an intermittent one, these losses may become a very considerable percentage of the total power consumed in performing the work. On the other hand, the money invested in feeder copper affects the operating cost only by the fixed charges, and as the salvage on copper is so very high, the use of plenty of feeder copper usually results in lowering the operating costs.

Another point brought out, and which I want to emphasize, is the question of the use of high voltages in mines. There used to be an idea prevalent among, I might say, the laymen that no high voltages should be taken into the mine at all. Of course, the contrary is the case. Unquestionably, voltages in contact with which the operators may come, if they do not exercise more than ordinary care, should be kept low, but there are a great many installations, such as Mr. Kingsland mentions, of hoists and pumps which may just as well be fed at a high voltage, and if properly installed will be quite as safe as if a low voltage is used.

Mr. Kingsland discusses the torque characteristics of shunt motors. Here again, I wish to caution against the wrong interpretation of what he says.

In his example he assumes the voltage reduced to one-half, presumably due to a high feeder drop, which drop should not be present if the mine is going to operate economically. If the drop is limited to 10 per cent or 15 per cent, the saturation of the fields considerably reduces this drop in the torque so that the shunt motor really shows up more favorably than is indicated.

With reference to the variable-speed drive of mine fans, I simply want to call attention to a type of alternating-current commutator motor which has been tried out sufficiently long to warrant claiming for it entire satisfaction, in fact, this is shown by repeated orders from the companies which originally installed them. We have recently brought through our testing room one of these motors which commutates at 100 per cent overload as well as any direct-current machine, in fact, the commutation being absolutely blank.

C. A. Adams: That is a polyphase motor?

K. A. Pauly: Yes. Of course, where constant speed is required, there is nothing better than the induction motor. I would like to ask, whether Mr. Kingsland has experienced any difficulty with squirrel-cage motors due to the belt being thrown off when they pass through the peak-torque point? Several cases have come to our attention where that has been the case, and we have been rather inclined to favor the polar-wound rotor for this reason, since it admits of getting up to full speed without passing through this excessive torque point.

There is one point in Mr. Sage's paper to which I wish to especially call your attention—the close agreement between expected and actual results.

If you will look at the curves of Fig. 3 which were calculated, and the curves of Fig. 9, you will note that the kilowatts direct current from the test are approximately 1900, perhaps 1850 as an average. Changing this to horse power, and correcting for the loss in the motor, the curves given in Fig. 3 being motor-output curves and those of Fig. 9 being motor-input curves, you will find that the test results show the power required at the peak of acceleration to be approximately 2350 h. p., while the calculated figures are approximately 2600 h. p. for this level. You will note by examining the test curves also that the period of acceleration is slightly longer than that assumed in making the calculations, which in part accounts for the difference.

How often we hear it said that this or that thing is all right in theory, but it will not work out in practise? Now, the trouble in such cases is usually not with the theory or the practise, but due largely or wholly to a failure on the part of the would-be prophet to include in his analysis of the problem all the factors affecting it.

Claims of absurdly high operating efficiencies and correspondingly low power consumptions have frequently come to my attention, in fact, in one instance an operator apologized for the size of his flywheel because it would not take the peak without too greatly reducing the speed of the generator, when in reality the trouble was due to an underestimation of the power required for making the trip.

Like every other problem of a similar nature, the calculation of the power required to perform a given hoisting cycle is made up of two factors; first, an accurate determination of the useful work to be performed, and second, the determination of the losses in the several pieces of machinery used. The calculation of the useful work in hoisting, as is frequently the case in problems of this kind, is comparatively simple and capable of great accuracy. The determination of the losses is not so simple, but even those with the electrically-driven hoist do not present any serious difficulties, if care commensurate with the problem is taken. In fact, we can limit the troublesome factors in the summation of the losses to two, namely, the losses beyond the drum and the friction and windage of the flywheel of the motor generator.

As the former varies widely in different localities and from time to time in the same shaft, due to movements of the rock through which the shaft passes, the power consumption should be based on as careful and accurate estimate of the efficiency of the mechanical hoist as can be made and this stated in giving estimated power consumptions.

So much test data are now available on which to base an estimate of the friction and windage of the flywheel that there is little excuse to miss this loss by any considerable percentage, certainly not enough to materially affect the power consumed per trip, a figure usually used in comparing different system equipments.

I have discussed this point rather at length, because power consumption is one of the very important factors influencing the choice of a system or equipment for meeting the conditions obtaining at any particular mine, and it is important that practical mining men should appreciate that our thorough knowledge of the characteristics of electrical machinery make it possible to predetermine power consumption very accurately. I think you will all agree with me that a careful study of this paper will convince the most skeptical engineer of the possibility of the accuracy of predetermined power consumptions for mine hoists, and I might say, although this is hardly necessary under the present state of the art, should convince him of the high economies possible with the electric hoist.

H. H. Clark: I agree with Mr. Pauly that the use of high voltage in coal mines is relatively safe, because of the fact that it is acknowledged to be dangerous, and consequently adequate safeguards are used, safeguards that are more adequate, I believe, than those applying to lower voltages, such as 300 and 650. The high voltage is usually brought in through three-phase cables, which are adequately insulated and protected and properly supported and shielded from injury, and are used for stationary apparatus, whereas the other voltages, the lower voltages, are used for portable apparatus, often times, and there the current is carried in and distributed over bare wires.

The introduction of high-voltage circuits brings up the problem of how they shall be taken into the mine, and frequently that is done through bore-holes, and I want to ask Mr. Kingsland his idea as to how cables in bore holes should be suspended. If I am correct, the depth of the bore holes in coal mines is comparatively small, from 150 feet to say 350 feet, so that the problem of suspending the three-phase cable is not as difficult as if the bore hole were of a greater depth. I should like to ask him the depths of the bore holes that he refers to in his paper, and also I would like to ask how the lead-armored cables are supported. If a cable is leaded, it adds materially to the weight that it is necessary to support, and the lead will not be self-supporting in any considerable degree.

I also ask Mr. Kingsland what he would think of using 30 per cent Para rubber for insulation, really, good 30 per cent Para rubber, without the use of armor and lead? It would seem to me that by using this 30 per cent rubber he could eliminate the armoring and lead and thus simplify the problem of the cable support.

E. H. Martindale: There is one point I would like to mention in the discussion of Mr. Kingsland's paper, and that is the under-cutting of mica on commutators, because it has a bearing on the maintenance costs and reliability of service. I do not believe that the practise of using under-cut commutators on mine locomotives and mining machines has developed as rapidly as warranted by the results obtained by those companies that have adopted the practise. Probably that is largely due to fear of short circuits between commutator bars on account of coal dust.

There are two very reliable methods for overcoming this—either painting the slots after under-cutting with an air-drying insulating varnish or filling the slots with a commutator cement. In the case of the ordinary U-shaped slot left by the milling saws either the commutator cement or the insulating varnish is satisfactory. With the V-shaped slot, such as is left by a commutator slotting file, the cement cannot be used, as it will not adhere properly to the slot. Personally, I favor the V-shaped slot because it leaves a very clean smooth slot, and leaves mica to protect the bars except at the very surface. In the U-shaped slot, on the other hand, the mica is cut from between the bars, leaving the bars exposed inside of the slot. Moreover the mica is torn, leaving holes in which dust is more apt to collect.

I would like to ask Mr. Kingsland what his experience has been with under-cutting commutators.

R. L. Kingsland: Mr. Martindale referred to undercutting commutators. We have tried this, in fact, in haulage locomotives we have had a good many under-cut commutators direct from the manufacturers which have given us a great deal of trouble. The principle seems to have been carried out in railway practise, and has proven highly successful, but in mining work, where it is necessary to work at lower speeds, the centrifugal action does not seem to free the slots of the undercut commutator from the collection of coal dust, oil and carbon to anywhere near the extent that it does in railway work, so that with the use of a milling machine for under-cutting we found the practise very unsatisfactory. We have done quite a little slotting with a three-cornered file, which has gotten us aw y from a great deal of the trouble.

The point brought up, of using an insulating varnish in the slots is a new one to me, and I have not considered it at all, but think it sounds very interesting. I would ask if Mr. Martindale has tried this out, and whether he would recommend an ordinary commercial air-drying varnish or any special varnish for that purpose?

E. H. Martindale: I know of two or three companies which have tried out the air-drying varnish, and it has proven very successful. I think the only precaution necessary is to get an insulating varnish which will dry so hard that when the commutator is cleaned afterward with a commutator stone, or sandpaper, the varnish will not gum. Such varnish can be secured, and it is a very easy matter to clean the surface of the commutator afterward. In the V-shaped slot there is the further advantage, that at the low speeds of mine locomotives the dust will fly out quite easily.

R. S. Sage: Does Mr. Kingsland in applying induction motors to his drives, such as pumps, fans and tipple machinery, have any line at which he departs from the use of squirrel-cage motors and goes to the use of the wound-rotor type of motor. Does he set the limit at any particular horse power? If not, what determines its selection?

R. L. Kingsland: In selecting motors, my preference has always been for the squirrel-cage type of motor, if it is possible to use it at all, and I have found that simplicity and ruggedness are the two things we cannot afford to sacrifice in mining work. Therefore, it is not a question of capacity, it is merely a question of the characteristics of the load, in deciding whether we would use the squirrel-cage or wound-rotor type of motor.

R. S. Sage: I had in mind the starting conditions determined by the capacity of the motor.

R. L. Kingsland: In the starting condition we never found that the peak load was serious enough to affect our power supply, and therefore it is merely a question of the application of the motor as having sufficient starting torque to start the load it is to drive. If we can do it with a squirrel-cage induction motor, we do.

C. M. Means (Abstracted by H. H. Clark): The paper as presented by Mr. Kingsland is a step in the right direction and touches on a field that has not received the intensive study that has been accorded many other lines of electrical engineering of relatively less importance. Considering the item of ventilation it appears to me that this phase could have been very materially elaborated upon because of the importance of proper ventilation in coal mines. The general trend throughout the bituminous coal fields is for more and better ventilation. Very many of the large mines require fan motors of capacities from 100 to 500 h. p. When the requirements of a mine are such as to demand the use of a large amount of power for ventilating purposes, it is important that this feature be studied very carefully. In many instances we find that the energy used for supplying ventilation equals or exceeds all other requirements for power in and about the mine. The energy required to operate a ventilating fan varies approximately as the cube of the speed. A motor that will allow speed reduction during such times as the mine is not in active operation will very materially lessen the amount of power required for ventilation. Inasmuch as nearly all large capacity fans are driven with alternating-current motors, the wound-rotor type will result in a very material saving in power even though considerable current is used up in heating resistance when operating at the reduced speed. When you consider that the average coal mine operates only eight hours per day and for something like 200 to 250 days per year, the saving because of reduced fan speeds is apparent. Several types of motors suitable for the operation of ventilating fans have been developed that show a relative high efficiency during the period when minimum air requirements exist. The two-speed motor is a very desirable scheme in the small sizes, but in general is not satisfactory for the larger installations because of the constantly changing conditions underground that affect the quantity and pressure of air required to properly ventilate the mine.

I fully agree with the author that a motor-generator set or synchronous converter will furnish equally satisfactory service under average conditions. Inasmuch as these sets can be located centrally with respect to load, the over-compounding of the motor-driven generator has little value. There are a few cases however, where there is a decided advantage in applying one or the other system, for the conversion of alternating current to direct current. In case it is desired to regulate power factor, the motor-generator set affords the proper solution with the use of a separate exciter and regulator. Where the charge for electrical energy is based on the kv-a. demand, the saving occasioned by the corrective characteristics of a motor-generator set may more than compensate for the inherent losses of the set by reducing the demand.

A. B. Kiser (Abstracted by H. H. Clark): In the near future most of the power that will be available in our coal fields will be in the form of alternating current, because coal companies during the past five years have awakened to the advisability of purchasing their power or of building comparatively large central stations of their own. In either case the power will be generated as alternating current.

The frequency available will, of course, be determined by the power companies and will probably be 60 cycles, since the power is primarily generated for lighting and industrial purposes. Recent developments, however, have made it possible to operate 60-cycle apparatus as satisfactorily as 25-cycle apparatus.

I question whether the statement as regards low voltage coming into more general favor can be verified in Pennsylvania.

Small individual operators working with a limited field may be making use of low voltage but the larger operators whose fields are more extensive are extending the use of medium voltage.

I note Mr. Kingsland favors the use of either d-c. or 3-phase, 60-cycle a-c. power at 330 volts or less for portable machines. 330 volts alternating current effective would at a maximum be necessarily classified as medium voltage.

The flexibility of variable speed d-c. motors has not as yet been approached by the a-c. motor. I say this having in mind every available speed-regulating device applicable to a-c. motors, slip-ring motors, brush-shifting motors, regulating sets and two-speed motors.

To my mind the use of belting has been proven the most satisfactory and reliable drive for mine fans. The modern high-speed fan, however, is entirely free from the disagreeable cut-off blows of the old style fan. With large motor-driven fans the necessity of operating large mine fans at reduced speed on idle days and at night could not be taken care of by the changing of pulleys.

As regards the elimination of commutators with attendant troubles I would state that I have seen motors in sizes from 100 h.p. to 250 h.p. where the commutation required less attention and concern than the observation of air-gap clearance would require in induction motors of the same size.

I believe that a drop of 10 per cent when using high-voltage alternating current is entirely too high. I am not in favor of the installation of substation equipment underground where it is possible to keep same above the ground. I cannot agree with the writer as regards the use of synchronous convertors for mine operation. The fact that Mr. Kingsland has control of the power plants, transmission lines and the various loads as applied has considerable bearing on his determination. If the writer were buying power from power companies where the voltage varies materially or it was necessary to make power-factor correction he would find synchronous convertors unsatisfactory.

The cost of transmission when included with convertors shows nothing in their favor as to their cost or efficiency.

Upon inquiry I find that one of the larger manufacturers is selling one convertor as compared with 24 motor-generator sets in the coal field.

Overloads can be carried with equal facility by motor-generator sets as I have seen 300 kw. motor-generator sets carrying 75 per cent overload without injury.

Using 500 volts as we do so largely in Pennsylvania we would be under the necessity of operating convertors at about 600 volts, direct-current. Whereas on light loads we want our voltage at 525 permitting same to over-compound to 575. This takes care of the drop in the transmission lines and permits our small 500-volt motors to operate at the proper speed. Convertors necessarily do not over-compound to compensate for line drop nor can the operator control the d-c. voltage as he sees fit.

Further in a motor-generator set the a-c. circuits are removed from all proximity to the d-c. circuits.

I also regard the control and operation of our motor-generator sets as much simpler and more satisfactory than the synchronous convertors.

Graham Bright (Abstracted by H. H. Clark): Mr. Kingsland's paper shows very clearly the modern trend of the application of electric power to bituminous coal mines. In comparing the characteristics of the various motors used about a coal mine, Mr. Kingsland lays most stress on the comparison between the shunt or compound motor and the a-c. induction motor. This is true when comparing the motors used on the tipple and outside of the mine. Most of the motors inside of the mine, however, such as those used to drive locomotives and mining machines are of the series type whose characteristics are quite different from the shunt type. As stated by Mr. Kingsland, the torque of an induction motor varies directly as the square of the voltage. The torque of a series d-c. motor, however, does not depend in any way on the voltage, but is a direct function of current. The only effect of lowering the voltage on a series motor is to reduce the speed, and the torque for a given current remains the same irrespective of voltage. With a series motor, a very large drop in voltage can be successfully used so that it is quite common to find conditions where a voltage drop of 50 per cent exists during the accelerating periods or in particularly heavy loads on a haulage or cutting system. This drop is entirely permissible when using a d-c. series motor but would be inoperative with an induction motor. For this reason the amount of copper required for an induction motor is greatly in excess of that necessary with a series-type direct-current motor at the same voltage.

When using a rail return with an induction motor the impedance of the rail is very many times the ohmic resistance. This frequently means the strengthening of the rail circuit by means of a copper feeder. The a-c. system has been used very successfully in cutting machines, but has not proved practical for locomotive haulage for the long hauls inside of the mine. A number of three-phase locomotives for industrial purposes are in operation for use outside of the mine where the capacity is small and the distance of the haul not great.

I do not agree with Mr. Kingsland that all underground distribution should be either bare wire or lead-covered steel-armored wire. Where 500 volts is used on the trolley, the feed wires should be insulated since a contact with an insulated wire, although it may result in a shock, is not so liable to produce fatal results as with a bare wire. Persons in the mine are always on the look out for a trolley wire, but are not so careful regarding feed wires and a contact with a bare feed wire will invariably produce a greater shock than in case the wire is insulated.

It is true that Tungsten lamps for 250 volts are not very satisfactory for mine illumination. This, however, can be gotten around by using two 110-volt lamps in series. The principal reason, however, in most mines for not using Tungsten lamps is that it is difficult to keep the miners from stealing them. This is not only true in mines, but has been found to be true also in manufacturing establishments. It is generally necessary to connect all lights inside of the mine to the trolley or cutting system while outside of the mine lights can be connected to the a-c. system in case of purchased power.

Mr. Kingsland mentions the advantage of a motor-generator set over a synchronous converter to be a high power factor and a closer voltage regulation. The power factor of the motor-generator set is only high at the lighter loads and at heavy overloads the power factor is generally very much less than with the synchronous converter. The voltage regulation of the converter is generally arranged so as to give flat compounding. That is, the voltage on the d-c. side is practically constant at all loads. The motor-generator set has the advantage that at the heavier loads it can be made to over-compound, thus partly compensating for the line loss.

I certainly agree with Mr. Kingsland regarding the use of the squirrel-cage motors for operating mine fans. The mine fan is a piece of apparatus which should operate over long periods with little or no attention, and the squirrel-cage motor is particularly adapted for this kind of operation. I also agree with Mr. Kingsland in that the belt drive is by far the best method of connecting the motor to the fan. There are few people who appreciate that some types of fans produce a decided vibration due to the cut-off blows of the fan blades as mentioned by Mr. Kingsland. These cut-off blows have been known to set up shocks in the motor windings which eventually cause motor trouble and necessitate rewinding of the motor. The use of a belt produces a cushioning effect so that the vibrations are smoothed out and are not communicated to the motor windings. Where the speed is to be changed daily, a two-speed motor provides the best solution.

Where the speeds are to be changed only occasionally due to mine development, the best method, as pointed out by Mr. Kingsland, is to change the size of pulley from time to time. This scheme is far superior to the accomplishment of variable speed by the use of auxiliary machines and complicated control which has been tried out to a limited extent in this country. This scheme is in use in Europe where high-class operators can be obtained at very low wages. In this country, however, it is very difficult to obtain any kind of operators for the mining regions and the type of equipment which will operate over long periods with little or no attention is the type of equipment which the mine operators are looking for.

Mr. Kingsland advocates the use of totally-enclosed motors in dusty places. Most operators do not realize just what totally inclosing a motor means to the size of the motor. The capacity of a motor depends on its ability to dissipate the heat losses.

The heat losses are largely taken care of by circulating air through the machine, the air taking up the heat as it passes through the various parts of the machine. If the machine is totally enclosed, then the heat must be dissipated largely by conduction to the surface and by radiation and convection from the surface. This means that for a given capacity a much larger motor must be used. When the average operator is told the difference in price between an open motor and an enclosed motor he is generally willing to risk the open motor rather than pay the difference. A scheme which is generally much cheaper than using an enclosed motor is to use an enclosed ventilated motor. In general, this will not increase the size of the frame over the open motor and the ventilation is taken care of by piping air from a blower located in a fairly clean place to the motor casing. The air is allowed to escape from the motor from one or more openings. Another method which does not involve enclosed parts for the motor is to build a small house around the motor and blow air into this house from a blower located in a fairly clean place. The advantage of this scheme is that a standard motor can be used which is much cheaper than the enclosed motor and can generally be obtained in much less time.

Considerable impetus has been given to the use of central-station power in mines due to the fact that the value of fuel has greatly increased in price during the last year or so, while the price of central-station power has practically remained constant. A few years ago, it was a very difficult matter to persuade a coal operator that he could purchase power as cheap as he could produce it. The great success of a large number of mining companies that have changed over to central-station power makes it no longer a question of doubt and there are very few operators today who will admit that they can produce power cheaper than they can purchase it from a central station.

F. J. Duffy (Abstracted by H. H. Clark): I agree with Mr. Kingsland in the use of 60 cycles in preference to 25 cycles. We have used a great many 60-cycle synchronous converters with excellent results. The only possible advantage I can see to the use of 25 cycles in the mining industries would be where very long transmission lines are used.

The use of 300 volts with a divergency factor of 10 per cent for portable machinery is used almost exclusively in the anthracite field. There are some instances where 500-volt direct current and 440-volt alternating current is used but I would prefer the use of the lower voltage from a safety point of view.

In regard to cables. We have had in service for about five years a number of low-tension and also high-tension cables up to 4000 volts in bore holes, using varnished cambric insulation with four braids of heavy twisted twine over the varnished cambric. This braiding is thoroughly impregnated with waterproof compound. We first used these cables to replace lead-covered steel-armored cables with which we had trouble, due to the acid water and the action due to electrolysis eating holes in the steel armor, after which occurrence water penetrated the cambric insulation and caused the failure of the cable. The cables which we installed with twine covering have given excellent satisfaction and we have had no failures to date.

I would like to ask Mr. Kingsland the highest voltage he would use on larger induction motors in the mines used for hoist and pump service. Our practise is to limit this to 2200 volts on account of getting the proper amount of insulation on the stator coils. We have, however, one 800-h.p. induction motor operating on 4000 volts and have had some trouble with this equipment. We do not hesitate, however, to use synchronous motors directly on the 4000-volt circuit.

As stated before, as a general rule we use synchronous converters but where we have a large induction-motor load and the question of power factor is involved, we use synchronous-motor-generator sets. At one colliery we have a number of large induction-motor hoists and the breaker operated by induction motors. We use a 440-volt synchronous motor driving the generator. This 440-volt service is taken from the secondaries of the same transformers that operate the induction motors in the breaker. A Tirrill regulator is used in connection with the synchronous motor and in this manner we get the advantage of power factor correction on the transformers used for the induction-motor load.

I understand that in the bituminous region a number of ventilating fans are being operated by commutator-type variable-speed induction motors. I would like to ask Mr. Kingsland how these are working out. Personally I prefer the belt drive and all our recent installations have been belt drive, using idlers, allowing a greater reduction in speed and shorter belt centers.

In the preparation of anthracite coal we have considerable dust to contend with but do not find that this gives any trouble other than clogging up the motor ventilating ducts, but we find it necessary, on account of high momentary peak loads, to install larger motors than would ordinarily be required, and notwithstanding the clogging of the ventilating ducts with dust the temperature of the motor is well within the limits. From the electrical side we find it necessary to use a motor of high torque in order to take care of the peaks. As a general rule our motors are designed for 250 per cent torque. Due to this peak condition considerable trouble was experienced with the screw fastened and soldered rotor bar. The welded or cast in rotor bar has eliminated these troubles. We did find, however, 80 per cent of our trouble was due to mechanical reasons and on all of the later-type motors changes have been made which have practically eliminated the electrical as well as the mechanical troubles.

The use of inside substations is oftentimes advisable. We have two such substations in operation. The only objection I can see to the inside station is the delay in changing apparatus in case extensive repairs are necessary. In regard to locating substations at various points in large mines this is often advisable where the workings are extensive and when two or more substations are used at one mine I find that where it is practicable the d-c. side of these substations should be tied together. This would not only decrease the voltage drop but would allow operation with a smaller substation capacity, due to taking advantage of the divergency factor. We are starting to do this extensively and at the present time have plans for the tying together of eleven different substations at the various points in the mines. The d-c. feeding system of these various substations is such that they can be tied together by a very small expenditure. As a protection in the substation we are using reverse-current relays and speed-limiting devices on the converters.

L. C. Ilsley (Abstracted by H. H. Clark): Mr. Kingsland's statement that "no practical means has been developed for heavy haulage by the use of alternating current and we are therefore limited to direct current" merits consideration, since the lack of suitable alternating-current motors for operating mining locomotives complicates many mining problems. Especially is this true when a new field is under development. Under such conditions it would be possible to distribute power over the field at a high voltage, stepping it down at the point where it is used to whatever value is required. Such a practise would reduce materially the cost of the feeder circuit; would replace synchronous-converter stations and their attendants with transformer stations without attendants; would materially reduce the length of the low-tension circuits, which would be attended by a correspondingly decreased secondary voltage drop; would facilitate the maintenance of the distribution system and decrease the cost of maintenance, since more of the system would be above ground where it would be less subject to injury and where it could be more readily inspected. The haulage locomotive is the only piece of mining equipment to which alternating current has not been successfully applied.

Pumps and hoists operated by alternating-current motors are already in universal use and in most cases are preferred to similar equipment operated by direct-current motors. The use of alternating current simplifies the lighting problem, since it provides a more flexible voltage. Alternating-current coal-cutting equipments have been on the market for some time and are in apparently successful use. On account of the excessive amount of dust which attends the mining of coal alternating-current motors are much to be preferred to direct-current motors for the various miscellaneous uses around a mine, both inside and out.

Therefore, if some kind hearted engineer would only solve the problem of providing a suitable alternating-current motor for operating the mine locomotive he would insure that the mine distribution systems of the future would be alternating current, and would incidentally permit a considerable saving in installation costs on the part of the mine operators.

R. L. Kingsland: Mr. Pauly mentioned the feeder copper inside of mines. From an engineering standpoint, feeder copper in the mine is a very simple problem, all you have to do is to work out how much is required and put it in. I have found it a very difficult problem to ever get a mine to install as much feeder copper as you want installed, they always try to discount it and talk of the high price of copper and the scarcity of labor, that they are there for getting out coal and not for putting in copper, and such arguments as that, so that as a matter of fact, it is not an unusual case to find 100 volts, or even below 100 volts, as the working voltage in a mine where the substation is generating the power at a voltage of 275. That is not a condition we want, but one we have.

Then as to the question of motors for driving fans. There was a little instance that came to my attention which may have given some designing engineers wrong information. I know of a case where they installed seven or eight fans, each belt connected to a 150-h.p. squirrel-cage induction motor. I was not at the works when it was done, in fact, did not go there for several months afterward. In the meantime we had a lot of trouble, could not keep the belts on, and were burning the belts. We reported this to the manufacturer, who supplied special switches so that we could have two starting taps. By closing the first switch a voltage high enough to turn over the motor was applied. Opening the first switch and closing the second reduced the voltage applied to the motor to a point where there was just enough to bring it to speed. The closing of a third switch placed line voltage on the motor. We tried out this apparatus but still had the same trouble with the belt, due to too rapid acceleration. A trouble man from the manufacturer investigated the trouble on the ground and made recommendations for further special apparatus. This was supplied by the manufacturers, and tried out without success. Soon after this the writer had occasion to visit these installations and discovered that they had the auto-transformer connections reversed: the starting leads were connected to the line and the line leads to the motor. This stepped up the voltage for starting instead of reducing it. These connections were changed and no further trouble experienced using the standard equipment as originally furnished. These fans are larger than usually used for motor drive, being 14 feet in diameter by 5 feet wide. The fans run at about 90 rev. per min. A heavy torque is necessary to start from rest.

Mr. Clark mentioned the question of suspending the cables in bore holes. We have suspended three-conductor varnished-cambric-insulated lead-covered steel-armored cables in bore holes of a maximum depth of 515 feet. For the armor we are using steel wire with a jute coating to protect it on the outside, and then we are suspending the cable by means of the steel wire drawn back on clamps and have found it satisfactory to that depth.

As to the suggestion of Mr. Clark to use a high quality of rubber insulation, we have not tried that, but I would be afraid of it, on account of the acid content in mine waters and the vapors to which the bare rubber would be subjected. Of course, if the rubber was acid proof, it would be all right, but it would not stand up indefinitely. It would also be difficult to suspend cable without the steel armor.

Mr. Means' statements regarding the paper can be explained, I believe, from the fact that he has looked at the case from quite a different angle than I have. He is considering, primarily, purchased power; whereas all the power that we are using is generated in our own central stations. On off-peak periods this means quite a difference in cost; that is, there is not the same advantage to us in cutting down a fan speed at the off-peak period that there would be to a man purchasing his power.

Mr. Kiser mentioned that 330 volts should be classed with the medium voltage. My interpretation of the standards is that the low voltage includes 300 volts, with a divergency factor of 10 per cent, which would allow 330 volts to be classed as low, not medium.

Mr. Kiser also states that he has seen motor-generator sets operating satisfactorily with 75 per cent overload. I have seen synchronous converters operating satisfactorily with 200 per cent overload.

Mr. Bright mentions the use of series motors for coal-cutting machines. I have never seen any coal-cutting machine using a series motor. They operate practically at no load before they come in contact with the coal, and therefore it is necessary to have some shunt winding on them to hold the speed down. My reason for not comparing the series motor with the alternating-current motor was that I knew of no practical use in coal mining work of the alternating-current motor, as against the series motor.

Mr. Bright also recommends the insulation of 500-volt wires. We have found that ordinary insulation on a wire is even more dangerous than a bare wire, on account of the fact that the men trust to the insulation underground where it will not last.

Mr. Bright also mentions the power factor on motor-generator sets as not being any better than converters. The power factor of the motor-generator set depends almost entirely on what it is designed for, and it can be regulated to suit conditions.

Mr. Duffy asked regarding the highest voltage we were using for underground motors. So far, the highest voltage we have applied direct to underground motors is 2300. We have taken 11,000 volts down a three-conductor cable, through a bore hole, and applied that to transformers for synchronous converters. Our principal reason for not using a higher voltage is that 2300 is the highest voltage for which a complete line of standard motors is manufactured.

In reply to Mr. Duffy's question I would say that we are not using any commutator type alternating-current motors. We are not experimenting with them at all.

Mr. Ilsley's discussion brings up the question of alternating-current haulage. As I said in the beginning of this paper, I tried to make it an entirely practical paper, and I know of no practical application of alternating current to mine haulage.

Graham Bright (Communicated after adjournment): Mr. Sage has presented a valuable paper on the electric mine hoisting industry and has given some very interesting operating data. The conditions of operation are rather fortunate for the hoisting equipment in that the usual operation is found to be far below normal and will probably be so for some time. This gives the equipment a chance to be broken into regular service and gives the operators a chance to become acquainted with the equipment and its limitations.

I believe that Mr. Sage is a little optimistic when he states that the Elm Orlu hoisting equipment is one of the two largest hoisting equipments in this country. There was at the time Mr. Sage's paper was written, one hoist which had been in operation for two years at the North Butte Mining Company of not only larger capacity but performing a very much heavier duty than the Elm Orlu hoist will be called upon to perform, even when the operating conditions of the mine are approaching the normal conditions as outlined in the maximum duty given in the original specifications. At this same time there were two other hoists of the same capacity as the North Butte about to be placed in operation, one at the Black Rock Mine of the Butte & Superior Copper Co. at Butte, Mont. and the other at the Canadian Copper Co. near Sudbury, Ont. Both of these hoists will be placed in operation some time in July or August of this year. The above hoists are the three largest in North America and the attached table gives a comparison of these three hoists with the Elm Orlu hoist.

In regard to the layout of the equipment, the writer believes that the flywheel motor-generator set should be placed in a separate room or a tight partition be placed between the motor-generator set and the hoist to eliminate the noise from the set. The hoist operator can accomplish his work much better if the room is quiet and he is not bothered with the continual noise made by the motor-generator set.

The WR^2 of the drum is given as 3,600,000 lb. ft. squared. It would be interesting to know if this is the original estimated value and if so, how close it checks with the actual value of the equipment when built. The WR^2 of the hoist parts of the North Butte hoist is about twice as great.

It might be interesting to compare the information given in Mr. Sage's paper with that given in a very similar paper by Mr. Sykes presented at the San Francisco meeting, September, 1915.

It would be also interesting to know just how the losses in mechanical parts of the hoist sheaves and guides are distributed

HOIST DATA

	Depth	Angle	Wgt. ore	Rope speed	Wgt. × Speed ÷ 1,000,000	D-c. motor cap. h.p.	Generator cap h.p.	A-c. motor cap. h.p.	Flywheel weight
North Butte	4000	90 deg.	15000	2700	40.5	2000	1500	1400	100,000
Canadian Copper	1500	55 "	18000	2500	37.0	1850	1500	1400	100,000
Elm Orlu	3500	90 "	10000	2500	25.0	1800	1300	1150	92,000
Butte & Superior	4000	90 "	16000	2270	36.3	2000	1500	2000	120,000

over the load cycle. This evidently cannot be taken by assuming a certain definite efficiency as the losses would be entirely too great at certain portions of the cycle and entirely too light at other portions. For instance, taking the cycle at 3500 ft., the point at which retardation begins shows a load on the hoist motor of about 50 h.p. At 85 per cent efficiency, this would mean that the hoist loss would be about 7½ h.p. which, of course, can not be correct. The scheme of assuming a certain torque as the hoist friction seems to be more reasonable than assuming a mechanical efficiency of a definite amount.

The use of a circuit breaker in the main circuit between the hoist motor and the generator is rather questionable if this circuit breaker is to be hand operated. When this circuit breaker opens, the operator loses the electrical control of his hoist and must depend entirely upon mechanical brakes. This is not the case when the circuit breaker is omitted and safety protection is obtained by placing the circuit breaker in the field of the hoist motor and generator, this circuit breaker to be reset by the operator with the controller in the off position.

The rate of acceleration and retardation and the time required at rest as shown in the tests are much greater than will occur when the hoist is operating up to full capacity. It would be interesting to compare a set of similar curves when working at full capacity to those as shown in Fig. 9.

A very ample margin has been chosen in selecting the equipment and it should operate over long periods with very low up-keep and very small amount of attention.

J. B. Morrill (Communicated after adjournment): In the seventh paragraph of Mr. Kingsland's paper is the statement "if we had only half rated voltage on a d-c. shunt motor we would have one-half torque, while with one-half voltage on a squirrel-cage induction motor we would only have one-fourth torque. This is a good point to keep in mind when considering a-c. motors for portable machines underground." I take it that the author intends to make a case against the induction motor by this statement, whereas theoretically the power in each case would be the same, for the theoretical speed of the shunt motor would be cut in two by the reduced voltage and the induction motor would run nearly at full speed. The induction motor would give then about one-fourth its rated output at nearly its rated speed while the shunt motor would give one-fourth its rated output at less than half speed, and it seems to me that in most cases the former would be preferred. Moreover the voltage for the induction motor could be boosted up to normal without introducing a rotating machine, making it far easier to obtain rated power than in the case of the d-c. shunt machine.

R. S. Sage: Referring to Mr. Graham Bright's communicated discussion would say, lest there be a wrong impression gained, that it was only a matter of a couple of weeks until the operators became as familiar with the control as with the steam hoist and without exception praised highly the superiority of control over the steam engine. At the end of this time there would have been no difficulty whatever had there been any occasion to hoist at the maximum rate.

The author agrees with Mr. Bright that it would be advantageous in installations of this size to place the fly-wheel motor-generator set in a room separated from the rest of the hoist house as there is always more or less noise attending the operation of high-speed machinery of the nature of this set.

As to the $W R^2$ of the hoist drums the value given in the paper is that estimated by the hoist builders and an analysis of the test results would indicate no great variance of the actual from the estimated value.

Referring to Mr. Bright's points relative to the mechanical efficiency of the hoist it was considered to be so generally understood by those engaged in this line of work that the author in his paper did not define the term "mechanical efficiency." As used, it was understood to mean the ratio of the net work done on the material in the shaft to the work represented by the duty cycle, and the resulting friction work was considered to have been done at a uniform rate during the entire trip. This efficiency cannot be exactly calculated but is arrived at by a consideration of the relative weights of the material being worked upon, of the skip and rope, the rope speed, etc. In the case of a first motion hoist with skips in balance, it is believed that the total friction load does not vary greatly in value from point to point in the trip, and that no great error results in taking it as uniform. It is obviously incorrect to assume the friction to be a fixed percentage of the net rope pulls, for at some points in the trip this may be zero.

Regarding the use of a hand-operated overload circuit breaker between the hoist motor and generator, experience has proven its use highly advantageous, it being understood of course, that overload tripping is confined to loads only slightly under the commutating limit of the machines. The dynamic braking effect obtained by simultaneously opening the motor and generator fields must necessarily be of a very indefinite and variable value. Depending as it does on the relative rate of upbuilding of the two field fluxes, its value can neither be predetermined nor pre-established with any practical degree of accuracy. Its effect can very readily be between the ranges of zero and a value sufficient to cause an arc-over of serious mechanical strains. Wherever the matter is considered of sufficient importance to warrant the additional cost involved, very simple means can be provided which will provide dynamic braking of a definite character and at the same time permit the use of the circuit breaker and retain the benefits which it affords.

In regard to the second paragraph of Mr. Bright's discussion would say that, inasmuch as the Elm Orlu and the North Butte hoists have been the only ones of comparable capacity in operation in this country to date, October 8th, there is no inaccuracy in the author's previous statement to the same effect. No claim was made of greater capacity for the Elm Orlu equipment.

In view of the operating experience at North Butte it is interesting to note that an 1850 h. p. continuous rating motor is required to perform, as Mr. Bright says, 60 per cent more duty than the Elm Orlu hoist, which has a motor capacity of 1800 h. p. continuous capacity.

Considered aside from these points the table given by Mr. Bright is a valuable bit of data covering noteworthy equipments of this character, three of which are located in this country and one in Canada.

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ECONOMICAL COMBINATION OF WATER POWER AND STEAM PLANTS AND A CONVENIENT METHOD OF SOLUTION

BY H. ST. CLAIR PUTNAM

ABSTRACT OF PAPER

A determination of the respective economic limits in steam and water power plant developments. Formulas are derived for determining total annual costs of each type of plant and the most economical combination. Certain general conclusions are reached as follows: Practically all water power plants should be developed beyond minimum power available and hence in combination with a steam plant. The economic limit depends largely on the value of money, fuel, labor, the increasing efficiency of steam turbines and the location of the enterprise.

IN THE development of an industry it is important to carefully consider possible improvements that may be made in methods and efficiency. Of equal importance is the prompt recognition of changes in conditions which affect the industry. During recent years these factors have been especially important in the development of power station practise. The perfecting of the steam turbine, the great increase in the practicable capacity of generating units, the larger boilers used, the use of higher steam pressures and higher superheat, and improvements in automatic stokers, have added thirty or forty per cent to the overall station efficiency within the past ten or fifteen years. Progress also has been made in the design of water turbines. The efficiency of water turbines has been increased from near 80 per cent to over 90 per cent of the potential water power available, and the size of the generating unit has been materially increased.

The improvement in power station efficiency, both in steam and hydraulic practise, has been great, but of greater importance is the change introduced by the change in conditions. Foremost among these changes in conditions is the fact that the modern power plant, whether water power or steam, is a highly efficient manufacturing plant producing electric power for sale in large blocks as well as for retail. The recognition of this function of the modern power plant to supply large blocks of power for industrial use has resulted in the development of large highly efficient power stations manufacturing electric power as a business. The industrial use of electric power has greatly increased in recent years, but there are large amounts of such industrial power still independently produced, and much of this can be more economically supplied from central power stations.

Probably the greatest change in conditions, however, has been introduced by the development and perfection of long distance high-voltage transmission lines. Transmission lines are now built of steel with factors of safety equivalent to those used in the construction of bridges and office buildings. This renders them practically immune from mechanical failure. Improved methods of insulation, the increase in voltage, and the development of methods of protection against short circuit and lightning have reduced electrical failures to mere surges, in most cases, so that records show that the interruptions to service resulting from the failure of high-voltage transmission lines are not in excess of those which occur in the ordinary operation of the modern power station. The comparatively unreasonable fear of the reliability of transmitted power is diminishing, and it may be expected to disappear when transmission lines are constructed with adequate factors of safety and when they are properly protected against electrical disturbances, by devices which have been already tried out and proved effective.

The practical development of the transmission line has opened wide fields for the economic development of power. Within reasonable limits, power stations can be constructed where power can be most economically produced. The successful transmission of power over distances of from one hundred to two hundred miles has given great impetus to the development of water powers on a large scale, and only recently, in this country, it has been applied not less effectively to the transmission of power from steam plants located at the coal mines, where cheap or waste coal can be obtained and utilized. The public has become accustomed to the transmission of electrical energy from water powers for long distances, but as yet the possibilities resulting from the transmission of energy from steam power produced directly in the coal fields have not been realized, and the advantages resulting from the combination of such cheap steam power with existing and undeveloped water powers are not fully appreciated.

The development of large steam and water power plants, and the improvements made in the transmission of power, have raised new problems as to the most economical method of operation and combination of the several power plants. It is no longer good engineering to regard a power project as self-contained, to be concerned only with its immediate power market, but it must be considered and studied in connection with other power plants, and its design and development should be planned to meet possible future connections with such plants.

This general subject was discussed by the writer in a paper on the "Conservation of Power Resources,"* presented at the first national conference called by President Roosevelt, in 1908, on the Conservation of Natural Resources. In this paper the economic development and combined operation of large water power and steam plants were strongly urged. It is the purpose of these notes to describe a relatively simple method of solving the different problems that arise in combining different power plants with different costs and different operating characteristics, and especially the use of the method proposed to determine the

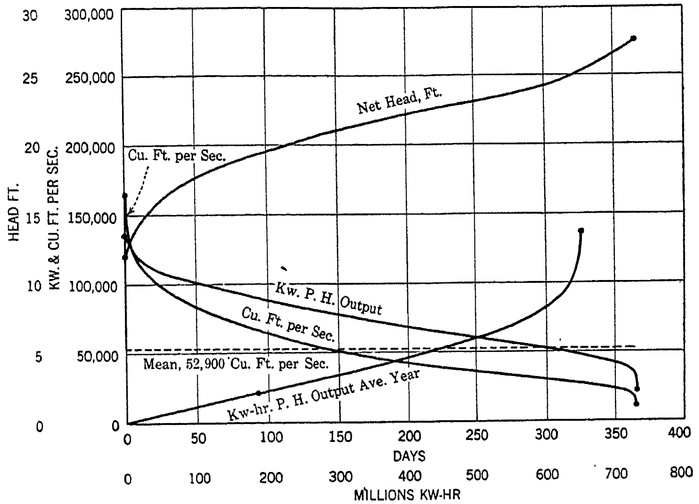

FIG. 1—DURATION CURVE OF FLOW—AVERAGE OF 15 YEARS

feasibility of water power developments, and the magnitude to which such developments should be carried.

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*"Conservation of Power Resources," by H. St. Clair Putnam. Presented at the Conference on the Conservation of Natural Resources, White House, Washington, D. C., May 13-15, 1908; reprinted in TRANSACTIONS A. I. E. E., Vol. XXVII, p. 1398.

Primarily, the proposed method of solution is based upon careful estimates of the cost of construction of such power plants as may be required initially, and a second estimate of the same plants extended to greater capacity, preferably to not less than double the size of the initial plant. Careful estimates are then made of the annual capital costs of each and the operating costs at two or more load factors. The method is equally applicable to water power plants and to steam plants, and naturally can be extended to transmission lines and substations and also to existing power plants and systems. As illustrating the general method of solution proposed, some estimates of cost which were prepared for a rather special case where special conditions existed are given. The purpose of these estimates was to determine the feasibility of a low-head water power development represented by the data plotted in Fig. 1, and also the economic capacity for which it should be designed. For the purpose of determining possible economic combinations, the results of the water power development were combined with the results which

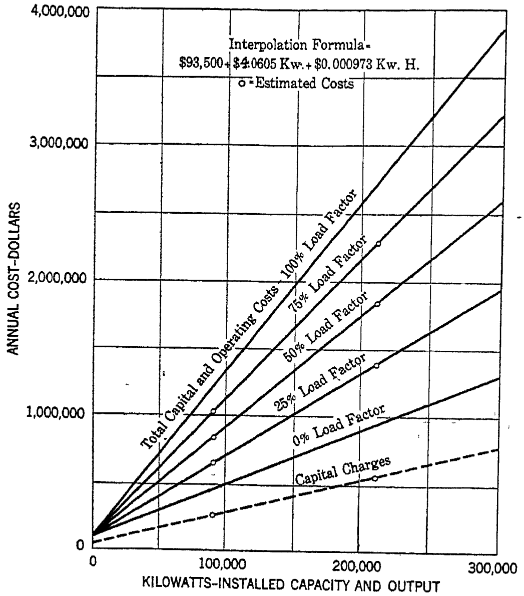

FIG. 2—STEAM POWER STATION—ANNUAL COSTS

could be obtained from a well located steam plant of modern design. These estimates are used only to illustrate the method employed, and while they are approximately correct, their accuracy is not essential to the purpose of this paper.

ESTIMATED COSTS

WATER POWER PLANT

Installed Capacity....	100,000 kw.		200,000 kw.	
Dams, dykes, locks, tail race and power plant, complete, including overhead costs.......	$12,200,000		$19,266,000	
Annual Cost:	*25% L.F.*	*75% L.F.*	*25% L.F.*	*75% L.F.*
Interest, depreciation and taxes	$522,350	$522,350	$872,500	$872,500
Operation and maintenance.....	96,080	157,180	161,890	284,090
	$618,430	$679,530	$1,034,390	$1,156,590

Formula—First cost: $5,134,000 + $70.66 per kw.

Capital cost...$172,250 + 3.5010 per kw.
*Operating cost 30,270 + 0.3526 per kw. + 0.0001395 per kw-hr.

Total annual cost $202,520 + 3.8536 per kw. + 0.0001395 per kw-hr.

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*The formula for operating cost is derived from the estimates of operation and maintenance, as follows:

100,000 *Kilowatt Plant.*

Operating cost	75% L.F.	$157,180
" "	25% L.F.	96,080
Increment...................	50% L.F.	$61,100 = 0.611 per kw

Increment cost 25% L.F. = $0.3055 per kw. = $0.0001395 per kw.-hr.

Operating cost, 25% L.F. (above)......................	$96,080
100,000 kw. @ $0.3055..............................	30,550
Plant constant, 100,000 kw..........................	$65,530

200,000 *Kilowatt Plant.*

Operating cost 25% L.F..............................	$161,890
200,000 kw. @ $0.3055 (above).......................	61,100
Plant constant, 200,000 kw...........................	$100,790
Plant constant, 100,000 kw. (above)....................	65,530
Increment..100,000 kw............................	$35,260

Increment cost $0.3526 per kw.

Constant.

Plant constant 100,000 kw. (above)....................	$65,530
100,000 kw. @ $0.3526...............................	35,260
Constant...	$30,270

Formula—$30,270 + $0.3526 kw. + $0.0001395 kw-hr.

The same general method is employed to derive the other interpolation formula, used in this paper, from the estimated costs.

The corresponding costs for a steam plant constructed in accordance with the best modern practise and located immediately at the coal mines where suitable coal can be obtained at an average price of $1.00 per ton, are as follows:

Steam Power Plant

Installed Capacity	90,000 kw.		210,000 kw.	
Ground, building equipment, coal tracks, and storage and condensing water facilities....	$5,062,350		$10,643,230	
Annual Cost:	*25% L.F.*	*75% L.F.*	*25% L.F.*	*75% L.F.*
Interest (3%), depreciation and taxes....	$265,430	$265,430	$566,640	$566,640
Operation and maintenance..........	385,293	768,850	827,058	1,722,023
Total annual cost	$650,723	$1,034,280	$1,393,698	$2,288,663

First cost—formula:	$876,720 +	$46.507 per kw.	
Capital cost.....	$39,530 +	$2.5100 per kw. +	
Operating cost...	53,970 +	1.5505 per kw. +	0.000973 kw-hr.
Total annual cost..	$93,500 +	$4.0605 per kw. +	0.000973 kw-hr.

The estimated annual costs, as given in this estimate, are plotted in Fig. 2, which is a graphic representation of the method used in obtaining the above interpolation formula.

Transmission Lines

Installed capacity....................	150,000 kw.	300,000 kw.
Steel tower line and right-of-way........	$1,484,700	$2,839,400
Annual Cost:		
Interest, depreciation and taxes.......	$62,496	$120,492
Operation and maintenance..........	21,770	36,040
Total........................	$84,266	$156,532

Formulas—First cost.......	$149,950 +	$8.965 per kw.
Capital cost..............	4,450 +	0.387 " "
Operating cost............	7,520 +	0.095 " "
Total Annual Cost............	$11,970 +	$0.482 " "

SUBSTATION—DISTRIBUTION

Installed capacity	90,000 kw.	240,000 kw.
Substation building, transformers, switching gear, lightning arresters, etc.	$600,000	$1,500,000
Capital cost	$33,000	$82,500
Operating cost	11,000	22,500
Total Annual Cost	$44,000	$105,000

Formula—First cost	$60,000	+	$ 6.00 per kw.
Capital cost	$ 3,300	+	$0.330 per kw.
Operating cost	4,100	+	0.077 per kw.
Total Annual Cost	$7,400	+	$0.407 kw.

The above estimates are based upon interest at 3 per cent and coal $1.00 per ton. In the table below, transmission losses are taken at 6 per cent and the load factor at 100 per cent. The market for power is assumed to be at the water power end of the line. The resulting costs expressed in terms of power delivered to the distribution system, in terms of the derived formulas, are as follows:

RESULTING FORMULAS—100 PER CENT LOAD FACTOR
6 PER CENT LOSSES ADDED.

Construction Costs.					
A.	Water Power Plant	$5,134,000	+	$70.660	per kw.
(1)	Steam plant	$ 876,720	+	49.297	"
(2)	Transmission line	149,950	+	9.503	"
(3)	Substation at W. P. end	60,000	+	6.000	"
B.	Total for Steam Plant	$1,086,670	+	$64.800	"
Interest (3%) *and Depreciation.*					
C.	Water Power Plant	$ 172,250	+	$3.501	"
(1)	Basis 8% interest, etc.	428,950	+	7.034	"
(2)	Steam plant	$ 39,530	+	$2.661	"
(3)	Transmission line	4,450	+	0.410	"
(4)	Substation	3,300	+	0.330	"
D.	Total Steam Plant	$ 47,280	+	$3.401	"
(1)	Basis 9% interest, etc.	$ 112,480	+	$7.289	"

Operation and Maintenance.

E. Water Power Plant.............	$	30,270	+	$1.5746	"
(1) Steam plant................	$	53,970	+	$10.6751	"
(2) Transmission line..........		7,520	+	0.1007	"
(3) Substation.................		4,100	+	0.0770	"
F. Total Steam Plant, $1.00 coal....		$65,590	+	$10.8528	"
(2) " " " 3.00 "		88,910	+	26.6368	"

The Problems to be Solved

1. *When the Development of a Water Power will Secure Better Results than the Construction of a Steam Plant.* One of the first questions to be determined in the development of a water power is whether or not it will pay, and if so, whether it is the cheapest and best method of producing the power required. In the case used as an illustration, the water power is compared with a steam plant of modern design, located at the coal mines, thus reducing the cost of fuel to a minimum. The comparison, therefore, is most severe. After the above described cost formulas are developed, the problem is quickly solved. On the basis that the power is to be delivered to a local distribution system at approximately 13,000 volts and 100 per cent load factor, the formulas derived above become as follows:

Total annual cost, water power,	\$202,520 + \$5.076 per kw. $(C+E)$
Total annual cost, steam power,	112,870 + 14.253 per kw. $(D+F)$

Then the point of equality of cost will be found as follows:

$$\$202{,}520 + \$5.076 \text{ per kw.} = \$112{,}870 + \$14.253 \text{ per kw.}$$

$$\text{kw.} = \frac{89{,}650}{9.177} = 9{,}769$$

That is, if the power required is in excess of 9,769 kw., it is better to develop the water power than to build the steam plant.

2. *When it is Better to Construct the Water Power Than to Extend an Existing Steam Plant.* If the steam plant is already built, and additional power is required, then the question arises whether it is better to extend the steam plant or to construct the water power.

This question is answered as follows:

$$\$202{,}520 + \$5.076 \text{ per kw.} = \$14.253 \text{ per kw.} \quad (C+E = \text{inc.} D + F)$$

$$\text{kw.} = \frac{202{,}520}{9.177} = 22{,}068$$

That is, if the increased power desired is in excess of 22,068 kw., it is better to construct the water power plant than to extend the steam plant. As stations of large capacity are here considered, and the steam station is assumed to be equipped with 30,000-kw. or 35,000-kw. steam turbines, it is evident that it is better to construct the water power than to install an additional unit in the steam station.

As has been stated, the estimates used in this memorandum are based upon the development of a water power with a low head, but with a large volume of water. The characteristic discharge curve for such a stream, together with the potential power that can be developed and the variations in head with different flows, are shown in Fig. 1. It will be noted that, with the exception of but two or three days in the period of fifteen years, the power during low-water periods does not drop below 30,000 kw. It is evident from the above deductions, therefore, that the water power should be developed under the conditions assumed, rather than that a steam plant and transmission system should be constructed, or the steam station extended.

Water Power with Steam Auxiliary

It has been found profitable to develop water powers much beyond the minimum flow of the stream when a market for the power is available. If there is no market immediately at hand, in many cases new industries grow up and absorb the power after such developments are made. Under such circumstances, as the demand for power in any industrial community constantly increases and the power must be supplied, it often becomes a serious problem to determine how best to obtain additional power when the power in the original development is absorbed.

It is of importance, therefore, to determine in advance the best water power development to make, and to determine as nearly as possible, the manner in which increased demands for power are to be met. There has been a tendency quite general among bankers and investors to regard a water power as limited by the minimum flow, and to look with suspicion upon the proposition to include an auxiliary steam plant as a necessary part of a water power development. As a matter of fact, practically all water powers have auxiliary steam plants in connection with them, but, unfortunately, these auxiliary plants are often uneconomically furnished by the customers. It is much better engineering, and much greater economy is obtained, if the question of auxiliary steam plants for a water power is met boldly and adequate provision is made for it in the beginning. The purpose of this memorandum is to suggest a convenient method for determining with accuracy the best development for a water power and the best combination that can be made with an auxiliary steam plant. The method can be used also to determine the best combination that can be made of existing steam or other power plants, and the manner in which all these factors are affected by changes in interest rates and the cost of coal, labor, supplies, etc.

The Most Economical Water Power Development. In the case used as an illustration, it is clear that if the power required exceeds 10,000 kw., the water power should be developed rather than a steam plant constructed, and this development can be carried to 25,000 or 30,000 kw. before the question of an auxiliary steam plant arises. Every water power development should be so designed that its maximum economical capacity can be conveniently developed without reconstruction, and it is important that this capacity should be determined when the original designs are made.

In the problem used as an illustration, the supplementary or auxiliary steam plant is located at the point where fuel can be obtained most economically. The analysis is based upon the annual cost of production, rather than upon earnings, for the reason that gross receipts should be approximately the same, independently of the manner in which the power is produced. We are concerned, therefore, with the single matter of ascertaining the manner in which the power can be produced at the lowest cost.

An inspection of the formulas on page 697 shows that they are materially different in the distribution of costs, and it is evident that the costs can be combined so as to obtain the point where power is produced at a minimum cost. The formulas at 100 per cent load factor, 3 per cent interest, coal at $1 per ton, and with power delivered to the low voltage distribution system, are as follows:

Water Power:	Capital cost,	$172,250 +	3.5010 per kw.	(C)
	Operation,	30,270 +	1.5746 per kw.	(E)
Steam Plant:	Capital cost,	$ 47,280 +	3,4010 per kw.	(D)
	Operation,	65,590 +	10.8528 per kw.	(F)

As it is assumed that the water power plant will be developed beyond the minimum flow, any increase in the installed capacity in the water power plant will require a corresponding increase in the capacity of the auxiliary steam plant, so the increase in capital cost in both plants must be offset by the saving effected, and the water power can be extended until the net result is zero. This minimum point is shown at 80,000 kw. in Fig. 3. The point of minimum cost can be determined by differentiation, but is most easily obtained by a direct estimate of costs at several points. This point of minimum cost, however,

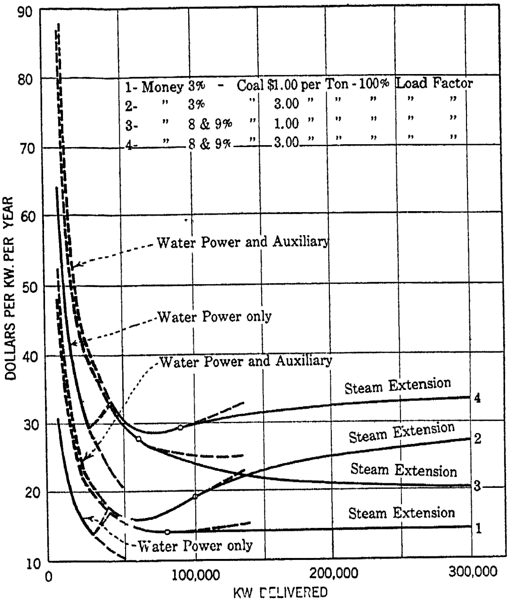

FIG. 3—LOCI OF MOST ECONOMICAL COMBINATION OF WATER POWER AND STEAM PLANTS

is not the determining factor in deciding upon the capacity to which a water power should be developed.

The Most Economical Combination Development. When the steam plant is larger than the water power, the water power is used to supplement the steam plant, when and as the water power is available. In this case the capital charges of the steam plant need not be considered, and the increase in the development of the water power must be justified by the saving in the operating costs of the steam plant, and the development of the water power should be stopped where the saving resulting from such extension amounts to zero. This relationship is expressed as follows:

$$3.501 + 1.5746X - 10.8528X = 0.$$

$$X = \frac{3.501}{9.2782} = 0.378 \times 365 = 138 \text{ days.}$$

From flow sheet Fig. 1, 138 days operation = 80,000 kw.

Under the conditions assumed, therefore, the water power can be developed in connection with a steam plant to the point where it utilizes the flow of the stream for 138 days, which corresponds in the present case to the development of the water power to 80,000 kw. (See Fig. 3.) It is merely a coincidence that the best points for the water power and for the combination plants happen to be the same in this case.

Effect of Change in Conditions. In the contemplated construction of a water power plant, it is impossible to foretell with certainty just when conditions will change, and how much that change will be. Within the limits of probability, however, possible changes in conditions should be given careful consideration in the design and construction of the power plants. The formula method of solving these problems permits other conditions than those assumed to be readily studied and the power plant designed to meet such possible contingencies.

In the case here used as an illustration, if the rate for money is increased from 3 per cent to 8 per cent for the water power and to 9 per cent for the steam plant, to cover interest, taxes and amortization, and coal increased in cost from $1.00 per ton to $3.00 per ton, the following tables show the effect of these changes on the most economical development for a water power for operation in combination with a steam plant under the conditions assumed.

POINTS OF ECONOMICAL DEVELOPMENT

3 per cent. Money; Coal $1.00 to $3.00 per Ton

	Water power and steam auxiliary		Combination of water power and steam plant	
	Days of utilization	Kw. installed	Days of max. flow	Kw. installed
Cost of Coal:				
$1.00 per ton..............	138	80,000	138	80,000
3.00 " "	258	60,000	52	100,000

Under the condition that interest, taxes and amortization cost 8 per cent for the water power and 9 per cent for the steam plant, the points of economical development become as follows:

8 per cent. and 9 per cent. Money; Coal $1.00 to $3.00 per Ton

	Water power and steam auxiliary		Combination of water power and steam plant	
	Days of utilization	Kw. installed	Days of max. flow	Kw. installed
Cost of Coal:				
$1.00 per ton..............	365	130,000	276	56,000
3.00 " "	185	71,000	102	88,000

In the above, with coal at $1.00 per ton and money at 8 per cent and 9 per cent, it is not financially practicable to develop the water power. In the other cases assumed, the development of the water power is feasible and profitable. These results are plotted in Fig. 3.

The cost of coal will remain low, as the contemplated steam plant is assumed to be located immediately at the coal mines. The price of coal will tend to rise, however, and this fact must receive consideration in the design of the power plants. The tendency of money rates will probably be downward as the country increases in wealth. An examination of the above tables and the curves shown in Fig. 3 indicates clearly that, under the conditions assumed, the power plant should be designed for an ultimate extension to 100,000 kw., and that under the probable changed conditions, as stated, it will not be advisable to exceed this amount.

Suggested Special Power House Design. In this connection, although it is removed from the purpose of these notes, I wish to suggest a power station design which may be of use in the development of water powers with low operating heads. Direct-connected water-wheel generators are now generally used in hydroelectric developments, but when the head is low, these generators become costly on account of the low speed and the large volume of water to be passed through them. The generators must have relatively small capacity as compared to those used in steam practise. Where the power development is large, the result is that the power house building and foundations, as well as the water wheels and generators, become serious factors in the cost of such a power development. In the case used as an illustration, the power station must be from one-quarter to one-half mile in length, according to the size of the generating unit used, and this is a serious factor in the cost of development. In the accompanying drawing, Fig. 4, the suggestion is made to place a simple protecting building over the operating room and to place the switching gear, transformers, etc. in a substation building on shore, the power house being used simply as a generator room. Instead of building a high and costly structure to support and contain the crane, an external gantry crane is used and the apparatus in the station is reached through removable transoms in the roof. This method of design greatly reduces the cost of the power station construction.

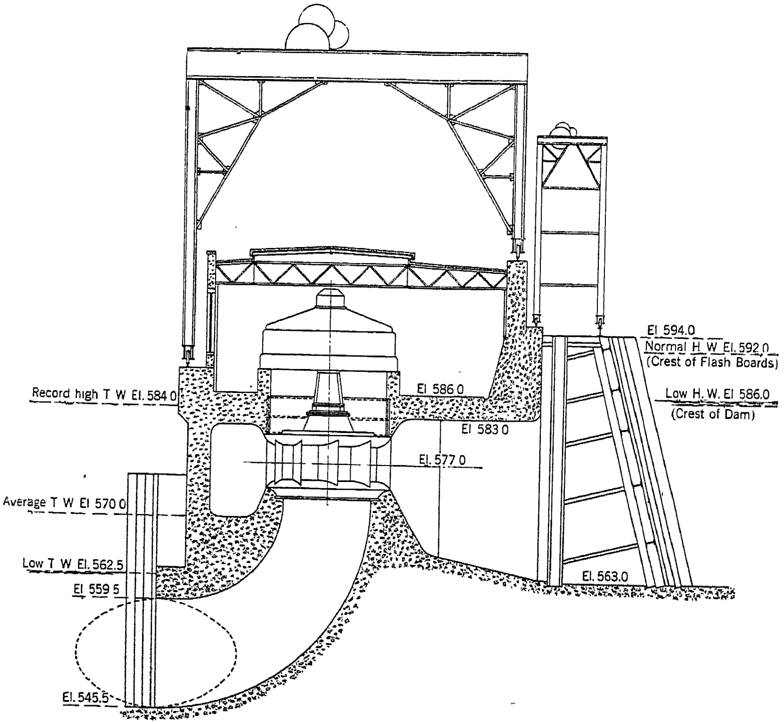

FIG. 4—CROSS SECTION OF POWER HOUSE—MISSISSIPPI RIVER POWER DEVELOPMENT—ROCK ISLAND RAPIDS

There are obvious objections to reaching the apparatus through transoms, especially in bad weather. But a unit out of commission is but a small proportion of the total, and during a large part of the year there are many spare units. It is believed that the reduction in the cost more than offsets any possible inconvenience in making repairs.

Load Factor. The effect of a reduction of the load factor below the 100 per cent used in this illustration will be to increase the number of days per year during which a water wheel must be operated in order to justify its installation. On the other hand, if pondage is available above the dam, the kilowatt capacity which can be installed will be increased. The object of this paper is to indicate a method of solution for such problems, rather than to solve the various problems connected with water power developments. The necessary changes are readily made to adopt the formulas to any load factor desired. The method of solution is then the same as has been used. The kilowatt capacity curve on the flow sheet must be reconstructed to adapt it to the storage capacity which may be available above the dam.

Reserves. The reserve generating units for obvious reasons should be placed in the steam plant. An entire spare circuit should be provided in the transmission line as a reserve, and a reserve set of transformers should be provided in the substation. These reserve factors will appear as constants in the formulas expressing costs, and hence do not enter into the problem of the determination of the most economical development of the water power.

Other Uses of Formulas. Formulas such as have been shown can be developed for water powers of high head, and also where storage reservoirs are available. The capacity that can be economically installed in such developments is frequently much above preconceived conceptions. Similar formulas can be developed for existing power plants of various kinds and efficiencies, and one of the most satisfactory results of a study of the possible combinations of such plants is, that unless the apparatus installed in a plant is unusually inefficient, practically all plants have their economical place and can be efficiently used. In extreme cases, however, the resulting economy is negligible. The formulas can be used to determine whether it is best to change the frequency of a generator or to use frequency changers to obtain the desired result; also whether it is better to change an inefficient generator for a more efficient generator or to use the inefficient generator on the peak. The solution of this problem gives the peak the generator should be allowed to carry and beyond which it should not be operated. In fact, these formulas can be used in determining practically all questions of power house design and operation which confront the designing and operating engineer. With the costs expressed in this form, the estimates of cost at other load factors and under changed conditions are quickly and readily made, and with the assurance that they are correct, for, in the last analysis, they are but interpolation, or pro rating, formulas obtained from established costs.

FINANCIAL EFFICIENCY

Financial efficiency might be defined as the greatest earning power of a dollar. This may or may not be a controlling factor in a water power development, although it is believed that as a matter of good policy it should not be. It is conceivable, for instance, that in the long run, it might serve better in an initial development to earn 10 per cent on $200,000 than to earn 20 per cent. on $100,000. Besides, the excess profit tax list might be the only beneficiary of the higher rate of return. The present notes, however, are concerned only with the explanation of a method to determine the best point of development, although this may not be the point where the dividend rate is the largest on the investment. It determines the point, however, where the enterprise is safe from competition, for the reason that the cost of production is a minimum.

BY-PRODUCT PLANTS

It is feasible to use gas instead of coal in the production of power in the auxiliary steam plant. In most cases gas can be burned under boilers with better results than in gas engines, on account of the low cost and high efficiency of the modern steam turbine. It is now possible to obtain from all bituminous coals, whether coking coals or not, by-products of great value, by the use of either producer gas or coking plants. In many cases the sale of the by-products will pay all the costs of the by-product plant, including the cost of coal. The gas for use in the steam power plant will be a surplus product, or an additional profit, which can be added to the profits of the enterprise. Such by-product plants should receive careful consideration in the design of such a power system as is here described.

CONCLUSION

From the above results, several general propositions can be laid down as to the development of water powers.

Where a market for the power is available or can be created, practically all water powers should be developed beyond the minimum power available, and hence all water power plants require steam plants in connection with them.

Water power plants are most valuable when developed in connection with steam plants, and the development should be carried to the point where the best results can be obtained in combination with such steam plants.

The point of economical development of the water power is increased in capacity as the operating costs of the steam plant are increased, such as are due to the increased cost of fuel and labor.

The point of economical water power development is decreased as the value of money increases. This is true in general, although there are exceptions in the case of water powers with high heads.

The general tendency of coal and labor and other items entering into the operating cost of a steam plant is upward, and as the world increases in wealth the tendency in the value of money will be downward. These factors should be kept in mind in planning the development of a water power and adequate provision should be made for the possible extension of its capacity.

The increase in the efficiency of steam turbines and the construction of steam plants at the coal mines both operate to make it difficult to profitably develop water powers, especially those with low head, where the first cost is relatively high. The development of all possible water powers should be encouraged, both for the conservation of the fuel supply and the economical advantage resulting from the use of power in our industries. It is, therefore, important that all rights essential to the development of water power and transmission projects should be given power companies, to the end that the cost of development should be reduced and many serious difficulties now encountered removed. Any policy under which additional burdens are imposed upon water power developments, whether by the government or other authority, is against the true interests of the public and contrary to the best utilization and preservation of our natural resources. On the contrary, if a plan could be devised so that government funds at low interest rates could be used for such developments, similar to the plan used in the construction of the New York subways, the development of water powers would be greatly facilitated and the financial values created would react to the great advantage of the government.

DISCUSSION ON "ECONOMICAL COMBINATION OF WATER POWER AND STEAM PLANTS AND A CONVENIENT METHOD OF SOLUTION," (PUTNAM), NEW YORK, JUNE 28, 1917.

H. R. Summerhayes: Some time ago Dr. Steinmetz brought out the idea that the induction generator on account of its simplicity could be used to a greater extent in the development of water power plants as auxiliary apparatus. That has actually been done in one or two cases, and there is one case with which I am familiar in the State of Washington in which a 1700-kw. induction generator has been installed directly connected to a water wheel, and perhaps a mile and a half or two miles from an already existing plant on the same river and using the same water. That machine operates without an attendant, it is connected to a step-up transformer that feeds into the same transmission line, and they set the gate at a certain point which will give the desired quantity of water at that time, and it runs at that load until they want to change. The patrolman starts it up when it is desired and then goes away. The machinery is very simple and operates without any attendant at all. There is no governor on it.

The machine is designed so that it will stand every speed, including maximum speed, which it may attain in case the load is pulled off, and I believe that while the induction generator is not suitable for all installations that it should be looked into more carefully for auxiliary plants of that sort, as it is very valuable in such cases. A good deal of money can be saved in that way. That machine, by the way, has been found to operate without any synchronous machines; that is rather a new point. When they start it up it runs in parallel with synchronous machines which supply the exciting currents, but it has been found, in some cases, when the synchronous machines have been pulled off the line by a momentary short circuit that the induction machine will excite itself from the capacity of the line—there happens to be plenty of line capacity—and will go on supplying the load. The frequency, of course, will be determined by the amount of load in reference to the gate opening.

There is another way of cheapening a water power plant. Mr. Putnam suggests a very simple construction of generator station. Now, it is possible to omit the roof altogether and make a generator that will run out of doors, just have the generators and no roof at all, just a crane and possibly a shed into which the crane can transport parts of the generator for repairs. Such a station has been planned, but I do not believe it has ever been constructed.

H. S. Putnam: It is not very generally appreciated that the load on an electric power system, such as we have in New York and most of the electric light companies, has a load factor in the neighborhood of 40 per cent. One-half of the load is at a load factor of probably 10 to 15 per cent, and for every load of 100,000 kw., about 50,000 kw.

will be operated with a load factor of approximately 15 per cent on the peak, and that some apparatus in the station will only be operated for one or two hours a day during the Christmas holidays. A water power plant can carry the short peak much more economically than a steam power plant can. In the next place, the annual interest on the capital investment is less in the case of a water power plant than in the case of a steam power plant, as shown in the case cited of the low-head development. The figures for the steam power plant are given under "Estimated Costs," and there the cost of the steam plant is shown to be $4.06 per kw. The cost is shown to be $3.85 per kw. for the water power plant, which shows that even in that case the short peak can be carried more economically on the water power plant than on the steam plant, and adding to that the cost per kw-hr., for each kw-hr. developed on that peak, it can be seen that the water power plant is more valuable for operation on the peak than it would be to use the steam plant there; in other words, these figures show at once that wherever water is available for the operation of a water power plant, that plant is to be operated rather than a steam plant.

In the case of the ordinary high-head development, the cost of water power is greatly reduced per kilowatt, and ordinarily runs from $1.50 to $2.00, instead of the $3.85.

In general I want to add,—that outside of Niagara Falls and a few of the Western developments, where large amounts of power are available, in most of the water powers in the East and especially those with low heads, the main function of the water power plant is to save coal. Every power plant requires a reserve capacity to be installed in order to make sure that the service is reliable, and every water power plant, when the water is available, should be operated to the fullest extent. I think the conclusion is, where there are reserves, the reserve should be in the steam plant. The steam plant may be large enough to supplement the water power plant at times of low water. But in times of flood the steam plant, to supplement the water power, must be very close to the same size as the water power development, so that the reserve steam plant will equal the water power plant in capacity in most cases. In that case the steam plant can carry the load by itself, and the water power is used to produce kilowatt hours for the economy in coal and labor.

Presented at the 332nd Meeting of the American Institute of Electrical Engineers, New York, June 28, 1917.

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COOLING OF OIL-IMMERSED TRANSFORMER WINDINGS AFTER SHUT-DOWN

BY V. M. MONTSINGER

Abstract of Paper

The 1916 A. I. E. E. Rules require that the temperature rise of transformer windings be observed by the resistance method. Since the measurement of resistance usually requires considerable time, there is always a drop in temperature between the instant of shut-down and the time of observing this resistance measurement. There are three general methods of determining the temperature at shut-down. These are:

1. To take a cooling curve and extrapolate back to the instant of shut-down.
2. To use an arbitrary correction.
3. To calculate the rate of cooling. The usual theoretical formula for calculating the cooling of a body is not in a convenient form for practical use. Furthermore, the conditions in a transformer are generally such that it would be difficult to apply. However, it is shown in the paper that the cooling of oil-immersed transformer windings, for a limited time (four or five minutes) after shut-down, is approximately a function of the watts per lb. of copper, and that when it is necessary to make calculations, more accurate results can be obtained by this partially empirical method than by attempting to use the theoretical formula.

Under "Conclusions", the general advantages and disadvantages of each method are given.

In the Appendix are developed certain formulas used in the paper.

Introductory

IT IS standard practise today to rate electrical apparatus at its maximum continuous capacity. When the rating is limited by temperature (as most transformers are) the insulation or some part of it is operating close to the danger point. For this reason, it is important to know more accurately the average temperature of the windings than it has been in the past when it was the practise to make the temperature guarantee for two different loads; namely, full-load followed by a short-time overload.

The A. I. E. E. Standardization Rules recognize three general methods of determining the temperature of electrical apparatus. These are:

Method No. 1: Thermometer method (Section No. 345).

Method No. 2: Resistance method (Section No. 348).

Method No. 3: Embedded temperature-detector method (Section No. 352).

Due to the fact that transformer windings are usually inaccessible for thermometers and less accessible for temperature detectors than are generators, motors, etc., the A. I. E. E. Rules (Section No. 351) specifies that the temperature of windings of transformers is always to be ascertained by method No. 2—resistance method. However, it is stated that in the application of method No. 2, careful thermometer measurements must also be made whenever practicable to increase the probability of revealing the highest observable temperature. The use of thermometers in determining the "hottest spot" in the windings of transformers applies almost exclusively to air-blast or natural draft rather than to oil-immersed transformers.

The foot note to Section No. 348 in the A. I. E. E. Standardization Rules states:

> Whenever a sufficient time has elapsed between the instant of shut-down and the time of the final temperature measurement to permit the temperature to fall, suitable corrections shall be applied, so as to obtain as nearly as practicable the temperature at the instant of shut-down. This can sometimes be approximately effected by plotting a curve, with temperature readings as ordinates and times as abscissas, and extrapolating back to the instant of shut-down. In other instances, acceptable correction factors can be applied. In transformers of 200 kv-a. and less, the measured temperature shall be increased one degree for every minute between the instant of shut-down and the time of the final temperature measurement, provided this time does not exceed three minutes.
>
> In cases where successive measurements show increasing temperatures after shut-down, the highest value shall be taken.

In observing the temperature of transformers by means of change in resistance, there usually is a sufficient lapse of time to permit an appreciable drop in temperature. In practise this drop ranges from a fraction of a degree to several degrees per minute, hence it becomes highly important to know the extent of it.

While the cooling-curve method is fairly reliable, commercially it is not always practicable. Furthermore, it is desirable, from the manufacturer's standpoint, to have some method whereby for designing purposes, one can predetermine the fall in temperature for various intervals of time which a cooling curve does not give until after the machine has been designed and tested.

It is the purpose of the writer to discuss from a theoretical standpoint the cooling of oil-immersed transformer windings after load has been removed, and to show that the cooling in degrees of the average type of oil-immersed winding for the first four or five minutes after shut-down is approximately a function of the loss expressed in terms of watts per lb. of copper. In other words, it is shown that for practical purposes, the cooling can be calculated for a limited time after shut-down without considering either the thermal capacity of the insulation or the initial temperature rise of the winding. If we can neglect these two factors, it is obvious that the process of calculating the cooling is greatly simplified as compared with the usual method of calculations by considering the thermal capacity of the mass and its initial temperature rise.

It is the difficulty, for the usual transformer winding complicated by oil ducts, etc., of determining accurately the above two factors, especially the initial rise, that makes it difficult to calculate the cooling by the usual theoretical formula. For instance, for a transformer winding with numerous oil ducts, the temperature of the surrounding medium or ambient temperature is that of the oil in the ducts and of the oil surrounding the coil stack. While the oil surrounding the coil stack no doubt remains fairly constant for the first five minutes after shut-down, the oil in the ducts is moving through, just after shut-down, at the same rate it was before shut-down. Since this moving oil is influenced by the temperature of the coils, and since the temperature of the coils is decreasing after shut-down, the temperature of this oil in the ducts is also decreasing.

In other words, if we consider the temperature of the oil in the ducts as the ambient temperature, we have here a constantly decreasing ambient temperature which would be troublesome to deal with in making calculations. On the other hand, if we consider the oil surrounding the coil stack (*i.e.*, neglecting that in the ducts) as the ambient temperature, we must take into consideration the thermal capacity of the oil in the ducts. This also would be difficult to do because as stated above, this oil is not stationary. However, in the case of a single coil immersed in oil where conditions are not complicated by oil ducts, fairly accurate calculations can be made by the use of the usual theoretical formula.

CALCULATION OF COOLING BY THEORETICAL FORMULA

When loss of heat energy is proportional to temperature rise the cooling of a body takes place according to a "die-away" curve which is expressed by the formula

$$\theta_t = \theta_0 \, \epsilon^{-\beta t} \tag{1}$$

in which

θ_t = temperature rise at any time t of the body over its ambient temperature.

θ_0 = initial temperature rise or rise at shut-down.

ϵ = base of Naperian logarithms.

β = time constant.

Equation (**1**) is based on the assumption that the rate of heat dissipation, *i.e.*, the cooling, is proportional to the temperature rise. It is shown later that this assumption is not quite correct for oil-immersed windings, for the reason that the rate of heat dissipation is seldom if ever a direct function of the temperature rise. A formula is developed in the Appendix for the condition where the loss of heat is not proportional to the temperature rise, but equation (**1**) is usually accurate enough for practical purposes.

Equation (**1**) may be put into a more convenient form by changing signs, adding θ_0 to both sides and then putting $(\theta_0 - \theta_t) = \theta$, where θ is the cooling in deg. cent. We now have

$$\theta = \theta_0 \, (1 - \epsilon^{-\beta t}) \tag{2}$$

Differentiating equation (**1**) with respect to time, we find that when $t = 0$ the time constant β is

$$\beta = \frac{\text{initial rate of cooling}}{\text{initial temperature rise}}$$

INITIAL RATE OF COOLING

The "initial rate of cooling" depends upon the thermal capacity of the body being cooled. Transformer windings consist mainly of copper and fibrous insulation. The thermal capacity or energy in joules required to raise the temperature of copper one deg. cent. equals the weight of copper times the number of grams in one pound times the specific heat of copper times the number of joules in one calorie $= W \times 453.6 \times 0.0935 \times 4.185 = 177.5\, W$, where W is the weight in pounds. The rate of heat storage in copper which is the same as the "initial rate of cooling" in degrees per minute is $\frac{60 \times \text{watts}}{177.5\ W} = 0.338\ W_c$ where W_c is the watts per pound of copper.

The thermal capacity of most insulating materials by *volume* ranges from about $\frac{1}{3}$ to $\frac{1}{2}$ that of copper. Tests indicate that for most impregnated insulations the value of $\frac{1}{2}$ is more nearly correct. For an insulated copper conductor or coil then, the

initial rate of cooling $= 0.338\ W_c \frac{2_a}{A + a} = \frac{0.676\ a\ W_c}{A + a}$ in which

a = the cross-sectional area of the copper and
A = the cross-sectional area of the copper plus the insulation.

We now have

$$\beta = \frac{0.676 \frac{a\ W_c}{A + a}}{\theta_0}$$

The above expression, of course, does not hold when the average temperature of the insulation is considerably lower than the temperature of the copper. This condition, however, seldom if ever exists except probably for a few end turns (exposed to line voltage) in the windings of a transformer.

Initial Temperature Rise

It has been shown* that the temperature rise (same as "initial temperature rise" when considering cooling curves following constant conditions) of a tank surface in a gas (air) cooled by convection varies as the 0.8 power of the loss through a limited range of temperatures. This, or a similar law, seems to be universally true for all types of oil-immersed windings, although the exponential value depends to some extent upon the thicknesses of insulation. The reason for this is that the temperature drop from copper to oil is composed of two components,— surface drop and drop through the insulation, the former of which changes more rapidly with a change in temperature than the latter. (Hereafter in referring to temperature drop through insulation, it will be expressed in deg. cent. drop when there is a flow of one watt per sq. in. through a distance of one inch of material. This is generally written "deg. per watt per in.^{3}"

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*Trans. A. I. E. E., 1916. Vol. XXXV, Part I, p. 599.

when considering the temperature drop through a definite thickness and "deg. per watt per in.^{2}" when considering the temperature drop of a surface.)

Referring to Fig. 1, it will be noted that as the temperature increases from approximately 33 deg. in lower curve to 75 deg. cent. in upper curve the surface drop from insulation to oil decreases from 6.7 deg. to 4.7 deg. per watt per in.^{2}. Practically the same decrease is noticed in the surface drop from copper to insulation, while the drop through the insulation, for a change in average temperature from about 38 deg. to 110 deg. cent., is only decreased from 274 deg. to 246 deg. per watt per in.^{3}. The thickness of this insulation, 72 mils (0.183 cm.) is considerably greater than that used on most transformer windings, so that this large drop through the insulation does not necessarily represent conditions as found in practise. The insulation, composed of 0.012-in. varnished-cambric strips one-in. in width, wound on butt joint, was made of sufficient

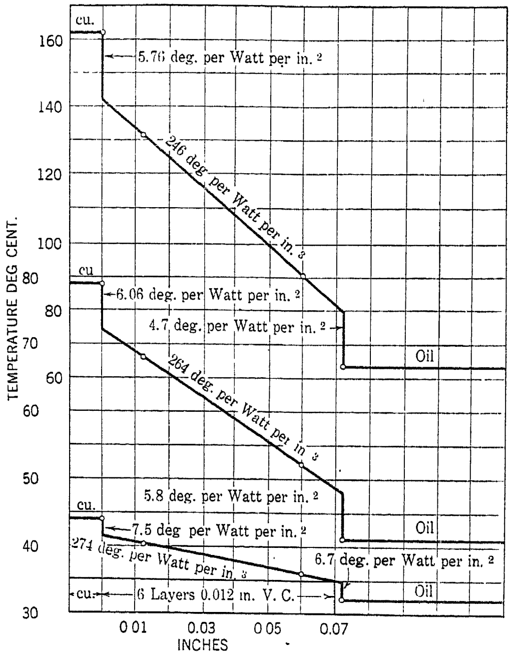

FIG. 1—SURFACE AND INSULATION TEMPERATURE DROPS

thickness to obtain an appreciable temperature drop. By drawing a straight line through two internal temperature points, obtained by thermocouples, the inner and outer surface temperatures were determined.

It is evident, therefore, that the value of the exponent in the equation of *temperature rise* vs. *loss*, will be less when the greater portion of the total drop is a surface drop than when the greater portion is a drop through insulation.

Fig. 2 shows the value of the exponent (slope of line) to be 0.7 when the suface of a coil is bare, and 0.9 when covered with six layers of 0.012-in. varnished cambric, and also 0.9 with nine layers of varnished cambric. It is shown later that the value of the exponent is approximately 0.7 when a coil has a light insulation such as is generally used in practise.

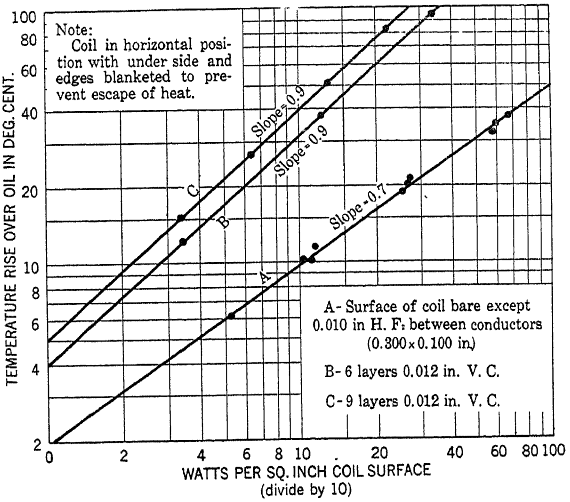

FIG. 2—TEMPERATURE RISE OF OIL-IMMERSED WINDING

It has been shown by Dr. Langmuir* that the "convection" of heat from a surface in a gas consists essentially of conduction through a film of stationary gas 0.177 in. (0.045 cm.) over the surface. A stationary film no doubt exists when heat energy is transferred from a surface to liquids. Very little, however, is known about this film, except that it exists, and for this reason

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*Physical Review May, 1912.
Am. Electro Chem. Soc. April, 1913.

we have to depend upon experimental observations for determining its thickness and its effect on the temperature rise.

THICKNESS OF OIL FILM

The best data at hand indicate that the thermal resistivity of transformer oil is approximately 250 deg. cent. per watt per in.^{3}. Accepting that this figure is correct (which may or may not be very accurate) according to the surface drops in Fig. 1, (approximately 6 deg. per watt per in.^{2}.) the indications are that the thickness of the stationary film of oil is $6/250 = 0.024$ in. (0.061 cm.) This value, however, should be considered as merely approximate, since it may vary somewhat for different conditions. Further work is required along this line to establish more accurate data.

THERMAL RESISTIVITY OF INSULATION

According to Fig. 1, the average thermal resistivity of the insulation is 261 deg. per watt per in.^{3}. However, the thermal resistivity of insulation as applied to electrical apparatus is by no means a constant value but may vary anywhere from about 200 deg. cent. for solid to 500 deg. cent. per watt per in.^{3}. for loosely wound insulation containing oil or air spaces between layers. Generally speaking, about 250 deg. per watt per in.^{3}. seems to be the value most commonly found* in practise for compact insulation coverings.

RATE OF COOLING AS AFFECTED BY THERMAL CAPACITY AND BY INITIAL TEMPERATURE RISE

It is obvious from the form of equation (2) that for a given initial temperature rise θ_0, for a given mass of copper and a given loss, the greater the mass of insulation, the slower will be the rate of cooling. However, it should be *remembered* that in order to have the same initial temperature rise when insulation is increased, on, say a transformer winding, it is necessary that the watts per unit area exposed to the cooling medium be decreased sufficiently to compensate for the increase in temperature due to the added insulation. Otherwise an increased temperature rise will result.

Insulation is usually used in transformer windings, in two ways; (1) by placing it on individual strands or conductors,

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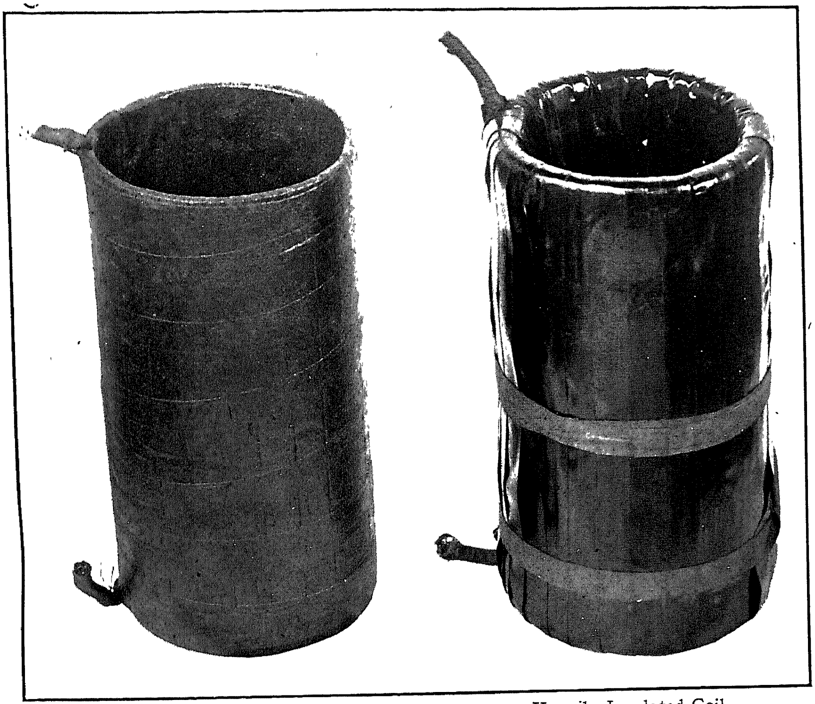
*See paper and discussion on Heat Paths in Elec Machinery, Symons and Walker, British Inst. of E. E., November, 1911.



Lightly Insulated Coil Heavily Insulated Coil

[MONTSINGER]

FIG. 3—COILS USED IN DETERMINING EFFECT OF COIL INSULATION ON COOLING AFTER SHUT-DOWN

See Figs. 4 and 5 for Cooling Curves

and (2) by placing it on individual coils. When the conductor insulation is increased, although the watts per unit area of surface are decreased, the temperature rise is generally increased. When the coil insulation is increased, the surface loss is not usually affected and hence it follows that the temperature rise is always increased.

On the whole, it seems fair to assume that when insulation is added to transformer windings, the surface loss per per unit area is not usually reduced sufficiently to neutralize the increased drop established in the insulation and an increased temperature rise results. Accepting this assumption, it is obvious that when insulation is added to a winding, (although the rate of cooling immediately after shut-down is decreased), there must be a time when not only the *rate* but the *actual cooling* in degrees of this winding is greater than that for a winding with a lesser amount of insulation. This same effect is had when a transformer has very complicated oil ducts of such a nature to restrict the oil circulation and cause an increase in the temperature rise. For instance, it is obvious that for a given loss per lb. of copper, any factor which makes it more difficult for the coil to cool, and hence causes an increase in the temperature rise, also causes the rate of cooling to be

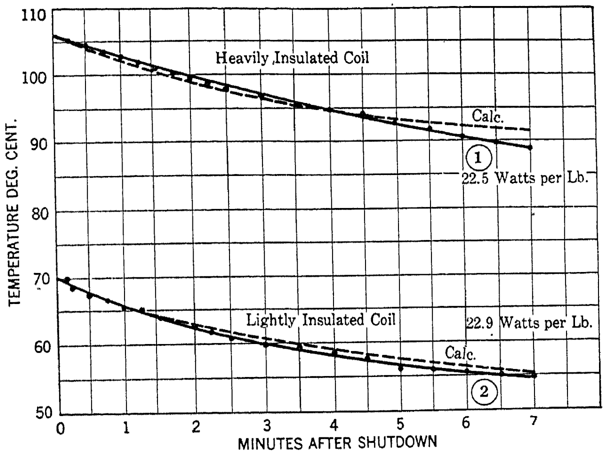


FIG. 4—COOLING OF OIL-IMMERSED WINDINGS SHOWN IN FIG. 3 (AVERAGE OIL APPROXIMATELY 55 DEG. CENT.)

less for a short time after shut-down. The duration of this time seems to be from one to three minutes. Finally, it may be stated as follows:

"*Any factor or set of factors which causes an increase in temperature rise for constant conditions, also causes a decrease in the rate of cooling for a short time after shut-down.*"

Of course, after a time, the cooling necessarily becomes greater.

For example, the effect of increasing the insulation on a coil (where the surface loss is not changed) on its rate of cooling is very well illustrated in Figs. 4 and 5. These curves give the cooling of two coils under identical conditions, excepting that one had heavy insulation and the other light insulation. Both coils wound on a foundation ring or cylinder, 40 mils in thickness and consisted of 0.2 × 0.055-in. edge-wound conductor with a 10.5-mil cotton covering, and over both coil surfaces were wound 15 mils of binding tape. Fifteen to eighteen layers of 0.012 in. v.c. was then wrapped on both the inside and outside surface of one coil, while the other coil was given no extra insulation (see Fig. 3). We might designate the insulation on the coils as "heavy" and "light". According to curves in Figs. 4 and 5 their cooling is as follows:

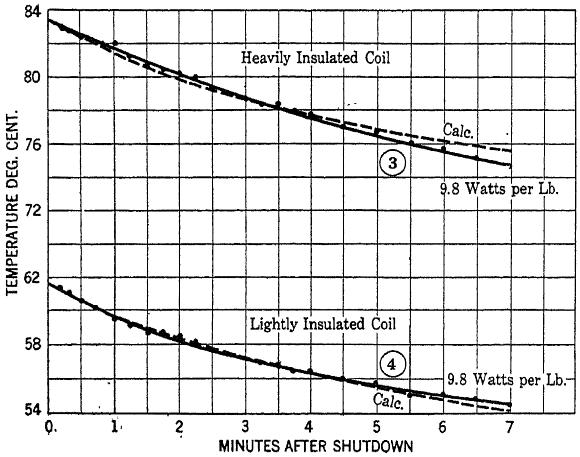


FIG. 5—COOLING OF OIL-IMMERSED WINDINGS SHOWN IN FIG. 3 (AVERAGE OIL APPROXIMATELY 50 DEG. CENT.)

Insulation on coil	Watts per lb. cu.	Approx. initial temp. rise	Cooling in deg cent.						
			1	2	3	4	5	6	7 min.
Heavy......	22.5	56°	3.6	7.2	10.3	12.9	15.2	17.3	19.2
Light.......	22.9	20°	4.9	8.1	10.8	12.8	14.1	15.3	16.0
Heavy......	9.8	33.5°	1.8	2.4	4.6	5.8	6.9	7.9	8.8
Light.......	9.8	11.5°	2.0	2.4	4.5	5.3	6.1	6.7	7.2

The above shows that the cooling of the heavily insulated coil is at first less and then finally becomes greater than the cooling of the lightly insulated coil. The cooling of both coils, however, is approximately the same until four or five minutes after shut-down. The amount of insulation on the heavily insulated coil is somewhat exaggerated as compared with practise, but it illustrates the point in mind.

DERIVATION OF PRACTICAL FORMULA FOR CORRECTING BACK TO SHUT-DOWN

The question, what is the rate of cooling for various watts per pound of copper, naturally arises. This can be calculated by the use of equation (2) providing the initial temperature rise and the thermal capacity of the mass is given for a typical coil. As stated before, the true initial rise of windings over oil is difficult to obtain in an actual transformer, but can be determined for a single coil operating under a condition that the surrounding oil temperature can be observed. According to the above reasoning, the cooling of this coil for a given loss and for a limited time should be approximately the same when operating singly as when operating in a group of coils.

In Fig. 6 is shown on logarithmic paper the temperature rise for various watts per square inch coil surface obtained by tests for constant conditions on a single transformer coil immersed in oil. A diagram is given showing the size of conductor (used in the coil) and its insulation. This line of temperature rise vs. loss, shows that the temperature rise varies as the 0.705 power of the loss. If we let θ_0 equal the temperature rise for the above referred to coil, the equation of the line is

$$\theta_0 = K\,W_s^{0.705} \qquad (3)$$

in which

K = constant = 5.2

W_s = watts per sq. in. of surface (divide by 10).

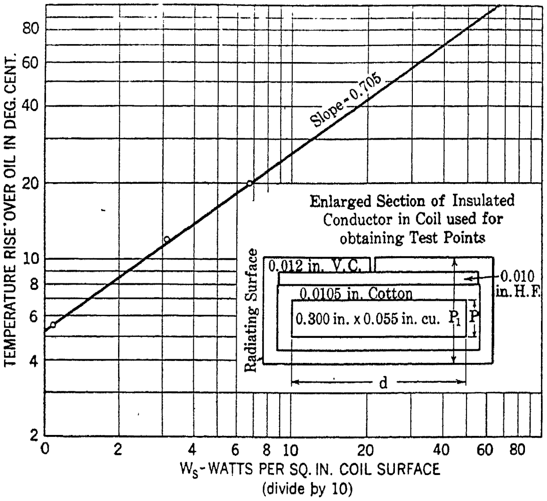
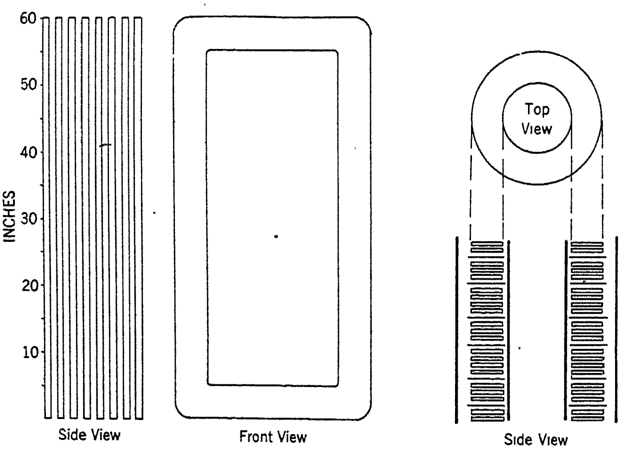


FIG. 6—TEMPERATURE RISE OF OIL-IMMERSED WINDING



RECTANGULAR COILS DISK COILS

FIG. 7—SKETCH OF WINDINGS USED FOR COOLING CURVES IN FIGS. 8 AND 9

At 25 deg. cent. the surface loss of a coil with rectangular or square conductors is expressed by

$$W_s = 3.47\, C^2\, d\, n\, s\, 10^{-7} \qquad \textbf{(4)}$$

in which

W_s = $R I^2$ watts per sq. in. surface (neglecting edges of coil but including conductor insulation).

C = current density in amperes per sq. in.

d = depth of bare conductor in inches, in direction of flow of heat.

n = number of conductors in direction of flow of heat.

s = conductor space factor at right angles to flow of heat.

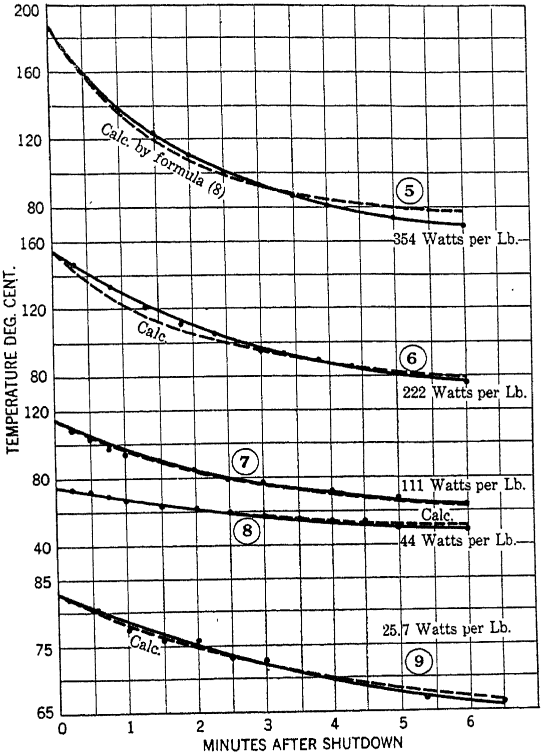


FIG. 8—COOLING OF OIL-IMMERSED DISK WINDINGS SHOWN IN FIG. 7

For coils with round conductors, the value of W_s in (4) is decreased in the ratio of 4 to π, or

$$W_s = 2.725 \times 10^{-7} C^2 d n s \tag{5}$$

At 25 deg. cent. the $R I^2$ watts per lb. of bare copper (W_c) is

$$W_c = 2.16 C^2 10^{-6} \tag{6}$$

Combining equations (**3**), (**4**) and (**6**), we have

$$\theta_0 = 7.27\ (W_c\, d\, n\, s)^{0.705} \tag{7}$$

Substituting the above value of θ_0 in equation (**2**)

$$\theta = 7.27\ (W_c\, d\, n\, s)^{0.705}\left(1 - \epsilon^{-\frac{0.0928\,\frac{a\,W_c\,t}{A+a}}{(W_c\,d\,n\,s)\ 0.705}}\right)$$

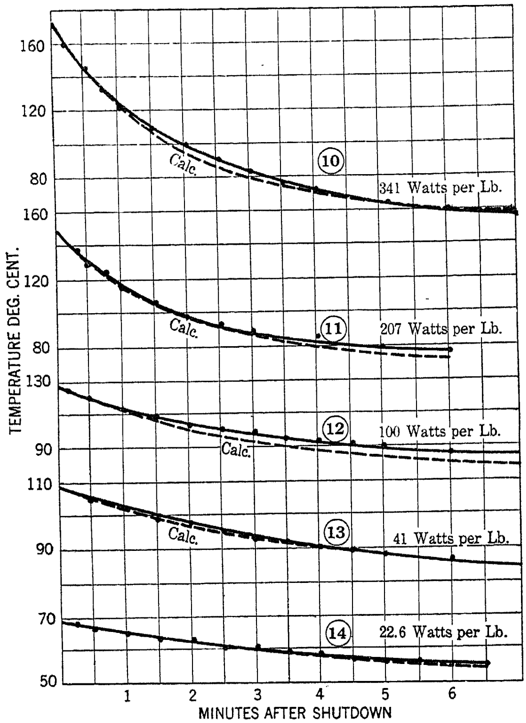


FIG. 9—COOLING OF OIL-IMMERSED RECTANGULAR WINDINGS SHOWN IN FIG. 7

Putting (from dimensions of conductor in Fig. 6)

d = 0.3
n = 1
s = 0.5
a = 0.0165
A = 0.038

and changing 0.705 to 0.7 and increasing the value 7.27 to 7.35 (to compensate for reduction of 0.705 to 0.7), we have

$$\theta = 1.95\, W_c^{0.7} \left(1 - \epsilon^{-0.106\, W_c^{0.3} t}\right) \qquad (8)$$

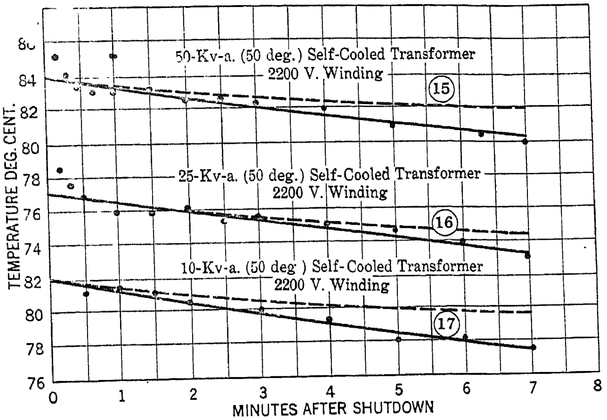


FIG. 10—COOLING OF DISTRIBUTING TRANSFORMERS

in which

θ = cooling in deg. cent.

W_c = watts per lb. of bare copper.

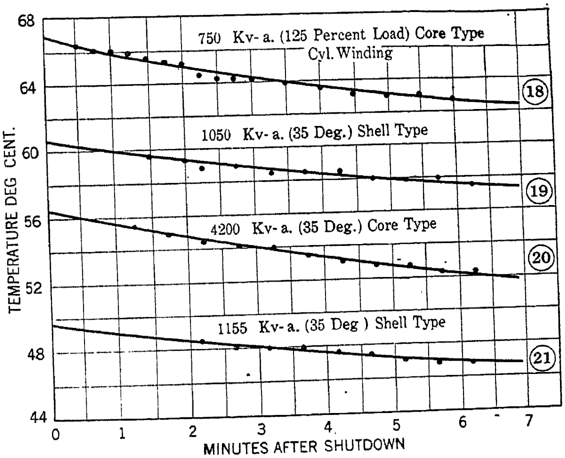


FIG. 11—COOLING OF AIR-BLAST TRANSFORMERS (AIR SHUT OFF)

at 75 deg. cent., $W_c = 2.577\, C^2 \left(1 + \frac{e}{100}\right) 10^{-6}$

e = eddy current loss in per cent of $R\, I^2$.

t = time in minutes after shut-down.

In Fig. 13 are plotted, from equation (8), the degrees cooling vs. different values of W_c for 1, 2, 3, 4, 5, 6 and 7 minutes after shut-down.

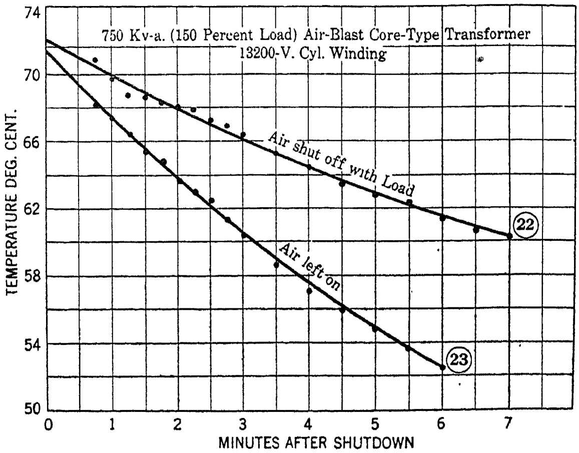


FIG. 12—COOLING OF AIR-BLAST TRANSFORMER

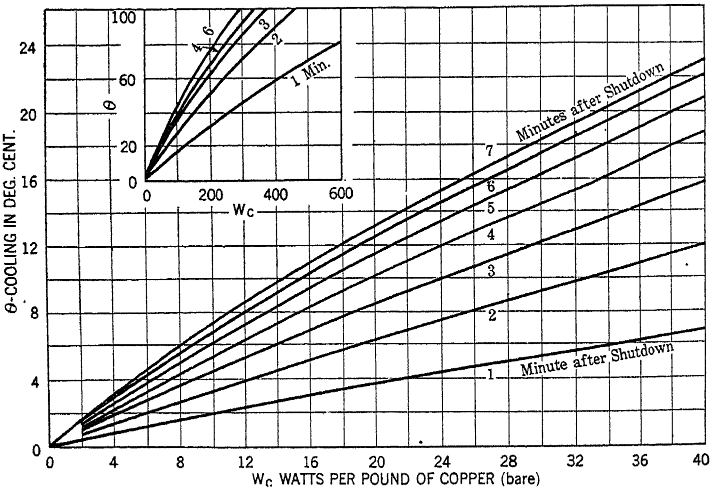


FIG. 13—COOLING OF OIL-IMMERSED WINDINGS—CALCULATED BY FORMULA

Table 1 gives the variation in degrees centigrade (+ or −) that these curves differ from the cooling found by tests, on various types of oil-immersed windings.

TABLE I.

COMPARISON OF COOLING BY TEST OF OIL-IMMERSED WINDINGS WITH COOLING CALCULATED BY FORMULA (8).

Self-or water-cooled	Kv-a.	Style of winding	Deg. cent.			
			Variation of test from calculated cooling			
			2 minutes after shut-down		4 minutes after shut-down	
			+	—	+	—
Self	200	Cyl.	0.5		0.8	
"	400	Disk		0.5		0.7
"	1000	Cyl.	0	0	0	0
"	750	Disk	0.5		0.8	
Water	3000	Disk	0		0.2	
"	2500	Cyl.		0.5		0.9
Self	200	Cyl.	0.8		1.1	
"	433	Disk	0.5		0.4	
"	400	Disk		0.7		0.8
"	135	Cyl.	0.1		0.1	
"	750	Disk	0	0	0	0
Water	450	Disk		0.5		0.4
"	2000	Disk	0.0		0.9	
Self	300	Cyl.	0.2		0.6	
"	300	Disk	0.4		1.0	
Water	1000	Disk		0.7		0.8
"	900	Cyl.		0.2		0.8
"	750	Disk	0.6		0.7	
"	750	Disk		0.2		0
Self	1000	Cyl.	0.3		0.6	
"	750	Cyl.		0.5	0.1	
Water	5500	Rectangular	0	0	0.5	
"	5500	"	0.2		0.3	
"	6000	"	0	0	0.5	
"	6000	"	0.3		0.5	
Self	46	Regulator	0.5		0.8	
"	62.5	"		0.1		0.2
"	8.6	"	0.3		0.7	
"	17.25	"		0.3		0.4
"	62.5	"	0	0	0.7	
"	8.6	"	0.2		0.9	
"	46	"	0.4		0.7	

GENERAL CONCLUSIONS

Regarding the three general methods of determining the temperature at the instant of shut-down, as mentioned in the "Abstract", the following may be said of each:

1. *Cooling-Curve Method.* Practise has shown that generally this method is fairly reliable. In some cases, however, it is liable to error, due to the difficulty of being able to extrapolate back correctly. This is true when considerable time is required in obtaining the initial readings or when these first readings are

influenced by inductance in the windings. At any rate, commercially, this method is not always practical due to the increased time required.

2. *Arbitrary Correction.* The advantages of this method are simplicity and ease of application. The disadvantage is that it is liable to error. Generally speaking, for maximum rated water-cooled transformers, the copper loss will vary from about 6 to 25 watts per lb. If an arbitrary correction of say 2 deg. cent. per minute is adopted, the following errors (plus or minus) are liable to be had for this particular type of apparatus.

Minutes after shut-down	Cooling in deg. cent. by curves in Fig. 13		Cooling by arbitrary correction of 2 deg. cent per minute	Max. error
	6 watts per lb.	25 watts per lb.		
2	2.0 deg.	8.0 deg.	4 deg.	4 deg.
4	3.5 deg.	12.5 deg.	8 deg.	4.5 deg.

In cases where machines operate on short-time over-loads, or on duty-cycle operation, the error would naturally be greater than the above.

Again this method might put a premium on using high current densities.

Providing the time does not exceed four minutes after the instant of shut-down, the following, however, seems to be about the general average rates of cooling for

(a) Air-blast transformers (air shut off)........	$\frac{1}{2}$ deg. cent. per min.
(b) Distributing transformers (100 kv-a. and less)	$\frac{1}{2}$ " " " "
(c) Self-cooled transformers (except (b))......	$1\frac{1}{2}$ " " " "
(d) Water-cooled transformers................	2 " " " "

3. *Calculation of Cooling.* (By formula (8) or by curves in Fig. 13). The advantage of this method is that it is fairly reliable and simple in application. In order to apply the formula, it is necessary only to know the watts ($R\ I^2$ + eddy loss) per lb. of copper.

The disadvantage of this is that in order to apply the formula or curves (Fig. 13) plotted from the formula, the current density and eddy loss must be known, which cannot be determined by test.

APPENDIX

A.—COOLING AFTER SHUT-DOWN WHEN LOSS OF HEAT IS PROPORTIONAL TO TEMPERATURE RISE

Assuming that the cooling is proportional to temperature rise, the following equation results

$$\frac{d\,\theta}{d\,t} = \beta\,\theta$$

The solution of this is

$$\theta\,t = \theta_0\,\epsilon^{-\beta\,t} \tag{1}$$

where

θ_t = temperature rise at any time t after shut-down.
θ_0 = temperature rise at shut-down.
β = time constant.

At the time of shut-down the rate of cooling in deg. cent. per minute of copper is 0.338 W_c, where W_c is the watts per pound. Differentiating (**1**).

$$\frac{d\,\theta_t}{d\,t} = \beta\,\theta_0\,\epsilon^{-\beta\,t} \quad \text{When } t = o, \qquad \epsilon^{-\beta\,t} = 1$$

$$\frac{d\,\theta_t}{d\,t} = \beta\,\theta_0 = 0.338\;W_c$$

or

$$\beta = \frac{0.338\;W_c}{\theta_0} = \frac{\text{initial rate of cooling}}{\text{initial temperature rise}}.$$

B.—COOLING AFTER SHUT-DOWN WHEN LOSS OF HEAT IS NOT PROPORTIONAL TO TEMPERATURE RISE

For this condition we have

$$\frac{d\,\theta}{d\,t} = \beta\,\theta^n$$

$$\int_{\theta_t}^{\theta_0} \frac{d\,\theta}{\theta^n} = \beta \int_0^t d\,t$$

$$\frac{\theta_0^{-n+1}}{1-n} - \frac{\theta_t^{-n+1}}{1-n} = \beta\,t + C$$

when $t = 0,\ \theta_0 = \theta_t\ \&\ C = 0$

$$\theta_t^{-n+1} = \theta_0^{-n+1} - \beta\,t\,(1-n)$$

Assuming that $n = \frac{1}{0.7} = 1.43$

$$\theta_t = \left[\frac{1}{\left(\frac{1}{\theta_0}\right)^{.43} + 0.43\;\beta\,t}\right]^{2.33} \tag{3}$$

C.—SURFACE LOSS OF COIL WITH RECTANGULAR (OR SQUARE) CONDUCTOR

Let W_s = Watts per sq. in. surface, (both sides).
d = Depth of conductor in inches (bare).
n = Number of conductors deep.
s = Space factor $= \frac{P}{P_1}$
P = Width of conductor (bare) in inches at right angles to flow of heat.
P_1 = Width of conductor (insulated) in inches.
I = Current in amperes.
C = Current density in amperes per sq. in. $= \frac{I}{d\,P}$
l = length of conductor.

Now

$$W_s = \frac{\text{Loss in watts} \times s}{\text{Area of coil in sq. in.}}$$

$$= \frac{R I^2 \times s}{2\,P\,l}$$

Based on 0.6935 microhm per in. cube at 25 deg. cent., resistance of copper (conductor) $= \frac{0.008322}{d\,p}$ ohms per 1000 ft. length.

Then

$$W_s = \frac{\dfrac{0.008322\ I^2\ s}{d\,p}}{\dfrac{2p\ 12000}{n}}$$

Multiplying and dividing by d

$$W_s = \frac{3.47 \times 10^{-7}\ I^2\,d\,n\,s}{d^2\,p^2}$$

$$= 3.47 \times 10^{-7}\ C^2\,d\,n\,s \tag{4}$$

D.—WATTS PER POUND OF COPPER

The weight in pounds of copper per 1000 ft.
= 3858 × cross section in sq. in. = 3858 $d\,p$.

$W_c = \frac{R\,I^2}{3858\,d\,p}$ where R = resistance in ohms per 1000 ft.

$$W_c = \frac{\dfrac{0.008322\,I^2}{d\,p}}{3858\ d\,p}$$

$$= 2.16 \times 10^{-6}\ C^2 \tag{5}$$

DISCUSSION ON "COOLING OF OIL-IMMERSED TRANSFORMER WINDINGS AFTER SHUT-DOWN" (MONTSINGER), NEW YORK, JUNE 28, 1917.

A. S. McAllister: The author speaks of the difficulty of obtaining current readings when attempting to measure the resistance of the windings shortly after the transformer cools down, on account of the presence of the magnetic circuit. With any given voltage impressed on the coil the current will reach 63 per cent of its maximum value in a time equal to the time-constant of the circuit. If the voltage at the end of that time, which necessarily is very short, be immediately reduced from the initial value to 63 per cent of the initial value, the current will be immediately steadied. I do not see why one cannot adjust the impressed voltage to give, say approximately 10 amperes, and when the current reaches say 7 amperes, reduce the voltage to seven-tenths value, thereby eliminating all delay. There must be some reason why that is not done.

V. M. Montsinger: That would necessarily involve a little more trouble in obtaining the resistance measurements than if we simply brought the current up to the 100 per cent point and let it become steady of its own account. I have never tried to see whether it would reduce the initial inductance if we brought the current up to 100 per cent immediately and then reduced it to exactly 63 per cent. In some cases, however, where the initial inductance was high, I have had the current brought up as soon as possible to some value above the chosen value for measurement and then reduced to this chosen value, but in these cases the inductance was so high that it took from one-half to one minute for the current to build up to say two-thirds of the final value, so that it does not seem that this method offers a complete solution of this trouble. I have found generally that it is an easier matter to close the switch and leave the current on about one minute before taking the reading and then correct back to shut-down time by the formula. Unless the windings are for very heavy current, with the time required for disconnecting and connecting on the resistance leads and allowing the one minute for the current to become steady, the resistance can be observed in about two minutes. In other cases, however, it will require considerably more than two minutes. By following this routine we are always sure that the measurement is accurate so far as the current value is concerned, and for this reason we do not attempt to hurry it along, but let it become constant of its own accord.

F. F. Brand: Mr. Montsinger's statement that transformer windings in oil do cool appreciably after shut-down is a very important fact, both for the manufacturer and purchaser. Very fortunately, Mr. Montsinger has developed a method which is very simple for the manufacturer, at least, to apply. It is a fact that in modern transformers, particularly those of the artificially-cooled type, the windings usually operate at a fairly high current density, and therefore if it takes one or two or three minutes to get an accurate reading of the resistance after shut-down, then the amount of cooling which the winding will go through in this time is quite appreciable, it may be five or ten degrees. If no correction were made, and a transformer gave a reading after two minutes which corresponded to 55 deg. rise, then actually that transformer, because it has cooled 5 deg. cent., is a 60 deg. transformer; that is, it is operating at a higher rise than the Rules of the Institute would allow, and furthermore, such a 60 deg. transformer should be 4 or 5 per cent cheaper than a 55 deg. transformer.

The designer of the transformer, as well as the purchaser is interested in getting the true operating temperature at the time of shut-down, and I think the Institute Rules are at present somewhat vague in specifying the method of correcting 'back, and not making the correction back to shut-down mandatory by some approved method. Personally, I should like to see this method of Mr. Montsinger's given in Fig. 13 approved by the Institute, and I would like to see the correction back to shut-down, after the load is taken off, made mandatory in the Rules by some approved method.

A. S. McAllister: Assuming that the cooling curve is logarithmic, the time-constant of that curve can be found from any section of the curve whatsoever. Taking any point as a starting point, follow down the curve to a point whose ordinate has a height which is 63 per cent of the height of the ordinate of the starting point. The distance between the two points expressed in time is the time-constant of that curve. The actual value is

$$1 - \frac{1}{\epsilon} = 1 - \frac{1}{2.7183} = 0.632$$

L. F. Blume (by letter): As increasing emphasis is being laid upon the influence of temperature and temperature rise in the determination of the rating of electrical machinery, Mr. Montsinger's paper showing that an error of 10 per cent or more may be introduced in the determination of temperature rise if care is not exercised in taking the temperature at the instant of shut-down, is worthy of careful consideration. Since in some instances it is very difficult to obtain an accurate cooling curve and extrapolate back to the instant of shut-down, a formula by which the shut-down temperature can be derived from an observed temperature at a definite time after shut-down, is very desirable.

The difficulty with the theoretical law of cooling (see formula 2), is that it is expressed in terms of the unknown factor, the initial temperature rise of the winding above the surrounding oil temperature. This rise is unknown, not only because the initial observable temperature is unknown, but also because the temperature of the cooling oil surrounding the winding is also unknown. However, by the use of formula 7, Mr. Montsinger expresses the initial temperature rise above oil in terms of the constant of the coil, that is, in terms of pounds of copper and the dimensions of the winding. This is quite permissible in view of the fact that equation 2 is a formula for obtaining a correction to an observed temperature, and considerable approximation may be introduced without materially affecting the accuracy of the final results. It is on this account that Mr. Montsinger can introduce into the formula the specific dimensions of an average transformer coil, thereby deriving formula 8, and apply this formula without great error to all kinds of oil-immersed transformer windings.

The only objection to using exclusively the method proposed by Mr. Montsinger, is the fact that its use requires a knowledge of the watts per pound of copper lost in the windings. From the point of view of the manufacturer, this offers no difficulty, but to the purchaser who wishes to determine by means of test, the operating characteristics of the transformer he is buying, it is objectionable, because this value cannot be obtained by test. In the latter instance, of course, the only resource would be the extrapolation method.

It is evident, therefore, that both the extrapolation method and the semi-theoretical method given by Mr. Montsinger will find useful application in practise, and it is to be hoped that the Institute will see fit to recognize both of them as acceptable methods of determining temperature at the instant of shut-down.

Walter C. Smith: The correction which Mr. Montsinger has derived, while, approximate, gives sufficiently accurate results for all practical purposes, and it is easy of application. He has pointed out that the arbitrary correction method leads to considerable error. Now, that is true, and particularly where high current densities are employed, such as in water-cooled transformers; it also naturally follows that an arbitrary correction leads to error where the elapsed time is considerable. In the case of distributing transformers, say 200 kv-a. and below, we rarely have either of these conditions. In the first place, we do not use the extra high current densities, and secondly, it is easy to obtain the readings within one or two minutes at the most. I, therefore, feel that the Standards Committee have very wisely chosen the arbitrary correction for distributing transformers in formulating the present rules. The arbitrary correction is much simpler of application and is very accurate, when the reading is taken a minute or two after shut-down.

Referring to Dr. McAllister's remarks, I will point out that while I believe his suggestion would no doubt prove of value, he should consider the fact that even though he cuts the time of reading the instruments down to practically zero, still it has taken a minute or two to make these connections on large transformers with heavy windings. Only those who have tried to accomplish this really appreciate how long it does take to disconnect the heavy leads, put on the resistance connections and take the proper readings. A minute or two on a high current density water-cooled transformer means several degrees drop.

V. M. Montsinger: Replying to Mr. McAllister's point regarding the constant in the logarithmic curve. I agree with him that if we determine two points on the curve, as it decreases, that enables us to determine the time constant. However, in order to do that, it is necessary to take at least two points of a cooling curve, and if we should take two points, generally, it would not be so very much more trouble to take a few more points and extrapolate back to shut-down and use the cooling-curve method.

There is one more point that I did not bring out in the presentation, which I shall mention here. If we plot the "rate of cooling" as the ordinates on logarithmic scale, vs. "time," divided into equal divisions (semi-logarithmic paper) and the base temperature does not change appreciably during the time of cooling, the points, if observed correctly, fall in a straight line. This method enables us to determine more accurately the true initial temperature than by plotting the "cooling in degrees" vs. "time" on co-ordinate paper. This is for the reason that if the first few points are erratic or high on account of inductance in the windings, it is almost impossible to extrapolate back correctly on co-ordinate paper, whereas in using the semi-logarithmic paper, if we have a few correct points, that is, enough to enable us to draw a straight line through them, it is an easy matter to extrapolate back.

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TRANSMISSION LINE DESIGN

BY F. K. KIRSTEN

ABSTRACT OF PAPER

This paper contains a complete mathematical analysis of the forces which determine the location in space of a cable suspended from points of equal elevation, and gives the designer and constructing engineer of a transmission line some useful working formulas based on this analysis.

Section A of the paper covers the derivation of the catenary formulas and contains a chart from which any catenary problem may be quickly and accurately solved.

Section B contains an analysis of the influences of changes in temperature and cable load, resulting in formulas by the aid of which the magnitude of these influences in regard to changes in position of the cable and changes in stresses at any point of the cable may be accurately computed. The formulas derived make it possible to plot a temperature-tension stringing chart to be used by the constructing engineer when stringing the cable at various temperatures.

Section C is an investigation of the economic features involved in the proper design of the cable supports. A working formula is derived enabling the designer to determine the most economical span and corresponding height of tower.

Solutions of typical design problems are given in each section of the paper demonstrating the use and manipulation of all formulas derived.

THE SPAN BETWEEN POINTS OF EQUAL ELEVATION

(*A*). MATHEMATICAL ANALYSIS OF THE PREREQUISITES FOR EQUILIBRIUM OF FORCES ACTING ON A FLEXIBLE CABLE ONLY SUPPORTED AT TWO POINTS OF THE SAME ELEVATION

The curve $P_1\ O\ P_2$ in Fig. 1 represents a cable suspended from points P_1 and P_2 of equal elevation.

Assumptions:

1. The suspended cable is a cylindrical solid with all elements of outer surface parallel.
2. The suspended cable is of uniform texture.
3. The suspended cable is perfectly flexible, that is, the internal forces acting on every element of any cross-sectional plane normal to these forces are equal in magnitude and their lines of action parallel to the axis of the cable.

4. No other external force besides the force of gravitation to be the active force on the cable.

5. The axis of the cable will assume the form of a curve $P_1\,O\,P_2$ in a vertical plane.

Since the active gravitational forces are proportional to mass which is uniformly distributed along the axis of the cable, and since the two points of support are on equal elevation, there must exist a condition of symmetry of the shape of the curve $P_1\,O\,P_2$ with respect to a vertical plane midway between points P_1 and P_2 perpendicular to the straight line $P_1\,P_2$. This plane will be selected as the reference plane Y-Y. The point of maximum deflection of the cable from the straight line $P_1\,P_2$ must lie in this plane. A horizontal plane tangent to the curve $P_1\,O\,P_2$ at the point of maximum deflection is chosen as the reference plane X-X. Thus, the point of maximum deflection of the cable is defined as the origin for the rectangular coordinates x and y.

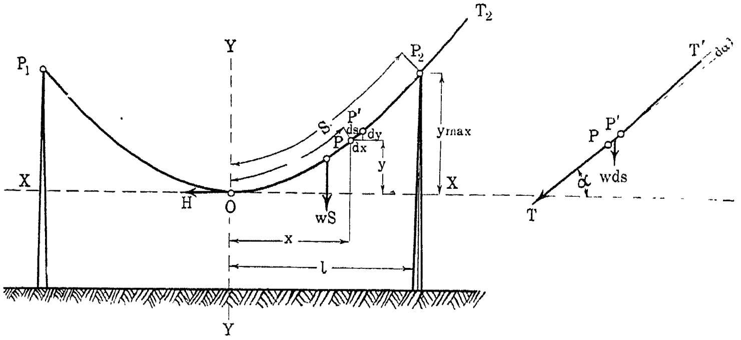

FIG. 1

Definitions with reference to Fig. 1

l = half distance between points P_1 and P_2 or half tower spacing.

S = half length of suspended cable or half length of span.

w = weight per unit length of suspended cable

x and y are rectangular coordinates of any point P on the axis of the cable.

s = length of curve between points O and P

ds, dx and dy are increments of s, x and y respectively, which at the limit zero will bear the relation $(dx)^2 + (dy)^2 = (ds)^2$.

H = tension on cable at point of maximum deflection.
T_2 = tension on cable at point of support P_2.
T = tension on cable at point P, coordinates x and y.
T' = tension on cable at point P', coordinates $x + dx$ and $y + dy$.
α = angle between T and the X axis.
$d\alpha$ = increment of α or angle between T and T'.

All stresses normal to the cross-sectional plane at any point of the cable are considered as concentrated on the axis of the cable and by assumption 3 act in the direction of the tangent to the axis at that point.

INVESTIGATION

For equilibrium of half span (see Fig. 1).

$$T_2^2 = H^2 + w^2 S^2$$

If the point of support were moved to P without disturbing equilibrium,

$$T^2 = H^2 + w^2 s^2 \tag{1}$$

For equilibrium of length ds between points P and P' (see Fig. 1),

$$T \cos \alpha = T' \cos (\alpha + d\,\alpha) \tag{2}$$

$$T \sin \alpha + w\, ds = T' \sin (\alpha + d\,\alpha) \tag{3}$$

At point P',

$$(T')^2 = H^2 + w^2 (s + ds)^2 \tag{4}$$

From the geometric relation (right angle triangle) of forces given by equations (**1**) and (**4**), it is evident that

$$\cos \alpha = \frac{H}{T} \;;\; \sin \alpha = \frac{w\,s}{T} \;;\; \cos (\alpha + d\alpha) = \frac{dx}{ds} \;;$$

$$\sin (\alpha + d\alpha) = \frac{dy}{ds} \tag{5}$$

Substituting (**4**) and (**5**) into (**2**),

$$H = \left[H^2 + w^2 (s + ds)^2 \right]^{\frac{1}{2}} \frac{dx}{ds}$$

Letting $H = wc$, where c is a constant, and simplifying,

$$\frac{dx}{ds} = \frac{c}{[c^2 + (s + ds)^2]^{\frac{1}{2}}}$$

But if ds approaches zero as a limit, at this limit $s + ds = s$, therefore,

$$\frac{dx}{ds} = \frac{c}{[c^2 + s^2]^{\frac{1}{2}}}$$

By integration,

$$x = \int_0^s \frac{c\,ds}{[c^2 + s^2]^{\frac{1}{2}}} = c \left[_0^s \log_\epsilon (s + [c^2 + s^2]^{\frac{1}{2}})\right.$$

$$= c\,[\log_\epsilon(s + [c^2 + s^2]^{\frac{1}{2}}) - \log_\epsilon c]$$

$$= c \log_\epsilon \frac{s + [c^2 + s^2]^{\frac{1}{2}}}{c} \qquad \mathbf{(6)}$$

Equation (**6**) may be written

$$\epsilon^{\frac{x}{c}} = \frac{s + [c^2 + s^2]^{\frac{1}{2}}}{c}$$

Or,

$$s = \frac{c}{2}\left[\epsilon^{\frac{x}{c}} - \epsilon^{-\frac{x}{c}}\right] \qquad (7)$$

$$= c \sinh \frac{x}{c} \qquad (8)$$

And for entire half span:

$$S = c \sinh \frac{l}{c}$$

Substituting (**4**) and (**5**) into (**3**),

$$ws + wds = [H^2 + w^2 (s + ds)^2]^{\frac{1}{2}} \frac{dy}{ds}$$

Letting $H = wc$, where c is a constant, and simplifying,

$$\frac{dy}{ds} = \frac{s + ds}{[c^2 + (s + ds)^2]^{\frac{1}{2}}}$$

But if ds approaches zero as a limit, at this limit $s + ds = s$, therefore,

$$\frac{dy}{ds} = \frac{s}{[c^2 + s^2]^{\frac{1}{2}}}$$

By integration,

$$y = \int_0^s \frac{s\,ds}{[c^2 + s^2]^{\frac{1}{2}}} = \Big[_0^s (c^2 + s^2)^{\frac{1}{2}}$$
$$= [c^2 + s^2]^{\frac{1}{2}} - c \qquad (9)$$

Substituting (**7**) into (**9**) and simplifying,

$$y = \frac{c}{2}\left[\epsilon^{\frac{x}{c}} + \epsilon^{-\frac{x}{c}}\right] - c \qquad (10)$$

$$= c\left(\cosh\frac{x}{c} - 1\right) \qquad (11)$$

Equations (**10**) or (**11**) are the equations of the catenary.

From equation (**1**) the tension along the axis of the cable at any point P is

$$T = [H^2 + w^2 s^2]^{\frac{1}{2}}$$
$$= [c^2 w^2 + w^2 s^2]^{\frac{1}{2}}$$

Hence:

$$\frac{T}{w} = [c^2 + s^2]^{\frac{1}{2}}$$

By substitution from equation (**8**),

$$\frac{T}{w} = c \cosh\frac{x}{c} \qquad (12)$$

For maximum deflection of entire span and maximum tension at the point of support of the cable, half the tower spacing l must be substituted for x in equations (**11**) and (**12**), giving:

$$y_{max} = c\left(\cosh\frac{l}{c} - 1\right); \qquad \frac{T_{max.}}{w} = c\cosh\frac{l}{c}$$

For preliminary computations of span characteristics or for computations of the characteristics of relatively short spans when maximum precision is not essential, approximations of the above formulas have found universal use. These approximations are obtained as follows:

From Maclaurin's Theorem:

$$\epsilon^{\frac{x}{c}} = 1 + \frac{x}{c} + \frac{x^2}{c^2\,\underline{|2}} + \frac{x^3}{c^3\,\underline{|3}} + \frac{x^4}{c^4\,\underline{|4}} + \cdots\cdots$$

$$\epsilon^{-\frac{x}{c}} = 1 - \frac{x}{c} + \frac{x^2}{c^2\,\underline{|2}} - \frac{x^3}{c^3\,\underline{|3}} + \frac{x^4}{c^4\,\underline{|4}} - \cdots\cdots$$

From the above series:

$$\sinh\frac{x}{c} = \frac{\epsilon^{\frac{x}{c}} - \epsilon^{-\frac{x}{c}}}{2} = \frac{x}{c} + \frac{x^3}{c^3\,\underline{|3}} + \frac{x^5}{c^5\,\underline{|5}} + \frac{x^7}{c^7\,\underline{|7}} + \cdots \qquad \mathbf{(13)}$$

$$\cosh\frac{x}{c} = \frac{\epsilon^{\frac{x}{c}} + \epsilon^{-\frac{x}{c}}}{2} = 1 + \frac{x^2}{c^2\,\underline{|2}} + \frac{x^4}{c^4\,\underline{|4}} + \frac{x^6}{c^6\,\underline{|6}} + \cdots\cdots \qquad \mathbf{(14)}$$

Since in practical line design the ratio $x \div c$ is less than unity, the terms $\epsilon^{\frac{x}{c}}$ and $\epsilon^{-\frac{x}{c}}$ form, when expanded, a rapidly converging series of which the sum of the first terms may be assumed to represent the sum of all terms in the series with sufficient accuracy.

Substituting the first two terms of (**13**) into (**8**) gives

$$s = x + \frac{x^3}{6\,c^2} \qquad \mathbf{(15)}$$

Substituting the first two terms of (**14**) into (**11**) gives

$$y = \frac{x^2}{2c} \qquad \mathbf{(16)}$$

Substituting the first two terms of (**14**) into (**12**) gives

$$\frac{T}{w} = c + \frac{x^2}{2c} = c + y \qquad \mathbf{(17)}$$

Equation (**16**) is the equation of the parabola.

The catenary equation (**10**) in its exponential form may seem very complex as compared with the equation of the parabola (**16**), and for that reason the parabolic forms have found great favor in designs which do not require extreme accuracy. However, the reluctance with which the practical engineer calls to his aid the catenary equations is entirely unwarranted for the reason that the exponential form may be converted into its equivalent expressed in hyperbolic functions by equation (**11**). A person, equally versatile in the use of hyperbolic functions or elementary algebra will undoubtedly make a choice in favor of equations (**8**), (**11**) and (**12**) where every possible variable appears without exponents, rather than choosing equations (**15**), (**16**) and (**17**) where x appears with exponents ranging between one and three. It would be absurd to use cumbersome equations resulting in approximations of actual conditions, if relatively simpler forms are available which yield correct results.

In the following analysis the catenary equations will be used exclusively.

SUMMARY

The length of cable suspended between the point of maximum deflection and any point of support P is, by equation (**8**)

$$s = c \sinh \frac{x}{c}$$

The maximum deflection of a suspended cable measured with reference to the elevation of any point of support P is, by equation (**11**)

$$y = c\left(\cosh \frac{x}{c} - 1\right)$$

The tension per w units weight of unit length of cable acting along the axis of the cable at any point of support P is, by equation (**12**)

$$\frac{T}{w} = c \cosh \frac{x}{c}$$

In the above three equations

x = projection upon a horizontal plane of the distance between the point of maximum deflection and point P.

c = length of cable upon which gravitation acts with a force equal to the tension H at the point of maximum deflection.

T = tension at point of support P } expressed in the same
w = weight per unit length of cable } unit.

x, y, s and c are all expressed in the same linear unit, which must also be the same as that used in connection with w.

Solution of Problems. These last three equations show the concepts s, y and $T \div w$ to be hyperbolic functions of x and c so that their magnitudes could be computed directly if both x and c were the given quantities in a problem of span design. Usually, however, c is not given since it is in reality a more or less fictitious concept and the solution of span problems, with any two of the remaining concepts given, is accomplished by trial methods.

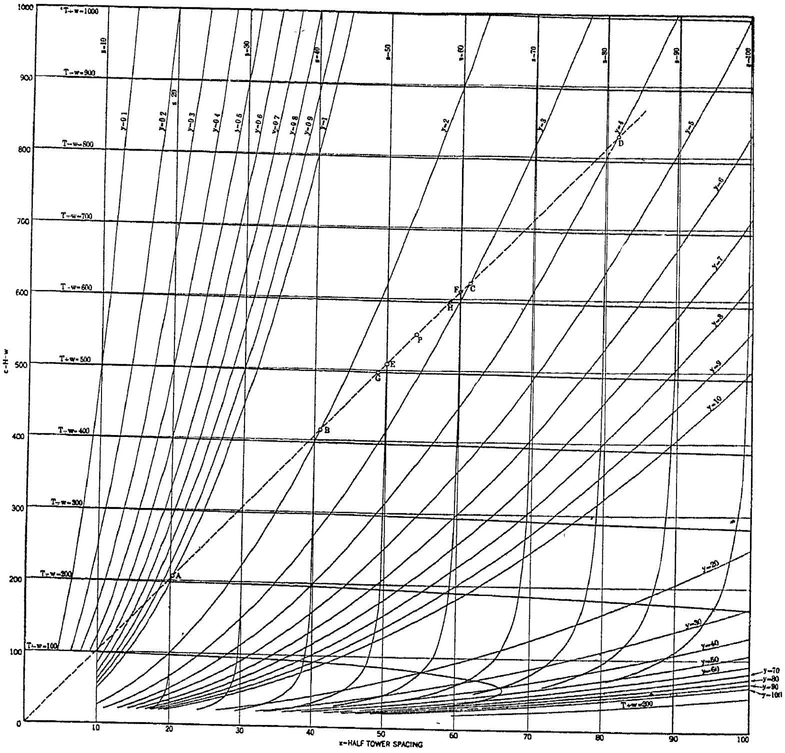

CHART I

In order to avoid the loss of time incurred by such methods, Chart I has been devised from which, with any two of the five concepts given, the remaining three may be found at once. The chart is laid out on the basis of the decimal system to permit of easy interpolation. The concepts x and c, which form the hyperbolic argument, are the abscissas and ordinates, respectively, of the s, y and $T \div w$ curves.

Interpolation on Chart I. It can be easily demonstrated that any straight line passing through the origin (point O) must be divided into intercepts of equal length by a set of hyperbolic curves the indices of which vary in arithmetic progression. For instance, the curves indexed $y=0.1$, $y = 0.2$, $y = 0.3 \ldots y = 0.9$, $y = 1.0$ have indices which increase progressively by 0.1, and in consequence any straight line passing through the origin will be divided by these ten curves into ten equal intercepts. It follows, then, that if each intercept of this straight line were again divided into ten equal lengths, the division points would be points on the curves $y=0.01$, $y = 0.02$, $y = 0.03 \ldots y = 0.99$, $y = 1.00$. The same reasoning holds true for interpolation between the curves $y = 1$, $y = 2$, $y = 3$, $y = 4, \ldots y = 9$, $y = 10$, and of the last set $y = 10$, $y = 20$, $y = 30$, $y = 40$, $\ldots . y = 90$, $y = 100$. Similarly this system of interpolation is correct for the s and $T \div w$ curves.

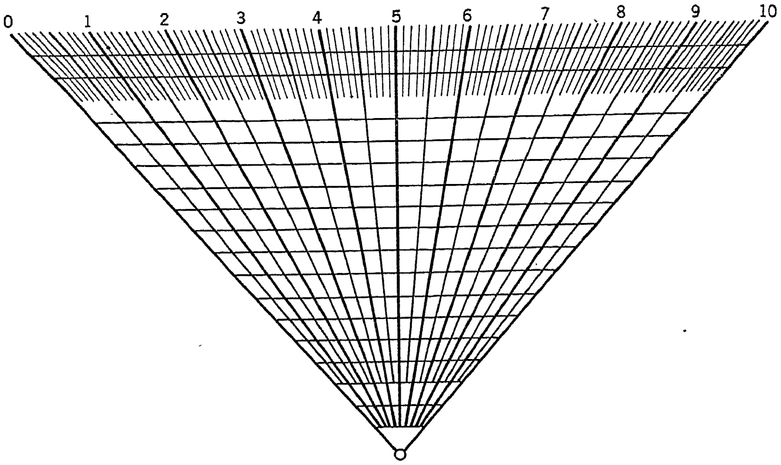

FIG 2

In order to be able to accomplish quick and accurate decimal subdivision of any length of line, it is suggested to trace Fig. 2 on transparent cloth or paper for use on Chart I. Any one of the parallel lines in Fig. 2 is divided into ten or a multiple of ten units of equal length.

Example of Interpolation. It is desired to interpolate for s, y and $T \div w$ a point P, coordinates $c = 550$, $x = 54$.

Through point P draw a straight line to the origin O. That this line $P\,O$ is actually subdivided into equal sections by the curves indexed in arithmetic progression is most clearly shown by the intercepts between the y curves. The intercepts $O\,A$, $A\,B$, $B\,C$ and $C\,D$ of the straight line formed by the curves $y = 1, y = 2$, $y = 3$ and $y = 4$ are exactly equal in length. The same holds true of the intercepts formed by the s and $T \div w$ curves.

By placing Fig. 2 on the chart so that the two outside radiating lines pass through points E and F and the parallel lines are at the same time parallel to line $E\,F$, point P is interpolated directly and corresponds to the magnitude $s = 54.1$.

Placing Fig. 2 so that the two outside radiating lines pass through points G and H and the parallel lines are parallel to $G\,H$, $T \div w$ is read directly to be 553.

In a similar manner y is read directly to be 2.65.

Chart I is independent of any fixed conventional unit of length, weight or force and may be used with equal precision for the English system of foot-pounds or the decimal system of centimeter-grams.

The use of the chart will now be demonstrated by the solution of two representative problems.

Problem 1. A number of 40-ft. (12.1 m.) poles are to be used for supports of a line consisting of No. 00 hard-drawn, bare copper wires. The points of support on the insulators are 30 ft. (9.1 m.) above the ground level when the poles are installed. The minimum clearance of the wire to the ground is to be 20 ft. (6.0 m.), and the tension on the wire at the point of support is not to exceed 200 lb. 90.7 kg.).

What is the maximum permissible spacing of poles?

The weight per 1000 ft. (304.8 m.), of wire is 402.8 lb. (182.6 kg.). If the pound is chosen as the unit of force and the foot as the unit of length, $w = 402.8 \div 1000 = 0.4028$ and $T \div w = 200 \div 0.4028 = 496.5$; $y = 30 - 20 = 10$.

By interpolation between curves $T \div w = 400$ and $T \div w = 500$ curve $T \div w = 496.5$ is drawn. The intersection point of this curve and the curve $y = 10$ has the abscissa $x = 98.3$, which is half the pole spacing in feet.

Hence the maximum permissible pole spacing is 2 × 98.3 = 196.6 ft. (59.9 m.).

Problem 2. A $1\frac{1}{2}$-in. (38.1 mm.) steel cable is to span a river. The foundations for the anchor towers on both shores are 10 ft. (3.0 m.) above the water level and a distance of 2000 ft. (609.9 m.) apart. The maximum tension on the cable is not to exceed 70,000 lb. (31,751.4 kg.). Weight of cable is 4700 lb. 2131.8 kg.) per 1000 ft. Clearance between cable and water surface is not to be less than 50 ft. (15.2 m.).

What is the minimum height of anchor towers?

What is the length of cable between points of support?

What are the vertical and horizontal components of the tension on the points of support in the plane of the suspended cable?

Half the tower spacing is 1000 ft. In order to bring this value within range of Chart I, the unit of length will be chosen as 20 ft. Hence $x = 1000 \div 20 = 50$. For the same unit of length $w = 4700 \times 20 \div 1000 = 94$. Therefore $T \div w = 70{,}000 \div 94 = 745$. This value is within range of the chart. If this value had exceeded the range of the chart, the unit of length would have to be taken greater than 20 ft.

Between the curves $T \div w = 700$ and $T \div w = 800$ a short length of curve $T \div w = 745$ is interpolated near the line $x = 50$. The point of intersection of this curve with the line $x = 50$ has the ordinate $c = 742$. Now a straight line is drawn from the origin through point $x = 50$, $c = 742$ and this point interpolated on the straight line for y and for s.

Interpolation between curves $y = 1$ and $y = 2$ yields $y = 1.68$. Interpolation between $s = 50$ and $s = 60$ yields $s = 50.1$. Hence,

Maximum deflection $= y = 1.68$ units $= 33.6$ ft. (10.2 m.)

Length of cable $= 2\,s = 2 \times 50.1$ units $=$ 2004 ft. (610.8 m.).

Minimum height of anchor tower $= 33.6 + 50 - 10 = 73.6$ ft. (22.4 m.).

Vertical component of tension on point of support $= 2004 \div 2 \times 4.7 = 4720$ lb. (2140.9 kg.)

Horizontal component of tension on point of support in plane of cable $= c\,w = 742 \times 20 \times 4.7 = 69{,}748$ lb. (31,637 kg.).

AVERAGE TENSION ON CABLE BETWEEN POINTS OF SUPPORT

The tension at any point P from equation (**1**) is

$$T = [H^2 + w^2\,s^2]^{\frac{1}{2}}$$

$$= w[c^2 + s^2]^{\frac{1}{2}}$$

For the average tension along the axis of the cable the above expression for the tension at any point must be integrated between the limits 0 and s and the result divided by s. Hence; the average tension is

$$\text{av. } T = \frac{w}{s}\int_0^s [c^2 + s^2]^{\frac{1}{2}}\, ds$$

$$= \frac{w}{s}\Big[\frac{s}{2}[c^2 + s^2]^{\frac{1}{2}} + \frac{c^2}{2}\log_\epsilon (s + [c^2 + s^2]^{\frac{1}{2}})\Big]_0^s$$

$$= \frac{w}{s}\left[\frac{s}{2}[c^2 + s^2]^{\frac{1}{2}} + \frac{c^2}{2}\log_\epsilon (s+[c^2+s^2]^{\frac{1}{2}}) - \frac{c^2}{2}\log_\epsilon c\right]$$

$$= \frac{w}{s}\left(\frac{s}{2}[c^2 + s^2]^{\frac{1}{2}} + \frac{c^2}{2}\log_\epsilon \frac{s + [c^2 + s^2]^{\frac{1}{2}}}{c}\right) \qquad \mathbf{(18)}$$

But by substitutions from equations (**6**) and (**9**), (**18**) becomes:

$$\text{av. } T = \frac{w}{2}\left[y + c + \frac{cx}{s}\right]$$

But from (**8**),

$$s = c \sinh \frac{x}{c}$$

and from (**11**),

$$y = c\left(\cosh \frac{x}{c} - 1\right)$$

Therefore,

$$\text{av. } T = \frac{w}{2}\left[c \cosh \frac{x}{c} + \frac{x}{\sinh \frac{x}{c}}\right]$$

$$= \frac{cw}{2}\left[\cosh \frac{x}{c} + \frac{\frac{x}{c}}{\sinh \frac{x}{c}}\right] \qquad \mathbf{(19)}$$

or,

$$\frac{\text{av. } T}{w} = \frac{c}{2}\left[\cosh\frac{x}{c} + \frac{\frac{x}{c}}{\sinh\frac{x}{c}}\right] \qquad \textbf{(20)}$$

Here it is shown again that the ratio of the average tension on a suspended cable to its weight per unit length is a hyperbolic function of x and c.

Attention is here called to the similarity of equations (**12**) and (**20**). For all positive magnitudes x and c,

$$c < \frac{c}{2}\left[\cosh\frac{x}{c} + \frac{\frac{x}{c}}{\sinh\frac{x}{c}}\right] < c\cosh\frac{x}{c}$$

However, the smaller the value x, the more equal will be the above expressions until

$$\underset{x=o}{\text{limit}}\left\{ c = \frac{c}{2}\left[\cosh\frac{x}{c} + \frac{\frac{x}{c}}{\sinh\frac{x}{c}}\right] = c\cosh\frac{x}{c}\right.$$

At this limit, then

$$cw = \text{av. } T = T$$

But $cw = H =$ tension at the point of maximum deflection.

A very close approximation to formula (**19**) is obtained by using the average between the tension at the point of support and the tension at the point of maximum deflection:

$$\frac{cw\cosh\frac{x}{c} + cw}{2} = \frac{cw}{2}\left[\cosh\frac{x}{c} + 1\right]$$

(*B*). EFFECT OF CHANGES IN TEMPERATURE ON THE CATENARY DESCRIBED BY A CABLE SUSPENDED BETWEEN TWO FIXED POINTS OF THE SAME ELEVATION

In the preceding investigation the concept w (weight per unit length of cable) has been treated as a constant for a given cable.

The above formula, however, does not take into account the change in cross-sectional area of the cable with the change in tension from T_0 to T. Since at constant temperature the volume of the cable remains constant, an increase or decrease in length, caused by a change in tension on the cable, is accompanied by a proportional decrease or increase, respectively, of the cross-sectional area of the cable. The concept A should be understood to represent the average cross-sectional area of the cable as its length changes from T_0 to T. Or, if

A_0 = cross-sectional area of cable under initial tension,
A_T = cross-sectional area of cable under final tension,

$$A = \frac{A_0 + A_T}{2}$$

Equation (**24**) would then become

$$s_T = s_0\left(1 + \frac{2\,[T - T_0]}{E\,[A_0 + A_T]}\right) \qquad \textbf{(25)}$$

In order to simplify equation (**25**) for future use, an approximation

$$s_T = s_0\left(1 + \frac{T}{EA_T} - \frac{T_0}{EA_0}\right) \qquad \textbf{(26)}$$

will be substituted which is more accurate than equation (**24**), but which does not describe conditions quite as precisely as (**25**).

If the strain is proportional to the stress, the total change in the length of a suspended cable is proportional to the change in average tension along the cable, or

$$s_T = s_0\left[1 - \frac{\text{av.}\,T_0}{EA_0} + \frac{\text{av.}\,T}{EA_T}\right) \qquad \textbf{(27)}$$

where

av. T_0 = initial average tension.
av. T = final average tension.

The relation

$$s_0 A_0 = s_T\, A_T \qquad \textbf{(28)}$$

expresses the constancy of volume of the cable subjected to different tensions av. T_0 and av. T.

Although a change in tension produces a deformation of the cable, its total mass remains the same and, in consequence its weight per unit length must change in inverse proportion to the length of the cable, that is,

$$\frac{s_0}{s_T} = \frac{w_T}{w_0} \tag{29}$$

where

w_0 = weight per unit length of cable under av. T_0.
w_T = weight per unit length of cable under av. T.

Influence of Ice and Wind Loading. Usually the gravitational forces acting on a suspended cable by virtue of its mass are augmented by a layer of ice on the cable at low temperatures and by the action of wind on the cable at all temperatures. The ice loading simply increases w which is a vertical downward force, whereas the wind pressure acts horizontally deflecting the cable from its position in a vertical plane to another position in a plane inclined to the vertical at an angle whose tangent is

$$\frac{p}{w + i}$$

where

$$\left.\begin{aligned} i &= \text{weight of ice per unit length of cable.} \\ p &= \text{wind pressure exerted on unit length} \\ &\quad\ \text{cable with ice coating.} \end{aligned}\right\} \text{expressed in the same unit.}$$

The resultant force per unit length of the cable acting downward in the deflected plane is

$$w_1 = [(w + i)^2 + p^2]^{\frac{1}{2}} \tag{30}$$

or the catenary described by the cable in an inclined plane under the influence of ice and wind loading is equivalent to a catenary described in a vertical plane by a cable whose weight per unit length is w_1.

With the knowledge of the influences which may act on a suspended cable in all climatic conditions, and with the knowledge of the laws which govern the response of the suspended cable to these influences, all possible catenaries described by the cable can be mathematically expressed.

Let it be required to find all possible catenaries that can be described by a cable, the characteristics of which are specified by the manufacturer as follows:

$$\left.\begin{array}{ll}\text{Weight per unit length free from stress} & = w \\ \text{Cross-sectional area free from stress.....} & = A\end{array}\right\}\begin{array}{l}\text{measured at}\\ \text{temperature } t\end{array}$$

Modulus of elasticity.................. = E
Coefficient of expansion................ = α
Maximum safe stress on cable.......... = T_{max}

Investigation of the records, furnished by the weather bureau located near the place where the cable is to be suspended, shows:

Minimum observed temperature.... = t_1
Maximum observed temperature.... = t_4
Maximum observed wind pressure.. = v units per unit area normal to direction of wind.

I. CATENARY COVERING CONDITIONS AT MINIMUM TEMPERATURE; CABLE UNDER ICE LOADING AND WIND PRESSURE

NOTE: All concepts characteristic of conditions specified in the heading of this section will appear with the subscript 1.

The maximum stress acts on the cable at the point of support, when the temperature is a minimum, and when at the same time the cable carries its maximum ice and wind load. From equation (**12**) this maximum stress is

$$\max T_1 = w_1 \, c_1 \cosh \frac{x}{c_1} \tag{31}$$

The magnitude max T_1 in equation (**31**) is given in the specifications and the equation could be solved for any argument $x \div c_1$ if w_1 were known. The magnitude of w_1, is determined as follows:

Due to a change in temperature from t to t_1, w changes according to equations (**21**) and (**23**) to

$$w' = \frac{w}{1 - \alpha\,[t - t_1]} \tag{32}$$

At the same time the cross-sectional area A changes according to equation (**22**) to

$$A' = A\,(1 - \alpha\,[t - t_1])^2 \tag{33}$$

Now let it be assumed that the cable receives its ice loading and is strung under a wind pressure v so that the tension at the point of support is maxT_1. On account of this tension the weight per unit length again changes according to equations (**27**) and (**29**) to

$$w'' = \frac{w'}{1 + \frac{\text{av.}\, T_1}{EA_1}} \tag{34}$$

where

av. T_1 = average tension along span corresponding to max T_1 at point of support.

In order to simplify computations for w'' the ratio max $T_1 \div EA'$ will be substituted for the ratio av. $T_1 \div EA_1$ in equation (**34**). max T_1 and A' are both slightly greater than av. T_1 and A_1, respectively, however, their ratios are practically identical. In the case of very long spans this approximation would slightly increase the factor of safety for cable stress, which is a desirable feature. Equation (**34**) is changed to

$$w'' = \frac{w'}{1 + \frac{\max T_1}{EA'}} \tag{35}$$

From equations (**28**) and (**29**)

$$A_1 = \frac{w'' A'}{w'} \tag{36}$$

Let the thickness of the layer of ice be a all around the cable, then the volume of ice per unit length of cable is

$$\frac{\pi}{4}\left(2\left[\frac{A_1}{\pi}\right]^{\frac{1}{2}} + 2\,a\right)^2 - A_1 = \pi a\left(a + 2\left[\frac{A_1}{\pi}\right]^{\frac{1}{2}}\right)$$

and the weight of ice per unit length of cable is

$$i = u\,\pi\,a\left(a + 2\left[\frac{A_1}{\pi}\right]^{\frac{1}{2}}\right) \tag{37}$$

where

u = weight per unit volume of ice.

The maximum force exerted by the wind on unit length of cable with ice envelope is

$$p = 2v\left(a + \left[\frac{A_1}{\pi}\right]^{\frac{1}{2}}\right) \tag{38}$$

With a given wind velocity the wind pressure acting on a cylindrical body of unit length is here assumed to be the same as the wind pressure acting on a rectangular plane of unit length and of width equal to the diameter of the cylindrical body, provided that the inclination to the direction of the wind of the axis of the cylinder is the same as that of the plane of the rectangular surface. The result of the slight error in this assumption is an increase in the factor of safety for cable stress.

The resultant, equivalent weight per unit length of catenary in an inclined plane is, according to equation (**30**)

$$w_1 = [(w'' + i)^2 + p^2]^{\frac{1}{2}} \tag{39}$$

where, from equations (**32**), (**33**) and (**35**),

$$w'' = \frac{w\,E\,A\;(1 - \alpha\,[t - t_1])}{E\,A\;(1 - \alpha\,[t - t_1])^2 + \max T_1} \tag{40}$$

and from equations (**33**), (**35**), (**36**) and (**37**),

$$i = u\,\pi\,a\left[a + 2A\;(1 - \alpha\,[t - t_1])^2 \left(\frac{E}{\pi\,[E\,A\;(1 - \alpha\,[t - t_1])^2 + \max T_1]}\right)^{\frac{1}{2}}\right] \tag{41}$$

and similarly:

$$p = 2v\left[a + A\;(1 - \alpha\,[t - t_1])^2 \left(\frac{E}{\pi\,[E\,A\;(1 - \alpha\,[t - t_1])^2 + \max T_1]}\right)^{\frac{1}{2}}\right] \tag{42}$$

The actual average cross-sectional area of the cable, not including ice covering, is from equations (**33**), (**35**) and (**36**),

$$A_1 = \frac{E\,A^2\,(1 - \alpha\,[t - t_1])^4}{E\,A\;(1 - \alpha\,[t - t_1])^2 + \max T_1} \tag{43}$$

Equation (**31**) can now be solved for any argument $x \div c_1$ since max T_1 and w_1 are given in terms of the manufacturer's specification and in terms of the minimum temperature and maximum wind pressure.

Equations (**41**) and (**42**) seem rather elaborate in view of the arbitrary assumption of the thickness of the ice envelope, but for the sake of accurate comparisons between the characteristics of cables of different size or material, exactly the same assumptions, however arbitrary they may be, should hold for all cables to be compared. So, if it is not a question as to what kind of a cable to use for certain span requirements, the application of approximations which greatly simplify equations (**41**) and (**42**) is justified. The substitution of A for A_1 in these two equations would yield a permissible approximation in the form of the simplest expressions for i and p.

From equation (**8**)

$$s_1 = c_1 \sinh \frac{x}{c_1} \qquad (\mathbf{44})$$

and from equation (**11**)

$$y_1 = c_1 \left(\cosh \frac{x}{c_1} - 1\right) \qquad (\mathbf{45})$$

Equations (**31**), (**44**) and (**45**) completely describe the catenary for minimum temperature with wind and ice loading.

II. CATENARIES COVERING CONDITIONS FROM MINIMUM TEMPERATURE TO THE FREEZING POINT; CABLE UNDER ICE LOADING AND WIND PRESSURE

NOTE: All concepts characteristic of the range of conditions specified in the heading of this section will appear with the subscript *2*.

It is assumed that the total weight of the suspended cable remains constant as the temperature increases from t_1 to the freezing point t_2. At the freezing point it is assumed that no wind is blowing, that, however, on account of the calmness of the atmosphere a greater accumulation of snow on the cable is possible, resulting in the same total weight of the cable. At the freezing point, then, the catenary lies in a vertical plane.

Let the temperature change from t_1 to t_2 without a change in tension and w_1, then, according to equation (**21**), s_1 changes to

$$s_1' = s_1 (1 + \alpha [t_2 - t_1]) \qquad (\mathbf{46})$$

and, according to equation (**22**),

$$A_1' = A_1 (1 + \alpha [t_2 - t_1])^2 \qquad (\mathbf{47})$$

On account of the change in temperature the average tension at temperature t_1 (av. T_1) changes to the average tension at temperature t_2 (av. T_2), and according to equation (**27**) the length s_1' changes to

$$s_2 = s_1'\left(1 - \frac{\text{av. } T_1}{E A_1'} + \frac{\text{av. } T_2}{E A_2}\right) \tag{48}$$

From equation (**8**), at temperature t_2,

$$s_2 = c_2 \sinh \frac{x}{c_2} \tag{49}$$

and the average tension, according to equation (**19**) is.

$$\text{av. } T_2 = \frac{c_2 w_2}{2}\left[\cosh \frac{x}{c_2} + \frac{\dfrac{x}{c_2}}{\sinh \dfrac{x}{c_2}}\right] \tag{50}$$

At temperature t_1 the average tension was

$$\text{av. } T_1 = \frac{c_1 w_1}{2}\left[\cosh \frac{x}{c_1} + \frac{\dfrac{x}{c_1}}{\sinh \dfrac{x}{c_1}}\right] \tag{51}$$

Although the tension has changed from av. T_1 to av. T_2, the volume of the cable remains constant, or,

$$s_1' A_1' = s_2 A_2$$

Hence,

$$A_2 = \frac{s_1' A_1'}{s_2} \tag{52}$$

From the assumption that the total weight of the suspended cable remains constant,

$$s_1 w_1 = s_2 w_2$$

Hence,

$$w_2 = \frac{s_1 w_1}{s_2} \tag{53}$$

Substituting equations (**46**), (**47**), (**49**), (**50**), (**51**), (**52**) and (**53**) into (**48**),

$$c_2 \sinh \frac{x}{c_2} =$$

$$s_1(1+\alpha[t_2-t_1])\left(1 - \frac{w_1\, c_1}{2\,E\,A_1(1+\alpha\,[t_2-t_1])^2}\left[\cosh \frac{x}{c_1} + \frac{\frac{x}{c_1}}{\sinh \frac{x}{c_1}}\right]\right.$$

$$\left. + \frac{w_1\, c_2}{2\,E\,A_1(1+\alpha\,[t_2-t_1])^3}\left[\cosh \frac{x}{c_2} + \frac{\frac{x}{c_2}}{\sinh \frac{x}{c_2}}\right]\right)$$

$$= s_1\,(1+\alpha\,[t_2-t_1]) - \frac{s_1\, w_1\, c_1}{2\,E\,A_1\,(1+\alpha\,[t_2-t_1])}\left[\cosh \frac{x}{c_1} + \frac{\frac{x}{c_1}}{\sinh \frac{x}{c_1}}\right]$$

$$+ \frac{s_1\, w_1\, c_2}{2\,E\,A_1(1+\alpha\,[t_2-t_1])^2}\left[\cosh \frac{x}{c_2} + \frac{\frac{x}{c_2}}{\sinh \frac{x}{c_2}}\right]$$

Dividing both sides of above equation by x:

$$\frac{\sinh \frac{x}{c_2}}{\frac{x}{c_2}} =$$

$$\frac{s_1}{x}(1+\alpha[t_2-t_1]) - \frac{s_1\, w_1}{2\,E\,A_1\,(1+\alpha\,[t_2-t_1])}\left[\frac{\cosh \frac{x}{c_1}}{\frac{x}{c_1}} + \frac{1}{\sinh \frac{x}{c_1}}\right]$$

$$+ \frac{s_1\, w_1}{2\,E\,A_1(1+\alpha\,[t_2-t_1])^2}\left[\frac{\cosh \frac{x}{c_2}}{\frac{x}{c_2}} + \frac{1}{\sinh \frac{x}{c_2}}\right] \qquad \textbf{(54)}$$

By substituting for t_2 in equation (**54**) values ranging between t_1 and the freezing point, any catenary within that range of temperature can be computed, provided that the equivalent weight per unit length of suspended cable remains constant.

III. CATENARIES COVERING CONDITIONS FROM MINIMUM TEMPERATURE TO MAXIMUM TEMPERATURE; CABLE NOT SUBJECTED TO EITHER WIND OR ICE LOADING

NOTE: All concepts characteristic of the range of conditions specified in the heading of this section will appear with the subscript *3* for the catenary at the minimum temperature and with the subscript *4* for all other catenaries.

It is now assumed that at the minimum temperature t_1 the ice and wind load is removed from the suspended cable, thereby changing the total weight of the span from $w_1\, s_1$ to $w_3\, s_3$. Under ice and wind loading the total equivalent weight of the cable is $w_1\, s_1$, while the total weight of the cable alone is $w''\, s_1$ which is the total mass of metal suspended between the points of support. Since the total mass of the cable does not change,

$$s_3\, w_3 = s_1\, w'' \tag{55}$$

The average tension under ice and wind loading was

$$\text{av. } T_1 = \frac{c_1\, w_1}{2}\left[\cosh\frac{x}{c_1} + \frac{\dfrac{x}{c_1}}{\sinh\dfrac{x}{c_1}}\right]$$

but has now changed to

$$\text{av. } T_3 = \frac{c_3\, w_3}{2}\left[\cosh\frac{x}{c_3} + \frac{\dfrac{x}{c_3}}{\sinh\dfrac{x}{c_3}}\right] \tag{56}$$

The change in tension produces a change in length of cable from s_1 to s_3. Expressing this change in length as per equation **(27)**:

$$s_3 = s_1\left(1 - \frac{\text{av. } T_1}{E A_1} + \frac{\text{av. } T_3}{E A_3}\right) \tag{57}$$

From equation(**8**),

$$s_3 = c_3 \sinh\frac{x}{c_3} \tag{58}$$

Although the cable changes its length, its volume remains constant, or,

$$s_1 A_1 = s_3 A_3 \tag{59}$$

Substituting from equations (**51**), (**55**), (**56**), (**58**) and (**59**) into (**57**) and dividing both sides of the equation by x, gives:

$$\frac{\sinh \dfrac{x}{c_3}}{\dfrac{x}{c_3}} = \frac{s_1}{x} - \frac{s_1 w_1}{2\,E\,A_1}\left[\frac{\cosh \dfrac{x}{c_1}}{\dfrac{x}{c_1}} + \frac{1}{\sinh \dfrac{x}{c_1}}\right]$$

$$+ \frac{s_1 w''}{2\,E\,A_1}\left[\frac{\cosh \dfrac{x}{c_3}}{\dfrac{x}{c_3}} + \frac{1}{\sinh \dfrac{x}{c_3}}\right] \tag{60}$$

Equation (**60**) describes the characteristics of the catenary at minimum temperature without either wind or ice loading. All other catenaries for higher temperatures may be found by the use of equation (**54**) which is also based on the assumption that the total weight of the suspended cable is constant irrespective of changes in temperature and tension. Substituting for s_1, w_1, A_1 and c_2 in equation(**54**) the values s_3, w_3, A_3 and c_4, respectively, which are found by the solution of equation (**60**), equation (**54**) changes to

$$\frac{\sinh \dfrac{x}{c_4}}{\dfrac{x}{c_4}} =$$

$$s_3\,(1+\alpha\,[t_4-t_1]) - \frac{s_3\,w_3}{2\,E\,A_3\,(1+\alpha\,[t_4-t_1])}\left[\frac{\cosh \dfrac{x}{c_3}}{\dfrac{x}{c_3}} + \frac{1}{\sinh \dfrac{x}{c_3}}\right]$$

$$+ \frac{s_3\,w_3}{2\,E\,A_3(1+\alpha\,[t_4-t_1])^2}\left[\frac{\cosh \dfrac{x}{c_4}}{\dfrac{x}{c_4}} + \frac{1}{\sinh \dfrac{x}{c_4}}\right] \tag{61}$$

According to the reasoning outlined by equations (**52**) and (**53**),

$$\frac{w_3}{A_3} = \frac{w''}{A_1}$$

which, if substituted into equation (**61**) simplifies this equation to the form

$$\frac{\sinh\frac{x}{c_4}}{\frac{x}{c_4}} = s_3\,(1+\alpha\,[t_4-t_1]) - \frac{s_3\,w''}{2\,E\,A_1\,(1+\alpha\,[t_4-t_1])}\left[\frac{\cosh\frac{x}{c_3}}{\frac{x}{c_3}} + \frac{1}{\sinh\frac{x}{c_3}}\right] + \frac{s_3\,w''}{2\,E\,A_1(1+\alpha\,[t_4-t_1])^2}\left[\frac{\cosh\frac{x}{c_4}}{\frac{x}{c_4}} + \frac{1}{\sinh\frac{x}{c_4}}\right] \tag{62}$$

The concept t_4 covers the range from the minimum to the maximum temperature.

In the design of a practical span the engineer must guarantee, first, that the suspended cable will not break under the influence of the most severe weather conditions recorded in the locality where the span is to be installed and, secondly, that the clearance of the cable from the ground or from a given reference plane will never be less than a given, permissible magnitude; provided that the extremes of the climate recorded for the past of the locality in question are not exceeded in the future.

The first mentioned requirement is met by the use of equation (**31**), if for max.T_1 the maximum safe stress on the cable is substituted and proper allowances are made in the determination of w_1 for ice loading and wind pressure. The guarantee for proper clearance from the ground or from a given reference plane is based upon the knowledge of the maximum deflection of the suspended cable for a given range of climate from extreme winter weather at minimum temperature to the maximum summer heat.

It is evident that the minimum deflection occurs at minimum temperature when the cable is free from ice load and wind pressure. Supposing the cable receives at this temperature a coating of ice being at the same time subjected to wind pressure, which is equivalent to an increase of the gravitational forces acting on the span, equilibrium of forces is re-established by an increase in tension on the cable and its consequent elongation [see equation (**27**)]. Since the distance between the points of support is constant, an increase in length of the cable suspended from these points means an increase in its deflection. Let the temperature increase toward the freezing point. An increase in temperature is accompanied by an increase in length of the cable. Although this increase in length is partly offset by a decrease in tension, as demonstrated in the derivation of equation (**54**), an increase in length does result from an increase in temperature for all materials used in practical transmission spans, and in consequence the deflection of the cable must increase also; provided that the ice coating and wind pressure do not diminish. Since the ice loading cannot exist above the freezing point, the deflection will naturally reach a maximum at the freezing point.

As soon as the temperature rises above the freezing point, the ice load and a large portion of the wind load vanish, resulting in a decrease in the deflection. If the temperature increases to its maximum, the deflection will also increase to a maximum which may either be smaller or greater than the deflection at the freezing point, depending upon the relative magnitude of the maximum temperature and the magnitude of the ice and wind loading which existed at the freezing point. With the help of equation (**54**) the maximum deflection of a given cable for any length of span may be accurately computed for conditions of ice and wind loading at the freezing point. Equations (**60**) and (**62**), solved in sequence, will yield the deflection at the maximum temperature. The one of the two catenaries thus found, which gives the greatest deflection will hereafter be called the *critical catenary.* In warm climates the critical catenary will tend to exist at the maximum temperature, whereas in cold climates the critical catenary is likely to be found at the freezing point. The diameter of the cable is also a factor in the determination of the critical catenary. The smaller the diameter, the greater is the ice load in comparison to the weight of the cable itself, so that at the same locality the critical catenary may occur at the maximum temperature for a large cable, whereas for a smaller cable it might occur at the freezing point. It is at all times advisable to calculate the catenaries for both the maximum temperature and the freezing point in order to be sure of accurate results.

At first sight equations (**54**), (**60**) and (**62**) will appear rather cumbersome and difficult to operate. If, however, equation (**54**) is written

$$\frac{\sinh\frac{x}{c_2}}{\frac{x}{c_2}} = F_1 - G_1 + H_1\left[\frac{\cosh\frac{x}{c_2}}{\frac{x}{c_2}} + \frac{1}{\sinh\frac{x}{c_2}}\right]$$

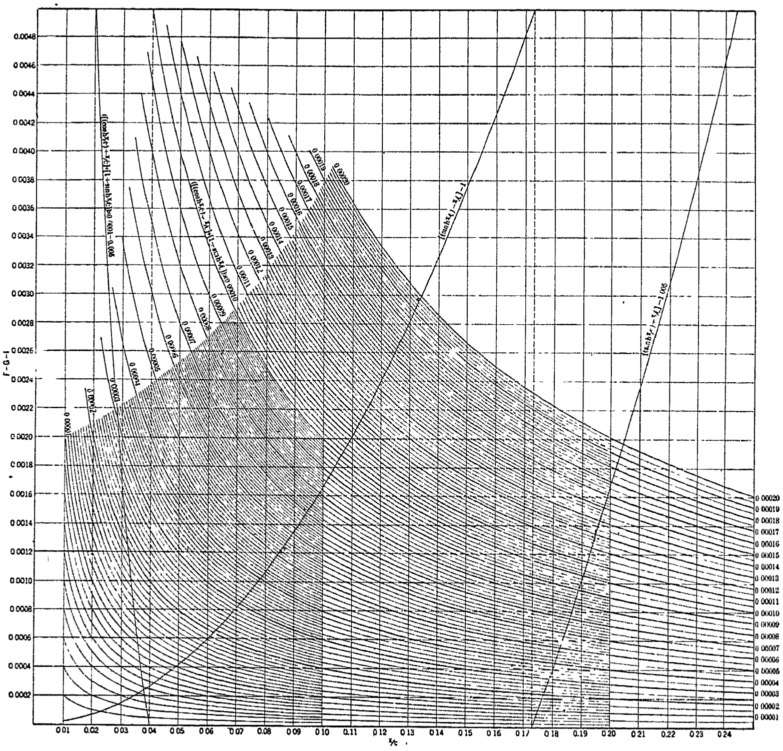

CHART II

where F_1, G_1 and H_1 are constants, the simplicity of the solution of the equation for the argument $x \div c_2$ is apparent.

Equations (**54**), (**60**) and (**62**) are easily and rapidly solved by the use of the curves of Chart II. The curve indexed

$$\left[\left(\sinh\frac{x}{c}\right)\div\frac{x}{c}\right]-1$$

gives values of the magnitude

$$\left[\left(\sinh\frac{x}{c}\right)\div\frac{x}{c}\right]-1$$

for arguments $x \div c$ ranging from 0 to 0.25. The curve indexed

$$0.0001\left[\frac{\cosh\frac{x}{c}}{\frac{x}{c}}+\frac{1}{\sinh\frac{x}{c}}\right]$$

represents magnitudes of

$$H\times\left[\frac{\cosh\frac{x}{c}}{\frac{x}{c}}+\frac{1}{\sinh\frac{x}{c}}\right]$$

for the same range of the argument, if $H = 0.0001$. Corresponding curves for values of H ranging from 0 to 0.0002 are indexed 0.00001, 0.00002,...0.00019, 0.0002. A straight line perpendicular to the $x \div c$ axis is divided into equal lengths by these curves indexed in arithmetic progression and in consequence direct interpolation can be effected along such lines perpendicular to the $x \div c$ axis.

For all positive arguments the ratio of the hyperbolic sine to the argument is always greater than unity. By subtracting unity from this ratio, the two sets of curves of Chart II are located nearer the $x \div c$ axis. The equation is again balanced by subtracting unity from the constants. Hence, for convenience equation (**54**) is changed in form to

$$\frac{\sinh\frac{x}{c_2}}{\frac{x}{c_2}}-1=F_1-G_1+H_1\times\left[\frac{\cosh\frac{x}{c_2}}{\frac{x}{c_2}}+\frac{1}{\sinh\frac{x}{c_2}}\right]-1$$

Supposing, $F_1 = 1.002821$; $G_1 = 0.000\,525$; and $H_1 = 0.0000315$, then

$$F_1 - G_1 - 1 = 0.002296.$$

The magnitude

$$H_1 \times \left[\frac{\cosh \dfrac{x}{c_2}}{\dfrac{x}{c_2}} + \frac{1}{\sinh \dfrac{x}{c_2}}\right]$$

$$= 0.0000315 \left[\frac{\cosh \dfrac{x}{c_2}}{\dfrac{x}{c_2}} + \frac{1}{\sinh \dfrac{x}{c_2}}\right]$$

is given by a curve interpolated between curves indexed 0.00003 and 0.00004 for any argument $x \div c_2$ between the limits 0 and 0.25. The value 0.002296 added to an ordinate of this curve must be, according to the above equation, an ordinate of curve

$$\left[\left(\sinh \frac{x}{c_2}\right) \div \frac{x}{c_2}\right] - 1$$

Hence, the correct argument $x \div c_2$ which gives the same length of ordinate for the curves

$$\left[\left(\sinh \frac{x}{c_2}\right) \div \frac{x}{c_2}\right] - 1$$

and

$$0.002296 + 0.0000315 \left[\frac{\cosh \dfrac{x}{c_2}}{\dfrac{x}{c_2}} + \frac{1}{\sinh \dfrac{x}{c_2}}\right]$$ can be found by

marking the length 0.002296 on the straight edge of a sheet of paper and by moving the point marked on the sheet along the curve 0.0000315, keeping the straight edge perpendicular to the $x \div c$ axis, until the other point touches the curve

$$\left[\left(\sinh \frac{x}{c}\right) \div \frac{x}{c}\right] - 1$$

The straight edge in this position indicates on the $x \div c$ axis the value 0.1294, which, if substituted for $x \div c_2$ in above equation will satisfy this equation.

The curves of Chart II have been drawn with great care from hyperbolic functions computed accurate to the twelfth decimal place. With a little practise in interpolation the argument of equations (**54**), (**60**) and (**62**) can be determined accurately to the fourth decimal place. Values of

$$0.0001\left[\frac{\cosh\frac{x}{c}}{\frac{x}{c}}+\frac{1}{\sinh\frac{x}{c}}\right]$$

with a given argument can be read directly to the sixth decimal place from the curve bearing this index.

OUTLINE FOR SYSTEMATIC DETERMINATION OF THE CRITICAL CATENARY

First Step. Determination of the catenary at the freezing point with ice and wind loading, using equation (**54**) changed for convenience to

$$\frac{\sinh\frac{x}{c_2}}{\frac{x}{c_2}}-1=F_1-G_1+H_1\times\left[\frac{\cosh\frac{x}{c_2}}{\frac{x}{c_2}}+\frac{1}{\sinh\frac{x}{c_2}}\right]-1 \qquad \textbf{(63)}$$

where

$$F_1=\frac{s_1}{x}\left(1+\alpha\,[t_2-t_1]\right)$$

$$G_1=\frac{s_1\,w_1}{2\,E\,A_1\,(1+\alpha\,[t_2-t_1])}\left[\frac{\cosh\frac{x}{c_1}}{\frac{x}{c_1}}+\frac{1}{\sinh\frac{x}{c_1}}\right]$$

$$H_1=\frac{s_1\,w_1}{2\,E\,A_1\,(1+\alpha\,[t_2-t_1])^2}$$

For any argument $x \div c_1$,

$$c_1=\frac{\max.\,T_1}{w_1\cosh\frac{x}{c_1}};\quad x=\frac{x}{c_1}\times c_1;\quad s_1=c_1\sinh\frac{x}{c_1}$$

For w_1 see equation (**39**); for A_1 see equation (**43**).

The hyperbolic coefficient of G_1 for the argument $x \div c_1$ is the direct reading from the curve of Chart II indexed

$$0.0001\left[\frac{\cosh\frac{x}{c}}{\frac{x}{c}}+\frac{1}{\sinh\frac{x}{c}}\right],\ \text{divided by } 0.0001.$$

After the constants F_1, G_1 and H_1 are computed, the argument $x \div c_2$ which satisfies equation (**63**) is read from Chart II. With the knowledge of $x \div c_2$,

$$c_2 = \frac{x}{\frac{x}{c_2}};\quad y_2 = c_2\left(\cosh\frac{x}{c_2} - 1\right);\quad s_2 = c_2 \sinh\frac{x}{c_2};$$

$$w_2 = \frac{s_1 w_1}{s_2};\quad \text{max. } T_2 = c_2 w_2 \cosh\frac{x}{c_2}.$$

Second Step. Determination of the catenary at minimum temperature without ice and wind loading, using equation (**60**) changed for convenience to

$$\frac{\sinh\frac{x}{c_3}}{\frac{x}{c_3}} - 1 = F_2 - G_2 + H_2\left[\frac{\cosh\frac{x}{c_3}}{\frac{x}{c_1}} + \frac{1}{\sinh\frac{x}{c_3}}\right] - 1 \qquad (\mathbf{64})$$

where

$$F_2 = \frac{s_1}{x}$$

$$G_2 = \frac{s_1 w_1}{2\,E\,A_1}\left[\frac{\cosh\frac{x}{c_1}}{\frac{x}{c_1}} + \frac{1}{\sinh\frac{x}{c_1}}\right]$$

$$H_2 = \frac{s_1 w''}{2\,EA_1}$$

For w'' see equation (**40**), other terms are the same as in equation (**63**).

With the knowledge of $x \div c_3$,

$$c_3 = \frac{x}{\frac{x}{c_3}}; \quad y_3 = c_3\left(\cosh\frac{x}{c_3} - 1\right); \quad s_3 = c_3 \sinh\frac{x}{c_3};$$

$$w_3 = \frac{s_1 w''}{s_3}; \text{ max. } T_3 = c_3 w_3 \cosh\frac{x}{c_3}.$$

Third Step. Determination of the catenary at maximum temperature, using equation (**62**) changed for convenience to

$$\frac{\sinh\frac{x}{c_4}}{\frac{x}{c_4}} - 1 = F_3 - G_3 + H_3\left[\frac{\cosh\frac{x}{c_4}}{\frac{x}{c_4}} + \frac{1}{\sinh\frac{x}{c_4}}\right] - 1 \quad \textbf{(65)}$$

where

$$F_3 = \frac{s_3}{x}(1 + \alpha[t_4 - t_1])$$

$$G_3 = \frac{s_3 w''}{2 E A_1 (1+\alpha[t_4 - t_1])}\left[\frac{\cosh\frac{x}{c_3}}{\frac{x}{c_3}} + \frac{1}{\sinh\frac{x}{c_3}}\right]$$

$$H_3 = \frac{s_3 w''}{2 E A_1 (1+\alpha[t_4 - t_1])^2}$$

With the knowledge of $x \div c_4$,

$$c_4 = \frac{x}{\frac{x}{c_4}}; \quad y_4 = c_4\left(\cosh\frac{x}{c_4} - 1\right); \quad s_4 = c_4 \sinh\frac{x}{c_4};$$

$$w_4 = \frac{s_1 w''}{s_4}; \text{ max. } T_4 = c_4 w_4 \cosh\frac{x}{c_4}.$$

The critical catenary is either

$$y_2 = c_2\left(\cosh\frac{x}{c_2} - 1\right) \text{ or } \quad y_4 = c_4\left(\cosh\frac{x}{c_4} - 1\right),$$

whichever equation yields the greatest deflection for a given tower spacing. The determination of the tower spacing and of the corresponding height of tower must be based on the maximum deflection of the critical catenary.

The catenary for any other temperature t_4' somewhere within the range of from t_1 to t_4 can be computed by substituting t_4' for t_4 in equation (**65**). Since this equation is based on the assumption that the cable is free from ice or wind load, the resulting catenaries including those for temperatures below the freezing point, describe the span in calm weather, without sleet deposits on the cable at the low temperatures. Such weather conditions usually prevail at the time of span construction. Hence, a cable strung at any temperature t_4' in calm weather, so that its axis forms a catenary obtained by the use of equation (**65**) for the temperature t_4', would under the most severe weather conditions be subjected to not more nor less than the maximum safe stress. The constructing engineer should have a means of quickly determining the required stringing tension for a given tower spacing and temperature, so that he may guarantee that his span is safe under the most severe weather conditions and that the margin of safety is not too great for economical span construction.

In order to furnish the constructing engineer this necessary information, catenaries for the temperatures t_1, t_4', t_4'' and t_4 are computed with the use of equation (**65**) for a number of arguments $x \div c_4$. The four curves obtained in terms of max T and $2x$ for the range of tower spacing covered by the range of the argument are plotted on a chart and all other curves corresponding to intermediate temperatures are interpolated. A chart of this kind, from which the engineer can directly read the required stringing tension for any length of span and any stringing temperature will hereafter be called a *temperature-tension stringing chart.*

The use of Chart II and of the formulas derived will now be demonstrated in the solution of the following problem:

Problem 3. It is required to draw the temperature-tension stringing chart for a 500,000 cir. mil hard-drawn copper cable.

If the cable is suspended from 50-ft. towers so that the minimum clearance of the cable from the ground is 30 ft., what is the maximum permissible tower spacing?

Extreme climatic conditions:

Minimum temperature = − 10 deg. fahr.
Maximum temperature = + 100 deg. fahr.
Maximum ice loading = ½-inch (12.7 mm.) layer enveloping cable.
Maximum wind pressure = 10 lb. per square ft. area normal to direction of wind.

Specification of Cable:

Number of strands in cable = 37.
Modulus of elasticity = 16,000,000.
Ultimate tensile strength of one strand = 673 lb. (305.2 kg.).
Temperature coefficient of linear expansion 0.00000922 (fahr. scale).

Diameter of strand = 0.1162 in. (2.9 mm.) Diameter of cable = 0.8134 in. (20.6 mm.) Weight per ft. of cable = 1.54 lb. (0.6 kg.)	Measurements taken at 75 deg. fahr. with cable free from mechanical stress.

The ultimate tensile strength of the cable to be not more than 0.9 the combined ultimate tensile strength of its strands.

$$673 \times 37 \times 0.9 = 22{,}411 \text{ lb. } (10{,}165.3 \text{ kg.})$$

The elastic limit to be taken at 50 per cent of the ultimate breaking stress.

$$22{,}411 \times 0.5 = 11{,}206 \text{ lb. } (5082.9 \text{ kg.})$$

The maximum tension on the cable should not exceed 75 per cent of the tension at the elastic limit.

$$11{,}206 \times 0.75 = 8400 \text{ lb. } (3810.1 \text{ kg.})$$

Since all preceding formulas describe the characteristics of a suspended cylindrical solid, it will be of advantage to substitute for the stranded cable a cylindrical solid which has the same characteristics as the actual stranded cable in regard to temperature changes, changes in tension, weight per unit length and ice and wind load. The ice and wind load on the equivalent cylindrical solid will be the same as the ice and wind load on the stranded cable, if the cylindrical solid has the same diameter as the outside diameter of the stranded cable. The inside area of a cylindrical envelope surrounding the stranded cable is, as well as the actual cross-sectional area of metal of the cable, directly proportional to the square of the diameter of the strand; hence, as far as the influence of temperature is concerned, both areas are affected in the same proportion. Similarly, the changes in both areas due to changes in tension are proportional to the changes in area of the strand; however, since the modulus of elasticity is measured on the basis of the cross-sectional area of metal, it must be changed in inverse proportion to the area for the same strain, so that

$$E' \times A' = E \times A$$

where,

E' = modulus of elasticity for stranded cable.
E = modulus of elasticity for equivalent cylindrical solid.
A' = cross-sectional area of metal of stranded cable.
A = cross-sectional area of equivalent cylindrical solid.

Hence,

$$E = \frac{E'\,A'}{A} = \frac{16{,}000{,}000 \times 37 \times 0.1162^2}{0.8134^2} = 12{,}082{,}000$$

Revised specification of cable, using the foot as the unit of length and the pound as the unit of force:

A = Cross-sectional area free from stress

$$= \frac{\pi \times 0.8314^2}{4 \times 144} = 0.00361$$

w = Weight per foot length free from stress = 1.54
E = Modulus of elasticity = 12,082,000 × 144 = 1,739,808,000
α = Coefficient of linear expansion = 0.00000922
max T = Maximum safe stress on cable = 8400
t = Temperature at which measurements of A and w were taken = 75

Other given data:

t_1 = Minimum temperature = $-$ 10
t_2 = Temperature at freezing point = + 32
t_4 = Maximum temperature = + 100
a = Maximum thickness of ice layer enveloping cable = 0.04167

u = Weight per cubic foot of ice = 57.37
v = Maximum wind pressure per square foot area normal to direction of wind = 10

For the temperature-tension stringing chart two other temperatures $t_4' = +30$ and $t_4'' = +60$ will be used in connection with equation (**65**).

From equation (**40**)

$$w'' = \frac{1.54 \times 1{,}739{,}808{,}000 \times 0.00361\ (1 - 0.00000922\ [75 + 10])}{1{,}739{,}808{,}000 \times 0.00361\ (1 - 0.00000922\ [75+10])^2 + 8400}$$
$$= 1.5391$$

From equation (**41**)

$$i = 57.37 \times 3.1416 \times 0.04167 \Bigg[0.04167 + 2 \times 0.00361\ (1 - 0.00000922\ [75 + 10])^2 \left(\frac{1{,}739{,}808{,}000}{3.1416\ (1739808000 \times 0.00361(1-0.00000922[75+10])^2+8400]}\right)^{\frac{1}{2}} \Bigg]$$
$$= 0.8212$$

From equation (**42**)

$$p = 2 \times 10 \Bigg[0.04167 + 0.00361(1 - 0.00000922[75 + 10])^2 \left(\frac{1{,}739{,}808{,}000}{3.1416[1739808000 \times 0.00361(1-0.00000922[75+10])^2+8400]}\right)^{\frac{1}{2}} \Bigg]$$
$$= 1.5100$$

From equation (**39**)

$$w_1 = [(1.5391 + 0.8212)^2 + 1.51^2]^{\frac{1}{2}} = 2.802$$

From equation (**43**)

$$A_1 = \frac{1{,}739{,}808{,}000 \times 0.00361^2\ (1 - 0.00000922\ [75 + 10])^4}{1{,}739{,}808{,}000 \times 0.00361\ (1 - 0.00000922\ [75+10])^2 + 8400}$$
$$= 0.003598$$

In the following five tables the computations necessary for the solution of the problem are carried out systematically. Tables I, II and III determine the critical catenary and Tables II, III, IV and V give the data necessary for the temperature-tension stringing chart. The columns of figures are indexed successively from 1 to 44. In the space below the index of a column is given the relation of the values in that column to the alues computed in preceding columns. For instance, in column of Table III indexed 31 are given values obtained by multiplying the values given in column of Table II indexed 24 by the constant 12,281 × 10^{-11}. The second space below the indices indicates the significance of the values in the column with respect to the working equation at the top of the table.

A comparison of the deflections y_2 and y_4 given in column 15 of Table I and column 39 of Table III shows that the critical catenary is the catenary formed by the cable at the maximum temperature + 100 deg. fahr. The difference between the deflection of the cable at the freezing point under maximum ice loading and the deflection at the maximum temperature is, however, very small which shows the necessity of performing the deflection computations for both weather conditions in every case.

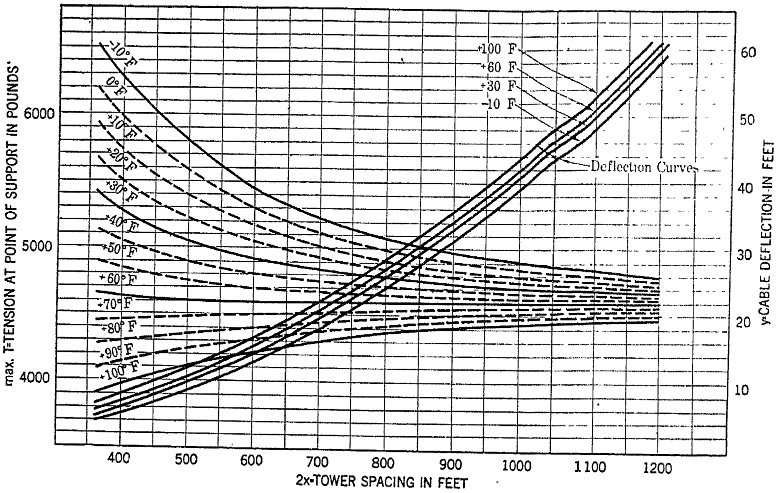

FIG. 3—TEMPERATURE-TENSION STRINGING CHART 500,000 CIR. MILS HARD-DRAWN COPPER CABLE

Maximum tension to be not more nor less than 3400 lb. at 10 deg. fahr. under wind load of 10 lb. per sq. ft. and ice load of one-half inch layer enveloping cable.

With a given height of tower and a given maximum cable deflection the critical catenary determines the tower spacing. The height of towers given in this problem is 50 ft., and since the deflection of the cable is not to exceed 20 ft. (50 − 30), the tower spacing is found directly from the curves of Fig. 3. The abscissa corresponding to the ordinate $y = 20$ of the deflection curve + 100 deg. fahr. (critical catenary) is 665. Hence the maximum permissible tower spacing is 665 ft. (202.6 m.)

TABLE I. CATENARY AT FREEZING POINT.

Working Formula. [See equation (**63**)]

$$\frac{\sinh\frac{x}{c_2}}{\frac{x}{c_2}} - 1 = F_1 - G_1 + H_1 \times \left[\frac{\cosh\frac{x}{c_2}}{\frac{x}{c_2}} + \frac{1}{\sinh\frac{x}{c_2}}\right] - 1$$

$$F_1 = \frac{s_1}{x} \times (1 + 0.00000922\,[32 + 10]) = \frac{s_1}{x} \times 1.0003872$$

$$G_1 = s_1 \times \frac{2.802}{2 \times 1{,}739{,}808{,}000 \times 0.003598 \times 1.0003872}\left[\frac{\cosh\frac{x}{c_1}}{\frac{x}{c_1}} + \frac{1}{\sinh\frac{x}{c_1}}\right]$$

$$= s_1 \times 0.00000022373\left[\frac{\cosh\frac{x}{c_1}}{\frac{x}{c_1}} + \frac{1}{\sinh\frac{x}{c_1}}\right]$$

$$H_1 = s_1 \times \frac{2.802}{2 \times 1{,}739{,}808{,}000 \times 0.003598 \times 1.0003872^2} = s_1 \times 0.00000022364$$

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	$\frac{2997.86}{\cosh (1)}$	(1) × (2)	(2) × sinh (1)	$\frac{(4)}{(3)}$	(5) × 1.000387	(4) × 22,373 × 10^{-11}	(1) & Ch't II
$\frac{x}{c_1}$ = Assumed Argument	$c_1 = \frac{8400}{2.802\cosh\frac{x}{c_1}}$	$x = \frac{x}{c_1} \times c_1$	$s_1 = c_1 \sinh\frac{x}{c_1}$	$\frac{s_1}{x}$	F_1	$\frac{G_1}{\frac{\cosh\frac{x}{c_1}}{\frac{x}{c_1}} + \frac{1}{\sinh\frac{x}{c_1}}}$	$\frac{\cosh\frac{x}{c_1}}{\frac{x}{c_1}} + \frac{1}{\sinh\frac{x}{c_1}}$
0.06	2992.47	179.548	179.656	1.000599	1.000986	0.00004019	33.36
0.08	2988.29	239.063	239.318	1.001068	1.001456	0.00005354	25.03
0.10	2982.94	298.294	298.791	1.001667	1.002055	0.00006685	20.03
0.12	2976.40	357.168	358.026	1.002402	1.002790	0.00008010	16.71
0.14	2968.72	415.621	416.980	1.003270	1.003658	0.00009329	14.33
0.16	2959.89	473.582	475.605	1.004272	1.004661	0.00010641	12.55
0.18	2949.94	530.989	533.861	1.005409	1.005798	0.00011944	11.17
0.20	2938.89	587.778	591.704	1.006680	1.007069	0.00013238	10.07

(9)	(10)	(11)	(12)	(13)	(14)	(15)
(7) × (8)	(6) − (9) − 1	$\frac{(7)}{1.0003872}$	(10), (11) & Ch't II	$\frac{(3)}{(12)}$	− 1 + cosh (12)	(13) × (14)
G_1	$F_1 - G_1 - 1$	H_1,	$\frac{x}{c_2}$	$c_2 = \frac{x}{\frac{x}{c_2}}$	$\frac{y_2}{c_2} = \cosh\frac{x}{c_2} - 1$	y_2
0.001341	− 0.000355	0.0000402	0.0696	2579.71	0.00242	6.24
0.001340	+ 0.000116	0.0000535	0.0889	2689.12	0.00395	10.62
0.001339	+ 0.000716	0.0000668	0.1082	2756.88	0.00586	16.17
0.001338	+ 0.001452	0.0000801	0.1275	2801.32	0.00814	22.80
0.001337	+ 0.002321	0.0000933	0.1468	2831.21	0.01080	30.57
0.001335	+ 0.003326	0.0001064	0.1662	2849.46	0.01384	39.43
0.001334	+ 0.004464	0.0001194	0.1857	2859.40	0.01729	49.43
0.001333	+ 0.005736	0.0001323	0.2053	2863.02	0.02115	60.55

TABLE II CATENARY AT TEMPERATURE – 10 DEG. FAHR.

Working Formula. [See equation (**64**)]

$$\frac{\sinh\frac{x}{c_3}}{\frac{x}{c_3}} - 1 = F_2 - G_2 + H_2\left[\frac{\cosh\frac{x}{c_3}}{\frac{x}{c_3}} + \frac{1}{\sinh\frac{x}{c_3}}\right] - 1$$

$$F_2 = \frac{s_1}{x} = \text{given in col. (5), Table I.}$$

$$G_2 = s_1\,\frac{2.802}{2 \times 1{,}739{,}808\,000 \times 0.003598}\left[\frac{\cosh\frac{x}{c_1}}{\frac{x}{c_1}} + \frac{1}{\sinh\frac{x}{c_1}}\right]$$

$$= s_1 \times 0.00000022381 \times \text{value given in col. 8, Table I.}$$

$$H_2 = s_1 \times \frac{1.5391}{2 \times 1{,}739{,}808{,}000 \times 0.003598} = s_1 \times 0.00000012294$$

(16)	(17)	(18)	(19)	(20)	(21)
(4) × 22,381 × 10^{-11}	(8) × (16)	(5) − (17) − 1	(4) × 12,294 × 10^{-11}	(18), (19) & Ch't II	$\frac{(3)}{(20)}$
$\frac{G_2}{\frac{\cosh\frac{x}{c_1}}{\frac{x}{c_1}} + \frac{1}{\sinh\frac{x}{c_1}}}$	G_2	$F_2 - G_2 - 1$	H_2	$\frac{x}{c_3}$	$c_3 = \frac{x}{\frac{x}{c_3}}$
0.00004021	0.001341	−0.000742	0.0000221	0.0424	4234.62
0.00005356	0.001340	−0.000272	0.0000294	0.0628	3806.74
0.00006687	0.001339	+0.000328	0.0000367	0.0846	3525.93
0.00008013	0.001338	+0.001064	0.0000440	0.1066	3350.54
0.00009332	0.001337	+0.001938	0.0000513	0.1282	3241.98
0.00010644	0.001336	+0.002936	0.0000585	0.1494	3169.89
0.00011948	0.001335	+0.004074	0.0000656	0.1704	3116.13
0.00013242	0.001334	+0.005346	0.0000728	0.1913	3072.54

(22)	(23)	(24)	(25)	(26)	(27)	(28)
cosh (20) −1	(21) × (22)	(21) × sinh (20)	$\frac{(4)}{(24)}$	(25) × 1.5391	(21) × cosh (20)	(26) × (27)
$\frac{y_3}{c_3} = \cosh\frac{x}{c_3} - 1$	y_3	$s_3 = c_3 \sinh\frac{x}{c_3}$	$\frac{s_1}{s_3}$	$w_3 = \frac{s_1}{s_3}\,w''$	$\frac{\max T_3}{w_3} = c_3 \cosh\frac{x}{c_3}$	$\max T_3$
0.00090	3.81	179.590	1.00037	1.5397	4238.43	6525.91
0.00197	7.50	239.216	1.00043	1.5398	3813.97	5872.75
0.00358	12.62	298.646	1.00049	1.5399	3538.55	5448.01
0.00568	19.03	357.838	1.00055	1.5399	3369.57	5188.80
0.00823	26.68	416.757	1.00061	1.5400	3268.66	5033.74
0.01118	35.44	475.325	1.00059	1.5400	3205.33	4936.21
0.01455	45.24	533.575	1.00057	1.5400	3161.47	4868.66
0.01836	56.42	591.372	1.00056	1.5399	3128.95	4818.27

TABLE III. CATENARY TEMPERATURE + 100 DEG. FAHR.

Working Formula. [See equation (**65**)]

$$\frac{\sinh\frac{x}{c_4}}{\frac{x}{c_4}} - 1 = F_3 - G_3 + H_3\left[\frac{\cosh\frac{x}{c_4}}{\frac{x}{c_4}} + \frac{1}{\sinh\frac{x}{c_4}}\right] - 1$$

$$F_3 = s_3/x \times (1 + 0.00000922\,[100 + 10]) = s_3 \times 1.001014$$

$$G_3 = s_3 \times \frac{1.5391}{2 \times 1{,}739{,}808{,}000 \times 0.003598 \times 1.001014}\left[\frac{\cosh\frac{x}{c_3}}{\frac{x}{c_3}} + \frac{1}{\sinh\frac{x}{c_3}}\right]$$

$$= s_3 \times 0.00000012281\left[\frac{\cosh\frac{x}{c_3}}{\frac{x}{c_3}} + \frac{1}{\sinh\frac{x}{c}}\right]$$

$$H_3 = s_3 \times \frac{1.5391}{2 \times 1{,}739{,}808{,}000 \times 0.003598 \times 1.001014^2} = s_3 \times 0.00000012268$$

(29)	(30)	(31)	(32)	(33)	(34)	(35)
$\frac{(24)}{(3)}$	(29) × 1.001014	(24) × 12,281 × 10^{-11}	(20) & Ch't II	(31) × (32)	(30) − (33) − 1	(24) × 12,268 × 10^{-11}
$\frac{s_3}{x}$	F_3	$\dfrac{G_3}{\frac{\cosh\frac{x}{c_3}}{\frac{x}{c_3}} + \frac{1}{\sinh\frac{x}{c_3}}}$	$\frac{\cosh\frac{x}{c_3}}{\frac{x}{c_3}} + \frac{1}{\sinh\frac{x}{c_3}}$	G_3	$F_3 - G_3 - 1$	H_3
1.000234	1.001248	0.00002206	47.16	0.001040	+0.000208	0.0000220
1.000640	1.001655	0.00002938	31.83	0.000935	+0.000720	0.0000294
1.001180	1.002195	0.00003668	23.67	0.000868	+0.001327	0.0000366
1.001876	1.002892	0.00004395	18.80	0.000826	+0.002066	0.0000439
1.002735	1.003750	0.00005118	15.65	0.000801	+0.002949	0.0000511
1.003680	1.004698	0.00005838	13.43	0.000784	+0.003914	0.0000583
1.004870	1.005889	0.00006553	11.80	0.000773	+0.005116	0.0000655
1.006114	1.007134	0.00007263	10.52	0.000764	+0.006370	0.0000726

(36)	(37)	(38)	(39)	(40)	(41)	(42)	(43)	(44)
(34), (35) & Ch't II	$\frac{(3)}{(36)}$	cosh (36) − 1	(37) × (38)	(37) × sinh (36)	$\frac{(4)}{(40)}$	(41) × 1.5391	(37) + (39)	(42) × (43)
$\frac{x}{c_4}$	$c_4 = \frac{x}{\frac{x}{c_4}}$	$\frac{y_4}{c_4} = \cosh\frac{x}{c_4} - 1$	y_4	$s_4 = c_4 \sinh\frac{x}{c_4}$	$\frac{s_1}{s_4}$	$w_4 = \frac{s_1}{s_4} w''$	$\frac{\max T_4}{w_4} = c_4 \cosh\frac{x}{c_4}$	max T_4
0.0710	2528.85	0.00252	6.37	179.690	0.99981	1.5388	2535.22	3901
0.0905	2641.58	0.00410	10.83	239.380	0.99974	1.5387	2652.41	4081
0.1095	2724.15	0.00600	16.34	298.894	0.99969	1.5386	2740.49	4216
0.1285	2779.52	0.00826	22.96	358.141	0.99966	1.5386	2802.48	4312
0.1475	2817.77	0.01089	30.69	417.143	0.99964	1.5385	2848.46	4382
0.1665	2844.34	0.01389	39.50	475.773	0.99963	1.5385	2883.84	4437
0 1865	2847.13	0.01744	49.65	534.065	0.99961	1.5385	2896.78	4457
0.2060	2853.30	0.02129	60.74	591.946	0.99960	1.5385	2914.04	4483

TABLE IV. CATENARY AT TEMPERATURE + 30 DEG. FAHR.

Working Formula. [See equation (**65**)]

$$\frac{\sinh\frac{x}{c_4'}}{\frac{x}{c_4'}} - 1 = F_3' - G_3' + H_3'\left[\frac{\cosh\frac{x}{c_4'}}{\frac{x}{c_4'}} + \frac{1}{\sinh\frac{x}{c_4'}}\right] - 1$$

$$F_3' = s_3 \times (1 + 0.00000922\,[30 + 10]) = s_3 \times 1.0003688$$

$$G_3' = s_3 \times \frac{1.5391}{2 \times 1{,}739{,}808{,}000 \times 0.003598 \times 1.0003688}\left[\frac{\cosh\frac{x}{c_3}}{\frac{x}{c_3}} + \frac{1}{\sinh\frac{x}{c_3}}\right]$$

$$= s_3 \times 0.00000012289 \times \text{value given in col. 32, Table III}$$

$$H_3' = s_3 \times \frac{1.5391}{2 \times 1{,}739\,808{,}000 \times 0.003598 \times 1.0003688^2} = s_3 \times 1.00000012285$$

(30) ′	(31) ′	(33) ′	(34) ′	(35) ′	(36) ′
(29) × 1.0003688	(24) × 12,289 × 10^{-11}	(31) ′ × (32)	(30) ′ − (33) ′ − 1	(24) × 12,285 × 10^{-11}	(34), ′ (35) ′ & Ch't II
F_3'	$\frac{G_3'}{\frac{\cosh\frac{x}{c_3'}}{\frac{x}{c_3'}} + \frac{1}{\sinh\frac{x}{c_3'}}}$	G_3'	$F_3' - G_3' - 1$	H_3'	$\frac{x}{c_4'}$
1.000603	0.00002207	0.001041	−0.000438	0.0000221	0.0510
1.001009	0.00002939	0.000936	+0.000073	0.0000294	0.0727
1.001549	0.00003670	0.000869	+0.000680	0.0000367	0.0936
1.002245	0.00004397	0.000827	+0.001418	0 0000440	0.1145
1.003105	0.00005121	0.000802	+0.002303	0.0000512	0.1354
1.004090	0.00005841	0.000785	+0.003265	0.0000584	0.1561
1.005241	0.00006557	0.000774	+0.004467	0.0000655	0.1766
1.006485	0.00007267	0.000765	+0.005720	0.0000727	0.1968

(37) ′	(38) ′	(39) ′	(40) ′	(41)′	(42) ′	(43) ′	(44) ′
$\frac{(3)}{(36)'}$	cosh (36) ′ − 1	(37)′ × (38) ′	(37)′ × sinh (36) ′	$\frac{(4)}{(40)'}$	(41) ′ × 1.5391	(37) ′ + (39) ′	(42)′ × (43) ′
c_4'	$\frac{y_4'}{c_4'} = \cosh\frac{x}{c_4'} - 1$	y_4'	$s_4' = c_4' \sinh\frac{x}{c_4'}$	$\frac{s_1}{s_4'}$	$w_4' = \frac{s_1}{s_4'} w''$	$\frac{\max T_4'}{w_4'} = c_4\cosh\frac{x}{c_4'}$	$\max T_4'$
3520.55	0.00130	4.58	179.618	1.00021	1.5394	3525.1	5427
3288.35	0.00264	8.68	239.268	1.00021	1.5394	3297.0	5074
3186.90	0.00438	13.95	298.730	1.00021	1.5394	3200.9	4927
3119.37	0.00657	20.49	357.948	1.00022	1.5394	3139.9	4833
3069.58	0.00918	28.18	416.880	1.00024	1.5395	3097.8	4768
3033.84	0.01221	37.05	475.494	1.00023	1.5395	3070.9	4727
3006.73	0.01564	47.03	533.755	1.00020	1.5394	3053.8	4701
2986.67	0.01943	58.04	591.600	1.00018	1.5393	3044.7	4687

TABLE V. CATENARY AT TEMPERATURE + 60 DEG. FAHR.

Working Formula. [See equation (**65**)]

$$\frac{\sinh\frac{x}{c_4''}}{\frac{x}{c_4''}} - 1 = F_3'' - G_3'' + H_3''\left[\frac{\cosh\frac{x}{c_4''}}{\frac{x}{c_4''}} + \frac{1}{\sinh\frac{x}{c_4''}}\right] - 1$$

$$F_3'' = s_3 \times (1 + 0.00000922[60 + 10]) = s_3 \times 1.0006454$$

$$G_3'' = s_3 \times \frac{1.5391}{2 \times 1{,}739{,}808{,}000 \times 0.003598 \times 1.0006454}\left[\frac{\cosh\frac{x}{c_3}}{\frac{x}{c_3}} + \frac{1}{\sinh\frac{x}{c_3}}\right]$$

$$= s_3 \times 0.00000012286 \times \text{value given in col. 32, Table III}$$

$$H_3'' = s \times \frac{1.5391}{2 \times 1{,}739{,}808{,}000 \times 0.003598 \times 1.0006454^2} = s_3 \times 1.00000012281$$

(30) ″	(31) ″	(33) ″	(34) ″	(35) ″	(36) ″
(29) × 1.0006454	(24) × 12,286 × 10⁻¹¹	(31) ″ × (32)	(30) ″ − (33) ″ − 1	(24) × 12,281 × 10⁻¹¹	(34) ″, (35) ″ & Ch't II
F_3''	$\dfrac{G_3''}{\dfrac{\cosh\frac{x}{c_3}}{\frac{x}{c_3}} + \dfrac{1}{\sinh\frac{x}{c_3}}}$	G_3''	$F_3'' - G_3'' - 1$	H_3''	$\frac{x}{c_4''}$
1.000880	0.00002206	0.001040	−0.000160	0.0000221	0.0595
1.001286	0.00002938	0 000935	+0 000351	0.0000294	0.0800
1.001826	0.00003669	0.000869	+0.000957	0.0000367	0.1005
1.002523	0.00004397	0.000827	+0.001696	0.0000440	0.1205
1.003382	0.00005120	0.000802	+0 002580	0.0000512	0.1405
1.004328	0.00005840	0.000785	+0 003543	0.0000584	0.1605
1.005519	0.00006556	0.000774	+0.004745	0.0000655	0.1805
1.006718	0.00007266	0.000765	+0.005953	0.0000726	0.2000

(37) ″	(38) ″	(39) ″	(40) ″	(41) ″	(42) ″	(43) ″	(44)″
$\frac{(3)}{(36)''}$	cosh (36) ″ −1	(37) ″ × (38) ″	(37) ″ × sinh (36)″	$\frac{(4)}{(40)''}$	(41) ″ × 1.5391	(37) ″ + (39)″	(42)″ × (43)″
c_4''	$\frac{y_4''}{c_4''} = \cosh\frac{x}{c_4''} - 1$	y_4''	$s_4'' = c_4'' \sinh\frac{x}{c_4''}$	$\frac{s_1}{s_4''}$	$w_4'' = \frac{s_1}{s_4''} w''$	$\frac{\max T_4''}{w_4''} = c_4'' \cosh\frac{x}{c_4''}$	$\max T_4''$
3017.61	0.00177	5.34	179.668	0.99993	1.5390	3023.0	4652
2988.30	0.00320	9.56	239.333	0.99994	1.5390	2997.9	4614
2968.10	0.00505	14 99	298.799	0.99997	1 5390	2983.1	4591
2964 05	0.00727	21.55	358.028	0.99998	1.5391	2985 6	4595
2958.16	0.00989	29.26	417.012	0.99998	1 5391	2987.4	4598
2950.67	0.01291	38.09	475.618	0.99998	1.5391	2988 8	4600
2941.77	0.01633	48.04	533.872	0.99999	1.5391	2989 8	4602
2938.89	0.02007	56.98	591.716	0.99999	1.5391	2997.9	4614

The temperature-tension stringing chart (Fig. 3) is plotted from the values given in columns 28 Table II, 44 Table III, 44′ Table IV and 44″ Table V. The dotted curves for intermediate temperatures have been interpolated. The deflection curves for the temperatures − 10 deg., + 30 deg., + 60 deg. and + 100 deg. have also been drawn on the same chart so as to give an effective check on the work of stringing.

Supposing the cable is to be suspended at a temperature of + 50 deg. fahr. from towers spaced 500 ft. (152.4 m.) apart, then, according to the chart, the tension at the point of support must be 4750 lb. (2154.5 kg.) with a corresponding deflection of 10.2 ft. (3.1 m.) If the same cable at the same temperature were to be strung between towers spaced 1000 ft. (304.8 m.) apart, the tension would have to be 4650 lb. (2109.2 kg.) with a corresponding cable deflection of 42.5 ft. (12.9 m.).

The computations for the Tables I to V consisting of 44 mathematical operations for each argument are all derived from the values given in the first columns of Table I, and a slight error made in those columns might be accumulative during each succeeding step leading to appreciable errors in the last computations. It is of importance, therefore, to begin the work with great precision using hyperbolic functions of the arguments in column 1 computed accurately to the eighth decimal. Below is given a table of natural hyperbolic functions of arguments which cover the range of ordinary span designs.

$\frac{x}{c}$	$\sinh \frac{x}{c}$	$\cosh \frac{x}{c}$
0.04	0.0400106675	1.0008001061
0.06	0.0600360065	1.0018005401
0.08	0.0800853606	1.0032017064
0.10	0.1001666750	1.0050041075
0.12	0.1202882070	1.0072086441
0.14	0.1404577820	1.0098160171
0.16	0.1606835410	1.0128270898
0.18	0.1809735759	1.0162437873
0.20	0.2013360025	1.0200667556
0.22	0.2217787996	1.0242977643
0.24	0.2423106127	1.0289385057

(*C*) THE MOST ECONOMICAL TOWER SPACING

The most important item in the cost of a transmission line is the cost of the supports for the line conductors. The following investigation will disclose the physical laws which govern the magnitude of this cost item and will evolve a method of its accurate computation in connection with the determination of some important mechanical features of the line.

The total cost of the line supports is directly proportional to the cost of each support and to the number of supports used for the entire line. For a minimum total cost, the cost of each support must be a minimum and the number of supports for the line must be a minimum, that is, the tower spacing must be a maximum. As demonstrated in the preceding section the critical catenary formed by the cable will determine the required height of support with a given tower spacing or it will determine the tower spacing with a given height of support, provided, the minimum clearance between cable and ground is fixed. The catenary equation also shows that the greater the tower spacing, the greater will be the height of the support. Since the cost of the support increases with the height of the support, an increase in tower spacing increases the cost of each support and at the same time decreases the total number of supports required for the given line. Hence, the total cost of the supports will be a minimum when the tower spacing is such, that if increased by an increment, the rate of increase in cost of the support multiplied by the number of supports is equal to the rate of decrease of the total cost of the supports due to a decrease of their number.

The above analysis shows that for the determination of the minimum total cost of the line supports it is necessary to segregate the cost items which are a function of the tower spacing only, from the cost items which are a function of the tower spacing and of the height of the support. The total cost must be expressed in terms of these two groups of items so that a mathematical solution for a minimum cost may be performed.

The cost of a line support comprises the following items;

1. Cost of tower at place of erection.
2. Cost of erection of tower.
3. Cost of lease or purchase of tower site.
4. Cost of foundation installed.

5. Cost of location and inspection of line support.
6. Cost of insulators at location of tower.
7. Cost of placing insulators and cable.

Item 1. The weight of the tower is proportional to the maximum cable stress for which it is designed. For the same stress, an increase in the height of the tower increases its weight in proportion to the square of its height. Since the cost of the tower is proportional to its weight, the magnitude of item 1 is directly proportional to the square of the height of the tower.

Item 2. The given stress for which the tower is designed practically fixes the weight per unit length and the lengths of its structural members. Hence, an increase in the height of the tower increases the number of its structural members, and in consequence the cost of erection, in direct proportion to the square of the height of the tower.

Item 3. Since in practical tower design the ratio of the height of the tower to the width of its base is a constant, the area of the tower site and in consequence the cost of its lease or purchase will also vary in direct proportion to the square of the height of the tower.

Item 4. The cost of the foundation is directly proportional to the tension for which the tower is designed and is practically independent of the height of the tower.

Items 5, 6 and 7 are independent of the mechanical features of the towers and are constants for given transportation rates and market conditions of materials and labor. The magnitude of item 6 is practically proportional to the transmission voltage and is fixed for a given transmission voltage.

The above analysis of the different cost items is based on the cost of a standard three-phase transmission line.

Let L = length of the transmission line.
$2x$ = tower spacing.
h = height of tower.
k_1 = minimum clearance of cable to ground.
y = maximum deflection of the catenary formed by the cable.

The number of line supports is, then

$$N = \frac{L}{2x} \tag{66}$$

and the height of the support is

$$h = k_1 + y \tag{67}$$

The total cost of the line supports is

$$M = N\,(h^2\,k_2 + k_3) \tag{68}$$

where:

$h^2\,k_2$ = sum of items 1, 2 and 3.
k_3 = sum of items 4, 5, 6 and 7.

Substituting equations (**66**) and (**67**) into (**68**):

$$M = \frac{L}{2x}(k_2\,[k_1 + y]^2 + k_3)$$

Substituting for y in above equation its equivalent from equation (**11**),

$$M = \frac{L}{2x}\left(k_2\left[k_1 + c\left(\cosh\frac{x}{c} - 1\right)\right]^2 + k_3\right) \tag{69}$$

For a minimum or maximum cost the derivative of equation (**69**) with respect to x must be equal to zero:

$$\frac{d\,M}{dx} = 0 = \frac{L}{2}\left(\frac{2\,k_2\,c\cosh\frac{x}{c}\,\sinh\frac{x}{c}}{x} - \frac{k_2\,c^2\cosh\frac{x}{c}}{x^2}\right.$$

$$\left. - \frac{2\,k_2\,c\sinh\frac{x}{c}}{x} + \frac{2\,k_2\,c^2\cosh\frac{x}{c}}{x^2}\right.$$

$$-\frac{k_2 c^2}{x^2} + \frac{2\,k_2\,k_1 \sinh \frac{x}{c}}{x} - \frac{2\,k_2\,k_1\,c \cosh \frac{x}{c}}{x^2} + \frac{2\,k_2\,k_1\,c}{x^2}$$

$$- \frac{k_2\,k_1{}^2}{x^2} - \frac{k_3}{x^2}\Bigg)$$

$$= 2\,cx \cosh \frac{x}{c} \sinh \frac{x}{c} - c^2 \cosh^2 \frac{x}{c} - 2\,cx \sinh \frac{x}{c}$$

$$+ 2\,c^2 \cosh \frac{x}{c} - c^2$$

$$+ 2\,k_1\,x \sinh \frac{x}{c} - 2\,k_1\,c \cosh \frac{x}{c} + 2\,k_1\,c - k_1{}^2 - \frac{k_3}{k_2}$$

But $x = c \times \frac{x}{c}$, hence,

$$0 = c^2 \left(2\,\frac{x}{c} \cosh \frac{x}{c} \sinh \frac{x}{c} - \cosh^2 \frac{x}{c} - 2\,\frac{x}{c} \sinh \frac{x}{c}\right.$$

$$\left. + 2 \cosh \frac{x}{c} - 1\right)$$

$$+ 2\,ck_1 \left(\frac{x}{c} \sinh \frac{x}{c} - \cosh \frac{x}{c} + 1\right) - k_1{}^2 - \frac{k_3}{k_2}$$

Simplifying:

$$c^2 \left(\cosh \frac{x}{c} - 1\right)\left(2\,\frac{x}{c} \sinh \frac{x}{c} - \cosh \frac{x}{c} + 1\right)$$

$$+ 2\,ck_1 \left(\frac{x}{c} \sinh \frac{x}{c} - \cosh \frac{x}{c} + 1\right) = k_1{}^2 + \frac{k_3}{k_2}$$

from which

$$c = -\frac{k_1\left(\frac{x}{c}\sinh\frac{x}{c} - \cosh\frac{x}{c} + 1\right)}{\left(\cosh\frac{x}{c} - 1\right)\left(2\frac{x}{c}\sinh\frac{x}{c} - \cosh\frac{x}{c} + 1\right)}$$

$$\pm\left[\frac{\left(k_1^2 + \frac{k_3}{k_2}\right)\left(\cosh\frac{x}{c} - 1\right)\left(2\frac{x}{c}\sinh\frac{x}{c} - \cosh\frac{x}{c} + 1\right) + k_1^2\left(\frac{x}{c}\sinh\frac{x}{c} - \cosh\frac{x}{c} + 1\right)^2}{\left(\cosh\frac{x}{c} - 1\right)^2\left(2\frac{x}{c}\sinh\frac{x}{c} - \cosh\frac{x}{c} + 1\right)^2}\right]^{\frac{1}{2}}$$

Simplifying:

$$c = -\frac{k_1\left[\frac{x}{c}\sinh\frac{x}{c} - \left(\cosh\frac{x}{c} - 1\right)\right]}{\left(\cosh\frac{x}{c} - 1\right)\left[2\frac{x}{c}\sinh\frac{x}{c} - \left(\cosh\frac{x}{c} - 1\right)\right]}$$

$$\pm\frac{\left[\frac{k_3}{k_2}\left(\cosh\frac{x}{c} - 1\right)\left[2\frac{x}{c}\sinh\frac{x}{c} - \left(\cosh\frac{x}{c} - 1\right)\right] + \left(k_1\frac{x}{c}\sinh\frac{x}{c}\right)^2\right]^{\frac{1}{2}}}{\left(\cosh\frac{x}{c} - 1\right)\left[2\frac{x}{c}\sinh\frac{x}{c} - \left(\cosh\frac{x}{c} - 1\right)\right]} \qquad (70)$$

Equation (**70**) yields the minimum cost if the positive sign before the radical is used.

For a given clearance of the cable to ground k_1 and a given hyperbolic argument, equation (**70**) contains only one variable, the ratio $k_3 \div k_2$. In practical line design the possible range of this ratio falls within the limits 500 and 1500. In the following table are given the solutions of equation (**70**) for $k_3 \div k_2 = 500$, $k_3 \div k_2 = 1000$ and $k_3 \div k_2 = 1500$ with arguments ranging between 0.06 and 0.20. The results are plotted in the form of curves of Fig. 4. The dotted curves were drawn by interpolation.

$k_1 = 30$						
	$k_3 \div k_2 = 500$		$k_3 \div k_2 = 1000$		$k_3 \div k_2 = 1500$	
$\frac{x}{c}$	c	$2x$	c	$2x$	c	$2x$
0.06	7661	919	9480	1138	11104	1332
0.08	4310	690	5330	853	6243	999
0.10	2755	551	3409	682	3979	796
0.12	1913	459	2366	568	2771	665
0.14	1403	393	1736	486	2034	570
0.16	1073	343	1329	425	1555	478
0.18	847	305	1048	377	1227	442
0.20	686	274	848	339	982	393

The most economical tower spacing is the abscissa of the intersection point between the curve $k_3 \div k_2$ corresponding to the given transportation rates and market prices of material and labor, and the curve of critical catenaries for the given cable plotted in terms of $2x$ and c; provided that the minimum clearance of the cable to ground is 30 ft. (9.1 m.). For any other clearance a different set of curves has to be computed with the use of equation (**70**).

The significance of the $k_3 \div k_2$ curves obtained by equation (**70**) will now be demonstrated by the solution of the following problem:

Problem 4. What is the most economical tower spacing for the cable specified in problem 3 of section B?

The cost data of a transmission line built under practically the same as the present conditions concerning transportation charges and prices of labor and materials are as follows:

1. Cost of tower at place of erection	$190.00
2. Cost of erection of tower	30.00
3. Cost of tower site	10.00
4. Cost of foundation installed	125.00
5. Cost of location and inspection of support	13.00
6. Cost of insulators at location of tower	30.00
7. Cost of placing insulators and cable	10.00

The height of the point of cable support above the tower footing is 37 ft. (11.2 m.). The tower is designed for a tension of 6000 lb. (2721.5 kg.) per cable. The transmission voltage is 150,000.

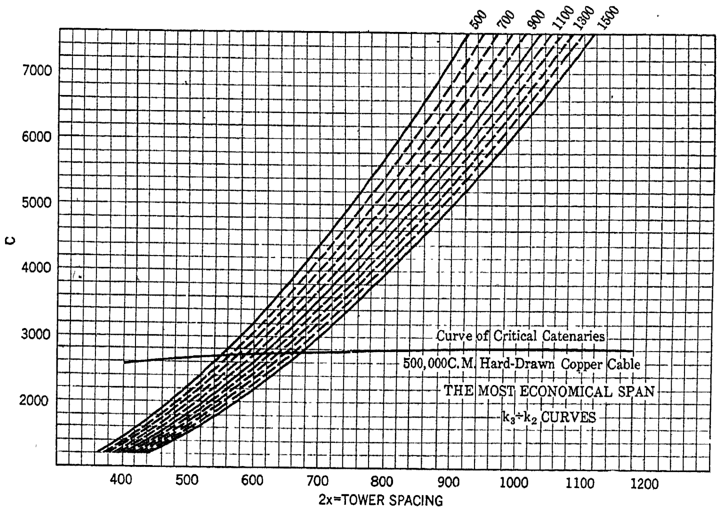

FIG. 4

The transmission voltage of the line formed by the cable specified in problem 3 is to be 110,000.

Solution. If the tower specified above were designed for a tension of 8400 lb. (3810.1 kg.). per cable, its cost would be

$$\$\ \frac{190 \times 8400}{6000} = \$266.00$$

and the cost of the foundation,

$$\$\ \frac{125 \times 8400}{6000} = \$175.00$$

For a transmission voltage of 110,000 the cost of the insulators will be approximately

$$\$\ \frac{30 \times 110{,}000}{150{,}000} = \$22.00$$

The remaining cost items are not affected by the tension and transmission voltage.

Cost data revised for different tension and voltage:

1. Cost of tower at place of erection	\$266.00
2. Cost of erection of tower	30.00
3. Cost of tower site	10.00
4. Cost of foundation installed	175.00
5. Cost of location and inspection of support	13.00
6. Cost of insulators at location of tower	22.00
7. Cost of placing insulators and cable	10.00

Determination of the constants k_1, k_2 and k_3 of equation (**70**).

k_1 = should not be less than 30.

k_2 = sum of items 1, 2 and 3 divided by the square of the height of the point of support above the tower footing.

$$= \frac{266 + 30 + 10}{37^2} = 0.2235.$$

k_3 = sum of items 4, 5, 6 and 7,

$= 175 + 13 + 22 + 10 = 220.$

Hence,

$$\frac{k_3}{k_2} = \frac{220}{0.2235} = 984$$

Interpolating the curve $k_3 \div k_2 = 984$ on Fig. 4 and plotting the curve of critical catenaries for the specified cable in terms of $2x$ and c from column 3 of Table I and column 37 of Table III, respectively, the two curves are found to intersect at the point $2x = 605$, $c = 2730$.

Hence, the most economical tower spacing for the cable and climate specified in problem 3 is 605 ft. (184.4 m.).

It will be remembered from the analyses of the preceding sections that the catenary at the minimum temperature under the most severe weather conditions is expressed by

$$\text{max. } T \div w_1 = c_1 \cosh \frac{x}{c_1}$$

The maximum tension max. T is proportional to the size of the cable of a given material, whereas the weight per unit length w_1 is not directly proportional to the size of the cable since the ice and wind load included in the term w_1 does not vary in proportion to the size of the cable. It will be found, however, that the variation of the ratio max. $T \div w_1$ decreases as the size of the cable increases, and that it will be practically negligible for large cables such as are used for long transmission lines. The influence of temperature on the length of the span is independent of the size of cable and the influence on the catenary formed by the cable due to changes of average tension along the span is also practically independent of the size of cable, remembering that the catenary changes due to changes in tension are proportional to the ratio av. $T \div w$. Therefore, the critical catenary is practically independent of the size of cable, and the curve on Fig. 4 representing critical catenaries for a copper cable in a given locality will be approximately fixed in location for any size of cable used in long transmission lines.

Since the critical catenary is a function of the range of temperature for a given locality rather than a function of the magnitude of the maximum or minimum temperature, and since the extreme range of temperature does not vary greatly for different localities in spite of the fact that the maximum or minimum temperatures may greatly differ, the critical catenary will be but little influenced by the location of the span. Of course, this generalization does not embrace span designs for the tropics and climates exclusively controlled by the trade winds.

The most extraordinary fluctuations of transportation charges and market prices of labor and materials do not influence the ratio $k_3 \div k_2$ to a very great extent since these factors jointly affect all the cost items which make up the total cost of a line support. The sensitiveness of this ratio to the influence of changes in the size of the cable is also very small since both k_2 and k_3 are affected practically in the same proportion. In comparing the magnitude of the item covering insulator cost with the total cost of the support, it will be conceded that variations in this item also have little influence on the magnitude of the ratio.

From the above it follows that for a given cable material the range of variation of the most economical tower spacing is very small, no matter what might be the size of the cable or the location of the transmission line in climates other than tropical or subtropical, or what might be the prices of commodities used in the make-up of the finished support.

Supposing a conservative estimate places the range of fluctuations of the ratio $k_3 \div k_2$ between the limits 800 and 1200, Fig. 4 will disclose the interesting fact that the most economical tower spacing for a copper cable, ranges between the narrow limits of 580 ft. (176.7 m.) and 630 ft. (192 m.) for a long transmission line.

The above analysis in connection with the curves of Fig. 4 will validate the statement that for a copper cable the most economical tower spacing is approximately 600 ft. (182.8 m.).

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DESIGN CONSTRUCTION AND TESTS OF AN ARTIFICIAL POWER TRANSMISSION LINE FOR THE TELLURIDE POWER COMPANY OF PROVO, UTAH

BY GEORGE H. GRAY

ABSTRACT OF PAPER

A description is given of an artificial power-transmission line which duplicates, in considerable detail, an actual transmission system. Each unit of the artificial line represents about ten miles of the actual line.

Methods are given of calculating the correct distribution of inductance between self and mutual, and of calculating the correct distribution of capacity between wire and wire, and wire and ground.

It is shown that this artificial system will duplicate very closely, even under extreme conditions of short circuits and grounds, many of the phenomena occurring on the actual system.

A description is given of some oscillographic tests, made on this line, of the magnitudes and phase relations of current and voltage at different points, when the line was subjected to various types of short circuits and grounds.

The data are presented in the form of vector diagrams; one for each point of the line where readings were taken. In addition to the diagrams, arrow-headed lines are shown at each station. These indicate the magnitudes and directions of power flow for each phase (as shown by star-connected wattmeters) and the total power flow.

It is shown that the total power flow is not always toward the short circuit (or ground). Hence, two-wattmeter principle relays will not always indicate toward the short circuit (or ground) and may sometimes operate the wrong switch.

With a three-phase short circuit the direction of power flow for each phase, and consequently the direction of total power flow, is always toward the short circuit.

With a two-wire short circuit, if the phase rotation is $A - B - C$, and the short circuit is between wires B and C the power flow indication of the B phase wattmeter is, at all stations, toward the short circuit. The wattmeters for the other phases may indicate away from the short circuit.

A ground on a single wire does not appreciably alter the power flow indication of any of the wattmeters, if the neutral is ungrounded.

If the neutral is grounded, the power flow indication for the grounded phase is toward the ground at all points.

If two wires are grounded at the same time, with the neutral grounded, the power flow indications are practically the same as for the two-wire short circuit.

INTRODUCTION

There are three ways in which an experimental study of problems connected with power transmission systems may be made:

first, by using the actual system; second, using an experimental line; and third, using an artificial line. By an "expermental line" is meant a short length of line of the same type of construction as the actual system. By an "artificial line" is meant one in which the constants of the actual system are obtained by means of inductances, resistances and condensers, usually lumped at frequent intervals.

The actual system would appear at first glance, to be the best place to conduct experiments, but it has certain disadvantages. In most cases it must be out of commercial service while the tests are being conducted; it is almost impossible to make simultaneous observations at different parts of the system: in case anything goes wrong there is a chance that expensive apparatus may be damaged; and lastly, there is always the possibility that a switch which is supposed to be closed may be open, and vice versa.

An experimental line is useful for studying the behavior of insulators, corona losses and similar problems, but is of no value for studying surges, voltage drop, or any problem which requires a long line.

The artificial line, however, has none of these drawbacks, and for most purposes seems to be the best of the three. Its greatest disadvantage is that, in order not to expend any great amount of power in the line, the voltage must be kept low. This eliminates the studying of such problems as corona losses, disturbances resulting from arc characteristics, etc. The following tables show the relations existing between the different quantities when a 40-kilovolt system is represented by an artificial system operating at 1000 volts:

	Kilovolts	Amperes	Kilowatts
Actual...........	40	100	20,000
Artificial.........	1	2.5	12.5

Because of their many advantages, several artificial systems have been constructed during the last few years. Most of them, however, have been built by engineering schools which, naturally, have not been interested in duplicating any particular system. Each artificial line, therefore, has been uniform throughout its entire length and has been built to represent a single line (or two parallel lines) of some standard type of construction.

The line described here is of a different character, and is, it is believed, the only artificial system which duplicates the whole, or the greater part of, an actual system. The line was built about six years ago by the Telluride Power Company (now a part of the Utah Power & Light Co.); a hydroelectric concern which operated, at that time (besides other divisions), about 500 miles of 44-kv. transmission line in northern Utah. This whole

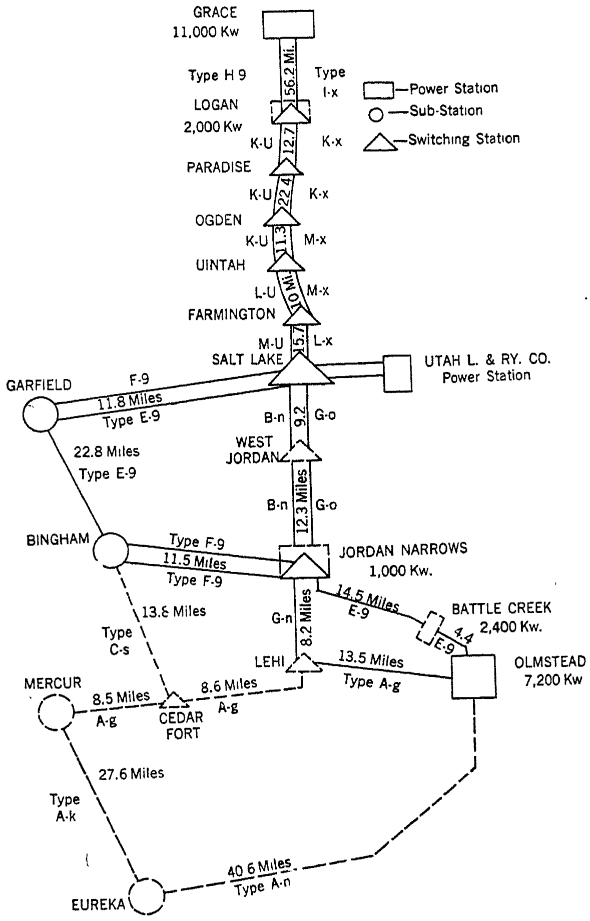

FIG. 1

Utah division, with the exception of one unimportant loop, was duplicated in the artificial system.

The artificial line was originally planned in 1908 by Mr. Edward Bennett (now Professor of Electrical Engineering in the University of Wisconsin). His original plans and computations were used later by Mr. L. N. Crichton and the author, who took

TABLE I.—TRANSMISSION LINE DATA.

Stations.	Miles	Type	Spacing. (Inches)		Mater-ial	Size Wire.	Equiv. Diameter. (Inch)	Lower wire to ground (Average.)	Constants per mile calculated								Number of units.	Assumed Dist. bet. stations	Miles rep by each unit.	Constants per unit.								Resistance per coil (average)	External Resistance (approx)
									Ind., 10^{-3} Henrys.			Capacity, 10^{-8} Farads.				Resist. per wire 10° C.				Ind., 10^{-2} Henrys			Capacity, 10^{-8} Farads.				Resist. per wire 10° C.		
			Side to Side	Center to Side					Self per wire.	Mutual Side to side	Mutual Cent. to side	Centre to Ground	Centre to side	Side to Ground	Side to side					Self, per wire.	Mutual Side to side	Mutual Cent. to side	Centre to Ground	Centre to side.	Side to ground	Side to side			
Grace to Logan, East	56.2	I–x	108	61¾	Cu.	7 st. No. 8	.363″	30′	2.70	.55	.73	.645	.305	.719	.188	.451	6	56.2	9.37	2.53	.52	.68	6.04	2.86	6.73	1.77	4.23	2.95	1.3
“ “ “ West	56.2	H–9	108	108	Cu.	No. 2	2576″	25′	2.80	.55	.55	.704	.204	.750	. 189	.777	6	56.2	9.37	2.62	.52	.52	6.59	1.91	7.03	1.78	7.27	3.00	4.25
Logan to Ogden, East	34.5	K–x	276	138	Cu.	7 st No. 8	.363″	35′	2.70	.25	.47	.724	217	.791	.114	.451	4	34.5	8.63	2.33	.22	.41	6.24	1.87	6.83	.98	3.89	2.80	1.10
“ “ Uintah, West	47.4	K–u	276	138	Al.	7 st. .16″	.451″	35′	2 64	.25	.47	.736	.228	.810	.115	.461	5	47.4	9.48	2.50	.24	.45	6.97	2.16	7.67	1.09	4.36	3.00	1.35
Ogden to Farmington, East	21.9	M–x	288	144	Cu.	7 st. No. 8	.363″	35′	2.70	.25	.47	.720	.214	.801	.099	.451	2	21.9	10.95	2.96	.27	.52	7.88	2.34	8.76	1.08	4.94	3.20	1.75
Uintah to “ West	9.0	L–u	2 Ground Wires 276	138	Al.	7 st. .16″	.451″	35′	2.64	.25	.47	.941	.165	.976	.070	.461	1	9.0	9.0	2.38	.23	.42	8.47	1.49	8.79	0.63	4.15	2.85	1.30
Farm. to Salt Lake, East	15.7	L–x	2 Ground Wires 276	138	Cu.	7 st. No. 8	.363″	35′	2.70	.25	.47	.912	.158	.955	.060	.451	2	16.0	8.0	2.16	.20	.38	7.30	1.27	7.64	0.48	3.61	2.70	.90
“ “ “ “ West	15.7	M–u	288	144	Al.	7 st. .16″	.451″	35′	.264	.25	.47	.733	.226	.819	.108	.461	2	16.0	8.0	2.12	.20	.38	5.86	1.81	6.55	0.87	3.69	2.70	1.00
S. Lake to Jor. Narrows, E.	21.5	G–o	72	60	Cu.	No. 4	.2043″	25′	2.88	.68	.73	.649	.251	.686	.214	1.24	2	20.0	10.0	2.88	.68	.73	6.49	2.51	6.86	2.14	12.4	3.15	9.25
“ “ “ “ W.	21.5	B–n	76	76	Al.	7 st. .0985″	.278″	25′	2.80	.67	.67	.682	.243	.714	.232	1.26	2	20.0	10.0	2.80	.67	.67	6.82	2.43	7.14	2.32	12.6	3.10	9.50
Jor. Nar. to Olmsted	18.9	E–9	76	74.5	Cu.	No. 2	.2576″	25′	2.80	.67	.67	.672	.241	.705	.229	.777	2	18.9	9.45	2.65	.63	.63	6.35	2.28	6.66	2.17	7.35	3.05	4.30
“ “ “ Lehi	8.2	G–n	72	60	Al.	7 st. .0985″	.278″	25′	2.80	.68	.73	.656	.273	.703	.229	1.26	1	10.0	10.0	2.80	.68	.73	6.56	2.73	7.03	2.29	12.6	3.10	9.50
Lehi to Olmsted	13.5	A–g	76	76	Cu.	No. 5	.1819″	25′	2.91	.67	.67	.656	.222	.691	.209	1.56	1	11.0	11.0	3.20	.74	.74	7.22	2.44	7.60	2.30	17.15	3.40	13.7
S. Lake to Garfield, South	11.8	E–9	76	74 5	Cu.	No. 2	.2576″	25′	2.80	.67	.67	.672	.241	.705	.229	.777	1	11.8	11.8	3.30	.79	.79	7.93	2.84	8.31	2.70	9.18	3.40	5.80
“ “ “ North	11.8	F–9	76	61	Cu.	No. 2	.2576″	25′	2.80	.67	.73	.654	.267	.701	.217	.777	1	11.8	11.8	3.30	.79	.86	7.71	3.15	8.27	2.56	9.16	3.40	5.80
Garfield to Bingham	22.8	E–9	76	74.5	Cu.	No. 2	.2576″	25′	2.80	.67	.67	.672	.241	.705	.229	.777	2	22.8	11.4	3.19	.77	.77	7.66	2.75	8.04	2.61	8.85	3.40	5.45
Bingham to Jor. Nar. South	11.5	F–9	76	61	Cu.	No. 2	.2576″	25′	2.80	.67	.73	.654	.267	.701	.217	.777	1	11.5	11.5	3.22	.77	.84	7.52	3.07	8.06	2.50	8.95	3.40	5.55
“ “ “ “ North	11.5	F–9	76	61	Cu.	No. 2	.2576″	25′	2.80	.67	.73	.654	.267	.701	.217	.777	1	11.5	11.5	3.22	.77	.84	7.52	3.07	8.06	2.50	8.95	3.45	5.50
“ “ Cedar Fort	13.8	C–s	76	76	Al.	7 st. .138	.370″	25′	2.69	.67	.67	.692	.261	.730	.249	.619													
Lehi “ “ “	8.6	A–g	76	76	Cu.	No. 5	.1819″	25′	2.91	.67	.67	.656	.222	.691	.209	1.56													
Mercur to “ “	8.5	A–g	76	76	Cu.	No. 5	.1819″	25′	2.91	.67	.67	.656	.222	.691	.209	1.56													
“ “ Eureka	27.6	A–k	76	76	Al.	7 st. .088″	.248″	25′	2.83	.67	.67	.671	.238	.705	.226	1.56													
Olmsted to “	40.6	A–n	76	76	Al.	7 st. .0985″	.278″	25′	2.80	.67	.67	.682	.243	.714	.232	1.26													

[GRAY]

up the work and completed the line during 1911, after making quite extensive additions to the plans.

The line was built mainly with the idea of using it to study the operation of some sectionalizing relays which were being developed by the company. These tests, however, are the subject of another paper being prepared by Mr. Crichton, and hence will not be treated here. Only the details of the line and some tests in which the relays were not used will be discussed.

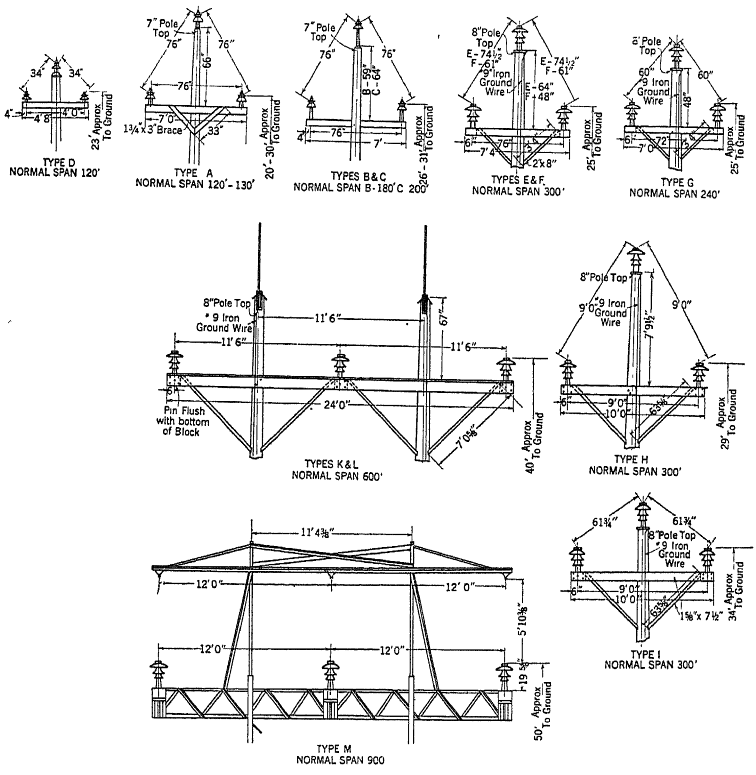

FIG. 2—TYPES OF 40,000-VOLT LINE CONSTRUCTION—THE TELLURIDE POWER COMPANY

The general layout of the Utah division of the Telluride Power Company in 1911 is shown by Fig. 1. The total generating capacity was a little under 24,000 kw., most of which was at the two ends of the system (11,000 kw. at Grace and 7,200 kw. at Olmsted). The load averaged about 14,000 kw. The total length of the lines was a little over 500 miles, covering about 340

miles of right of way. About 410 of the 500 miles were represented in the artificial system.

In Fig. 1 the duplicated part of the system is shown in full lines and the balance in dotted lines. The duplicated part, as will be seen, consisted of the Grace and Olmsted power stations, the parallel trunks from Grace to Olmsted through Salt Lake City (about 160 miles), the loop from Jordan Narrows through Bingham and Garfield to Salt Lake City (double except between Bingham and Garfield), the substations at Bingham and Garfield and the connection with the Utah Light & Railway Company at Salt Lake City. The lengths of the lines, the size and spacing of conductors, and other such data are given in Table I. Fig. 2 shows the different types of line construction.

DESIGN AND CONSTRUCTION

Transmission Line

The features of the transmission line which must be duplicated or approximated are:

Distributed inductance and capacity.
Leakage currents.
Electromagnetic coupling of phases.
Electrostatic coupling of phases.
Resistance of line conductors.

Distributed Inductance and Capacity. The most feasible way in which the large inductance and capacity of the transmission line may be obtained within the limits of an ordinary room is by the use of coils of wire and plate condensers. This, of course, does not result in distributed constants, but it has been shown that, for the frequencies used for the transmission of power and for the harmonics ordinarily encountered with those frequencies, an artificial line constructed with the capacities lumped at frequent intervals represents, to a very close degree of approximation, the actual line.

For example, if the spacing is such that there are ten condensers per wave length, most of the phenomena will be duplicated within 1 per cent. Since the ordinary 60-cycle power-transmission line has a wave length of about 3000 miles, a spacing of 300 miles is allowable. Even for the ninth harmonic the spacing may be as great as 33 miles without serious error. As this artificial line was composed of units, each representing only about ten miles of transmission line, errors due to lumping may be considered negligible.

Each unit consisted of three air-core inductance coils, electromagnetically interlinked, to represent inductive effects, with condensers connected between the coils, and also between each coil and ground, to represent capacity effects. In addition, resistance wire was used in series with each coil to represent the line resistance.

Leakage Currents. The resistance between each line wire and ground on the actual system was found to be of the order of 300 megohms per mile under average conditions. On a 40-kv. system, 500 miles long, having a peak load of 14,000 kw. and current per wire of 250 amperes, this means a total leakage current of only 0.038 amperes per wire, and a total watt expenditure of only about three kilowatts. These are negligible quantities and no attempt was made to approximate them on the artificial system. The resistance of the condensers from line to ground was about 300 megohms per ten-mile unit, or 3000 megohms per mile, when cold.

Electromagnetic Coupling of Phases. When the three conductors of a three-phase line are arranged at the vertices of an equilateral triangle, and the currents in the three phases are balanced, the sum of the currents in any two wires is equal to the current in the third wire, and flows in the opposite direction. Therefore, the only magnetic flux which is effective in producing the inductance of any wire is the flux due to the current in that wire and which lies between it and the other two wires. This flux, in absolute units, is

$$2\,I\,l\left(\log_e \frac{d}{r} - \frac{1}{4}\right) \qquad (1)$$

in which l = length of wire in centimeters.
d = distance between conductors.
r = radius of conductor.
I = current in absolute units.

The usual practise in line calculations is to consider this magnetic flux as causing the self inductance of the wire, thus eliminating the mutual inductance between wires or phases from the calculations; in which case the self-inductance of each wire, to neutral, is

$$L = 2\,l\left(\log_e \frac{d}{r} - \frac{1}{4}\right) \qquad (2)$$

If the system were always balanced, it would be satisfactory to represent the three line wires in the artificial system by three separate coils, each having a self-inductance as determined above and arranged so as to have no mutual effects upon each other. It is evident, however, that such an artificial line would fail to represent unbalanced conditions. For instance, if the artificial system were operated with grounded neutrals and a heavy short circuit should occur between one wire and neutral, it would have no effect upon the voltage between the other wires and neutral; whereas, owing to the mutual inductance between conductors on the actual system, this voltage should be affected.

It seemed advisable, therefore, to use coils having the correct inductance to represent the self-inductance of the wires, and to arrange these coils so that the correct mutual effects would be obtained between them. This necessitated the determination of the actual distribution of self and mutual inductance.

A method of doing this, which seemed at first to have some merit, was to use the formula* for the self-inductance of a single straight conductor. This formula is

$$L_S = 2\,l\left(\log_e \frac{2\,l}{r} - \frac{3}{4}\right) \qquad (\mathbf{3})$$

The corresponding formula for mutual inductance is

$$L_M = 2\,l\left(\log_e \frac{2\,l}{d} - 1\right) \qquad (\mathbf{4})$$

where l, d and r have the same significance as in formula (**1**). However, in the derivation of formula (**4**) all of the flux, due to wire 1, between d and infinity, is considered to be effective in producing the mutual inductance; while, in the actual system, in a case like the one cited above (with the grounded wire) only the flux between d and the path of the current in the ground would be effective. Another objection to this method was that a large amount of copper would be required to produce the large self-inductance. As mutual inductive effects were to play, at most, only a small part in the operation of the line, it did not seem wise to go to this extra expenditure for copper.

Hence it was decided to approximate, as closely as possible, the distance from the wire to the path of the return current in the ground. The accurate calculation of this distance was an almost impossible problem because of the unknown and variable resistances of the different layers of soil. Fifty feet was finally selected as this distance, and all of the calculations of self-inductance and mutual inductance were made on the assumption that the three wires had a common return, located fifty feet from the conductors.

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*See Bulletin of the Bureau of Standards, Vol. IV, page 301. "The Self and Mutual Inductances of Linear Conductors," by E. B. Rosa.

The self-inductance of each wire was then found from the formula

$$L_S = 2\,l\left(\log_e \frac{50 \times 12}{r} + \frac{1}{4}\right)$$

and the mutual inductance from:

$$L_M = 2\,l\left(\log_e \frac{50 \times 12}{d}\right)$$

The values given by these three methods for ten-mile sections (the average length of the sections used) of No. 2 solid copper, spaced nine feet on a side, are given in the following table.

	Inductance of 10 miles. (10^{-2} henrys)		
	L_S	L_M	$L_S - L_M$
Neglecting mutual inductance..	2.25	0.00	2.25
Return at infinite distance.....	4.95	2.70	2.25
Return 50 feet from wires......	2.80	0.55	2.25

The resulting inductance for balanced conditions is seen to be the same, regardless of the method used, but the other values vary widely in the different methods. The first and second methods may be considered as setting lower and upper limits, respectively, for the self and mutual inductances. The true values lie somewhere between and probably not very far from those given by the third method.

Reducing these formulas (assuming the return 50 feet away) to practical units, we obtain, (for the inductance of each wire to neutral)

$$L_S = 0.322 \times 10^{-3}\left(2.303 \log_{10} \frac{50 \times 12}{r} + \frac{1}{4}\right) \text{ henrys per mile.}$$

and

$$L_M = 0.742 \times 10^{-3} \log_{10} \frac{50 \times 12}{d} \text{ henrys per mile.}$$

From these formulas the self and mutual inductances were calculated for each different type of construction. These values are given in Table I, in the column headed "Constants per Mile, Calculated".

In calculating the inductance and capacity for the stranded wires, a value midway between the diameter of the stranded wire and that of a solid wire having the same cross section was used for the diameter. This is the value given in Table I in the column headed "Equivalent Diameter".

The line was divided into sections, each approximately ten miles in length. The values for the self and mutual inductances of each of these sections are given in Table I, under the heading "Constants per Unit".

In proportioning the coils, Stefen's Formula* for the self-inductance of coils was used, and the measured values of the self-inductance were found to agree with the computed values to about one half of one per cent.

The coils were made about six inches in mean diameter, and were wound with No. 17 double-cotton-covered copper wire. The number of turns varied from 360 to 435. A further means of adjustment was provided by bringing out several taps on each coil. The coils were wrapped with one layer of half-lap lin-o-tape, but were not shellacked.

The three coils to represent one section were then mounted in a wooden frame, as shown in Figs. 5 and 6. The correct self-inductance was obtained by means of the taps referred to. The correct mutual inductance between "side wires" was obtained by adjusting the coils in the horizontal slots; and that between the "center wire" and the "side wire" by adjusting the center coil in the vertical slot.

The range of inductance is illustrated by the values given in Table II for a few of the coils.

Tests showed that the coils could carry 2.2 amperes continuously with a 40 degree rise, and that 3.5 amperes could be carried for 15 minutes without dangerous heating. This current (3.5 amperes), at 1000 volts, corresponds to a current on the 44-kv. system of 154 amperes, which was never exceeded for any length of time.

Electrostatic Coupling of Phases. The only arrangement which will exactly represent, under all conditions, the capacity effects between the wires of a transmission system and ground, is the exact geometrical equivalent, on a small scale, of the line and earth immersed in a dielectric of very high inductive capacity. To keep such a line down to a reasonable length, a dielectric would be needed having a specific inductive capacity of 5000 or more. No such dielectric exists, so an arrangement was adopted in which properly proportioned plate condensers were connected between the conductors, and between conductors and ground, as shown in Fig. 3A. This arrangement, as will be seen later, approximates very closely the true distribution of capacity.

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*Bulletin of the Bureau of Standards, Vol. 5 No. 1. " Formulas and Tables for the Calculation of Mutual and Self Inductances of Coils" by Rosa and Cohn.

TABLE II.

Coil data. Inductance in 10^{-2} henries.

Inst. No.	Coil No.	L_1	L_2	L_3	L_4	L_5	L_6	R_6
53	15	2.57	2.61	2.66	2.71	2.78	2.83	3.04
	44	2.57	2.61	2.66	2.71	2.78	2.83	3.13
	61	2.56	2.61	2.66	2.71	2.78	2.83	3.14
54	47	2.56	2.60	2.65	2.70	2.76	2.84	3.06
	65	2.58	2.61	2.66	2.70	2.76	2.83	3.19
	64	2.56	2.60	2.65	2.70	2.76	2.84	3.18
55	128	2.83	2.89	2.95	2.99	3.06	3.17	3.29
	129	2.85	2.90	2.96	3.00	3.07	3.17	3.29
	131	2.83	2.88	2.94	2.98	3.05	3.15	3.28
56	114	3.00	3.05	3.11	3.15	3.22	3.31	3.43
	111	3.01	3.07	3.14	3.18	3.25	3.36	3.43
	113	2.97	3.02	3.09	3.12	3.20	3.30	3.43
57	117	2.93	3.00	3.06	3.10	3.18	3.29	3.42
	112	2.95	3.01	3.07	....	3.19	3.29	3.47
	115	2.95	3.02	3.08	3.12	3.20	3.30	3.40
58	120	2.78	2.83	2.89	2.93	3.01	3.12	3.41
	121	2.78	2.83	2.89	2.93	3.01	3.11	3.40
	126	2.81	2.87	2.94	2.98	3.05	3.15	3.41

The capacities of these condensers were calculated as follows:

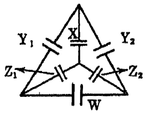

Fig 3A

Since the earth may be treated as an infinite equipotential plane, the actual transmission system (consisting of the earth's surface, conductors and ground wires) is the exact duplicate of the upper half of the electrical distribution which would be found between the actual conductors, with their charges, and the images of those charges with reference to the earth's surface.

Replacing the equipotential plane, then, by the images of the conductors and their charges, the potential above ground of any wire (say No. 1) is found to be

$$V_1 = 2q_1 \log_e \frac{D_1}{r_1} + 2q_2 \log_e \frac{D_2}{d_2} + 2q_3 \log_e \frac{D_3}{d_3} + \ldots\ldots$$

in which there is one term for each wire. q_1, q_2, q_3, etc. represent the quantities per unit length on the various conductors. r_1 is the radius of wire No. 1. D_1, D_2, D_3, etc. are the distances from conductor No. 1 to the images, and d_1, d_2, d_3, etc. are the distances from No. 1 to the other conductors. (For the derivation of this formula, see Appendix A).

By obtaining as many of these equations as there are unknown quantities q, solving for the q's, the quantity q on any conductor (and its phase relation) can finally be expressed in terms of the potentials of the conductors.

These calculations were made for wires No. 1 and No. 2, with the system ungrounded and for wire No. 1 with the center wire grounded, assuming the voltages to remain balanced in both cases.

Letting x, y, z and w represent the values of the capacities (connected as shown in Fig. 3A) the displacements CV across the three condensers connected to one wire were added (vectorially) and the sum put equal to the quantity q_1 or q_2 found by calculation for that wire. From these equations the values for the condensers x, y, z, and w were easily found.

These condenser values were calculated for each different type of construction, and are given in Table I, under the heading "Constants per Mile, Calculated". The complete set of calculations for one type is given in Appendix B.

The following values are taken from the calculations in Appendix B. They represent, in 10^{-8} coulombs, the quantities on the different conductors per mile; assuming that the voltages to ground are: $A \sin \alpha$, $A \sin (\alpha - 120 \text{ deg.})$, and $A \sin (\alpha - 240 \text{ deg.})$, for the three conductors.

Left	$1.339\, A \sin (\alpha + 0.55 \text{ deg.})$
Center	$1.316\, A \sin (\alpha - 120 \text{ deg.})$
Right	$1.339\, A \sin (\alpha - 240.55 \text{ deg.})$

Since the potential of the left conductor to ground is $A \sin \alpha$ and that of the center conductor $A \sin (\alpha - 120 \text{ deg.})$, the capacities of these conductors are seen to be 1.339 and 1.316 $\times 10^{-8}$ farads per mile. The usual formula, $C = \dfrac{l}{2 \log_e \dfrac{d}{r}}$, gives the capacity as 1.329×10^{-8} farads per mile. This is between the true values given above. (1.339 and 1.316), and is very nearly equal to the average of the capacities of the three conductors.

How closely this artificial arrangement approximates the actual conditions is seen by considering several extreme cases. For example, if both outside conductors are earthed, the capacity of the center conductor to earth is 1.112×10^{-8} farads. In the artificial system it is 1.112×10^{-8}. If all three conductors are connected together, their capacity to earth is 2.209×10^{-8} farads. In the artificial system it is 2.204×10^{-8}. The capacity of the right conductor to earth, the other two being insulated, is 1.058. The artificial system gives 1.059. For the center conductor the values are 1.023 and 1.024.

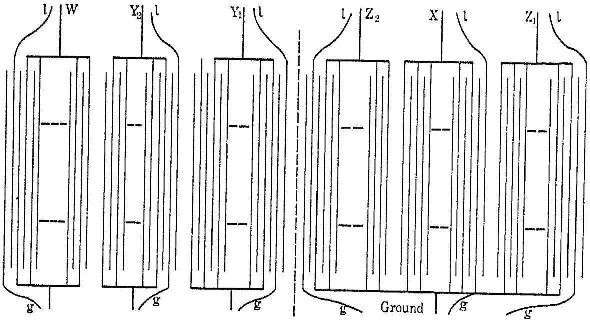

FIG. 3—CONSTRUCTION OF CONDENSERS X, Z_1, Z_2, W, Y_1, Y_2 CONNECTED AS SHOWN TO LEFT— l AND g REPRESENT EXTRA PLATES USED IN ADJUSTING

The condensers consisted of lead foil plates, separated by three thicknesses of ordinary letter paper (Colorado Bond.) Extra sheets were added between the different groups and a ¼ in. pine board was used between the line-to-line and the line-to-ground groups.

The plates were of 0.0015-in. lead foil, each 4 in. by 8 in., with a tab for connecting alternate plates together. A few plates were left unconnected at the end of each group, to be used in adjusting the condenser to the correct value.

Each condenser was tested at 2000 volts for one minute. They will operate continuously at 1000 volts, 60 cycles, without dangerous heating.

Fig. 3 shows, diagrammatically, the construction of the condensers.

The capacity of each group of plates and the possible adjustment obtainable with the extra end plates, is shown by Table III for a few of the condenser units.

TABLE III.

Condenser data. Capacities in 10^{-8} farads

Cond. No.	56	57	58	59	60
With stand number.	36	27	52	26	40
Z_1	8.01	6.66	6.76	7.09	7.86
$Z_1 + g$	8.01	6.88	6 76	7.37	8.10
$Z_1 + g + 1$	8.68	7.11	7.36	7.57	8.34
$Z_1 + 1$	8.33	6.66	6.91	7.09	7.86
X	7.85	6 55	6.34	6.41	7.12
$X + g$	7.85	6.55	6.34	6.41	7.27
$X + g + 1$	8.55	7.05	6.91	6.98	7.53
$X + 1$	8.29	6.90	6.71	6.80	7.21
Z_2	8.68	6.78	6.71	6.80	7.65
$Z_2 + g$	8.68	7.02	7.06	7.09	7.86
$Z_2 + g + 1$	9.30	7.29	7.30	7.34	8.08
$Z_2 + 1$	9.01	6 78	6.71	6.80	7.65
Y_1	1.84	1.84	1.83	1.98	1.89
$Y_1 + g$	1.95	1.84	1.83	1.98	1.97
$Y_1 + g + 1$	2.19	2.30	2.31	2.52	2.21
$Y_1 + 1$	1.84	2.10	2.04	2.28	1.89
Y_2	2.15	1.84	1.89	2.02	1.82
$Y_2 + g$	2.25	2.07	2.14	2.32	1.92
$Y_2 + g + 1$	2.62	2.22	2.28	2.43	2.14
$Y_2 + 1$	2.36	1.84	1.89	2.02	1.82
W	0.62	1.67	1.89	1.74	0.60
$W + g$	0.86	1.89	1.89	1.74	0.81
$W + g + 1$	1.08	2.07	2.45	2.28	1.04
$W + 1$	0.62	1.67	2.18	2.02	0.60

Resistance of Line Conductors. The resistance of the inductance coils was approximately three ohms, or 0.3 ohm per mile. Therefore, by connecting an additional resistance in series with the coils, any conductor up to, and including number 000 copper could be represented. This was larger than any in use on the system. These external resistances were made of such a value that the combined resistance of the coil and external resistance, when carrying normal current, was equal to the resistance of the corresponding section of the actual line at the average temperture, 20 deg. cent.

The external resistances were of No. 18 Nichrome wire, wound in the form of spiral springs about $\frac{5}{8}$ in. in diameter. Each coil was connected between a binding post of the coilstands and the correct tap of an inductance coil, as shown in Fig. 5.

These coils, or "spirals," were found to have only a very small inductance, (less than the difference between adjacent taps of the coils) so that their effect on the inductance was negligible.

The resistances of the inductance coils shown in Table II are given in the column headed R_6. These are the total resistances, at 20 deg. cent, not the resistance of the particular tap which was used. Table I, column headed "External Resistance, (Approx.)" shows approximately the amount of resistance required per line wire per section. This is only an average value, since the resistance of the different coils in the same section was found to be different.

Assembly of Line. Fig. 5 shows the method of assembling the units. The condensers were tapped in between the resistance and the inductance of each unit, and placed to one side of the coilstands.

Tests were made to determine the mutual inductance between units, and also the effect on the inductance of the iron straps with which the condensers were bound. Both of these were found to be negligible.

The boards shown between tiers were inserted in order to facilitate the removal of any unit for repairs or alterations.

TRANSFORMERS

The artificial system was operated at about 1000 volts from the 230-volt, three-phase, 60-cycle laboratory mains, by means of two banks of step-up transformers; one connected to the Grace and the other to the Olmsted station of the artificial system. Step-down transformers were used at the three substations and at the Salt Lake station two V-connected potential transformers were also used.

The actual transformers were connected Δ-Y at Grace and Salt Lake and Δ-Δ at Olmsted. As only one three-phase source was available for the artificial system, these two different connections could not be used. The Olmsted transformers were, therefore, connected Δ-Y, to correspond with the connections at the other two stations. Taps were provided at the proper points on the high-tension side to adapt them to Δ-Δ operation, in case a second supply should become available. Additional taps were provided on the low-tension side of all transformers.

Two series transformers were used in the lines of the artificial system at each of the three substations, to supply current to the relays.

GENERATORS

The generators were, as stated before, the supply mains of the laboratory. It was thought advisable, however, to provide a gradual decrease in the short-circuit current, similar to that brought about in the actual system by the demagnetizing action of the short-circuit currents on the generator fields. To accomplish this, the "artificial generators" shown in Fig. 5 were constructed, and connected to the system at Grace and Olmsted and at the Salt Lake substation. Each generator consisted of three coils, one in series with each line wire. An iron core was held by a spring above each coil and with its end just entering the coil. When the line current became excessive, the core was pulled into the coil, thus considerably decreasing the current. The desired rate of decrease was obtained by means of the dashpots shown. When the disturbance was removed the core returned automatically to its former position.

LOAD

The loads at Bingham and Garfield were taken care of by the shop motors in combination with some carbon lamps. A power factor and load at each place equivalent to those on the actual system could thus be obtained.

The situation at the Salt Lake substation was rather peculiar. The Utah Light and Railway Company bought considerable power at that point, but also ran their generators in parallel with those of the Telluride system. Thus, under short-circuit conditions, power would sometimes flow back into the system. This situation was met in the artificial system by raising the voltage on the low-tension side of the Salt Lake transformers and connecting them to the supply mains. Thus power was made to circulate around the system, coming in at the Grace and Olmsted stations and flowing out at Salt Lake. Wattmeter measurements made during one of the tests showed a load of 2.1 kw. at Salt Lake with only 0.3 kw. coming from the laboratory mains. Of course, with a short circuit near the substation, the direction of power flow was reversed.

SWITCHBOARD

The artificial system was controlled almost entirely by means of the switchboard shown in Figs. 4 and 5. On the front of this board was painted a map of the system, giving the location of each switch, power station, substation, etc. The high-tension windings of the transformers and the ends of the transmission line sections were brought to their proper places on the back of this board, terminating in small, spring clips. The switches were of the plug type and consisted of wooden blocks with three spikes driven into each. By inserting these from the front of the board the desired clips could be connected together behind the board. All high tension was thus kept behind the board, which made the operation of the line much safer. A hole drilled into each spike, where it entered the wooden block made possible the insertion of plugs (attached to lamp cord) for obtaining the voltage at any desired point. To obtain the current, plugs with split tips were used. Thus, instead of passing directly from clip to clip through the plug the current could be led out through an ammeter and back to the other side of the plug tip. The same two types of plugs were used for obtaining current and voltage for oscillograph work. Shelves, as shown in Fig. 4, could be readily attached to the board, for holding meters or other apparatus.

All of the wiring behind the board was done with No. 19 three-conductor telephone wire, having red, yellow, and green tracers. As the corresponding wires on the front of the board were painted the same colors, any trouble could be traced very easily.

The neutrals of all of the condensers were connected together, and used as the ground of the system.

Pilot lamps and ammeters were placed in the low-tension side at each station, to indicate, roughly, the load and voltage conditions.

In Fig. 4 a three-pole switch, with three braided leads attached, is shown to the right and above the Salt Lake substation. By connecting either two or three of these leads to any desired point, and closing the switch, a single or three-phase short circuit could be thrown on the system at that point.

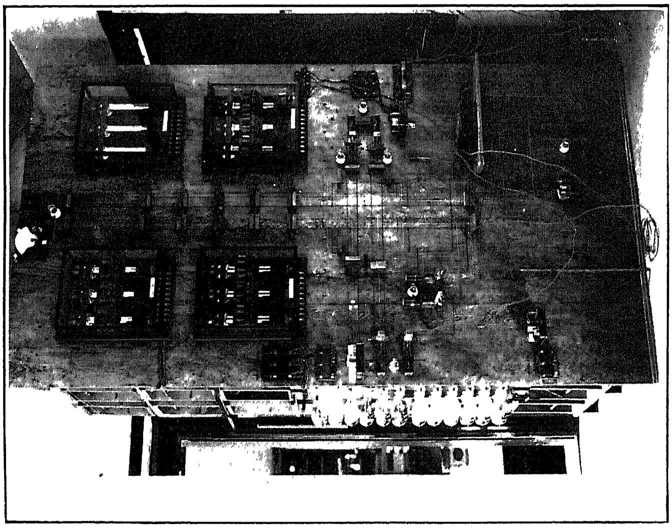

FIG. 4—FRONT VIEW OF ARTIFICIAL LINE

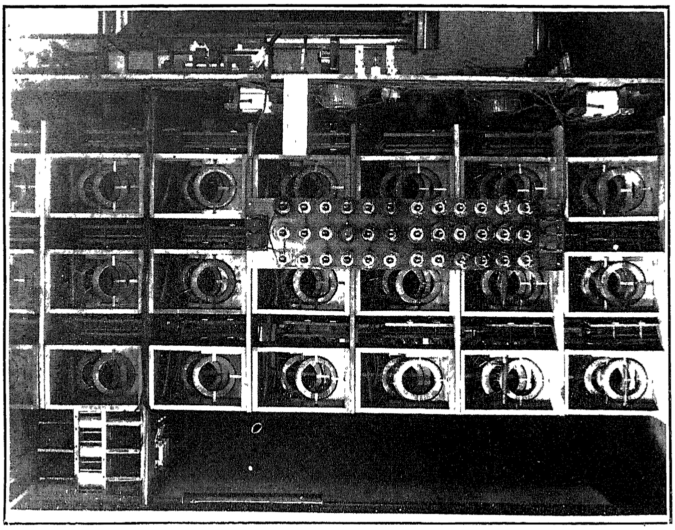

FIG. 5—SIDE VIEW OF ARTIFICIAL LINE

[GRAY]

PLATE XXXIV.
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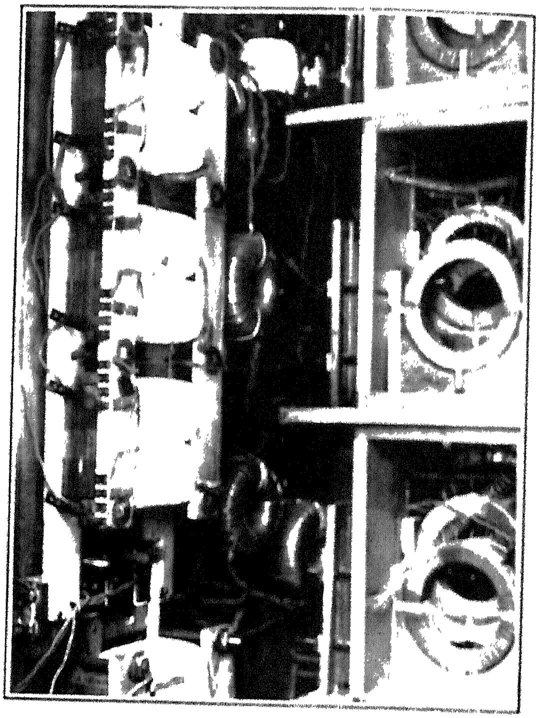

FIG. 6—TRANSFORMERS [illegible]

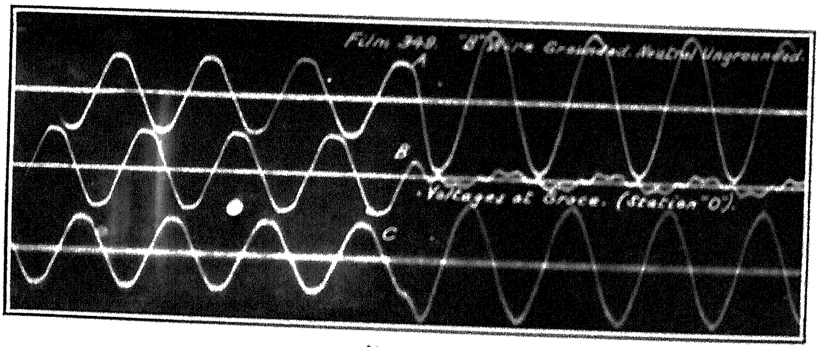

FIG. 8A [GRAY]

A ground could be studied by means of the grounding device shown below the short-circuiting switch. This consisted of three carbon arcs, the front carbons of each being connected to ground (the condenser neutrals), and the back, by means of lamp cord and a plug, to any wire at the point where it was desired to have a ground. Either a solid or an arcing ground could thus be represented, and the character of the arc could be varied by changing its length or by using different electrode materials.

The switches at Bingham, Garfield, and Salt Lake were made automatic and were worked from the relays, previously referred to, by means of a trigger and spring.

TESTS ON ARTIFICIAL LINE

Description of Tests

On a power-transmission system where each line is a unit in itself, simple overload relays are, in general, satisfactory for tripping off grounded or short-circuited lines. If the lines are interconnected, however, as in a network, some other device must be employed in order that only the section upon which the trouble is located may be disconnected. For this purpose some form of differentially selective relay is generally used. All such relays depend, for their action, upon conditions in the power lines, which show the direction, and in some cases the approximate location, of the disturbance.

Probably the most common form of differentially selective relay is the wattmeter relay. This is provided with two current and two potential coils, and is connected to the circuit according to the well known "two-wattmeter method" for measuring power. Two wattmeters, it can be shown, will correctly measure the total amount of power flowing in a three-wire circuit and will also indicate the direction in which that power is flowing. Hence, if a "two-wattmeter relay" is properly connected to a circuit its moving element should swing in one direction or the other according to the direction in which power is flowing in that circuit.

It is generally assumed that, with a short circuit in a given section of a network, power is flowing toward the short circuit from both ends of that section. Therefore two-wattmeter relays located at the ends of the section should indicate toward the short circuit, and if combined with suitable overload devices' should disconnect that section of line.

A little consideration will show, however, that it might be possible for the short-circuit power passing a given point toward a single-phase short circuit to be less than the load power flowing in the other direction, over the other two phases. Under these conditions a two-wattmeter relay placed at that point would indicate *away* from the short circuit and, if it operated at all, would operate the wrong switch.

This is something which has frequently happened in practise, and it was also verified by preliminary tests upon the artificial line. It seemed desirable, therefore, to determine the actual phase relations and magnitudes of the currents and voltages at several points along a line, under different conditions of short circuits and grounds, in the hope that such tests would bring out a method of connecting relays so that they would in all cases indicate toward the disturbance.

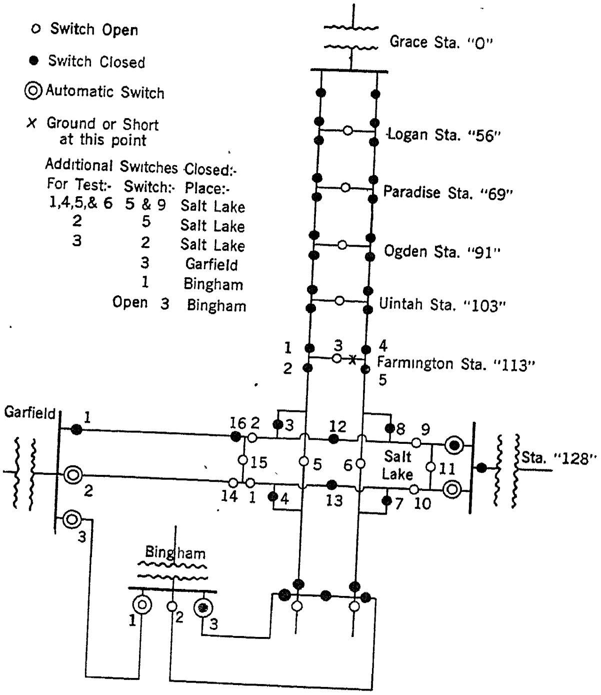

FIG. 7—CONNECTIONS FOR SHORT-CIRCUIT AND GROUND TESTS

For such tests the artificial line, combined with an oscillograph, was admirably suited, as simultaneous observations could readily be made at any stations desired. To simplify matters, only a part of the artificial line was used, connected as shown in Fig. 7. For all of the tests power was supplied through the Grace transformers and the two lines between Grace and Salt Lake were used, paralled at both ends. The load was in all cases assumed to be at Salt Lake. When the Bingham and Garfield transformers were used, the line losses between Salt Lake and these stations were treated as part of the load. The load consisted of incandescent lamps combined with the shop motors, running idle, to give a lagging power factor. Attempts to use the motors alone did not work well for it was found impossible to load them with brakes and keep the load steady. In some of the tests, two banks of transformers were used: one carrying load and the other unloaded. The ground or short circuit (indicated by the cross on Fig. 7) was put on the East line at Farmington, in such a position that the currents could be obtained, at Farmington, on both sides of the trouble.

In what follows "ground" always refers to the midpoints of the condensers (all of which were connected together), and "neutral" to the midpoint of the star-connected transformers. The stations where the measurements were taken will hereafter be designated by giving their distance, in miles, from the generator end of the line, instead of by the actual name of the station. Thus Grace, the generating station, is called Station 0, Farmington, which is 113 miles from Grace, is called Station 113, etc. This gives more meaning to the relative magnitudes of the current and voltage vectors at different points.

The phase relations between the currents and the voltages at a station were determined at a number of points for six different conditions of the line, namely: normal condition, three-wire short circuit, two-wire short circuit, ground on one wire with neutral of transformers ungrounded, ground on one wire with neutral grounded, and grounds on two wires with neutral grounded. Using the current and voltage plugs, previously described, oscillograms of the three currents and the three voltages to ground (the midpoint of the condensers) were obtained at each station desired. Measurements on these gave the magnitudes and phase relations of the three currents and voltages at that station. The vector representing the voltage between the *A* wire and ground at Station 0 was then taken as the reference vector. By taking additional oscillograms showing this voltage and one of the currents and voltages at another station, the lag of any current or voltage behind the reference voltage was obtained. The measurements were made in most cases on the second cycle after the short circuit or ground occurred. When only one or two voltage waves were being photographed, the other vibrator circuits were replaced by equal resistances.

Discussion of Oscillograms

Several typical oscillograms, taken during these tests, are shown and discussed.

On most of these oscillograms the waves are shown for a few cycles of normal operation. Then the short circuit or ground occurs and the waves may be seen changing to their final magnitudes and phases. At the instant the change takes place, the waves are very irregular. This is largely because of an uncertain contact in the short-circuiting or grounding switch at the moment of closing. As the transient effects were not being studied, no attempt was made to correct this.

In all of these oscillograms time is increasing from left to right. Unless otherwise noted, the top wave shows the current, or voltage, for the *A* phase, the center wave for the *B* phase, and the bottom wave for the *C* phase. It will be noted that the order in which the waves attain their peak values is top—center—bottom, making the phase rotation, *A*-*B*-*C*. In most cases the zero lines are also shown.

The lettering on each oscillogram shows the conditions under which it was taken so that it is necessary here merely to point out some of the interesting features.

The first five oscillograms, Figs. 8A, 8B, 8C, 8D and 8E, show conditions which exist when the *B* wire is grounded, with the neutral of the transformers ungrounded.

In Fig. 8A the voltages of the ungrounded wires are seen to rise almost to delta voltage, while the *B* voltage decreases very decidedly. This voltage does not fall to zero because of the drop in the wire between the point where the oscillogram was taken, and the point where the wire was grounded. (The voltages shown, it will be remembered, are those between each wire and the mid-points of the condensers; or those which would be obtained on an actual system between wires and ground). Fig. 8B shows a slight increase in the magnitude of the current in the grounded wire, and a change in wave shape for all three currents. There is a very peculiar distortion in the waves the instant the ground occurs, and also a slight shift in phase. Fig.

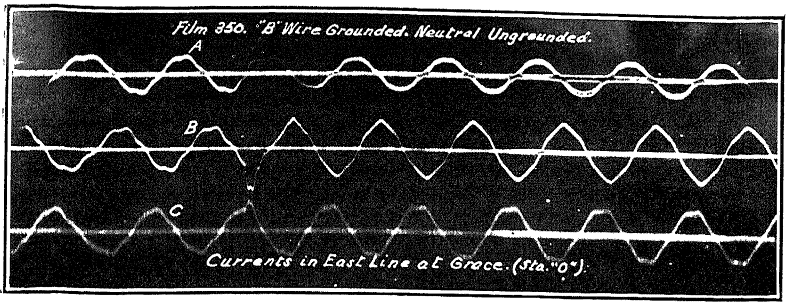

FIG 8B

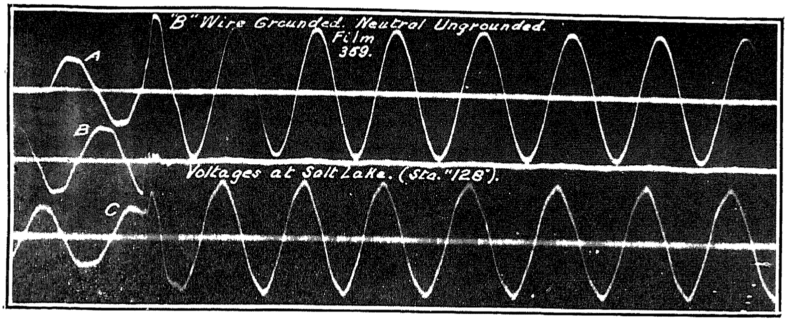

FIG 8C

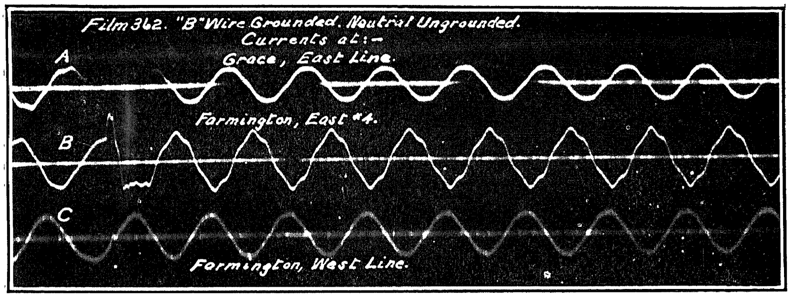

FIG. 8D

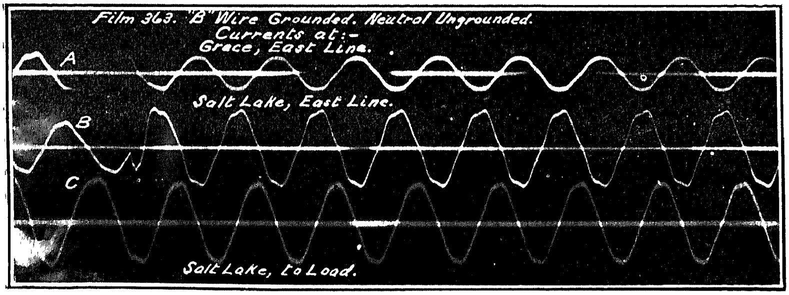

FIG. 8E

[GRAY]

PLATE XXXVI.
A. I. E. E.
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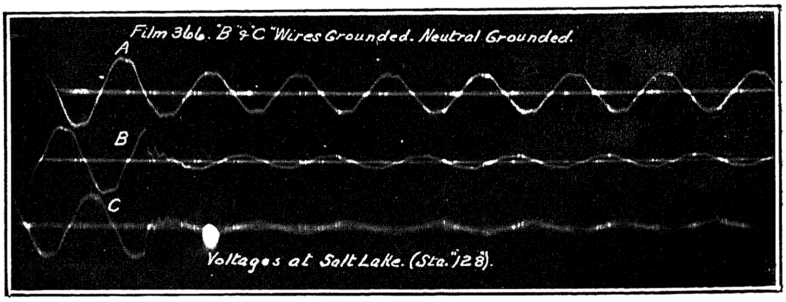

FIG 9A

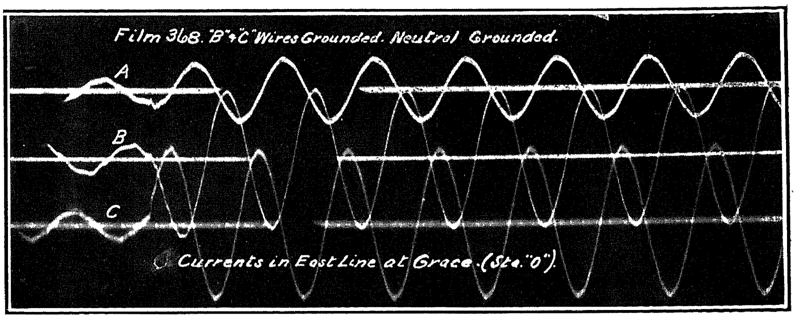

FIG. 9B

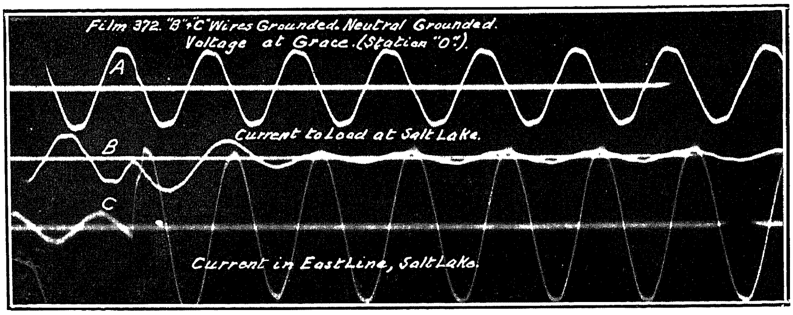

FIG. 9C

[GRAY]

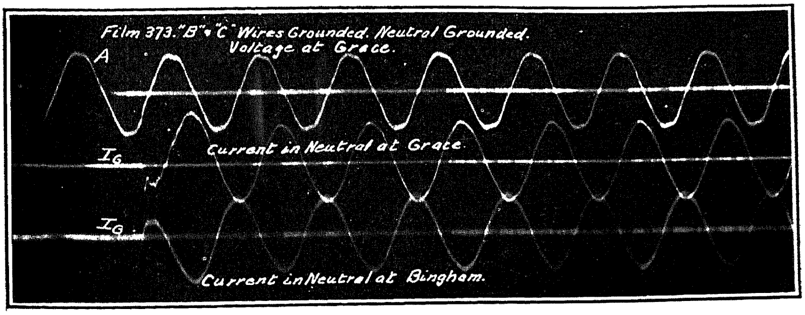

FIG. 9D

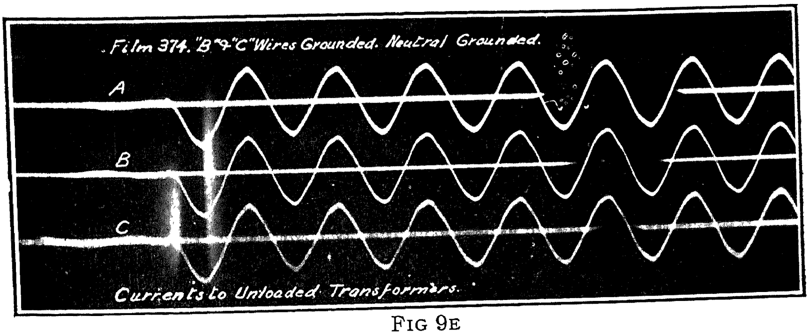

FIG 9E

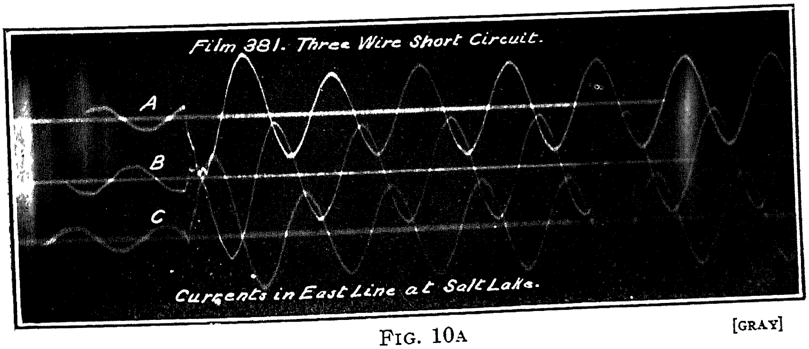

FIG. 10A

[GRAY]

PLATE XXXVIII.
A. I. E. E.
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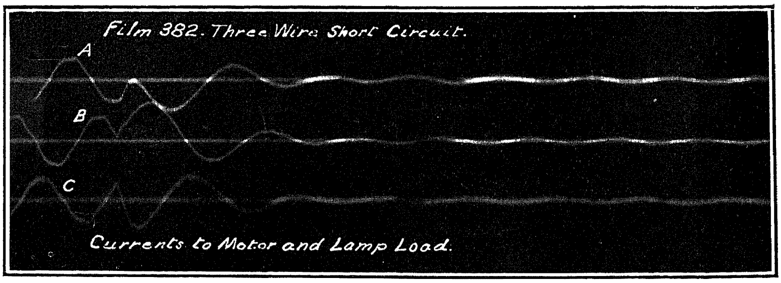


Fig. 10B

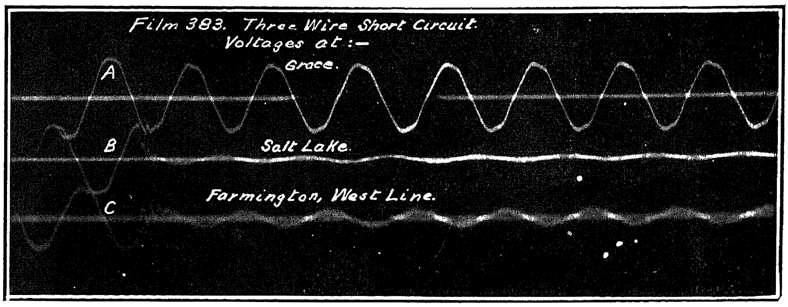


Fig. 10C

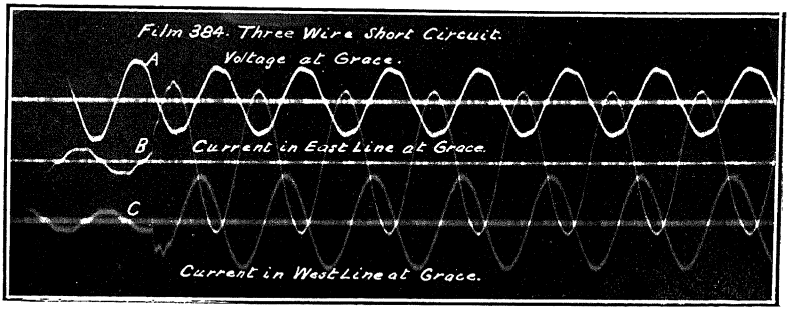


Fig. 10D

[GRAY]

8c shows much the same effects as were brought out in Fig. 8A. They are more pronounced, however, because this oscillogram was taken much nearer to the grounded point of the line. The comments given for Fig. 8B apply also to Figs. 8D and 8E.

Figs. 9A, 9B, 9C, 9D and 9E illustrate the grounding of the *B* and *C* wires when the neutral of the transformers is grounded. Fig. 9A shows the three voltages at a point near where the wires were grounded and Fig. 9 B shows the currents flowing out from the generating station over the East line, upon which the ground was placed. Both of these oscillograms show the effects which would be expected, and require no discussion. The middle wave on Fig. 9C shows the *B* current at Station 128, flowing to the motor load. At the instant the ground is thrown on this is seen to shift in phase by about 180 degrees, due to the fact that the motor pumps power back into the line. The current then slowly decreases in magnitude and changes in phase as the motor slows down. The lower wave, (showing the current at the load end of the East line) also reverses, but this is because the current in that wire now flows toward the grounded point instead of toward the load. The two lower waves on Fig. 9D show the currents flowing in the neutrals at the generator and load. There is, of course, no current until the wires are grounded.

Fig. 9E is a very interesting one. This shows the currents flowing to a bank of unloaded transformers, connected star-delta. Before the wires are grounded the three currents are very small and differ in phase by 120 degrees. With the grounds on the system these currents all increase to about the same magnitude and are all in phase with each other. The probable explanation of this is given later under the discussion of Fig. 16.

Figs. 10A, 10B, 10C and 10D show the current and voltage waves for a few cycles before and after a three-phase short circuit is thrown on the system. Fig. 10A shows the currents in the load end of the East line. When the short circuit occurs the direction of power flow is reversed: becoming toward the short circuit. Hence, the direction of current flow reverses and the currents are seen to change in phase by approximately 180 degrees. Fig. 10B shows the currents taken by the motor load. These reverse because of the pumping-back action of the motor, and then decrease toward zero as the motor slows down. Fig. 10C shows the voltage waves at three different points on the system. The magnitudes of the waves are roughly proportional to the distances from the short circuit. The center and bottom waves of Fig. 10D show the current at the generator end of the East and West lines, respectively.

DESCRIPTION OF FIGURES

Figs. 11, 12, 13, 15, 16 and 17 were drawn from measurements made on these oscillograms and on others not shown here. These figures show the magnitudes and phase relations of the currents and voltages at both ends of the line and at several intermediate points.

In explanation of the figures, it may be said that the vectors are assumed to be rotating *counter* clockwise, or that the phase rotation is *A*-*B*-*C*. The lines from *G* to the vertices of the triangles represent, in magnitude and phase, the peak voltages between each wire and ground; the sides of the triangles consequently represent the line voltages. The peak currents in the *A*, *B* and *C* wires are represented by the vectors marked I_A, I_B, and I_C respectively. At Station 113 (East Line) two diagrams are shown; one for the conditions on each side of the short circuit or ground. The diagrams marked "Generator" were obtained by adding vectorially the currents for the two lines, as no oscillograms were taken at this point. In Figs. 15, 16 and 17 dotted lines are drawn from the vertices of the triangles to a point marked *O*. These lines are the medians of the triangles, and represent the voltages which would be impressed on three equal impedances connected in star between the wires, with the neutral free: such, for instance, as the potential coils of three star-connected wattmeters or relays.

For Figs. 11, 12 and 13 (normal condition, three-wire and two-wire short circuits) these medians coincide with the vectors representing the voltages to ground, since the latter are voltages impressed on three equal impedances connected in star.

In all of the tests the voltages to ground were obtained from the oscillograms, and the voltages represented by the medians were used in computing the wattmeter readings, now to be discussed.

The four straight lines shown at each station between the diagrams for the East and West lines represent (to scale) the readings which would be given by three star-connected wattmeters at that point, and the sum of the three readings. These were obtained by multiplying each star (median) voltage by the "in phase" component of its corresponding current. These computations are shown in Tables IV-IX, inclusive. Numerous small discrepancies appear in these data; such, for instance, as the generator wattage not being equal to the sum of the wattages of the two lines. These discrepancies are easily explained, however, when it is recalled that the calibration of an oscillograph is, at best, somewhat doubtful; that the maximum deflection on the oscillograms rarely exceeded

CALCULATION OF WATTMETER READINGS FOR EACH STATION

The column headed "Total Watts" shows the magnitude and direction of the flow of power at each station. (Power which flows from the generator to the load is assumed to be positive. Power flowing toward the generator is assumed to be negative.)

The column headed "Watts per Phase" shows the readings which would be given by three star-connected wattmeters having the neutral of the potential coils free.

TABLE IV.

Test No. 1. Normal condition.

Station	Line or switch	Wire	Peak Voltage	Peak current		Watts per phase.	Total watts.
				Actual	"In-phase" Component		
Generator		A	931	2 05	2.00	931	
		B	894	2.10	2.07	925	2840
		C	875	2.30	2.24	980	
0	West	A	931	1.00	0.97	450	
		B	894	1.05	1.03	460	1380
		C	875	1.10	1.07	469	
0	East	A	931	1.05	1.03	480	
		B	894	1.15	1.15	515	1500
		C	875	1.25	1.15	503	
113	West	A	...	1.05			
		B	...	1.10			
		C	...	1.20			
113	East	A	780	1.10	0.97	378	
		B	780	1.30	0.93	362	1080
		C	740	1 20	0.97	359	
128	West	A	798	1.05	1 03	411	
		B	780	1 10	1.03	401	1170
		C	740	1.20	0.97	359	
128	East	A	798	1.10	0.95	379	
		B	780	1.30	0.95	370	1130
		C	740	1.25	1.03	381	
128	Loaded transformers	A	798	1.85	1.79	715	
		B	780	2 05	1.90	741	2120
		C	740	2.00	1.79	661	
128	Unloaded transformers	A	798	0.30	0.14	56	
		B	780	0.25	0.14	55	163
		C	740	0 30	0 14	52	

one half of an inch; and that after the currents had been scaled and plotted, their "in phase" components (often very small) had to be scaled again from the drawing. Hence a high degree of accuracy should not be expected.

TABLE V.
TEST NO. 2. THREE-WIRE SHORT CIRCUIT.

Station	Line or switch	Wire	Peak Voltage	Peak current		Watts per phase	Total watts
				Actual	"In-phase" Component		
Generator		A	780	9.55	5.66	2210	
		B	770	10.95	6.14	2360	6450
		C	703	10.15	5.35	1880	
0	West	A	780	4.15	2.55	995	
		B	770	4.50	2.86	1100	2920
		C	703	4.20	2.34	825	
0	East	A	780	5.50	3.04	1180	
		B	770	6.40	3.31	1270	3530
		C	703	5.80	3.07	1080	
113	West	A	171	4.20	2.83	242	
		B	171	4.40	3.14	268	730
		C	171	4.25	2.59	221	
113	East	A	...	....	....	...	
		B	...	....	....	...	...
		C	...	....	....	...	
128	West	A	95	4.15	3.07	146	
		B	95	4.40	3.51	167	450
		C	95	4.25	2.97	141	
128	East	A	95	3.95	−2.69	−128	
		B	95	4.50	−3.04	−144	−410
		C	95	4.00	−2 86	−136	
128	Loaded Transformers	A	95	0.30	0.28	13	
		B	95	0.30	0.28	13	40
		C	95	0.30	0.28	13	

DISCUSSION OF FIGURES

Fig. 11. *Normal Condition.* This figure shows the relations existing in the line under normal operating conditions. The phases are slightly unbalanced here because of an unbalanced load which was connected to the laboratory mains. An interesting point to be noted in these diagrams is the increase in the

TABLE VI

TEST NO. 3. TWO-WIRE SHORT CIRCUIT

Station	Line or switch	Wire	Peak voltage	Peak current		Watts per phase	Total watts
				Actual	"In-phase" Component		
Generator		A	872	3.40	3.04	1326	4860
		B	698	10.20	8.84	3080	
		C	829	8.85	1.10	456	
0	West	A	872	1.64	1.48	646	2310
		B	698	4.11	3.78	1320	
		C	829	3.61	0.82	340	
0	East	A	872	1.64	1.38	600	2520
		B	698	6.11	5.16	1800	
		C	829	5.30	0 28	116	
91	West	A	741	1.60	1.56	578	1490
		B	457	4.15	4.14	946	
		C	490	3.60	−0.14	−34	
91	East	A	730	1.65	1.61	589	1540
		B	435	5.90	5.61	1221	
		C	435	5.25	−1.24	−270	
113	West	A	665	1.60	1.56	519	1140
		B	360	4.05	4.05	730	
		C	392	3.55	−0.55	−108	
113	East No. 4	A	720	1.80	1.47	529	840
		B	381	5.95	4.87	929	
		C	381	5.75	−3.22	−614	
113	East No. 5	A	720	1.80	1.47	529	760
		B	381	4.85	−3.31	−630	
		C	381	5.15	4.50	858	
128	West	A	675	1.60	1.60	544	1030
		B	349	3.95	3.59	626	
		C	349	3.70	−0.83	−145	
128	East	A	675	1.95	1.65	557	530
		B	349	5.10	−3.95	−689	
		C	349	4.95	3.78	660	
128	Loaded Transformers	A	675	3.80	3.31	1118	1660
		B	349	1.65	0.74	129	
		C	349	2.35	2.35	410	

current lag in going from the generator to the load. This is, of course, due to the fact that the lagging current of the load is partly neutralized at the generator by the leading charging current.

TABLE VII

TEST NO. 4. ONE WIRE GROUNDED. NEUTRAL UNGROUNDED

The voltages are the dotted lines shown on Fig. 15.

Station	Line or switch	Wire	Peak voltage.	Peak current		Watts per phase	Total watts
				Actual	"In-phase" component		
Generator		A	975	1.75	1.72	839	3210
		B	895	2.80	2.72	1218	
		C	904	2.55	2.55	1150	
0	West	A	975	0.90	0.90	448	1500
		B	895	1.30	1.17	524	
		C	904	1.30	1.17	529	
0	East	A	975	0.85	0.83	405	1660
		B	895	1.55	1.48	661	
		C	904	1.40	1.31	592	
113	West	A		1.05			
		B		1.35			
		C		1.10			
113	East No. 4	A	865	1.15	1.03	446	1490
		B	789	1.55	1.45	571	
		C	780	1.40	1.21	472	
113	East No. 5	A	865	1.00	1.00	432	1200
		B	789	1.45	0.93	366	
		C	780	1.05	1.03	402	
128	West	A	865	1.00	1.00	432	1280
		B	808	1.20	1.14	460	
		C	751	1.15	1.03	387	
128	East	A	865	1.00	0.96	415	1170
		B	808	1.55	0.90	363	
		C	751	1.10	1.03	387	
128	Loaded transformers	A	865	2.10	2.07	895	2500
		B	808	2.10	2.05	829	
		C	751	2.25	2.07	777	

The data for these diagrams were obtained from measurements made on the oscillograms for Tests 4 and 6, using the two or three cycles shown previous to the occurrence of the ground.

Fig. 12. *Three-Wire Short Circuit.* This figure is not of much importance, for a three-wire short circuit very seldom

TABLE VIII

TEST NO. 5. ONE WIRE GROUNDED. NEUTRAL GROUNDED.

The voltages are the dotted lines shown on Fig. 16.

Station	Line or switch	Wire	Peak voltage	Peak current		Watts per phase	Total watts
				Actual	"In-phase" component		
Generator		A	813	4.25	4.25	1730	
		B	721	8.80	6.18	2230	4790
		C	870	3.80	1.90	826	
0	West	A	813	2.00	1.93	785	
		B	721	3.40	2.69	970	2210
		C	870	1.75	1.04	452	
0	East	A	813	2.35	2.21	940	
		B	721	5.45	3.38	1220	2510
		C	870	2.05	0.80	348	
91	West	A	650	1.95	1.52	495	
		B	314	3.45	2.97	466	1330
		C	694	1.70	1.07	371	
91	East	A	655	2.25	2.18	715	
		B	290	5.40	4.25	616	1660
		C	675	2.00	0.97	327	
113	West	A	608	1.95	1.59	483	
		B	285	3.40	2.76	393	1170
		C	650	1.70	0.90	292	
113	East No. 4	A	550	2.20	2.07	570	
		B	123	5.40	3.80	234	980
		C	580	2.45	0.59	171	
113	East No. 5	A	550	2 25	2.21	609	
		B	123	7.60	−5.00	−308	430
		C	580	2.25	0.45	130	
128	West	A	575	1.85	1.38	396	
		B	204	3.40	3.00	306	1010
		C	608	1.85	1.00	304	
128	East	A	575	2.30	2.21	635	
		B	204	7.75	−5.49	−560	310
		C	608	2.50	0.76	231	
128	Loaded transformers	A	575	2.85	2.76	793	
		B	204	1.70	−0.66	−67	1520
		C	608	2.85	2.62	795	
128	Unloaded transformers	A	575	2.60	1.35	388	
		B	204	2.60	−2.28	−232	20
		C	608	3.00	−0.45	−137	

occurs in practise. When it does occur, relays generally work properly, since the currents and the voltages are balanced. The voltages are seen to decrease from the generator to the short circuit. The power put into the line during a short circuit is not the tremendous amount sometimes supposed. In this case it is only a little more than twice that supplied under normal operation, in spite of the fact that the generator voltage is

TABLE IX

TEST NO. 6. TWO WIRES GROUNDED. NEUTRAL GROUNDED

The voltages are the dotted lines shown on Fig. 17.

Station	Line or switch	Wire	Peak voltage	Peak current		Watts per phase	Total watts
				Actual	"In-phase" component		
Generator		A	860	5.35	3.80	1633	5630
		B	707	10.00	6.50	2290	
		C	803	10.60	4.25	1708	
0	West	A	860	2.55	1.86	800	2520
		B	707	4.05	2.76	975	
		C	803	4.20	1.86	748	
0	East	A	860	2.75	1.86	800	3190
		B	707	6.00	3.80	1341	
		C	803	6.40	2.62	1051	
113	East No. 4	A	352	3.05	2.28	401	740
		B	176	6.10	5.60	492	
		C	176	6.30	−1.69	−149	
113	East No. 5	A	352	2.65	2.07	364	−190
		B	176	5.80	−5.80	−510	
		C	176	6.70	−0.55	−48	
128	West	A	370	2.80	2.28	421	800
		B	176	4.00	4.00	352	
		C	199	4.20	0.28	28	
128	East	A	370	2.70	1.93	357	−290
		B	176	5.85	−4.90	−431	
		C	199	6.70	−2.17	−216	
128	Loaded transformers	A	370	2.75	2.21	409	450
		B	176	0.40	0.10	9	
		C	199	1.10	0.28	28	
128	Unloaded transformers	A	370	2.80	1.79	331	0
		B	176	2.80	−0.83	−73	
		C	199	2.80	−2.59	−258	

maintained almost at normal value. The current, however, is over four times as large as the normal current.

Fig. 13. *Two-Wire Short Circuit.* These oscillograms were taken before the short-circuiting switch was arranged to catch the actual short circuit on the oscillogram. Hence the measurements were made on about the tenth or twelfth cycle after the short circuit occurred. The currents and voltages are, naturally, very much unbalanced. All three currents are larger than normal, the current in the good or *A* wire increasing because of the fact that the motor load takes more current when the voltage

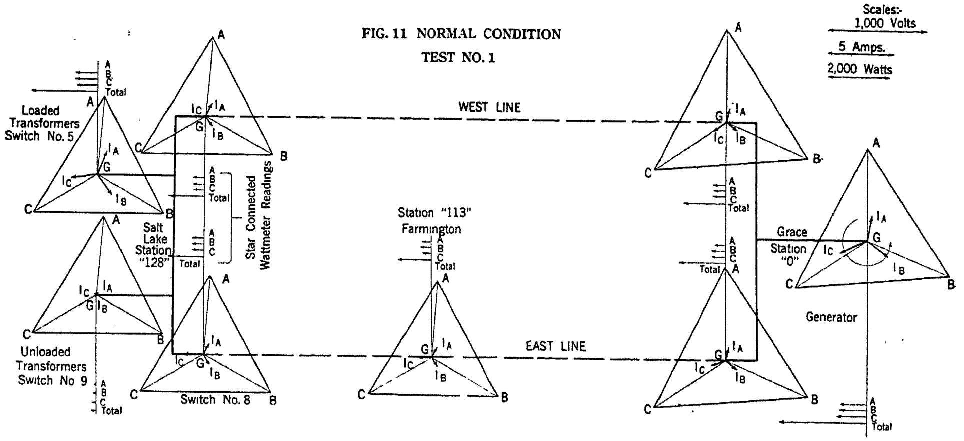


FIG. 11 NORMAL CONDITION
TEST NO. 1

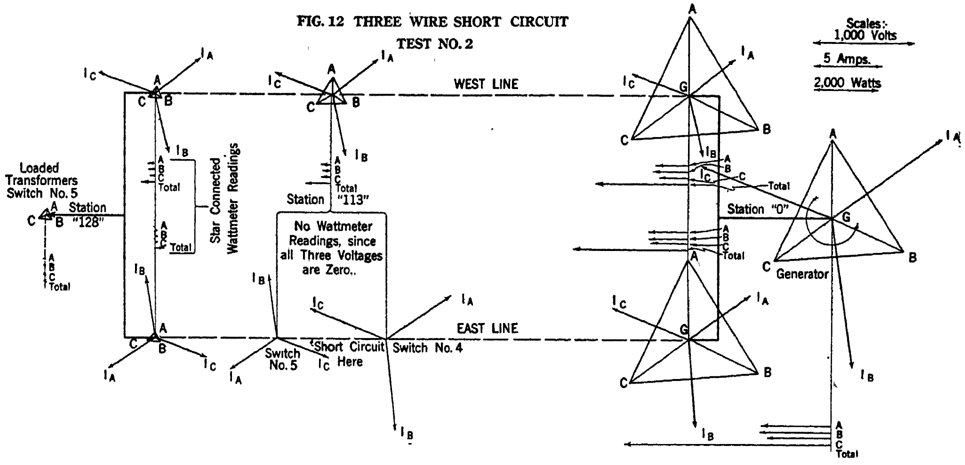


FIG. 12 THREE WIRE SHORT CIRCUIT
TEST NO. 2

triangle is unbalanced. The power delivered to the load is about 75 per cent of the normal amount. The power given out by the generator is about 70 per cent greater than normal, and about 25 per cent less than that for the three-wire short circuit.

The most interesting thing in this figure is the wattmeter readings. The A wattmeter is seen to indicate toward the load in every case. The B wattmeter indicates *toward* the short circuit at all points. The C wattmeter indicates *away* from the short circuit, except at stations some distance away. The B wattmeter is, thus, the only one which can be depended upon to show the direction of the short circuit. The arrow showing the total power, it will be noticed, points toward the load at every station. This means that relays connected according to the two-wattmeter principle can not work properly except on a system where the load is small compared with the power expenditure during short circuit.

The explanation of the backward reading of the C meter may be given briefly with the aid of Fig. 14. The triangle represents the delta voltages at some point near the short circuit, and I_B and I_C represent the currents in wires B and C. The phase rotation is assumed to be A-B-C, and the short circuit is between wires B and C.

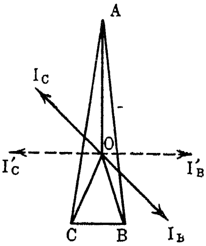


FIG. 14

With a non-inductive line and with no load current flowing, the short-circuit current would be in phase with B-C, or would take the position I_B'-I_C'. Since, however, the inductive reactance of a line generally exceeds the resistance, the current vector will lag by a considerable angle, and will take some such position as I_B-I_C. This makes the voltage E_{CO} and the current I_C more than 90 degrees out of phase, and causes the C wattmeter to read backwards. It may readily be seen that if the phase rotation is reversed the B meter will be the one which reads backwards.

Fig. 15. One Wire Grounded. Neutral of Transformers Ungrounded. For this test the B wire at Station 113 on the East line was connected to ground through the grounding device shown in Fig. 4. (Ground, as stated at the beginning of this chapter under *Description of Tests* means the midpoints of the condensers, all of which were connected together. **Neutral** refers to the midpoints of the star-connected transformers.)

In these diagrams the voltage triangle is only slightly distorted, but is greatly displaced with respect to ground potential. This point (ground potential) comes completely outside of the triangle. The ratio of delivered power to generator power is greater than

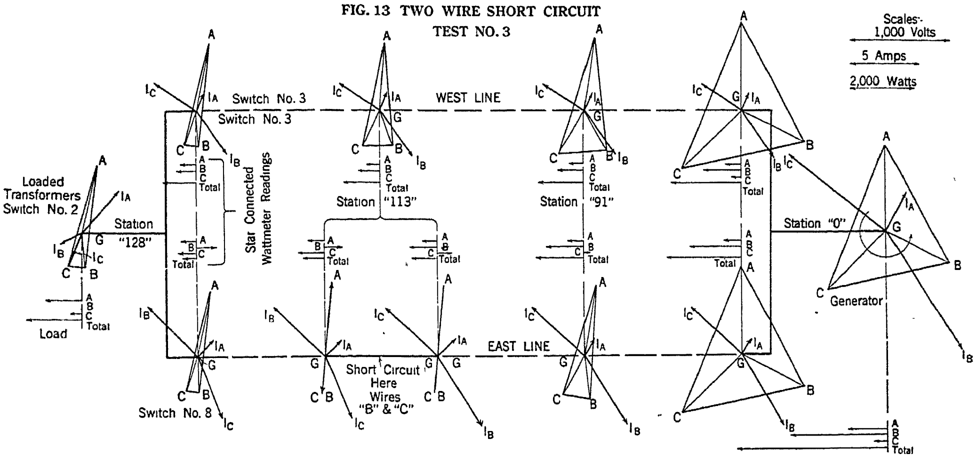


FIG. 13 TWO WIRE SHORT CIRCUIT TEST NO. 3

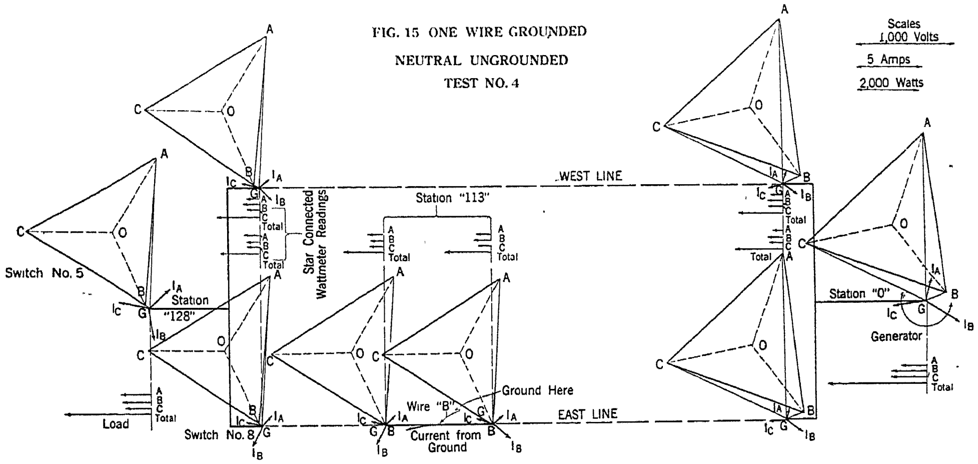


FIG. 15 ONE WIRE GROUNDED NEUTRAL UNGROUNDED TEST NO. 4

under normal conditions, probably because the higher potential above ground produces a greater charging current. This results in a higher power factor in the line, and hence a lower line loss. The wattmeter readings in this case are nearly the same as under normal conditions. Therefore, wattmeter relays can not be used to show the direction of such a ground. A dead ground of this kind, however, is not serious, except on very high-voltage systems.

Fig. 16. *One Wire Grounded. Neutral of Transformers Grounded.* The neutrals of all three banks of transformers were

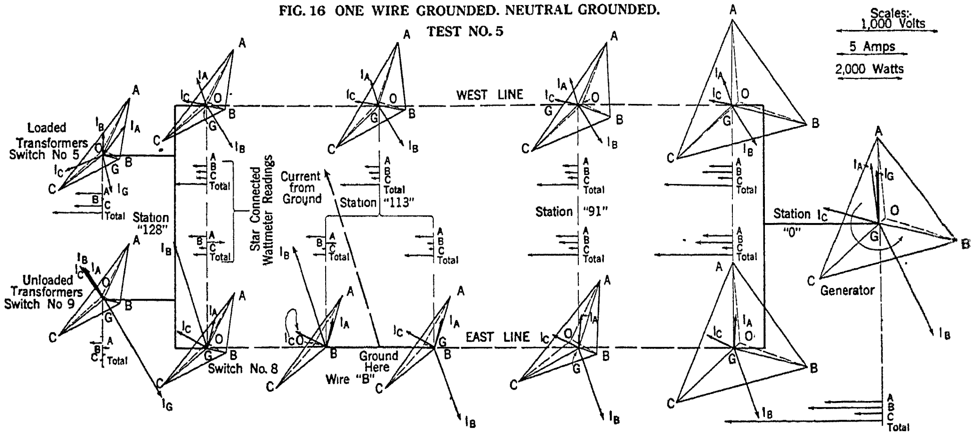


FIG. 16 ONE WIRE GROUNDED. NEUTRAL GROUNDED. TEST NO. 5

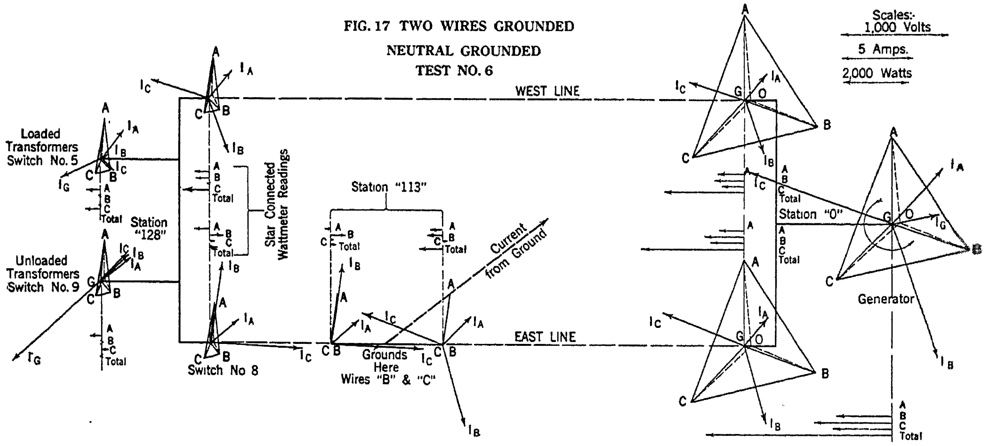


FIG. 17 TWO WIRES GROUNDED NEUTRAL GROUNDED TEST NO. 6

connected to ground, and a ground thrown on the system at the usual place (the East line at station 113). The voltage triangle, of course, becomes very much unbalanced. The wattmeter readings are again seen to behave in a rather unexpected manner, the *B* wattmeter being the only one which correctly shows, at

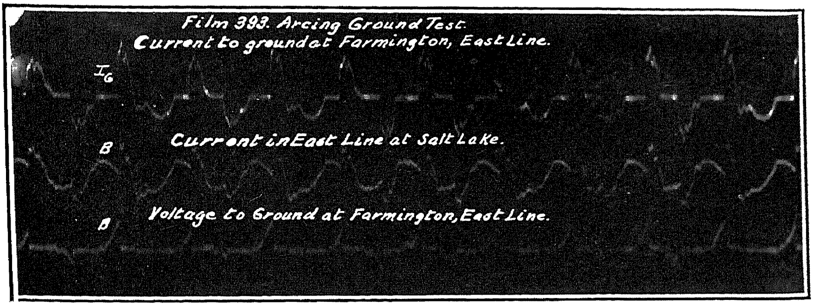


FIG. 18A

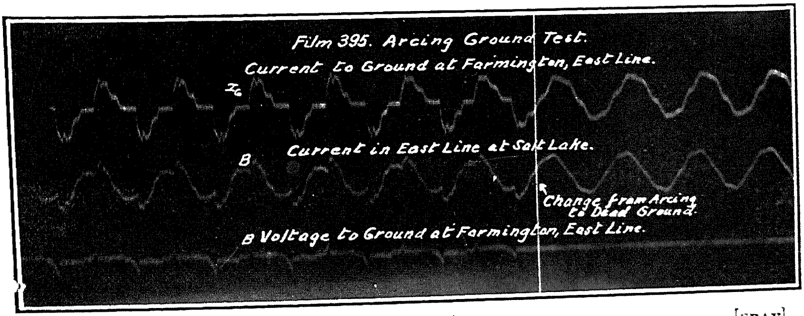


FIG. 18B

[GRAY]

all stations, the direction of the ground. The vectors marked I_c show the currents flowing into the system through the grounded neutrals. The difference in the magnitudes of this current for the loaded and unloaded transformers is due to the large line impedance between the loaded transformers and the Salt Lake busbars.

The currents in the three wires to the "Unloaded Transformers" are nearly equal and in phase. This results from the star-delta connection, and may be explained as follows: The voltages from wires A and C to ground are somewhere near normal, and this tends to keep up the voltage of the B transformer on the low-tension (delta) side. This induces a voltage in the high side of this transformer and sends a current out over the grounded wire. As the only current which can flow in the low-tension side must circulate around the delta, the three line currents must be approximately equal and in phase. For this reason the currents in all three wires will be abnormal when one wire is grounded; and the nearer the ground is to a bank of load transformers, the greater will be the current in that bank. Hence, a bank of small star-delta load transformers is liable to serious injury if its neutral is grounded.

The single vector at Farmington (station 113) between the two diagrams represents the current flowing into the system at the point where the wire is grounded. It is equal to the sum of the currents (flowing in opposite directions) in the B wire at that point.

Fig. 17. *Two Wires Grounded. Neutral of Transformers Grounded.* This case resembles the preceding one, except that it is more severe. The B and C wires were grounded at station 113. The voltages and power at the load are much less than in Fig. 16, and some of the meters have reversed. The B meter is again the only one which can be depended upon to read correctly. As practically all of the discussion given for the preceding case applies to this case, it will not be repeated here.

DISCUSSION OF ARCING GROUND TESTS

The following test has no connection with the preceding, but is presented as a matter of interest. This test was made with the regular connections shown in Fig. 7 for Tests 1, 4, 5, and 6. Wire B of the East line was grounded at station 113 using the grounding device shown in Fig. 4. Fig. 18A shows only the steady condition (if it can be called such) of the arcing ground. The top wave shows the ground current at station 113, the center one the current in the B wire at station 128 (East Line) and the bottom one the voltage from the B wire to ground at station 113. The arc is seen to break at the end of each half cycle of the voltage wave, and the current remains at zero until the voltage has risen considerably. It then increases rapidly to almost its final value. The current in the center wire is, of course, a combination of the arc current and the load current. Hence it has a value while the arc is broken.

Fig. 18B shows the waves while an arcing ground is being displaced by a dead ground. The most surprising feature here is that the arcing ground takes more current than the dead ground. This is because of the fact that, during each alternation, the electrostatic capacity of the system must be charged to a certain value. As quantity is equal to the product of current and time, it follows that if the beginning of the charge is delayed, the current must increase to a higher value in order to get the required charge into the system in the given time. Planimeter measurements on Fig. 18B show that the area under the irregular curve due to the arcing ground, is approximately equal to the area under the curve due to the dead ground. Hence, the quantities, which are proportional to the areas, are about equal in the two cases, and the average values of the currents are equal. The peaks and effective values, however, are not.

These observations were made using carbon electrodes. The effect of metallic electrodes is to make this phenomenon more pronounced. It should be remembered, however, that an arc on a 1000-volt system may have entirely different characteristics from one on a 44,000-volt system.

APPENDIX A

DERIVATION OF FORMULA USED IN CALCULATING THE CAPACITIES

Given wire 1 of radius r, length $2\,L$, $\frac{d}{2}$ units above the ground, and having a quantity of electricity q per unit length. To find its potential above ground.

As the potential of one point with respect to another is defined as the work which must be done in carrying a unit positive charge from the second point to the first, it is necessary first to find the force acting, perpendicular to the wire, on a unit charge at any distance from the wire.

This force, at any point P is equal to:

$$2\int_0^L \frac{q\,d\,l}{l^2+x^2} \times \frac{x}{\sqrt{l^2+x^2}} = 2\,q\,x^{-1}\frac{l}{\sqrt{l^2+x^2}}\bigg|_0^L = \frac{2\,q\,L}{x\sqrt{L^2+x^2}}$$

If x is small in comparison with L, this may be written:

$$\text{Force} = \frac{2\,q}{x}$$

Using the method of images in Fig. 19 let 4 be the image of 1 with respect to ground. The potential, $V_{1,4}$, between 1 and 4 is then equal to twice the potential, V_1, between 1 and ground. From the definition given above, $V_{1,4}$ is seen to be equal to the integral of the force acting on a unit positive charge over the distance between the two wires, or:

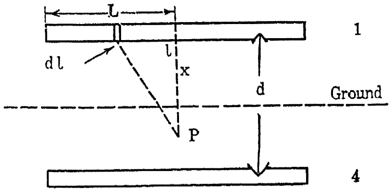


FIG 19

$$V_{1,4} = \int_{d-r}^{r} \frac{2\,q\,(-dx)}{x} - \int_{d-r}^{r} \frac{2\,(-q)\,(-dx)}{d-x}$$

the first term being for wire 1 and the second for wire 4. Integrating:

$$V_{1,4} = 2\,q\Big[-\log_\epsilon x + \log_\epsilon (d-x)\Big]_{d-r}^{r}$$

$$= 2\,q\log\frac{d-r}{r} + 2\,q\log\frac{d-r}{r}$$

$$= 4\,q\log_\epsilon\frac{d}{r}$$

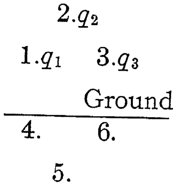
since r is negligible in comparison with d.

Hence, the potential from 1 to ground (which is one-half of that between 1 and 4), is:

$$V_1 = 2\,q \log_\epsilon \frac{d}{r}$$

It may readily be seen that the effect of having other charged conductors in the field, is to add a similar term for each conductor; in which q is the charge per unit length on that conductor. The logarithmic part of each term is the ratio of the initial to the final distance of the unit charge from each conductor, as the unit charge is carried from the image of the conductor whose potential is being found, to the conductor itself. Thus in a three-phase system, the potential of wire 1 above ground is

$$V_1 = 2\,q_1 \log_\epsilon \frac{d_{1,4}}{r_1} + 2\,q_2 \log_\epsilon \frac{d_{2,4}}{d_{1,2}} + 2\,q_3 \log_\epsilon \frac{d_{3,4}}{d_{1,3}}$$



$d_{1,2}$, $d_{1,3}$ etc., represent the distances between wires 1 and 2, 1 and 3, etc.

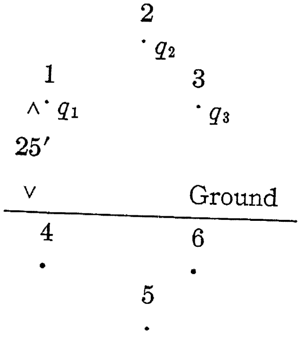
APPENDIX B.

Calculation of Capacities for West Line, Grace to Logan

Derivation of General Formula for Calculating Capacities

The wires are No. 2 copper, 0.2576 in. in diameter, at the corners of an equilateral triangle 9 ft. on a side. The average height of the lower wires above ground is 25 feet.

Using the method of images, let q_1, q_2, q_3 be the charges per unit length on the conductors. Let the voltages from wires to ground be



$$V_1 = A \sin \omega t$$
$$V_2 = A \sin (\omega t - 120°)$$
$$V_3 = A \sin (\omega t - 240°)$$

From the proof given in Appendix A,

$$V_1 = 2\, q_1 \log_\epsilon \frac{d_{1,4}}{r_1} + 2\, q_2 \log_\epsilon \frac{d_{2,4}}{d_{1,2}} + 2\, q_3 \log_\epsilon \frac{d_{3,4}}{d_{1,3}} = A \sin \omega t$$

Referring to the sketch above,

$d_{1,4} = 50$ ft. or 600 in. $d_{1,2} = 9$ ft.
$r_1 = 0.1288$ in. $d_{3,4} = 50.8$ ft. $d_{2,5} = 65.6$ ft.
$d_{2,4} = 58.0$ ft. $d_{1,3} = 9$ ft.

Substituting in the formula above:

$$A \text{ sin. } \omega t = 2\, q_1 \log_\epsilon \frac{600}{0.1288} + 2\, q_2 \log_\epsilon \frac{58.0}{9.0} + 2\, q_3 \log_\epsilon \frac{50.8}{9.0}$$

In the same way:

$$A \sin (\omega t - 120°) = 2\, q_1 \log_\epsilon \frac{58.0}{9.0} + 2\, q_2 \log_\epsilon \frac{787.2}{0.1288} + 2\, q_3 \log_\epsilon \frac{58.0}{9.0}$$

and

$$A \sin (\omega t - 240°) = 2\, q_1 \log_\epsilon \frac{50.8}{9.0} + 2\, q_2 \log_\epsilon \frac{58.0}{9.0} + 2\, q_3 \log_\epsilon \frac{600}{0.1288}$$

From these equations:

$$A \sin \omega t = 2 \times 2.303\, (3.668\, q_1 + 0.809\, q_2 + 0.752\, q_3)$$
$$A \sin (\omega t - 120°) = 2 \times 2.303\, (0.809\, q_1 + 3.786\, q_2 + 0.809\, q_3)$$
$$A \sin (\omega t - 240°) = 2 \times 2.303\, (0.752\, q_1 + 0.809\, q_2 + 3.668\, q_3)$$

Now let $B = \dfrac{A}{2 \times 2.303 \times 0.809}$

$$B \sin \omega t = 4.534\, q_1 + q_2 + 0.929\, q_3$$
$$B \sin (\omega t - 120°) = q_1 + 4.680\, q_2 + q_3$$
$$B \sin (\omega t - 240°) = 0.929\, q_1 + q_2 + 4.534\, q_3$$

Solving these equations gives

$$q_1 = B\,\frac{1.006 \sin \omega t + 0.0095 \cos \omega t}{3.605}$$

$$q_2 = 0.2742\, B \sin (\omega t - 120°)$$

To find the maximum value of q_1, differentiate with respect to ωt.

$$\frac{d\,q_1}{d\,\omega t} = 1.006 \cos \omega t - 0.0095 \sin \omega t = 0$$

$$\tan \omega t = \frac{1.006}{0.0095} = 105.6 \text{ or } \omega t = 89.45° \text{ when } q_1 \text{ is a maximum.}$$

Hence,

$$\text{Maximum } q_1 = B\,\frac{1.006 \sin 89.45° + 0.0095 \cos 89.45°}{3.605}$$

$$= 0.2790\, B.$$

and $q_1 = 0.2790\, B \sin (\omega t + 0.55°)$

Substituting for B its value, $\frac{A}{4.606 \times 0.809}$, and changing to coulombs per mile gives

$$q_1 = 0.2790\,\frac{A \times 161{,}000}{4.606 \times 0.809 \times 9 \times 10^{11}} \sin (\omega t + 0.55°)$$

$$= 1.339 \times 10^{-8} A \sin (\omega t + 0.55°) \text{ coulombs per mile.}$$

Hence $q_2 = 1.316 \times 10^{-8} A \sin (\omega t - 120°)$ coulombs per mile.
$q_3 = 1.339 \times 10^{-8} A \sin (\omega t - 240.55°)$ " " "

The next step is to get the quantities on each wire with the center wire grounded. Assuming that the potentials remain balanced, the voltages from wires to ground will be

From wire 1...... $\sqrt{3}\,A \sin (\omega t + 30°)$
" " 2...... 0
" " 3...... $\sqrt{3}\,A \sin (\omega t - 270°)$

Carrying, as before, the unit charge from wire 4 to wire 1:

$$\sqrt{3}A \sin(\omega t + 30^\circ) = 2 \times 2.303 \left(q_1 \log_{10} \frac{600}{0.1288} + q_2 \log_{10} \frac{58}{9} + q_3 \log_{10} \frac{50.8}{9.0} \right)$$

$$0 = 2 \times 2.303 \left(q_1 \log_{10} \frac{58.0}{9.0} + q_2 \log_{10} \frac{65.6 \times 12}{0.1288} + q_3 \log_{10} \frac{58.0}{9.0} \right)$$

$$\sqrt{3}A \sin(\omega t - 270^\circ) = 2 \times 2.303 \left(q_1 \log_{10} \frac{50.8}{9.0} + q_2 \log_{10} \frac{58}{9.0} + q_3 \log_{10} \frac{600}{0.1288} \right)$$

Let $B = \dfrac{\sqrt{3}A}{2 \times 2.303 \times 0.809}$

Then,

$$\begin{aligned} B \sin(\omega t + 30^\circ &= 4.534\, q_1 + q_2 + 0.929\, q_3 \\ 0 &= q_1 + 4.680\, q_2 + q_3 \\ B \sin(\omega t - 270^\circ) &= 0.929\, q_1 + q_2 + 4.534\, q_3 \end{aligned}$$

Solving:

$$q_1 = 0.2209\, B \sin(\omega t + 21.12^\circ)$$

$$q_2 = \frac{\sqrt{3}B}{23.57} \sin(\omega t - 120^\circ)$$

$$q_3 = 0.2209\, B \sin(\omega t - 261.12^\circ)$$

Substituting the value of B and changing to coulombs per mile, the equations become

$$q_1 = 0.2209 \frac{A\sqrt{3} \times 161{,}000}{4.606 \times 0.809 \times 9 \times 10^{11}} \sin(\omega t + 21.12^\circ)$$

$$= 1.061 \times 10^{-8} \sqrt{3}A \sin(\omega t + 21.12^\circ)$$

$$q_2 = 0.353 \sqrt{3} \times 10^{-8} A \sin(\omega t - 120^\circ)$$

$$q_2 = 1.061 \times 10^{-8} \sqrt{3}A \sin(\omega t - 261.12^\circ)$$

Having obtained the quantities on each wire under these two conditions, the next step is to determine the values to be assigned to each of the six condensers. Denote these values by x, y, z, and w as shown in Fig. 3 A. Let q_1 be the charge on wire A, q_2 the charge on wire B and q_3 the charge on wire C. The potentials to ground will then be

$$\begin{aligned} V_1 &= A \sin \omega t \\ V_2 &= A \sin (\omega t - 120°) \\ V_3 &= A \sin (\omega t - 240°) \end{aligned}$$

and the potentials between wires will be

$$\begin{aligned} V_{1,2} &= \sqrt{3} A \sin (\omega t + 30°) \\ V_{2,2} &= \sqrt{3} A \sin (\omega t - 90°) \\ V_{3,1} &= \sqrt{3} A \sin (\omega t - 210°) \end{aligned}$$

The quantity in any condenser is equal to the capacity of that condenser multiplied by its voltage. The quantity on any wire is equal to the vectorial sum of the quantities on the three condensers connected to that wire. Hence;

$$V_{1,2}\, y + V_1 z + V_{1,3}\, w = q_1$$

Substituting,

$$y \sqrt{3} A \sin (\omega t + 30°) + z A \sin \omega t + w \sqrt{3} A \sin (\omega t - 30°)$$
$$= 1.339 \times 10^{-8} A \sin (\omega t + 0.55°)$$

As this equation is true for all values of ωt, it must hold when $\omega t = 0$.

Then $$y \frac{\sqrt{3}}{2} - w \frac{\sqrt{3}}{2} = 0.01284 \times 10^{-8}$$

or $$y - w = 0.01485 \times 10^{-8} \qquad \textbf{(A)}$$

When $\omega t = -30°$

$$-\frac{z}{2} - 1.5\, w = -1.339 \times 10^{-8} \times 0.4917$$

or $$z + 3\, w = 1.317 \times 10^{-8} \qquad \textbf{(B)}$$

In the same way,

$$V_{2,1}\, y + V_2\, x + V_{2,3}\, y = q_2$$

$$y \sqrt{3} A \sin (\omega t - 150°) + x A \sin (\omega t - 120°)$$
$$+ y \sqrt{3} A \sin (\omega t - 90°) = 1.316 \times 10^{-8} A \sin (\omega t - 120°)$$

When $\omega t = 210°$,

$$1.5\,y + x + 1.5\,y = 1.316 \times 10^{-8} = 3\,y + x \qquad \textbf{(C)}$$

With the centre wire grounded,

$$V_{1,2}\,y + V_{1,2}\,z + V_{1,3}\,w = q_1$$

$$(y + z)\ \sqrt{3}\,A \sin(\omega t + 30°) + w\ \sqrt{3}\,A \sin(\omega t - 30°)$$
$$= 1.061\ \sqrt{3}\,A\ 10^{-8} \sin(\omega t + 21.12°)$$

When $\omega t = -\,30°$,

$$-\frac{\sqrt{3}}{2}\ w = 1.061 \times 10^{-8} \sin(-\,8.88°) = -\,0.164 \times 10^{-8}$$

or $\qquad w = 0.189 \times 10^{-8}$ farads per mile.

Substitute in (**A**) $y = 0.189 + 0.015 = 0.204 \times 10^{-8}$ farads per mile.

Substitute in (**B**) $z = 1.317 - 0.567 = 0.750 \times 10^{-8}$ farads per mile.

Substitute in (**C**) $x = 1.316 - 0.612 = 0.704 \times 10^{-8}$ farads per mile.

These are the values for the individual condensers, to be connected as shown in Fig. 3A.

Because of the large number of calculations to be made, the preceding work was shortened by means of the following formulas.

DERIVATION OF GENERAL FORMULA FOR CALCULATING THE CAPACITIES

After substituting B, in the equations for A (page 825), they have the following form: (for the ungrounded case)

$$a\,q_1 + q_2 + d\,q_3 = B \sin \omega t$$
$$q_1 + c\,q_2 + q_2 = B \sin(\omega t - 120°)$$
$$d\,q_1 + q_2 + a\,q_3 = B \sin(\omega t - 240°)$$

Solving these equations gives

$$\textbf{(A)}\ q_2 = \frac{(a + d + 1)}{(a + d)\,c - 2}\ B \sin(\omega t - 120°)$$

$$\textbf{(B)}\ q_1 = \frac{B}{2\,(a - d)\ [\,(a + d)\,c - 2]}$$

$$[(2\,a\,c + d\,c + a - d - 3) \sin \omega t + (a - d - d\,c + 1)\ \sqrt{3} \cos\ \omega t]$$

With the center wire grounded,

$$\begin{aligned} a\,q_1 + q_2 + d\,q_3 &= B \sin(\omega t + 30^\circ) \\ q_1 + c\,q_2 + q_3 &= 0 \\ d\,q_1 + q_2 + a\,q_3 &= B \sin(\omega t - 270^\circ) \end{aligned}$$

from which,

$$\text{(C)}\ q_1 = \frac{B\sqrt{3}}{2\,(a-d)\,[\,(a+d)\,c-2]}\,[\sqrt{3}\,(a\,c-1)\sin\omega t + (a\,c - 2\,c\,d + 1)\cos\omega t]$$

By differentiating (**B**) and (**C**) with respect to ωt, to get maximum q_1, equations of the following form are obtained:

$$\begin{aligned} q_2 &= M\,A \times 10^{-8} \sin(\omega t - 120^\circ) && \text{(From (A))} \\ q_1 &= N\,A \times 10^{-8} \sin(\omega t + \alpha^\circ) && \text{(From (B))} \\ q_1 &= P\,A \times 10^{-8} \sin(\omega t + \beta^\circ) \times \sqrt{3} && \text{(From (C))} \end{aligned}$$

From these, by the method previously given, is obtained:

$$y - w = \frac{2}{\sqrt{3}}\,N \sin\alpha \qquad 3\,y + x = M$$

$$z + 3\,w = 2\,N \sin(30^\circ - \alpha) \quad w = \frac{2}{\sqrt{3}}\,P \sin(30^\circ - \beta)$$

which are easily solved for x, y, z, and w, giving the values to be assigned to the different condensers.

ACKNOWLEDGEMENT

The writer wishes to express his thanks to Messrs. P. N. Nunn, Markham Cheever, C. S. Ruffner, Edward Bennett and L. N. Crichton (all formerly with the Telluride Power Company) who made the construction of the line possible and who gave very valuable assistance during the work.

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INDUSTRIAL RESEARCH AND THE COLLEGES

BY A. E. KENNELLY

ABSTRACT OF PAPER

An outline of the proper relationship that should exist in the industrial research field between the pure science college, the technical college and the industries, themselves.

IN ANY general discussion of the influence of research on the development of electrical industries, the share of the colleges in this influence deserves to be formulated and defined. The more clearly we can realize the ways in which the colleges can assist in this work, and the more closely we can agree upon the methods by which this assistance can be rendered, the more likely we are to be able to bring about the result desired. In what follows, certain propositions are ventured, in the hope of eliciting discussion, and of thereby reaching a consensus of opinion.

To begin with, it will probably be admitted that industrial research is scientific investigation directed economically towards improvements in production.

It has been known for centuries that discovery and invention have made the fountainhead of new industries. For example, it is said that the starting point of the wealth of Holland—and in the seventeenth century, the Dutch republic was the first exporting country in the world—was due to the discovery by the Dutch fishermen, on their remote islands, of a process of preserving fish for storage and transportation. Moreover, throughout all industrial history, men have devoted their time and labor to inventing or designing improved processes in manufacture. Industrial research, in a broad sense of the term, has therefore been at work ever since industry existed. The difference between industrial research in the modern sense, and in the mediaeval sense, lies in an emphasis upon applied science, and upon the application of scientific methods. Under the mediaeval regime, industry was understood to be handicraft. Individual skill in handicraft was aimed at, through the manual training of apprentices by master workmen. Improvements in production had to come slowly, through the gradual development of skill in successive generations of workmen and apprentices.

The advent of the steam engine and of steam-driven factory machines, overthrew, as we all know, the processes of production. These became mechanically stereotyped. The province of individual manual skill changed to the design, direction and management of machinery. The charm of individuality in specially skillful workmanship had to be relinquished, in favor of the enormously increased and standardized production of the humanly guided machine. Under the new conditions, mechanical or chemical improvements could be introduced much more easily, partly because of the absence of intellectual inertia in the producing machinery, and partly because of the more definite predetermination and regularization of machine-made over hand-made products.

The introduction of industrial research in the recent and more restricted sense, has come about partly materially through the opportunity provided by large-scale mechanical or chemical processes, and partly psychologically through the change in the general attitude of mind towards applied science. Formerly, there was an attitude of distrust towards scientific training in industry. Now it is coming to be realized that in all industry, applied science can be made to pay. In fact, it is coming into general apprehension, that sooner or later any industry will be outdistanced competitively, unless maintained not only by eternal vigilance, but also by eternal scientific study.

The part of the colleges has been and is to train young men to learn a business studiously and to carry it on systematically. In the colonial days, the colleges educated men almost exclusively for the professions. Today, the men educated in the colleges enter general business in as great numbers as they enter the professions.

The non-technical scientific departments of the colleges and universities include such fundamental subjects as mathematics, physics and chemistry. The duties of these departments of so-called pure-science *i.e.* science contemplated apart from applications, are two-fold; namely

(*a*) Teaching, by carrying on instruction of the students in those sciences.

(*b*) Learning, by carrying on researches in those sciences.

The teaching and the learning have to be carried on interdependently and perpetually.

It is probably undesirable that the pure-science departments of the colleges and universities should continuously undertake industrial research. There is nothing in their work to prevent their teachers or students from undertaking such research. Many on their staffs are excellently qualified for conducting industrial research; but there is so much other important work to be done, which these departments are alone able to do. In all times, the progress of the sciences has been largely left to the college teachers of those sciences. Physics and chemistry are subdivided into so many branches, that no one man can do justice by his efforts to more than a very small number of studies. There is a limitless field of undreamed of knowledge everywhere along these various lines. The expenses of modern scientific laboratories are so great, that few private individuals can expect to conduct researches in pure science, and the instructing staffs in the college laboratories have special facilities for experimental work. The bulk of new scientific material, on which industries may ultimately depend, has to be worked out and discovered in the pure-science laboratories of the world. Again, mathematics is a science covering a range of knowledge, thought and inquiry, so vast that no one mind can compass it, and yet its knowledge is woefully backward and deficient, even for purely utilitarian purposes; so that there are immense fields to be developed, even on the unlikely hypothesis that no new general discoveries will be forthcoming. Of the indefinitely great number of mathematical functions that may be mentally reviewed, only a few dozens have yet been tabulated and arranged for practical use The progress of knowledge in mathematics is far less restricted to the college faculties, than the progress of subjects like physics or chemistry; because the necessary plant is so inexpensive,—white paper, black pencil and grey matter—yet even here the influence of the college atmosphere for mutual teaching and learning is relatively so productive, that a large share of the world's mathematical pioneer work emanates from the colleges.

While, therefore, under extraordinary circumstances, such as those due to the present World-German war, the pure science departments may advantageously take up industrial research; yet, as a general rule, the best contribution of those departments to industry is by restricting their attention to pure science.

When we come to the engineering and technical colleges, or to corresponding university departments, the opportunities for their sharing in industrial research are greater. In the first place, they train men directly for work in applied science, and, in the second place, their contact with manufacturing industries is greater. By comparison with the pure-science departments, their field of activity is, on the one hand, intellectually narrower; but on the other hand, they come into closer relations with the needs and problems of industries. The applied-science college laboratory staffs are very frequently occupied on engineering and constructive problems of the industrial world. Such activities are valuable, both to the colleges and to the industries, provided that the amount of industrial research work undertaken does not swamp the regular teaching. There are various ways in which the technical colleges can assist in industrial research, and among them the following:—

(1) Training industrial-research investigators to go out into the industries.

(2) The taking up of particular industrial-research problems by particular members of the laboratory staffs, under individual private agreement with industrial concerns, the work being carried on in the college laboratories.

(3) The same as in (2): but with the work carried on in the workshops of the industrial concerns, instead of in the college laboratories.

(4) By the industries formulating their problems directly to the college as a corporation, and entering into an agreement for the maintenance of research work, in the college laboratories or elsewhere, towards the solution of those problems.

All of the above methods are in vogue, as well as various combinations of them. Each has its particular advantages and disadvantages, in view of the conditions of any individual case. So long as the total amount of industrial research work carried on in a technical college does not exceed a certain amount, depending on the size of its plant, it is not very important as to which method is followed. Where, on the contrary, the total demand for such industrial work exceeds that limit, it becomes a serious problem as to how it should be conducted, not only from the standpoint of the college; but also from the standpoints of the industries and of the public.

The only essential difference between scientific researches carried on for an industry, and general scientific researches carried on without reference to any industry, is that the former are expected to bring in results of economic value, and must therefore be safeguarded by secrecy, while the latter, being expected to bring in results of scientific and engineering value only, naturally demand publication so far as they are successful. It is manifest that no industry could afford to undertake research, unless it expected the results to pay a profit on the expense, and such profits would be jeopardized if the successful results were communicated to competitors. The watchword of industrial research is therefore loyalty to the industry, which makes the venture in investigative effort, and this entails watchful protection as to the secrecy of the investigation and its results.

So far as the large industries are concerned, they install their own experimental laboratories, and employ their own experimental investigators. It is natural and proper for them to protect the results of their inventive and experimental efforts, either by patents or by concealment, as may best suit the industrial need. The colleges can best serve these auto-research concerns, by supplying them with well trained graduates. The difficulties lie in the paths of the smaller industrial concerns, which desire to make progress by scientific effort and systematic study, but which cannot afford the expense of a special private research department. These younger industries must have recourse either to the services of a consulting specialist of the researcher type; or to the services of some specialist college laboratories.

The call of the younger industries on the technical colleges for help, is one which the colleges naturally desire to meet, so far as they can do so without disrupting their regular work of teaching and learning. On the other hand, the task is rendered difficult by the need for secrecy, and for the discrimination in dealings which that secrecy may involve. The whole atmosphere of any healthy college is one of intellectual freedom. The ideas and knowledge of any individual in the college community are placed at the academic disposition of all of the rest who may be inquisitively inclined. A recognized exclusive monopoly of information or knowledge must interfere with the best qualities of any institution of learning. Consequently, the maintenance of any considerable amount of industrial research in a college becomes a burden and a difficulty. For this reason, either method (3) or a combination of methods (3) and (4) is perhaps the best for a technical college to follow, when a considerable amount of technical research is desired to be undertaken for those industrial concerns which are not in a position to maintain their own research laboratories.

In the long run, therefore, it seems most desirable that the technical colleges should always carry on general researches in their laboratories, of such a nature as may advance applied science, stimulate careful observation on the part of students, contribute to the published fund of available technical information, supply new knowledge to the teaching staffs, and train students for entering industrial research. To this end, research fellowships and research endowments are of the greatest aid both to the colleges and to the industries. A limited amount of industrial research work may also be advantageously carried on in the college laboratories, along with the general research work. As, with the growth of new industries, more and more demand for such industrial research comes to the colleges, the desirability increases of dealing with it in a systematic way, through the college as a corporate body, by maintaining, in the workshops of the industrial concerns, special assistants under the direction of specialists on the teaching staff. In this way, perhaps, the most effective service may be given by the technical colleges to the junior industries, with the maximum of economy. It may not be too much to expect that, as time goes on, the junior industries may lean more and more upon the technical help of the colleges, without having to enter into any mutual agreement or combination, in order to avail themselves of such aid.

Reference has been made above to the continuous duties of a college, both as to teaching and learning. A third important duty also exists, although its influence on industrial research is less direct; namely, the maintenance and propagation of ideals, or of those underlying habits of thought and action which are sometimes summed up as "character", and at other times are referred to as the working philosophy of life. Colleges foster and transmit such ideals, more or less unconsciously perhaps, to those who study within their walls, partly by tradition, partly by contact between teachers and pupils, partly by the study of history, philosophy and a host of other horizon-enlarging subjects, and partly by the interchange of student opinions. The transmission of such subconscious motives may not have direct bearing upon the technical attainments of college trained men, and many men who have never gone to college are actuated by ideals as fine as those which any college can claim, nevertheless, the indirect stimulating effect of such ideals on the college-trained researcher is manifest to the careful observer on every hand.

An ideal international system would be one in which the pure-science colleges should lay the foundations of future industries by enlarging and disseminating the world's knowledge of the basic scientific principles, the technical colleges should do the same for applied science, while at the same time taking a share in the economic applications of science to industry. The industries themselves should undertake their own researches, under the guidance of qualified research specialists. Vocational schools would train industrial foremen in the elements of the same principles of science, art, technique, economy, thrift and hard work, as applied to those particular industries in which each nation, is, by its peculiar circumstances, specially adapted to excel.

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INDUSTRIAL RESEARCH
WITH SOME NOTES CONCERNING ITS SCOPE IN THE BELL TELEPHONE SYSTEM

BY F. B. JEWETT

ABSTRACT OF PAPER

The interrelation of industrial and "pure science" research and the restrictions which must be imposed to insure mutual existence. A brief outline of the scope and progress of industrial research in the Bell Telephone System.

IT IS with a feeling half of diffidence and half of apology that I venture to present a paper on industrial research at this time to the members of this great engineering organization. The diffidence arises from the fact that for the past few years, and particularly since the outbreak of the great world war, the terms, the emblems and the forms of industrial research, if not always the realities, have been so much a part of the industrial daily life of the nation that the working of them together in a paper seems trite and somewhat of a reflection on the intelligence of the Institute membership. The feeling is akin to that which many of us have experienced when, after numerous courses in mathematics which have made its general processes seem rather second-nature to us, we begin the study of logic and find there a detailed explanatory discussion of these processes. The query at once arises as to why anyone should care to write about matters so obvious, let alone why anyone should waste his time in the reading and studying of such writings. Under these conditions it is clear that to be of interest and value any paper on industrial research should be most carefully thought out and worked over. My apology to the members of the Institute results from the conditions under which I have labored for the past four or five months. The calls upon my time and energies for war work have been so great as to make the careful preparation just referred to impossible. In other words, the doing of industrial research has, for the time being, shouldered out the thinking and writing about industrial research. What follows must therefore be read with the understanding that it is the hurried jotting down of ideas which have been scattered and relegated to the background by the ceaseless day by day pressure for the results of industrial research in the grim game of human destruction.

Much has been said and columns have been written on the wonderful achievements of industrial research in the past and on the stupendous effects which are to follow from industrial researches not yet undertaken. In fact, it seems to be the exception rather than the rule for a present day conversation to run its course without some reference to industrial research or its results. Under the circumstances it is sometimes surprising to find how little actual knowledge seems to exist as to what really constitutes industrial research or of the relations which it bears to research which is not industrial. On the other hand, we are frequently surprised by the evidences of real industrial research in hitherto unsuspected places. That industrial research has taken a firm place in shaping the destinies of our economic future, no one who is at all cognizant of the facts would for a moment deny. A partial picture of present conditions in the United States is interestingly shown in a recent paper entitled "Industrial Research in the United States of America" by Mr. A. P. M. Fleming, M. I. E. E.

As might be expected in a departure so radical and relatively so new, the processes of industrial research have not as yet been applied to the various arts in anything like a uniform manner. A survey of present day conditions in the United States discloses the greatest progress in and the greatest results from industrial research in large electrical, chemical and metallurgical industries, where real scientific research methods were first applied and where its application has been carried to the greatest extreme. The electrical industries have been particularly fortunate in this respect, due largely to the fact that they had no generation-long traditions to overcome and to the further and more important fact that a large proportion of the leaders have been men educated in technical schools and universities where the spirit of scientific research prevails. Under these conditions it is not strange that the value of industrial research should early have been recognized as a vital necessity in the electrical industries, nor that the methods of scientific research should have become so closely interwoven in all its processes. So effectually has this dependence on research methods been established in the larger electrical manufacturing organizations that no question is ever raised as to the proper method of attacking a problem or a difficulty. Not only do we look to the established research departments for the production of most of that which is radically new in the art but they have become and are, either directly or indirectly, our haven of succor when some particularly difficult and knotty problem arises. It is difficult for those of us who have lived and worked long in this environment to appreciate any attitude of mind but the one which seeks the answer to a radically new problem in the processes of scientific research. Thus far have we progressed from the era of the cut and try method—an era productive of much that was of inestimable value to the world. In comparing the present and prospective future with the past, we should also not lose sight of the fact that our so-called scientific methods would be of little avail were it not for the accumulation of facts laboriously dug out by empirical methods at a time when such methods were the only ones that could be applied.

While the development and beneficial results of industrial research are most apparent in the larger industries, where they were first applied and where funds have been available for a continuation of the work during the period when scientific methods were on trial, the signs are numerous which indicate the rapid extension of scientific research to the smaller industries. These signs are shown principally in the growth of industrial research laboratories established for the purpose of assisting clients with research facilities in much the same way that consulting engineers have long assisted in engineering matters; also in the formation of centralized research establishments destined to serve an association of individual manufacturers engaged in the same or similar lines of work. That this growth of industrial research will continue at an ever-increasing rate seems beyond question. The results thus far obtained leave no doubt as to the value of scientific methods when applied to industry and the truth of these results is becoming each day more clearly evident to the rank and file of those engaged in the production of material things. The process of this evolution of thought and application is well illustrated by Macauley in his "Science of Government," where he says "First come hints, then fragments of systems, then defective systems. The sound opinion, held for a time by one bold speculator, becomes the opinion of a small minority, of a strong majority, of the majority of mankind. Thus the great progress goes on.'

That the great war on which we are embarked is exerting an enormous, stimulating influence on the extension of industrial research is clearly evident. The limits, both in time and extent, seem now to be determined merely by the ability to obtain specially trained men and women and the ability of the present industries and their workers to absorb properly the serum of the new inoculation. While very great benefits will undoubtedly accrue from this extension of scientific research methods into the arts and industries where it has not hitherto penetrated, very grave dangers undoubtedly attend the present forced growth. These dangers are two-fold in character. In the first place, with the enormous demand for research workers and with the glowing prospects of hitherto undreamed of accomplishments, many men will undoubtedly undertake so-called research work for which they are essentially unfitted, both as to mentality and training. At the same time, alleged research activities will be started on a large scale in places where there is little or no present call for such activities or where the nature of the problems is such as to necessitate a gradual building up of the research function. These two correlated factors, which are not at all unique to industrial research, since they in one way or another attend the publication of every new and useful achievement, will undoubtedly result in temporary reactions against the scientific method and in favor of the so-called "practical method" The extent to which these reactions will prove deleterious to the general early extension of scientific methods to the various industries will depend very largely on the control and guidance which is exercised in the next few years and on the ability of our universities and technical schools to meet the large demands for capable, scientifically trained men which will be thrust upon them.

This brings us to the second of the dangers, namely, the possibility that the pressure of industrial research may react on universities and technical schools in such a way as to cripple them in their essential functions as a part of the general scheme of national economic development.

In the chaos attendant upon the growth and development of this new element of our industrial fabric there has been a great deal of uncertainty as to the relations between so-called "pure" and industrial researches and the relations of the university to the industrial research laboratory and its problems. A part of this chaotic condition is inherent in the nature of a situation which requires much experimentation before the road to the answer is clearly marked out. A part, however, is undoubtedly due to a failure properly to analyze the situation and assign to each element in the research program its real sphere of utility.

The relations between pure and industrial research and between the university laboratory and the industrial laboratory were very fully considered and very clearly set forth by Colonel J. J. Carty in his Presidential Address at the annual meeting of this Institute in Cleveland in 1916. Colonel Carty pointed out that much of the confusion now existing was due to the fact that the *methods* employed in pure or academic research and industrial research were identical and that from this had arisen a confusion of ideas based on the assumption that identity or lack of identity in the method was the yard-stick by which research was to be designated as pure or industrial. Colonel Carty pointed out that the real distinction lay in the *motive* behind the research and not at all in the methods employed. In other words, two investigators starting from essentially the same basis of fact and pursuing practically identical paths to practically identical conclusions could be, the one a performer of industrial research with a purely utilitarian motive, and the other a delver in pure research for the purpose of enlarging the bounds of human knowledge and with no immediate utilitarian aim in view.

If, as Colonel Carty points out, the question of motive is the essential determining factor which differentiates pure and industrial research, a thorough appreciation of this fact should tend to clarify the situation as concerns the functions of academic institutions and their research laboratories in the industrial research problem.

Viewed from the standpoint of motive, there would appear to be but little place in an academic institution or its laboratories for real industrial research. Per contra, there would appear to be but little place for so-called "pure" research in a strictly industrial organization. It does not necessarily follow from this that an academic institution should engage in no industrial research, nor that an industrial institution should never undertake problems of pure research, for, on the one hand a certain direct and limited participation in industrial research may be the best way to advance pure scientific research and the development of men, while on the other hand, limited excursions into the realms of pure science may react to the best interests of the industrial laboratory and its staff and be productive of real and worth-while extensions of the realm of knowledge. In such cases, however, there should be no misunderstanding as to the governing motives.

Viewed in the broadest possible way and considered as a part of a complete national organization, industrial and academic or pure science research laboratories are, of necessity, very closely associated and must grow together and with the same relative vigor if the best interests of civilization are to be served. It is difficult to see how either could completely supplant the functions of the other. On the one hand, the academic institution, with its pure research laboratories, is engaged primarily in the education and development of men trained in scientific methods and in extending the realms of knowledge for the benefit of mankind and without any particular utilitarian motive. On the other hand, the industrial research organization is engaged primarily in the utilization of trained scientists for the solution of utilitarian problems; in other words, in the cultivation of the regions discovered and mapped out by the experimenters in pure science. The industrial research organization is not a general educational institution and from its very nature cannot take over that function of the university and technical school which has to do with the fundamental training of men. Further, since it is part and parcel of an institution, whose primary object is avowedly commercial production for the sake of monetary gain, and all of whose funds are derived from the sale of its wares, it cannot usurp in any large measure the functions of the university or the pure science research foundation without raising a grave question of ethics involving the body politic.

If, therefore, the university and technical school are primarily what we have long considered them to be, namely, institutions for the training of men and for the carrying on of researches designed to enlarge the bounds of knowledge, and if, on the other hand, industrial research organizations are institutions which utilize the human product and the advances of the universities in utilitarian adaptations, it follows that there is and can be no serious competition between the two, for, on the one hand, if the university or technical school should attempt to enter too largely into the realm of industrial research it would run the very grave risk of destroying its primary functions by the establishment of an atmosphere highly prejudicial to the broad training of men or to pure scientific research. On the other hand, the industrial research organization is not equipped to undertake the general work of the university either in the training of men or the conducting of pure research, nor can it afford to do so. From the very nature of its genesis, the utilitarian object is the motive controlling it. To be sure, occasional pieces of pure scientific research may from time to time emerge from the industrial laboratory, but this work will be small in comparison with the sum total of accomplishment in its proper field.

But to succeed in its proper field industrial research must receive a continual stream of capable men and women thoroughly trained in methods of scientific research, thoroughly grounded as to the geography of knowledge, and competent to appreciate any extensions in its boundaries and capable of immediately cultivating such extensions for the benefit of the particular industrial research organization with which they are connected. Any failure in the supply of men or any failure to advance the bounds of knowledge by pure scientific research will result inevitably in a strict limitation of industrial research. In referring to the work of the early chemists and alchemists, Sir William Tinney remarks that until men began to observe and interrogate nature for the sake of learning her ways and without concentrating their attention on the expectation of useful applications of such knowledge, little or no progress was made. In other words, until a sufficient foundation of pure science has been successfully laid there can be no applied science.

In considering the part which the educational institutions should play in the solution of research problems for industrial organizations, there are other factors which should not be lost sight of. As already indicated, there would appear to be a very real and imminent danger to its pure research function if the educational institution were to embark largely on the solution of industrial problems for commercial organizations. Such a course would establish and attempt to maintain in the same institution two sets of research activities propelled by antipodal motives. On the one hand would be the group of pure scientific investigators, whose impelling motive was the desire to explore unknown regions and extend the bounds of knowledge without any thought of the immediate practical application of the results of their work. The measure of their achievements would be the number and extent of the new facts brought to light. On the other hand, the industrial research group would be involved solely in the practical results of their work and the measure of their success would be an amount of money made or saved.

This state of affairs, involving the existence in the same institution of two groups using the same methods but impelled by different motives, would not itself be serious if the financial situation of the individual workers was relatively the same. Unfortunately, this is never the case. Having a monetary motive, success in industrial research is measured largely by a monetary reward and there would be, therefore, a strong tendency for the industrial research workers in the educational institution to command and obtain larger emoluments than their confreres in the pure research field. Since academic salaries are, in the main, notoriously low, any scheme which would raise the general income of the industrial research worker above that of his co-worker in the pure research field would produce dissatisfaction and discontent and what is worse, a tendency for the major part of research ability to gravitate into the industrial field. Occasional exceptional instances of men so imbued with the love of scientific research that the question of monetary return is wholly secondary cannot outweigh the fact that in the great majority of cases the needs of wife and family for the material necessities and comforts of life are a far stronger force in determining a man's actions than any other.

There is another factor of quite a different character which must be reckoned with. This is the element of human nature in man as at present constituted and as exemplified in the legal rules which he has evolved for the proper guidance of his affairs. Roughly speaking, in the material aspects of life human beings expect to enjoy, for a time at least, the exclusive results of their efforts. This is certainly true in the realm of industrial research where those who advance money for its promotion expect to enjoy the proceeds which flow from its results. This is neither an unnatural nor an unworthy aspiration, but it does raise a considerable question as to the policy of an educational and pure research institution entering upon a relation with the providers of funds which would tend to limit definitely the freedom of the institution in certain directions. For centuries early and full publication of the results of scientific research has been a cherished prerogative of the research worker and of the institution with which he was connected. If the institution is now to enter into a commercial relation with individuals and groups, whereby it is to undertake the solution of their problems with the help of funds which they provide, it must expect to become, in essence, merely a part of their organization and be governed to a large extent by the rules which govern in the commercial field. Not the least of these rules is the one which ordains that the results of industrial research shall be the exclusive property of those who accomplish it, either by being maintained in secret or by being protected by patent.

The welfare of the university and the technical school in their proper functions should therefore be a matter of the gravest concern to all who are engaged or interested in industrial research and we should be continually on the alert to see that their facilities are ample and functioning to full efficiency. From the standpoint of the writer, these facilities should be mainly those which will enable them to attack broadly problems of fundamental scientific research—the facilities for industrial research being provided either by the individual industries themselves, by associations of manufacturers, or through the medium of industrial research laboratories equipped to undertake the solution of problems for clients. While these latter would, of course, be self-sustaining, they would not necessarily be money-making institutions in the ordinary sense.

The second of the dangers of the present situation referred to above is that under the enormous pressure for industrial research, engendered by war conditions, too large a proportion of the men capable of pure scientific research and the instruction of other men may be drawn away from their proper field of endeavor. If this withdrawal is but temporary and for the duration of the war, no great harm will probably result. If, on the other hand, there is danger that the withdrawals will be permanent, the results for the next decade will be serious indeed, since we will have erected the skeletons of huge industrial research organizations and find ourselves confronted with an inadequate supply of properly trained men to carry on the work. Further, there will be a long continued cessation of explorations in those regions where exploration is most needed. For the good of the nation no stone should therefore be left unturned to provide suitable research facilities in a suitable research environment for every man who shows that he has that element of originality, without which progress is impossible.

From what has been written in the foregoing paragraphs it occurs to me that I may possibly have trenched too largely on the scope of the papers covered by Mr. Skinner and Profesor Kennelly. If so, my excuse is that I have not had opportunity to consult with them and also that it is difficult for me to set forth my ideas on industrial research without, at the same time, stating my viewpoint on the relations of industrial to pure or institutional research. Further, as my point of view may not be identical with that of the other writers, some useful purpose may be served by its presentation.

Having set forth certain conditions which seem to me fundamental and certain situations which appear to confront all of us who are interested in the progress of industrial research, it may not be uninteresting to outline briefly the scope and progress of industrial research in the Bell Telephone System.

The aim of this research is frankly utilitarian in character. The main and never-lost-sight-of aim of the work is the advancement of intelligence communication telephonically and telegraphically by the application of the best scientific methods to the solution of the innumerable problems which arise. In the present stage of development the arteries of industrial research permeate a very large part of the organization. This considerable utilization of scientific research methods is not the result of a mushroom growth but is a gradual evolution through the experience of many years. All of this experience has tended to confirm our conclusions that real advances in the art of telephony and telegraphy are to be looked for only in the results of broad-minded and painstaking research work. For many years, therefore, the constant effort has been to provide an ample staff of capable men, specially trained in scientific methods, and to place at the disposal of this staff whatever was necessary in the way of material facilities or human assistance. In common with many other industries engaged in the development of industrial research, the principal difficulty that has been experienced has been the securing of a sufficient number of trained workers. While not necessarily a reflection on the colleges, universities and technical schools, since their problem was a huge one, it is undoubtedly true that they have not in the past been able to secure, train and supply a sufficient number of skilled men and women to meet the evergrowing demands of industrial research. The result is that we of the Bell Telephone System are today confronted with a very large number of problems of the utmost importance in the advancement of national and international communication work, and it is only through the greatest exertions that we have been able to secure and organize the specially trained force needed for their solution.

A common and not unnatural mistake concerning industrial research is that its field is limited very largely to the solution of laboratory problems. I venture to say that if a hundred educated and intelligent people in the ordinary pursuits of life were asked their ideas as to what constituted industrial research the great majority of them would picture some sort of a building or buildings in which men were experimenting with bottles and glasses and chemicals or with mechanical or electrical instruments of some sort. Although the work which goes on in laboratories such as these is a vital and essential part of the industrial research problem, it is by no means the whole of its sum and substance. In order to make the work of the laboratory investigator of maximum efficiency it is necessary in some way or other to formulate the requirements of his problems so that he may direct his energies along the straightest path to the desired goal. In many cases it is further necessary in some way to investigate carefully the products of laboratory work from the standpoint of their adaptability to commercial requirements and to the re-actions which they may bring about in more or less unexpected places. All of this follows directly from the fact frequently lost sight of, that what should be done depends in large measure on what can be done, and what can be done in turn on what it is essential to do. In other words, it is the old problem of the gun, the projectile and the armor in a slightly modified form. While a little thought indicates the necessity for a proper correlation between a study of requirements, the formulation of the problems to be solved, the carrying on of the research, and a test of its results in their practical application, it is clear that no hard and fast scheme of organization, applicable alike to all industries, can be laid down. In fact, it seems that the proper methods are almost as numerous as the types of industry to be served and that they range from a condition, on the one hand, where the entire problem is undertaken by the man who actually performs the research to the other extreme, where each phase of the problem is in the hands of separate individuals, no one of whom encroaches in the slightest degree upon the field of his co-worker.

As exemplified in the Bell Telephone System, the carrying on of industrial research does not lie at either of the above extremes, although possibly it may approach slightly nearer to the latter than to the former condition.

Broadly speaking, the organization of our industrial research can be divided into two main subdivisions:

1. That part which is performed in the central Engineering Department of the American Telephone and Telegraph Company; and
2. That part which is performed in the centralized laboratories of the Western Electric Company.

In the first of these two divisions originate many of the requirements which call for the application of industrial research. This group, which constitutes in a way the central research department of the Bell System, is under the immediate direction of the Chief Engineer of the American Telephone and Telegraph Company, who has likewise general direction of the research problems undertaken by the second of the groups. The equipment of this group consists not of laboratories and apparatus but of highly trained human beings, the whole forming an organization capable of passing expert judgment on any of the vast number of problems continually arising in the system. In this central analytical research department are conducted not only the studies tending to correlate the requirements for the specific research work needed by the Bell Companies, but also those broader studies of fundamental requirements of growth in an ever-expanding electrical intelligence communication system. The necessity for a group of this kind, equipped in the manner indicated, is the result of our years of experience with other and what now appear to be essentially faulty alternatives.

From this first division of the research organization originate in large measure the sailing directions which govern the course of the physical and chemical researches carried on in the centralized laboratories of the Western Electric Company. Without these sailing directions it would be impossible to carry on much of the work of the centralized laboratories, which is now productive of results of great value. Before passing to an outline of the work of this second division, there is one point which should be clearly understood. This is the fact that while the research work of the Bell System is divided into the two main and essentially non-overlapping divisions noted above, it is not to be understood that the work of each of these divisions is carried on wholly independently of that of the other. Each group is in constant and direct communication with the other group and there is nothing of the idea of a separate cell organization in the arrangement. In other words, there is no thought that the work in the first division will be entirely completed without consultation with those in the second division, or that of the second division without consultations with those in the first division. For example, in the formulation by engineers of the first division of a problem to be investigated in the physical or chemical research laboratories, the research men of these laboratories are drawn into the work in a preliminary way to avoid, so far as possible, any chance of such formulation as would result negatively or involve a large expenditure of time and money in essentially useless work. Very frequently this preliminary participation of the laboratory research workers modifies the requirements which were originally thought proper.

As indicated above, the physical, chemical and mechanical research laboratories of the entire Bell System are centralized in the Western Electric Company and are under the immediate direction of its Chief Engineer. This centralization of laboratory facilities was adopted after a number of years' trial of other arrangements, involving at one time three large separate laboratories. The experience of the past ten years has indicated clearly the desirability of the present arrangement in the System as now constituted. First and foremost, it makes available an extent of plant and staff not otherwise economically possible. It also tends to a more efficient utilization of this plant and staff on the major problems of the System by eliminating practically all duplication of effort, which previous experience indicated to be inevitable in a non-centralized laboratory organization. Finally, and not by any means least in importance, is the material advantage of having the research laboratories closely associated with the manufacturing part of the System. This advantage is the direct consequence of the fact that practically all of the results of our research work find their immediate fruition in physical apparatus which is to be constructed in large quantities under modern manufacturing methods. Any failure, therefore, to co-ordinate closely the work of the industrial research department with that of the manufacturing department would result in considerable delays and inconveniences and great monetary loss.

For purposes of administration and the more efficient carrying on of the work, the laboratory is divided loosely into a number of principal divisions, each concerned with some general phase of the industrial research problems peculiar to the telephone and telegraph industry. Thus we have the chemical laboratory, dealing with the class of problems its name indicates; the transmission laboratory, dealing with those researches which are peculiarly involved with telephone transmitters, receivers and the efficient transmission of speech and telegraph currents; the fundamental physical research laboratory, which deals with the broader physical problems of the industry; the general physical laboratory, which deals with the more specific and immediate problems; the manual and machine switching circuit laboratories, which deal with those problems peculiar to the complicated networks which form such an integral part of modern communication systems; and finally, the various telegraph, mechanical and metallurgical laboratories, which deal with specific problems in other spheres of the work. All of these various laboratories are organized to work in the closest harmony and without unnecessary duplication. Very frequently the solution of some research problem involves the co-operation of three or four of the groups, each group handling the particular element of the problem for which its personnel and equipment particularly adapts it. Very frequently also problems arise which necessitate the formation of a special group selected from the various parts of the organization, with particular reference to the work in hand—this group being disbanded on the completion, or re-organized as the work progresses.

As indicated above, the laboratories of the Western Electric Company handle the physical, chemical and mechanical researches for the entire Bell System. In large measure these problems are those formulated as a result of the analytical researches of the first group noted above, namely, that under the immediate supervision of the Chief Engineer of the System. In the carrying on of this work with a staff of expert research investigators, many collateral problems arise. These problems, after preliminary investigation, are discussed with and studied by the members of the first group and if found to be of essential value in the art of communication they are put in the way of full and proper solution.

In conclusion, and judging the future by the evidence of the recent past, it seems clear that our industrial research problems are destined to grow enormously, provided an adequate supply of capable and highly trained investigators can be secured. This, and this alone, seems to mark the only practical limitation to the size of our research organizations, since experience has shown that where the proper type of trained mind can be applied, the financial results invariably return many times the cost of the research. To secure the necessary men, those charged with the direction of industrial research must look to the universities and higher technical schools. We must see that in our zeal to obtain immediate results we do not jeopardize our future by taking from the institutions the instructors and research men who can be of more service to the industries of the countries in their academic pursuits than they could be in industrial occupations.

DISCUSSION ON "INDUSTRIAL RESEARCH AND THE COLLEGES" (KENNELLY) AND "INDUSTRIAL RESEARCH WITH SOME NOTES CONCERNING ITS SCOPE IN THE BELL TELEPHONE SYSTEM," (JEWETT), PHILADELPHIA, PA., OCT. 8, 1917.

V. Karapetoff: Industrial research is in a much more favorable position than governmental research or college research, because industrial research has for its purpose the promotion of the sales, or the bettering of performance or economies of certain apparatus. Colloquially speaking, it means dollars and cents, and therefore in progressive companies it finds its due place; it becomes a matter of wise management to guide this research so that it is not wasteful, but is useful for the purpose in hand.

But governmental research, or the research in endowed or state institutions, has more general distant aims; therefore it is more difficult to convince those who hold the purse strings as to its necessity or usefulness. This is, perhaps, the greatest obstacle in the path of governmental or institutional research.

Speaking more specifically about the difficulties in conducting university research, I should like to point out a few disadvantages. First, it is not sufficiently correlated. Usually a researcher is a self-centered fellow by nature, and he is the last person to determine what is being done by others or to confer with other investigators in the field. That is a harmful and wasteful tendency.

I have urged once or twice before this Institute the necessity and the advisability of forming a Research Committee, a committee that would help to guide young individual investigators and strive for a better co-ordination of their efforts. Such a committee, if organized, should preferably consist of suitable members of other technical committees, so that the Research Committee would be in close touch with the rest of the technical work of the Institute.

Another way of correlating the efforts of individual investigators is by unofficial research centers. A man of national reputation, a specialist, say, in magnetic testing, could naturally become a center of information and stimulation for all other workers in magnetic research. Some one should bring him forward, and urge him to become such a center, so that students and beginners might write to him for guidance, references and advice on apparatus. Whatever is done, I make a plea for a better oversight of the efforts of individual investigators, otherwise in this age of efficiency and strenuous effort, we waste too much time on repetition in research.

There is a number of controversial questions in electrical engineering problems which investigators in colleges and in government institutions should go at, because the progress of our art can be promoted better if problems which are under controversy at present are settled. For instance, there are two or three theories of commutation in d-c. machinery; now is the time to settle them, as far as possible, and to eliminate perhaps nine-tenths of the historical material that is of importance no more or is even misleading. We can never hope to advance rapidly if we drag behind us a great mass of accumulated historical data and views that are no longer of value. The colleges of engineering with their libraries are probably better adapted than any other agency to do this work of scavenging and eliminating the accumulated materials.

Out of ten men who undertake research, perhaps only one is gifted enough to do original work. The rest of them fail, or get discouraged, because they attempt to go into new unexplored regions, while their proper sphere of work is to clean out the old stuff and to prepare the path for better investigators, more skillful men, than they are. That is the second plea I am making; namely, for cleaning up the old data and getting the results systematized and digested so that we can say "this is approximately known," or "this is unknown or controversial, but would be settled if such and such tests were performed."

Another difficulty that the engineering colleges are confronted with is this—A student or a young instructor comes to you and says that he would like to work on this or that subject, of which he is practically ignorant. You let him work for a year, and then he graduates and disappears. He has spoiled some apparatus, wasted some materials, and that is about all he has accomplished. Now, I make a plea that each institution of learning become a center of research for a few definite lines, and not duplicate the work done in other institutions. Having selected a few lines of research they should be conducted year after year so that in the end not only tangible results have been produced, but a group of investigators have been trained who are capable of tackling certain advanced problems.

Another plea that I wish to make is that more attention be paid to theoretical and mathematical research as against empirical experimental work. It is perhaps true that in Germany too much emphasis is laid upon mathematical relations, but in this country I would say that too much emphasis is laid upon unrelated experiments. We have a prodigious maze of curves and data that cannot be used for any purpose because the results have not been put into mathematical form and generalized. It is a matter of efficiency and economy to try to perfect formulas and rational theories that would make a vast number of experiments unnecessary.

Finally, I make a plea for more spare time to be allowed teachers and students for research. For a teacher research should be recognized not as a luxury, but as a necessary part, a function of his calling; also gifted students should not be burdened with traditional required courses, but should be given time for research work after they have shown their aptitude for it.

If all these pleas that I have made should be heeded. and if they be gradually incorporated into college curricula, we would create in our colleges that intangible thing that is known as an atmosphere or background of research. With this solid background in an institution, every student going there, and long before he is ready to undertake any work himself, would come in contact with men versed in research. He would hear scientific discussions, he would see mysterious tubes and coils, and he would be inspired by them. And when his time comes he would step into a group of more mature and inspired workers, and he be given a simple definite problem, a task to prove his worth.

Alexander M. Gray: The average student is immature when he enters college and the first two years of his course are spent on mathematics, physics, chemistry and drawing so that only two years are available in which to train him to fit into the industry and, because the engineering profession has asked the universities for everything but research men, thesis work has gradually been dropped in favor of commercial courses on specifications and business law.

If the industry cannot now find a sufficient number of research men, the industry is entirely to blame. It must let it be known that such men are needed and must offer a living wage in order to attract students of native research ability away from commercial engineering work, it must also be willing to take a college man after four years of training and not turn down applications from all except those who have been able to stay at college for eight years necessary to obtain the doctor's degree.

I do believe that the universities should have what Prof. Karapetoff has called a background of research in order to obtain results, not that we expect to get results from the students but we do expect to get some good work from the staff. In order however that the student may get research training, he should work on some problem in which he is interested, and in most cases such a problem is not one that appeals to any member of the staff.

E. P. Hyde: There are two aspects of the interrelation of pure and industrial research to which I see no reference in the papers under discussion, but to which, I think consideration should be given.

If I understand Dr. Kennelly correctly, and I believe his is the common opinion, fundamental research in pure science should be undertaken in the universities. It must be recognized, however, that the research worker in the department of pure science in the university is more or less sporadic in his method. He follows the lines of his immediate interest. When Roentgen rays were discovered, many men in this country put aside what they were doing to pursue the investigation of Roentgen rays. The same thing happened when radio-activity was discovered and its importance recognized. This is as it should be; for in this way the limits of science are carried forward.

At the same time it is necessary that there should be intensive investigation of problems which have already been skimmed of their cream by these sporadic investigators in the universities. Such intensive investigation lies midway between the kind of work undertaken ordinarily in the universities and the application in the industries, and consideration of it should be included in a general discussion of research. Possibly the university is the place for this kind of investigation, but the fact is it is not being done there, and it is my opinion that under the present circumstances the industry that is particularly interested in seeing an intensive cultivation of some particular aspect of pure science is not only justified, but even obliged to provide for research of this kind in order to furnish a basis on which the industrial application may proceed.

The second aspect of the subject to which I refer is that of correlated research in different sciences. Perhaps an illustration will make clear the point I wish to make. The science of illumination is not physics or physiology or psychology or simple engineering, as some think; it is a combination of all these. In order that the science of illumination may be developed, it is necessary that correlated research in all these subjects should be undertaken. I think I am correct in saying that very few, if any, such correlated research is carried on in the universities. In the present organization of university departments such correlation of research work to cover the intermediate fields is not likely to be obtained. It is, therefore, necessary that the industry which is interested in research that is on the border line between various sciences should undertake it in its own laboratories.

It is my judgment that in both of these two aspects mentioned above it is legitimate for the industry and even incumbent upon it to equip laboratories for research in pure science.

L. F. Morehouse: I am glad that Major Jewett has called our attention to the reaction on technical schools and colleges that may result from the pressure for men to take up industrial research.

Industrial research has advanced so rapidly within the last few years that this pressure for men is now tremendous, and our technical schools and colleges are holding their own in the way of men with great difficulty. The problems of industrial research as presented to the individual members of the faculty, the graduate students, and to a certain extent, the undergraduates, are most attractive. The remuneration for the industrial researcher compared with that paid the members of the faculty is relatively large and but few of these, who are competent are, able to resist the temptation to go into the industrial field.

Scientific methods are no longer on trial, and there is much evidence that there is now a wide appreciation of the benefits from their employment in the industrial field. The demand for men trained to appreciate and utilize the results of pure scientific research is heavier than ever before, and, as Major Jewett pointed out, the welfare of the technical and university schools must be a matter of the greatest concern, for unless the universities and the technical schools can perform their functions properly, industrial research cannot accomplish what it should, because there cannot be made available sufficient men who have been properly trained.

What is the answer? Is it not to recognize that the home of pure scientific research is the university and the technical schools, and for industry, transportation, communication and commerce to appreciate the debt they owe to pure science, and give that "sympathetic appreciation" and generous financial support which is so much needed.

I am not losing sight here of the fact, as Col. Carty pointed out in his Presidential Address before the 33rd Annual Convention, that much can be accomplished without money, that the most important and fundamental factor in scientific research is the mind of a man suitably endowed by nature, for unless the scientific investigator has the proper genius for his work, no amount of financial assistance, no laboratory, however complete, would enable him to discover new truths or to inspire others to do so. On the other hand, there are many things required in the conduct of pure research which can only be done with the aid of money, of which there is not sufficient available.

Major Jewett has mentioned the importance of a proper direction of industrial research, particularly at this time, when the war is stimulating industrial research so much. That there should be proper direction, and a proper understanding of the objective in industrial research is important at all times. There is always the danger that men will undertake work for which they are not fitted and that much work will be started for which there is little or no cause. The result is not only misdirected effort, and a failure to accomplish that which most needs accomplishing, but sometimes the attaining of results which are unsound, uneconomical and wasteful.

I can do no better, I think, than to illustrate this by a problem of great importance which came before the great research staff of the Bell system. This was the problem of accomplishing efficient long distance transmission without economic waste and within certain economic limits, *i.e.* the problem of working out efficient long distance service to permit of the lowest possible rate to the public. The problem was being studied simultaneously in Europe and America. Certain groups of distinguished European engineers who had devoted much attention to the matter had reached the conclusion that the answer lay in the production of a powerful transmitter, and such transmitters were being developed.

The Bell system had studied the problem, but had concluded that it was not to be solved by using loud speaking transmitters, but rather in the construction and design of the line and the apparatus that was associated directly with it. That is the line along which our research was directed, a line which was followed out with such complete success that efficient long distance telephone transmission stands as an accomplished fact today, without appreciable reconstruction, and the junking of the existing plant.

To have attempted this result with high-power transmitters would have required extensive changes at all of the ten-million substations connected to the Bell system in the United States, substantial reconstruction of switchboards, and perhaps the cable plant. While greatly increasing the cost of the service to the public, the results would not have been satisfactory, nor indeed more than a partial solution for the limitations imposed by the use of high-power transmitters on any plant that could be constructed would have prevented ever attaining the result accomplished by the other method.

Without proper direction, and without a proper understanding of the objective to be accomplished, this great achievement could not have been accomplished.

The importance to the telephone art of industrial research carried along scientific lines was long ago recognized by the officers of the Bell system. The research departments then founded, of which Major Jewett has spoken, has grown until now there are at work under the general direction of Col. Carty more than six hundred engineers and scientists, carefully selected, with due regard to the practical and scientific nature of the problems encountered. Among these are former professors and instructors of our universities, post graduates and other graduates, holding various engineering and scientific degrees from more than seventy scientific schools and universities. How these departments are related and how the work is carried on is touched on sufficiently, I think, by Major Jewett.

Clayton H. Sharp: Dr. Kennelly has made a plea for undertaking and carrying out of a certain limited amount of industrial research in the universities.

I am surprised that Dr. Kennelly has not emphasized more fully the requirements of the situation as far as the interests of the universities or technical schools themselves are concerned. Undoubtedly, it would be a fine thing for the industries if a large number of the technical problems requiring research were to be solved for them by the technical schools, but what about the technical schools? It may not be to their best advantage. Dr. Kennelly has outlined, and has pointed out, some of the serious disadvantages under which the technical school or university labors in undertaking this work, and these very disadvantages impose quite strict limitations upon the amount and the character of such work, which can be done. He mentions the atmosphere of intellectual freedom and frankness which must pervade the scientific department of a university, and he shows that that is directly inimical to the conduct of many classes of research. This restriction is important both to the university and to the industry involved and cannot be undervalued.

Dr. Kennelly says in another place: "the watchword of industrial research is therefore loyalty to the industry which makes the venture in investigative effort, and this entails watchful protection as to the secrecy of the investigation and its results." On the other hand, loyalty to the university and the interests which it represents may directly exclude all industrial research of this character, and loyalty to the university certainly is an element which is deserving of a good deal of emphasis.

In another place Dr. Kennelly says: "A limited amount of industrial research work may also be advantageously carried on in the college laboratories, along with the general research work. As, with the growth of new industries, a greater demand for such industrial research comes to the colleges, the desirability increases of dealing with it in a systematic way, through the college as a corporate body, by maintaining, in the workshops of the industrial concerns, special assistants under the direction of specialists on the teaching staff. In this way, perhaps, the most effective service may be given by the technical colleges to the junior industries, with the maximum of economy." In connection with this proposition the fact must be taken into consideration that if the college as a corporation undertakes to conduct industrial research, it puts itself in a position of responsibility to the industry which may prove an embarrassment to the college in its more legitimate function of teaching.

It occurs to me also in view of the preceding discussion that as things are the college pretty nearly has its work cut out for it in the provision and proper training of research men as such.

Dr. Kennelly brings up another point, the case of these junior industries, the younger industries which are desirous of having research done, where he says: "These younger industries must have recourse either to the services of a consulting specialist of the researcher type; or to the services of some specialist college laboratories." There is still another alternative, and that is in having recourse to other laboratories which are in existence, or which may be brought into existence, where research may be carried on a commercial basis under all of the limitations as to secrecy and protection of patents which the industry may desire, and where the advantages of continuity of work may be enjoyed, which, as has been pointed out, it is sometimes difficult to get in connection with college work.

Such independent industrial laboratories equipped for research work, are proper and legitimate adjuncts to the development of industrial research for the benefit of junior industries, and undoubtedly will prove to be of great value in the future.

W. I. Slichter: It is interesting to note that of the three papers on the subject of research presented today, two are by members of the research laboratories of commercial organizations and one is by a university professor. This indicates the recent trend of research to the laboratories of the manufacturing companies, as fifteen years ago practically all contributions on research came from the universities.

The enormous resources in wealth and apparatus of the large industrial companies, and the fact that their employes are right in touch with the problems of the art as they arise, gives them a great advantage over the universities. The most important factor in making research worth while is to have a problem that is worth while, and the universities are at a disadvantage in this respect.

The first duty of a university and a technical school is to train men and the major portion of the energies of the staff should be devoted to this purpose, but it is very desirable that a proper atmosphere of science and a proper attitude of mind, both among the instructing staff and students, be maintained by carrying on a considerable amount of research. An instructor who does nothing but teach will almost certainly become narrow minded and unprogressive.

In former times the problems for research were general and the equipment required, simpler and not very expensive, but as the art develops the problems become more special and the necessary apparatus becomes more special and expensive and it is quite difficult to get the thousands of dollars necessary for good research at the present time.

To have the university undertake such work for a commercial company for pay introduces undesirable complications such as patent rights, trade secrets and commercial rivalry, which are out of place in a university. The most practical alternative to this is to allow the individual member of the instructing staff freedom to associate himself with commercial organizations and to carry on work in the laboratories of the school. This is a quite prevalent practise and is satisfactory providing he does not become so engrossed in his outside work as to neglect his teaching. However, the ideal research is that which is performed by the individual for the love of the work. This type has its greatest benefit in its effect upon the worker and by reflection, upon the students, but is very slow in the production of material results.

We have been urged at this meeting to train students to take up research work as an avocation but the demand for such men is not great enough to warrant adapting the work of the school for this purpose. The greatest demand is for men to become operating or commercial engineers. The best research men come from that small class of students who, during their technical course develop an interest in advanced physics and remain for an additional year after receiving their engineering degree. These men readily find employment in research laboratories at much better salaries than those paid to the usual engineering graduate. But such research men are born and their natural talents have merely been brought out and developed by the university. After all, the ability to do fine research is a gift of nature, and one of the most valuable things an instructor can do for research is to recognize this talent in individual students and to direct their interests in such a way that they may develop into big research men of the future.

Harold Pender: The question of the pay of research men is an interesting one, and I must say I cannot agree with some of the statements that have been made here. A research man who is a *good* research man is well paid. However, you cannot expect a man who has had a three or four years' course in a technical school to be an expert research man, nor can you expect him to obtain a large salary at the start.

In the regular technical school curriculum of four years the best that can be done is to give a man a reasonably good knowledge of fundamental principles. If you want him to develop into an investigator, he must spend more time and he must go into things more deeply than he could possibly do in the four years of regular instruction.

To my mind, the work of the technical school or the university is primarily to train men, not to do research, except as research is incidental to the training of the men. Industrial research cannot be carried on economically except by trained research men. To engage men to carry on industrial research in a university, results in taking away from the industries just the men which the industries need.

To my mind, what is primarily needed, is more advanced study in the engineering departments of our universities. Not, however, for the majority of students, nor in the regular four year courses, but for those men who have natural ability along research lines, and who can profit by taking one or two years of graduate work. What is needed, therefore, is some practicable scheme whereby such men may be encouraged to take this advanced training.

One scheme, which seems to me entirely practicable, would be for a company which needs a steady influx of research men, to cooperate with some university in securing and training these men. For example, the company might give the university an annual appropriation of $5000, on condition that the university offer, say, four fellowships, each carrying free tuition and, say, $500 in cash; the appointments to these fellowships to be subject to mutual agreement on the part of the university and the company. Such a scheme is certainly worth trying, and I feel confident that, in the long run, the company would find its investment a paying one.

H. A. Hornor: There seems to be a curious confusion regarding this subject. The engineer wishes to know the real function of industrial research, and what is meant by a junior industry. As a matter of fact, many industries have done and are doing research work. For many years this work has been designated by them under the caption "development." Are the two terms synonymous?

At the present time we have a great demand for ships, and as I happen to be connected with that industry, let us consider the question of research in that particular industry. I should say as recently as three years ago, there would not be a man in this room who was actively interested in the question of shipbuilding. Today you have one million people keenly alive to the problem of shipbuilding. You have here an industry in which there has been a great amount of research. This research dates back, I suppose, many centuries, and yet it was only a hundred years ago that applied science turned to that industry. A half century ago naval architecture secured its independence as a specialized science because the problem of ship design grew to such degree that it was necessary to coordinate all the concrete questions surrounding the building of the ship.

Naval architecture like other branches of science is not an exact science. Admiral Taylor, Chief Naval Constructor of the United States Navy, in a recent speech called attention to the fact that naval architecture, this science of the designing of ships, was an inexact science. If you say, "I want a ship," you say nothing. You do not specify the kind of ship you want. If you wish a ship to sail on the Delaware River you will adopt the construction suitable to that particular trade. If you desire a ship to cross the North Atlantic, you should not use one that is built for the Delaware River.

Chief Naval Constructor Taylor further states that though there have been many observations made on sea waves it cannot be predicted what the proportions of a wave will be at any given time when encountered by a given ship, and therefore the ship must be designed with that great comprehensiveness of view which men have gained by a life time of research. Taking into account these numerous observations, the ship is designed to be seaworthy. This is the crux of the situation in regard to shipbuilding. This knowledge has been gained much the same way that the manufacturer of armor has learned to observe the law of tolerance.

Another point of this problem deals with the moulding of the mind for vocational occupation. Many of us here in this room are probably college graduates—I am one myself—could you say to your parents or anybody in the world, when you were twenty or twenty-one years of age, that you would undertake a certain line as your life work? I very much doubt it. I am of the opinion that the human mind is so constituted that quite readily some other factor may enter the problem after you leave college, and thus change your plans for the future. I do not desire to inject myself into the discussion as an example, but I recall that I intended first to be a doctor, then either a lawyer or a minister, and today I am in the electrical business. I do not presume that all the research I might have done during college days on the subject of banking would have made me a banker.

I imagine it is rather a difficult thing for a college professor to exactly determine how he will form or train the mind or inclination of a student so as to develop him into what we might call a research man. It is a question in my mind whether it is not better for him to give the student such a liberal education, that when the student enters any industry, such as the one I have mentioned, he will have a mental equipment that will put him in the exact place where he will do the most good for this industry, and perhaps fit him to be a more powerful factor in that great research—the development of great men.

Ralph W. Pope: In reading over the papers of today, it brought to my mind the thought that they lack one thing, they do not present a concrete problem.

Four or five years ago when I was in Washington, a New York Central train was wrecked by a defective rail. I then remembered that a prominent member of the Institute was engaged in the problem of ascertaining defects which existed in iron, by an electrical or magnetic process. I took it up with the Bureau of Standards, and found that a man had been working on the problem a little, but had laid it aside on account of pressure of other business, or the lack of money or researchers to carry on the work, so that nothing was being done.

That is one problem, ascertaining flaws in defective rails, and it is a little difficult to understand exactly who is the most interested to the extent of paying for the processes of research, whether it is the manufacturer of the rails, the railroad people, or the Federal Government.

Another problem which is very prominent on the Pacific coast is the question of high-tension insulators. I visited the laboratory of Prof. Ryan, at Stanford University, where he is provided with high-tension current from the mains of the Pacific Gas & Electric Company, for the purpose of investigating the qualities of material for high-tension insulators. The most prominent material at that time was quartz, and the question was the possibility of making quartz insulators. I merely mention these as examples of some of the problems to be encountered.

Charles F. Scott: I have been wondering in listening to this discussion what research really is, and what do we mean by it? We all have an idea, but maybe it is an indefinite one—we may think of it as original discovery, or the acquiring of new data, or the making of new combinations of known things. Is it a problem which always requires advanced and abstract scientific knowledge, or is it somewhat of an ordinary engineering problem with a definite engineering purpose? The discussion has not yet made it clear.

A few years ago, just after the opening of the trans-continental telephone line on the day previous, there was in New York a complimentary dinner to the President of the Institute, President Mailloux, if I remember rightly, at which the ex-presidents and some others were favored guests—a dinner for some thirty or forty persons. Our eloquent past-president, Mr. Dunn, was the toastmaster. Presently he referred to the great achievement of the trans-continental telephone demonstration the day before, and said we had present with us the man who made it possible. We all recognized immediately that the originator of the Pupin coil was present and that this coil represented the outcome of a great research problem in telephone engineering. Dr. Pupin in a modest way said he was very sorry that Mr. Carty could not be present at the dinner. Mr. Carty had not had his clothes off for some fifty or sixty hours before the trans-continental line demonstration, and he was therefore hardly ready to attend a formal dinner. He particularly regretted this absence because of Mr. Carty's early work on the transmission problem. Dr. Pupin went on to say that several years before he had talked with Mr. Carty with regard to the distance to which the telephone could be operated. Mr. Carty knew the problem. He stated what the difficulties were. He indicated that the real problem was in the transmission circuit, and he indicated the nature and the character of the difficulty which was encountered. Then he (Dr. Pupin) took up the problem and worked out a solution. When the engineer has formulated the problem it is more than half solved.

Again it may be asked, what is research,what was the important factor in the solution of this great problem in telephone engineering? Was the real research work in that problem the initial investigation and determining what the difficulties were, so that Mr. Carty could formulate the problem; or was it in the working out of specific method and apparatus by which those difficulties might be overcome, that is, the work of Dr. Pupin? The great result here was obtained by a coordination of the engineer and the scientist. The ability to see a new problem, to get the data, and place them in their proper relations and to definitely formulate the problem, as well as the application of the scientific knowledge and the ability to work it out in suitable form for useful service, are both elements which enter broadly into what we commonly understand as research. Sometimes these various elements are directed by a single man, sometimes they are the coordination of many.

A. E. Kennelly: Major Jewett's paper very rightly points out that care should be taken as to the amount of emphasis to be laid on the industrial research movement. As various speakers have pointed out in this discussion, very little demand was made in the industries for technically trained researchers until recently. Now, partly owing to the pressure of the present war, much demand has been created, and there is a danger, as Major Jewett indicates, of over-emphasizing research work. We ought not to suppose, or lead students to suppose, that everyone should undertake research work. We ought to make it clearly understood that men of natural aptitude for scientific investigation may properly undertake research work as a business, while men who have no special abilities in that way, should be advised to take up some other direction of engineering work for which they may have ability. It would be as great a disaster if all of the engineering profession were engaged in research, as if none of that profession were engaged in research.

It seems likely that any electrical engineering student, who is interested in experimental investigation, may profitably spend a year in postgraduate research after completing his regular four years of professional study, and that there is no danger of his being too much specialized by such a postgraduate year of research, even although his subsequent commercial work should be directed in entirely different lines. This is for the reason that any and all of the problems that present themselves to the engineer may properly be dealt with in the scientific attitude of mind of the research investigator. On the other hand, however, if a student expects to enter engineering work in other directions than that of research, it is probably undesirable for him to spend more than one year in postgraduate laboratory work, even although he may be attracted by such work, since a prolonged period of laboratory training may tend to specialize him too highly in a direction different from that of his expected career.

In the same way, men who are not attracted by experimental work in the laboratory should take up practical engineering work immediately after graduation and not attempt postgraduate electrical laboratory study. It is fortunate that engineering offers activities for great varieties of talent, such as salesmanship, design, construction, operation, invention, administration, and investigation, so that only the men who have special abilities in the last named direction should be encouraged to specialize that way.

The graduating thesis of a student is an excellent means for estimating his aptitude for research work. As is well known, there are various kinds of undergraduate theses, such as stati tical theses, design theses, machinery-test theses and experimental theses. If all of these types of theses are left open for the students' selection, it is likely that only those who have some natural ability for investigation will select the experimental thesis, which affords an excellent test of his student powers in that direction, as well as providing him an excellent opportunity to develop them. Dr. Sharp is undoubtedly correct in pointing out the desirability of a *tertium quid*, or a third alternative, for junior industries who may want to conduct research, namely, the special testing laboratory, which offers its services and facilities for particular commercial investigations. It seems likely that this type of laboratory will show a marked development in the future, perhaps through the work of research consultants, that is to say, a junior industry not having a laboratory of its own, but desiring to carry on some technical research for developing its manufacture, would be likely to call in a research consultant, who might employ the facilities of the commercial laboratory with great advantage.

Prof. Karapetoff has very properly pointed out the need of correlating the research investigations of the various colleges, technical schools and laboratories, whereby unnecessary reduplication of work may be avoided. In the past, electrical engineering colleges have naturally tended to make investigations along somewhat parallel lines. It would seem that any specialties which may be developed in particular laboratories should be encouraged, and recognized for the mutual benefit of all; so that a research specialty in any particular laboratory may be recorded as an asset, not only to that laboratory, but also to the whole engineering profession.

We will probably all agree with Professor Pender that if the industries demand a certain number of technically trained research graduates, such men, picked from the graduating students for their apparent abilities in this direction, as indicated by their theses, should be given an additional postgraduate year of laboratory work, and that industries may well realize that it is to their advantage to provide special scholarships in the colleges for enabling such postgraduate work to be carried on. Such scholarships, if unhampered by restrictions, would accomplish good work for the students, for the industries and for the colleges.

Capt. E. B. Craft: It does seem to me in the material brought out there is a practical agreement of ideas and that there is a demand for researchers. I personally can attest to that fact, and the only place we can look to for our supply is the university and technical school, and we only hope that these expert men will be forthcoming.

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INDUSTRIAL RESEARCH AND ITS RELATION TO UNIVERSITY AND GOVERNMENTAL RESEARCH

BY C. E. SKINNER

ABSTRACT OF PAPER

1. Introduction emphasizing the great activity in all branches of research at the present time.
2. A division of research activities into three principal classes, university, governmental and industrial.
3. A statement that the principal function of university research should be to train research men and some suggestions as to the type of training required.
4. A brief statement of the function of governmental research which has to do with such things as the development and conservation of our natural resources, the national defense and the promotion of those things which are for the benefit of the whole people.
5. A brief description of the Research Division of the Westinghouse Electric & Manufacturing Company, its rise and its relation to other departments.
6. A statement of the desirability of co-operation among all agencies engaged in research.

RESEARCH has been defined as "diligent protracted investigation, especially for the purpose of adding to human knowledge." If we accept this definition we find that it covers an extremely wide field of activities. It applies equally well to the work of the pure scientist, whose primary aim is to extend our knowledge to the remotest bounds of the universe, and to the work of the industrial chemist, who by careful painstaking analyses maps out the limits of impurities which may be allowed in a manufacturing process. Between these extremes lie possibilities for research in every imaginable field, with pure and applied science so closely interlinked that it is impossible to say where the one ends and the other begins.

That there are possibilities of adding to human knowledge in innumerable ways, is evidenced by the status of research today. Never before were there so many interested in research in all its branches; never before were there so many forces at work to promote research. Changing economic conditions, even before the war now in progress, made it necessary that research be carried on in almost every field of endeavor. The war has emphasized this necessity many fold. Many industrial corporations which were not provided with research facilities heretofore are making such provision and many of those having such facilities are making considerable additions, both for increased amounts and for an increased variety of research. Research departments and scholarships are being provided for in universities and colleges. Commercial research laboratories are being promoted to serve groups of industries where individually they cannot economically provide their own facilities. Scientific and engineering organizations are appointing research committees, and research organizations, such as the National Research Council, are being formed. We have research provided for by private gifts and by governmental grants. The Federal Government is constantly increasing its research facilities in its various departments, such as the Bureau of Standards, the Bureau of Mines, the Forest Products Laboratory, the Agricultural Experimental Stations, etc. This movement is not only national; it is international. Commissions from abroad have studied research conditions in the United States and the United States have sent men to our Allies to study their progress in research in connection with the war. For the immediate present, progress in pure scientific research is perhaps much less than prior to the war, but nevertheless the status of research today is a condition of great activity. The slowing up of pure scientific research at this time is due to the fact the workers the world over are in a very large measure engaged either in actual warfare or in research in connection with problems arising through the war. Fortunately for the future, many of the results of these war investigations will be of great value in connection with the peaceful pursuits to come after the war.

What vast fields for investigation are opened up by the problems relating directly to the war! These are physical, chemical, electrical, metallurgical, physiological, psychological, mechanical, medical, problems of undersea noises, problems of high altitudes, problems in wave propagation, problems in medicine, in surgery, food values, agriculture, transportation, and a host of others. It is merely the hundredth repetition to say that the present war is one of scientific methods and machinery, but this repetition emphasizes the place which research now necessarily occupies and will occupy after the war. It should be of value therefore to discuss some of the phases of the problems of research as a whole and especially industrial research and its relations to other lines of research.

For the purpose of this discussion, research may be conveniently divided into three,—possibly four—general classes; this division depending upon the agencies involved and the purposes for which the work is done. These are university research, industrial research, and governmental research,—and possibly philanthropic research. The first includes the pure scientific research, which naturally finds its home in the university, and all other research done there for the purpose of training men. The second includes all that done by industrial concerns or for them with the purpose of advancing industry. The third includes all that carried on by the Federal or the State Governments for the purpose of benefiting the people as a whole. Philanthropic research differs from governmental only in the fact that its funds are provided by individuals, instead of by the Government, as for example the work of the Rockefeller foundation. No sharp dividing lines can be drawn between these classes. They all have much in common. All should be scientific. Much industrial research is philanthropic, and many times university research leads to industrial progress. Governmental research is for the purpose of serving the whole people and it therefore does much for education and for industry and certainly could be classed as philanthropic. It is just as difficult, therefore, sharply to maintain these classifications as it is to maintain a clear-cut distinction between pure and applied science, as has been so well discussed by Past-President Carty in his presidential address.

It may be well, however, to consider something of the primary function of each of these four classes, their distinctive fields and their relation to each other. The discussion which follows naturally will be from the viewpoint of one whose experience lies wholly within the field of industrial research.

The primary function of the university in research should be the training of research men. If the conditions outlined in the introduction of this paper are accepted, then it is evident that men with the broadest and best training possible are going to be required in ever increasing numbers for every possible phase of the work. It is self-evident that the fundamental training of these men must be gotten in the university and universities should be equipped to turn out research men just as they are now equipped to turn out men with academic and engineering degrees. It is to the university we must look for the broad fundamental training so necessary in research. These trained men will in accordance with their training and their fitness find their places in pure scientific research, in industrial research, in governmental research and in teaching positions where more and more research men are to be trained. In this connection it is rather a sad commentary that the university has not been able to make its teaching positions more attractive by providing compensation more in line with that obtainable in other lines. While writing this paper, its author received letters of inquiry for several men for teaching positions,—some in connection with research—with specifications which could be met only by men of considerable experience. The compensation named was very much less than that which meeting the same specifications could command in other lines. While it is generally understood, at least by those outside the teaching profession, that it has many compensations, such as short hours and long vacations, the average salary is certainly not one of these special attractions. We have splendid endowments for buildings and sometimes for equipment, but we have practically no endowment for men of genius who might make notable progress in pure research and at the same time serve as teachers and examples for the training of men for the whole field. If it takes a genius to recognize a genius yet undeveloped and properly to stimulate and direct that genius, how necessary it is that we place men of genius at the head of the research departments of our universities. A few are already so equipped, but this is a plea for a larger and a more definite program of research training and the development of that judgment which will be able to direct these trained men to the line of work for which they are best suited.

In training research men, the university will naturally become the custodian and the promoter of pure scientific research. Properly equipped and properly manned, here as nowhere else can go on with ever increasing acceleration that wonderful work leading to the discovery of new elements, new laws and new phenomena. It is here that studies can be made of phenomena which are of absorbing interest, but which apparently have no commercial value whatsoever. It is here that we must look for studies of such things as the photoelectric effect, the penetrating power of alpha, beta and gamma rays, the character of the emanations of radium, the study of vortex action, and a thousand other studies,—all of which have value in advancing our knowledge of the unknown. All have value in the training of men and it may not be too much to say that all will ultimately be of value in advancing the well being of the human race. Contact with pure scientific research of this kind is of the utmost value to the research worker in whatever field if for no other reason than the stimulus it gives his imagination, and a research worker without imagination is sure to be a failure.

In the training of research men facilities and problems must be provided and the question arises as to the relation of these problems to going industrial problems. One should not be dogmatic on the subject, but the following suggestions may be worth consideration and discussion. Training should be aimed first at a broad general education. This should be such as to give culture as well as a thorough grounding in general science. Specialization should begin only after the broader foundation is laid. This specialization should be aimed at developing the research spirit, which is simply a desire for knowledge of the truth. To this end problems should be set which will develop the latent abilities to observe facts and to overcome obstacles. For such training the exact problem matters little just so it is in the field of chosen endeavor. It may be of a highly theoretical nature with no possible immediate industrial application. It may be for the establishment of a law, the proving of a principle or the finding of the limits of a known law. It may be a very restricted problem, as the proving of a single phase of one otherwise well worked out. The problem should be so chosen that the really essential apparatus is available for its solution or the problem should require that the necessary apparatus be devised. It is really curious to see the attitude of different men with regard to apparatus when a problem is set for solution. At the one extreme a worker will demand the most elaborate and accurate apparatus known to the art before he feels that he can begin his work. At the other extreme the worker uses whatever means are at hand or devises crude makeshift apparatus. The former is interested more in his apparatus than in his problem; the latter more in the principles involved than in the finish of his instruments. Both may fail,—the first due to too much elaboration of method and apparatus and the second due to the use of apparatus not adequate for the purpose. A part of the desirable training is therefore the training of the judgment as to how little and how simple apparatus can be used and still cover all the necessary points in research. The maximum of success often comes to the worker who uses the simplest apparatus both as to the time required to get results and their values. It is of course impossible in many cases to get the required results without the use of the most accurate and delicate apparatus available. Larger and more accurate telescopes make new discoveries possible and more sensitive oscillographs give accurate wave forms whose existence would not even be discovered by less sensitive apparatus.

Industrial research problems cannot as a rule be given over to university research departments for solution. There are many reasons for this,—the patent situation being one of the prominent ones. So long as industry depends so much upon patents or upon secret processes for the protection of its business it must carry on its vital research within its own laboratories and under its own control. Many industrial problems are also so closely related to the work daily being performed in producing the particular commodity that any research in connection therewith must be done on the ground and by men familiar with all phases of the problem.

There are, however, many problems which could be turned over to university research laboratories for solution and these would be very valuable for the training of men for the industry. These are problems having to do with such things as methods of measurement or fundamental laws such as the laws of heat transferences or the laws of hysteresis.

There should be much closer cooperation between the university and industrial research. Industry should recognize that it must depend primarily upon the universities for its trained research men and cooperate to the fullest possible extent to the end that properly trained men be turned out. Pure scientific research should be recognized as the legitimate field of work of the university. The application of science to industry should be recognized as the field of the industrial research organization and it should be realized that no sharp dividing line can be drawn between the two. The university should do research primarily to train men, and the industry to ensure dividends to its stockholders.

Governmental differs from industrial research mainly in the fact that it is financed and directed by the Government and its results should therefore be directly available to all the people. It has many and varied fields such as those relating to the development and conservation of our resources, the national defense and the establishment of standards of weights and measures.

Much governmental research is undertaken for the benefit of industry in general and its methods and equipment are necessarily much the same as those used in connection with industrial research. Like industrial research, it must look to the university for the training of its men. As in industry the present war has brought very greatly increased demands on our governmental research departments and has also shown the desirability of increased cooperation between all the forces having to do with research, both at home and abroad. Here again it is difficult to draw a sharp dividing line between the work to be undertaken by the Federal or the State Governments and that which should be left to industry or to the university. It is safe to say, however, that as in other lines research carried on by the Government will increase in amount and in the kinds of work undertaken, and that, further, there will be increased cooperation with other agencies carrying on research.

While philanthropic research has been mentioned as a possible class, its function and relation to the other classes mentioned are so obvious that it will not be further discussed here.

Industrial research has been described as having for its primary function the securing of dividends to the industry. Fortunately for those engaged in it, its ideals can be just as high as those of pure scientific research, for its work usually results in direct and lasting benefits to mankind. This is certainly true of those researches which have given us commercial wireless telegraphy, transcontinental telephony, half-watt incandescent lamps, giant turbo-generators, modern tool steel and a host of other things.

No single description will apply to the rise of research departments in different industries. In the older industries, the early work was usually carried on by more or less "rule of thumb methods" with a certain amount of experimentation directly on the product. Later, test laboratories were established, and finally these were either combined into a research department or a separate and distinct research department was formed to take care of the necessary research work. In many of the newer industries, the foundation on which the industry has been built is the result of research, and in such cases research work is almost always continuously maintained throughout the life of the industry. This is particularly true of our chemical and electrochemical industries.

In most industries having research departments, we can trace their rise from the beginning of the work, even though done under the guise of test work, and experimental work and without any thought of its being research in the sense of the present use of the term. This is particularly true of the specific industry and organization with which the writer is connected.

It is impossible to describe the rise of research in the Westinghouse Electric & Manufacturing Company without giving something of the part taken in it by the great founder of the company,—Mr. George Westinghouse. A critical study of his life work will show that he personally carried on research of a very high order and in many fields. In his personal work, however, Mr. Westinghouse rarely used ordinary laboratory methods. His experimental work was done full size scale. If he was interested in a rotary engine he did not build a model, he built an engine to suit the commercial conditions he had in mind. If interested in the development of producer gas no mere laboratory experiment would suffice; he built a complete producer gas plant. His work on the development of air brakes, switches and signals, the steam turbine and many other devices was all conducted in the same way. He was an experimental rather than an analytical research worker. He undoubtedly had many failures in his experimental work, but the commercial success of the companies which bear his name shows that in the end the balance was very greatly in his favor. The foundation of each was largely his own personal work. This type of experimental work to be successful requires a breadth of vision, a soundness of judgment, an optimism and a dynamic energy to carry it forward, which are possessed by but few men. Without these qualities so abundantly possessed by Mr. Westinghouse, research work done in his characteristic way is more liable to lead to failure than to success. Throughout his active life Mr. Westinghouse always had in hand a number of investigations of the type referred to and only those who have had the good fortune to work in close contact with him can appreciate the amount of thought, energy and attention to detail which he gave to all of his experimental work. It was characteristic of him that he so drove his work that he would get results in days where the average worker would require weeks or months.

He early recognized the merits of the alternating-current system of distribution and personally gave much attention to its development in the formative period. This system he pushed to a commercial success, overcoming all obstacles including attempts at legislation against the use of alternating current in many states. Not the least of his triumphs in connection with this system was the lighting of the World's Columbian Exposition in 1893 using a type of incandescent lamp necessitated by the patent situation which was considered by nearly every one familiar with the subject as a hopeless substitute for the then accepted type. The modern history of alternating current and its place in the electrical field today is too well known to merit further comment.

At a very early age Mr. Westinghouse became interested in the rotary engine and he carried on experimental work in the development of the rotary engine continuously for many years until the advent of the steam turbine. He immediately recognized the merits of the turbine principle and casting aside the work of half a lifetime on the rotary engine he at once transferred his allegiance and his energy to the steam turbine, with the result that the steam turbine has become the predominating prime mover in connection with the use of steam today. He was the first to combine the alternating-current generator and the steam turbine in large units,—these being the forerunners of the giant 70,000 kv-a. turbo-generators of today. In passing it may be said that the development of the turbo-generator to its present state of perfection is perhaps one of the most notable pieces of research work in modern times and while the finished result is the combined work of many minds, very much of the initiative can be ascribed to the personal work of Mr. George Westinghouse. Both the steam turbine and the turbo-generator are so different in type, construction and all essential details from the reciprocating engine and the slow-speed alternator that all previous experience had to be cast aside in the development of these machines. The problems of ventilation, insulation, stresses in materials, and many others had to be studied from entirely new angles. Experimental units as large as 10,000 kv-a. have been built for the sole purpose of studying the best method of ventilating turbo-generator units.

While much research of the kind carried on by Mr. Westinghouse is still necessary in the development of the company's product it is coordinated with an analytical study of all the data available and supplemented by research in the laboratory, the factory and the field. The laboratory was early recognized as a necessary part of the engineering equipment and several were provided for different purposes in which much research was done. The present organization has resulted from the combining of these laboratories and certain other activities into a division of the engineering department to which additional facilities for research have been added from time to time. Being a part of the engineering department and working in close cooperation with the works department the organization provides for the most intimate possible contact with the problems and requirements of the business and for the most direct applications of the results of research.

The research division is divided into sections and embraces work from the purely theoretical side of the problems presented to the practical application of principles and materials in the factory and in the field. It has under its control various laboratories, such as an electrical laboratory, a process laboratory, a molded-material laboratory, a materials testing laboratory, a chemical laboratory, a ceramic laboratory and a more recently acquired research laboratory proper especially set aside for the work of a more theoretical nature. This latter laboratory has provision for studies in organic chemistry, electrometallurgy, metallography, illumination, general physics, electrochemistry, insulation, etc., etc. The theoretical and the practical research men while in separate sections of the same unit work in close harmony. The result of a theoretical research, such as the production of a new device or material or the application of a new principle, can be tried in a practical way in another section of the division.

The division does all the engineering in connection with the purchase of materials and carries on the experimental work in connection with the establishment of shop processes. The men working on materials' specifications keep in the closest possible touch with the method of production of these materials, their characteristics, their engineering application and their shop use. The process men work between the engineering department and the shop in connection with processes of every sort and prepare all specifications with the cooperation of the shop men under which the process is carried out. The division has the technical direction of the brass foundry, the scrap recovery plant and the copper refinery and is thus in daily contact with the multitude of metallurgical problems arising in these processes. The control laboratory as well as the theoretical laboratory for these processes being under the direction of the division, the best possible cooperation between the two is ensured.

The above clearly shows that the division has been so organized that it can be a leader in the development of new things and at the same time keep in closest possible touch with their application and use. Problems for solution come to it from every possible direction and not the least fertile source of theoretical problems is the grist of troubles from the daily routine work in the factory. We believe that the research division should both lead and follow the practical end of the work. There should be work going on which has for its object the securing of information or the perfecting of devices which will be required in future years. At the same time old processes should be improved, old devices should be reviewed and new treatments should be provided for old material. Research should lead, should follow and should parallel engineering and factory work. Many times the results of researches cannot be taken advantage of at once and there should be a constant accumulation of information available when it is needed. No research can ever be considered as complete. Each new advance in contemporary lines, whether new instruments, new materials, new methods, new applications or new laws, will make further advances possible though not always profitable.

What of the future of industrial research? What lines will be profitable? Should there be greater centralization and greater co-operation between industrial, governmental and university research? The answers to these questions might well be given with counter-questions. What industrial subject is so far advanced that further advances are impossible? If co-operation is profitable in a single corporation, why should it not be profitable between greater forces. It should be evident from what has gone before that the writer believes implicitly in a constantly increasing field for research in every industry. If we can show advances in any material or process or device by proper research, why should not every industry profit by research. It is not expected that every individual study will be profitable nor that every successful result can be at once applied, but careful painstaking and persistent research should benefit any industry. It is expected of course that the research undertaken will have a direct bearing on the particular industry and not be a study of some unrelated subject. It would hardly be profitable for the steel industry to make an elaborate study of the life history of the yeast plant, but the progress in alloy steels has so profoundly changed many industries in the last few years that further research in this line will surely be of value.

It is probable that more and more industries will establish research departments or enlarge their research work (often carried on under some other name) and that these will in a large measure for the present work independently of each other. We must expect competition in this line as well as in other industrial lines. We may confidently predict, however, that we will have an increasingly better general cooperation of all research forces and such agencies as the National Research Council, which bring together for a common purpose men engaged in every phase of research, should have a profound influence on bringing about this cooperation. If such organizations are a benefit in times of national stress and need, surely they can be of similar value in times of peace. The university, the government and industry each has its distinctive field and each needs the cooperation of the others, and only when we secure complete cooperation among all these forces can we make our best advances as an industrial people.

DISCUSSION ON "INDUSTRIAL RESEARCH AND ITS RELATION TO UNIVERSITY AND GOVERNMENTAL RESEARCH," (SKINNER), PHILADELPHIA, PA., OCT. 8, 1917.

B. A. Behrend: It gives me particular pleasure to be able to discuss Mr. Skinner's paper. For ten years I have been closely associated with him, and I merely want to add my voice in praise of the most interesting paper which you have heard Mr. Skinner present this evening.

Mr. Skinner said that "the development of the turbo-generator to its present state of perfection is perhaps one of the most notable pieces of research work in modern times." It has occurred to me, therefore, to devote fifteen minutes of your time to describing in some detail the steps and the process through which the turbo-generator has reached its present development. I hope you will overlook any personal note or touch of egotism, as I am relating the work with which I have been associated and which I have directed for the past fifteen years.

In 1902, the first radial-slot turbo-generator was designed at the Bullock works in Cincinnati, a unit of 1500 kv-a., operating at 1500 rev. per min. This unit was later exhibited at St. Louis, where it received a Grand Prize. It is interesting that only comparatively small changes have been made since in the vital features of design, and that it has become the prototype adopted by all manufacturers of the entire family of turbo-generators of today, which differ in no material degree from this early type. The following enumeration will give an idea of the most salient new problems which were encountered and in which research and resourcefulness had to be developed.

(1) It is clear that a unit of small diameter and large capacity and high speed contains a very much larger loss per cubic inch of material than does a unit of large diameter and low speed. To aggravate this condition, the air passages, on account of differences in diameter inside and outside the punchings, are restricted and tend to the development of eddies in the flow of air. The most successful earlier machines, therefore, did not circulate the air through the cores radially, but made it enter at the bottom, circulating it circumferentially through the laminations, and letting the heated air escape at the top. In later designs, it has been possible to return successfully to carrying out the air radially, while equally successful results have been obtained by passing the air through axial holes in the stationary punchings. The working out of these new and radical methods of ventilation required much thought. Even in the present state of the art, it requires approximately a quantity of air equal to the full weight of the turbo-generator passing through the machine every forty to sixty minutes. In other words, if a large turbo-generator unit weighs 300,000 pounds, 150 tons of air have to be sent through it in a little less than an hour, in order to carry off the heat generated by the losses. How important under such circumstances it appears to pass the air

through in such a manner that part of the heated volume of air does not return into the circuit, and how carefully the flow of air should be guided so that obstacles in its path do not produce aerial eddies, is obvious. On the other hand, there was nothing at the time this problem had to be solved by the engineers to guide them in their designs. It is true that in 1868 the great Helmholtz published a solution of the hydro-dynamical problem of the flow of an incompressible perfect liquid past a barrier placed at right angles to the direction of flow, and thus laid the foundation of what is now called the "Theory of the Discontinuous Stream Line." (Fig. 1.) Kirchhoff and Lord Rayleigh developed this theory for the case of the barrier forming an angle with the direction of flow, (Fig. 2) indicating that behind the obstacle exists an area under pressure in which there is no flow. In the practical problem of the flow of air, which is a viscous fluid, this otherwise calm area becomes the seat of violent eddies, which carry back into the system the heated portions of the air which should have been carried out. It is not unlike the disastrous condition existing in so many of our cities on the Great Lakes, in which the sewage from the city becomes a hydrodynamic sink, whence stream lines radiate towards the intake crib of the water supply. As we all know, this has been a veritable Minotaur of typhoid fever, to which we have sacrificed so grievously through the years.

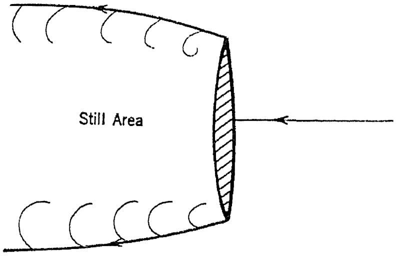


FIG. 1

But even the numerous experiments conducted by Eiffel, Langley, and others in connection with the development of the aeroplane have aided little in the practical improvement of the fan construction and the ventilation in large turbo-units. Nevertheless, here is a field in which the practical engineer should receive aid from the theoretical student, to whom he applies so often for bread only to receive a stone.

(2) The mechanical strength of the new rotors was a matter of much thought. Peripheral speeds of 10,000 ft. per min. were considered high, but they had to be doubled at a stroke in order to meet the new conditions, and this at enormously increased angular velocities. Thus, in the early turbo-generators to which I refer, the mass of an ounce produced a centrifugal force equal

to one ton. It will give you an idea of these peripheral speeds when you realize that the distance from New York to London could be traversed in twelve hours at this rate. Referring to the balancing of the completed rotor, it is now amusing and interesting to remember that it was not possible to induce the mechanics to approach the rotor while rotating at full speed in its bearings, and the present speaker balanced the first units referred to. Later, a sense of confidence developed among the machinists. The stresses in rotating bodies and the requirements of the material were all new problems. Approximate ideas could be obtained for the stresses in toroids and hollow cylinders. From an examination of the numerous mathematical solutions, a most confusing result became evident. The great Clerk Maxwell himself had examined this case, finding the stresses a maximum at the outside circumference. Prof. Carl Pearson pointed out that there was an error in these conclusions. My associate at the time, Mr. H. A. Burson, and myself then tried to work out an approximate solution, the results of which are graphically shown in the accompanying diagrams published here for the first time. We first attempted to apply the beautiful mathematical analysis of Barré de St. Venant and of Lord Kelvin to this problem, but we failed on account of its complexity. Even now, Dr. Chree of Cambridge has not succeeded in giving a correct solution of the stresses in a rotating disk. The elementary solution which we obtained, complex as it seems, is yet simple and clear, but it requires for its accuracy the assumption of a system of forces on the boundary planes of the disk which do not exist in reality. Dr. Chree's solution, which is extremely complicated and quite useless for practical purposes, also involves the existence of forces which do not have reality. Thus we decided to submit to test our problem, as we reasoned that the lateral deformation would give us a clew to the radial and tangential stresses of the disk. We selected lead disks which we spun in a cast steel motor housing, which acted as a bomb proof. The results of these experiments bore out our anticipations completely. We found that the disk thinned out in the center, the material flowing to the outside, showing the maximum

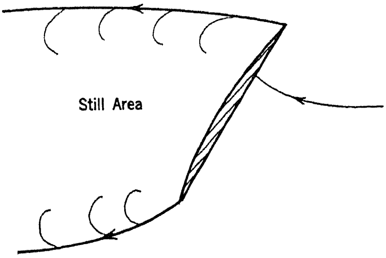


FIG. 2

stress in the center of the disk where the radial stress equals the tangential stress if the disk has no hole at the center. If the disk has a hole at the center, our theory and our test showed the tangential stress to be doubled, which is natural as there is no radial stress to help share the load. Figs. 3 and 4 illustrate these results. These results led us immediately to the following conclusions of tremendous importance, which have become the base of the testing of materials. All test bars have to be taken from the material in the direction in which the stresses occur, and it is futile to make indiscriminate tests on bars removed indiscriminately from the material. In other words, if hollow forgings or castings are to be used for turbo-rotors, test bars must be taken radially and tangentially from the material and not axially. For many years after these experiments were made, it was almost impossible to convince the engineers or the testing laboratories or the steel mills that such procedure had to be followed, and I record it as one of the important achievements, due to fifteen years of persistence, that this is now recognized as a matter of course.

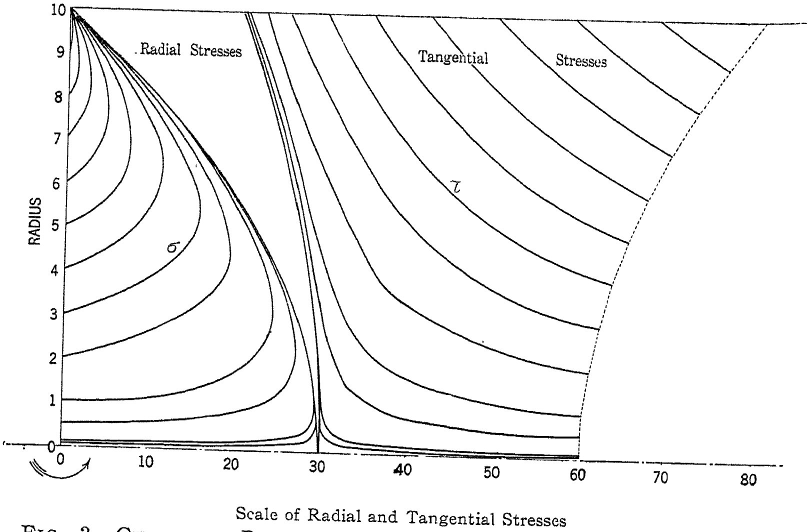


Fig. 3—Chart of Radial and Tangential Stresses in Rapidly Rotating Disk or Toroid

Figs. 1, 2, 3 to 10 represent inner radii of toroids. At cylindrical boundaries radial stresses σ must be zero. Tangential stresses τ are a maximum at the inner boundary. $\tau = \sigma$ at the center of disk. $2\sigma = \tau$ if there exists a small hole in the center of disk

The experiments also showed that it was of primary importance that metal should be able to flow. In other words, that the ultimate strength and the elastic limit of the material are not the factors which should determine its usefulness for high speed work. It is the ductility, namely the percentage elongation

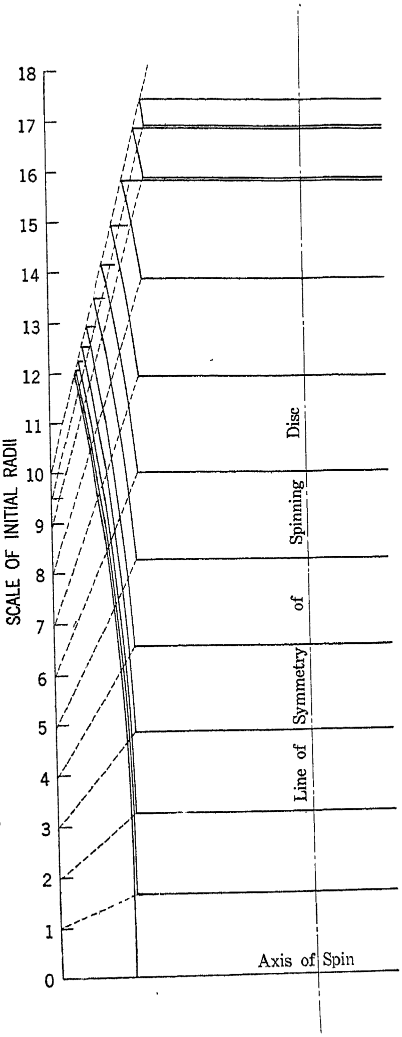


Fig. 4—Deformation and Strain Chart of Rapidly Spinning Disk and Toroid

Figs. 1, 2, 3 to 10 indicate initial inside and outside radii of disk before deformation. These points by virtue of the strain distribution travel along the dotted lines according as the inside radius of the toroid is small or large. The disk or toroids develop concave surfaces as boundaries perpendicular to the axis of spin.

The thecry makes the cylindrical boundary generated by a line parallel to the axis of rotation, while in reality the generating line is a curve with its center on the line of symmetry, making a convex surface for the disk

and the reduction of area, which determines the fitness of material. We demanded for the nickel steel disks which we used 25 per cent elongation, 40 per cent reduction of area, 50,000 lb. per sq. in. elastic limit, and 75,000 lb. per sq. in. for the ultimate strength. Disks were made 2.25 in. thick in the armor plate department of the Carnegie Steel Company, at Homestead, in 1907, under the speaker's supervision, and under the personal direction of Superintendent S. S. Wales and Managing Director A. R. Hunt of the Carnegie Steel Company, to whose patience and skill were due the remarkable results obtained. From these disks, strips an inch or two wide were taken and bent under the hammer or the press flat upon themselves, showing no indication of seams on the outside, although the hole at the center (Fig. 5) was only large enough for the size of a knitting needle, an instrument with which we are now all acquainted.

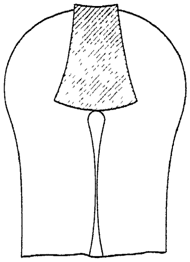


Fig. 5

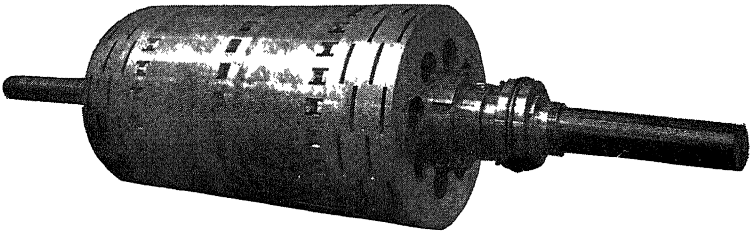
(3) For the retaining end rings which support the rotor coils, an entirely new material had to be developed. It was

found that by treating the nickel-chrome armor-plate steel, which is extremely hard, for ductility, a material could be obtained with 100,000 lb. elastic limit, 125,000 lb. ultimate strength, 22 per cent elongation, and 40 per cent reduction of area, which bent beautifully cold upon itself, and whose only drawback was its high magnetic permeability. Such forgings were made for us fourteen years ago by Krupp, Sir William Armstrong, and later by the Bethlehem Steel Company, under the able direction of their metallurgist, Mr. Edward O'Connor Acker. With these new materials, an entirely new era of safe construction has been entered leading to a discard of the use of cast steel and unreliable forgings of great size in which internal stresses are unavoidable.

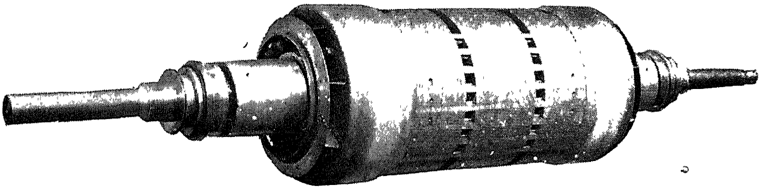
(4) The design of the plate rotor as shown in Fig. 6, thus took its origin in a clear understanding of fundamental phenomena. The building up of the rotor in such a manner that no hole was required for the shaft, bolting on the ends has been a complete success. The construction follows the idea presented in the steam turbine of that creative engineer De Laval, to whom we owe so much in connection with the advancement of engineering.

(5) In the development of the electrical part of the generator, the greatest difficulty is caused by eddy currents generated in the windings. The subject was most ably investigated by the speaker's long-time associate, A. B. Field, A. M., Cantab. By bringing to bear upon this problem powerful mathematical analysis, he showed new means and ways to deal with the extremely important problem of reducing the eddy current losses.

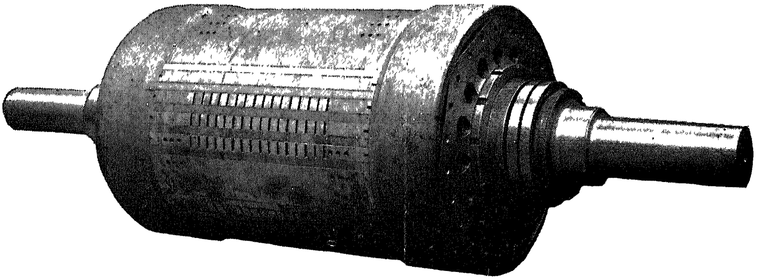
In summing up our labors in the development of this piece of apparatus now so essential in the engineering life of the world in our great power houses, and, owing to Mr. Emmet, applied to our battleships and cruisers, upon which the defence and safety of this nation depends, it is with regret that I have to record how little assistance was obtained by consultation with scientific men or their works. Right here would seem to be the opportunity for aid to be rendered the engineer by those who know more mathematics and physics than he does, but little of that aid has been forthcoming. On the other hand, such men as Sir George Greenhill, Prof. Hele-Shaw, Prof. Horace Lamb, Sir Joseph John Thomson, Prof. Perry, Prof. Blondel, Prof. Bouasse represent investments to their nations which cannot be counted even in millions of dollars. This nation must not become dependent altogether upon the scientific thought of others. Without the realization of the importance of original thought and investigation, the cultivation of science, and the theory of its application, solely for the fascination and interest they exercise inherently upon some minds; without the realization of the importance of the modification of the department store system of university education, of the fallacy of expecting great results from men "told" to make discoveries rather than creating an atmosphere in which the especially gifted may devote themselves to investigation; without the appreciation of such almost obvious



THE ORIGINAL 1500-KV-A.—1500-REV. PER MIN.—RADIAL-SLOT TURBO ROTOR, BUILT IN CINCINNATI IN 1902-1903—THE FIRST STEP IN THE EVOLUTION OF THE TYPE



THE SAME TYPE FOR 600 KV-A.—3600-REV. PER MIN.—BUILT IN CINCINNATI IN 1904



[BEHREND]

RADIAL SLOT PLATE ROTOR WITH BOLTED FORGED SHAFT ENDS—25,000 KV-A.—1500-REV. PER MIN.—BUILT IN PITTSBURGH IN 1916

premises, we cannot attain the scientific eminence which is the glory of England and of France, and the source of human industry, and which we should covet for the greatness of America, our country.

V. Karapetoff: I will say a few words regarding Mr. Skinner's proposed course on the ethics of employment. We teachers hear so much from practicing engineers—"Why don't you teach the ethics of employment? "Why don't you teach your students to do a soldering job decently?" "Why don't you teach the theory of determinants? That is really the most important thing in our work." These unreasonable demands come from a lack of conception of what college life is. A prominent educator has well said: "School must not be a preparation for life, school should be life itself."

In the kindergarten stage we are approaching this ideal, due to the efforts of Mme. Marie Montessori, who has showed us how to arrange the child's life in such a way that it is not a preparation for any future activity, but is a harmonious and spontaneous manifestation of child's functions, abilities and interests "now and here." The great problem with young men in college is to arrange their school life so that they live a professional life in the present, and are not "drilled" for some future employment or future conditions. If college life could be so organized, and I think it could, then we would not have to teach the ethics of employment. The student would be employed right there in an ethical way. Every time I hear a suggestion of a new discipline to be taught, I ask myself at the expense of what other discipline it should be taught, because the time at our disposal for teaching various things is limited.

Another of Mr. Skinner's statements also impressed me—he said that the primary function of the college is to develop men, and the primary function of an industry is to earn dividends. Am I right about the statement?

C. E. Skinner: Yes, referring to research.

V. Karapetoff: Dr. Pender has stated this afternoon that it makes no difference what research subject the student is working on, even if it be a repetition of some old experiment. That is not psychologically correct, because the element of genuine interest of a true investigator would be lacking. That again would be a preparation for life—calisthenics, so to say, mental gymnastics, rather than actual work. The problem must be new and original, no matter how modest, and the inspiration should come from the teachers, which means that the teachers must also do research. To inspire their students for research, they themselves must be investigators.

Men in the industry have told us, teachers of engineering subjects, to keep away from practical problems, because we cannot keep in sufficiently close touch with these problems, and may go astray and work on things that are not needed. They say that our sphere and our limit should be pure science. But are there only two kinds of research, industrial and pure scientific? Is there not an intermediate kind, which out of deference to my colleagues I will not call impure. Let me call it semi-industrial research, and it is precisely that kind of research that we teachers of electrical engineering are interested in. We are not physicists nor mathematicians, so that pure research is closed to us, or else we would be encroaching upon other branches of university instruction. It is this semi-industrial research that is our proper field. Perhaps the most brilliant representative of this semi-industrial research was the late Prof. E. Arnold of Karlsruhe, who left several volumes of writings on the subject of electrical machinery, volumes highly prized by designers and scholars. He was a man who combined a good ability for original research with the rare ability of a compiler and expounder of other people's work.

Between the Scylla of pure science and the Charybdis of industrial research, we teachers are forced into the middle path of work on the theory of electrical machinery and other devices, a subject that is neither pure physics nor industrial research. But how can we successfully carry on this, our apportioned task, unless the men of the industry tell us the relative value and importance of different topics? The poor teacher, separated from the inspiring guidance of the large manufacturing and operating companies, works for example on the condition for the maximum efficiency of a machine, and then he hears the sarcastic remark —Yes, but the cost of manufacturing and the reliability of operation are of much more importance than a gain of a few per cent in the efficiency. So the poor fellow may have wasted a year or two on that problem.

I trust we all drew a valuable lesson from Mr. Behrend's masterly presentation, though a lesson, perhaps not intended by Mr. Behrend. He said—here is a woeful picture of what colleges are not doing. And I say, here is a woeful picture and example of the secrecy of manufacturers, out of whom for many years we could not pull any information as to what research problems were needed in the development of the turbo-generator.

Harold Pender: Prof. Karapetoff has questioned my suggestion that the repetition of classical researches is worth while in the training of a man to be a "researcher." I wonder if a musician writes a symphony before he learns to play some that have been written by other musicians. I would like to hear such a symphony. I think it would be like some of the theses that are done in college.

I think what we need, as I said this afternoon, is some means whereby the men who have research talent may be encouraged to continue their work at the university for one or two years more, preferably two years. The reason men do not do that, I think, is due to the fact that a man who has an inclination for research work is also under the necessity of making his living. He has been four years in college, it has cost him or his father a considerable sum of money, and naturally he does not feel justified in spending two years more at his own expense, for he knows that if he goes immediately into active service in the industry, he can at least earn his bread and butter.

If the manufacturing companies need these research men, they should be willing to do something to encourage them. Giving a man a job in a research department before he has been properly trained for research work is more likely to discourage the man, rather than encourage him. Moreover, from the standpoint of the company, this procedure is also uneconomical. Whether or not the scheme of research fellowships, referred to earlier in the discussions would prove economical may be open to question, but it is certainly worth trying.

Oberlin Smith: I do not think that we can value too much the immense importance of scientific research especially along engineering lines, but what we want is a collaboration of the different researchers, so to speak, so that their work all comes together and is available, and not the present scattered knowledge. If we take a dozen, or more, of the so-called engineering pocket-books, excellent as they are and want to find the tensile and compressible strength of steels of various kinds, there is no trouble in getting the information from these books. There is much of it which comes into play in bridge work and other forms of construction, and it is important that we can be able to secure it at a glance. These books will tell you the tensile strength of cast-iron and sometimes its compressive strength, but when it comes to other metals, such as copper etc., there is a sad dearth of information. Some of these books give tables showing the tensile and compressive strength of a half-dozen or so metals, but they do not give many data regarding the less common metals, such as aluminium, silver, gold, etc. I do not know where you can find in these books the strength, elongation, hardness and annealing qualities of alloys of aluminum, now so much used. The designers of machinery in these days, following along new lines, are using newer metals, like the various alloys of aluminum. In our machinery designing we want to know the tensile, compressive and shearing strength of many materials, not only the different metals and their commoner alloys, but of hard rubber, glass, fiber and various other things. We want to know these little ordinary facts, and know them quickly.

There is no question but what in our college laboratories all over the country there is a great deal of such research work going on. No doubt if we could collect it, we would find most of what we want, but there is no way now of knowing what one particular university or another has done, and many of them are duplicating the work done by others. What we need is some committee to correspond with the laboratories of the colleges, with private laboratories and the laboratories of manufacturers, to ascertain what lines of work they are carrying on. Doubtless most of them would gladly give information pertaining to these things, thus a list of all of the items of information required by designers of modern machinery could be sent out and answers obtained, the whole thing then being tabulated for mutual use.

L. W. Chubb: There have been several views put forth in the discussion regarding the training of an industrial research man, and I would like to add what I can from my more or less limited experience with the men I have been associated with.

I believe that the research man, with an investigative turn of mind, is born and that if the college educators try to develop such men they will develop scientific research men but in most cases spoil the man that has the qualification for industrial research. If a man specializes at college during his first four years he will not have the broad education that is desirable in solving industrial research problems. A good grounding of fundamentals, an imagination, a physical conception of science and mathematics, good judgment, open mindedness and the power of observation are the things to be acquired in the undergraduate college course. The right man will develop all in the drilling which is given in the courses of the leading engineering colleges, after which specialization will do no great harm.

Industrial research differs from scientific research. In an industrial organization there is a greater variety of work to be done; there is usually a definite purpose and the method of attack is necessarily more direct and practical. On account of the variety of problems which arise, the industrial research man must have a working knowledge of several related subjects of science and engineering and go ahead and get a quick and practical solution for the problem assigned to him, rather than follow his bent and work out in minute detail a problem which is of great interest and scientific value. It is common for a man in the research work of an electrical manufacturer to successively solve problems in such branches as magnetics, electro-chemistry, metallurgy, oscillating circuits, mechanics, etc., and the useful man is the one with a broad engineering and scientific education, who keeps the practical limitation in mind and works rapidly and accurately.

We are getting the right kind of men today from the colleges. They have a good store of fundamentals and enough mathematical knowledge to do good industrial research.

Prof. Karapetoff said today that it would be attractive to the student to have rooms set off with signs "Research Work Inside" or "Secret" or something of that kind. During the college course what is wanted is something that will keep the men away from rather than attract them to research work, until they have the regular four year course completed, and from industrial research work until they have had experience in commercial engineering design and test. We have found that the work, rather than the sign, attracts the average man.

It has been very interesting to follow the college men who come to the works. They usually want railway, project or design work, or wish to fit themselves to be electrical salesman. Fortunately, practically none of them wish research work at first. In an organized apprenticeship course, a good many men are placed in the Research Division and very seldom one, after taking up research work, desires to make a change. I have not a case in mind of a man who has gone into the research laboratory who did not like research work. The able apprentices who are desired as regular investigators are often sent out of the work again for more shop, design or test experience, so as to acquire practical knowledge which will help in their future research work. Other inquisitive men with imagination usually hang around the investigation work on the test floors, have ideas, make suggestions and are spotted as good research men.

We feel that such men, if they had taken up research work in college before obtaining some commercial experience, would in many cases be failures at industrial research, because their enthusiasm would narrow their training and result in specialization too early.

Comfort A. Adams: A good deal which has been said about the difference between pure research and industrial research, and also about the education of the researcher. I must confess that I agree in part with nearly everything that has been said, but it is difficult to average it up, and get any very definite result.

I think those who have not been close to the teaching profession perhaps may be confused in the matter, so I am going back to just one very elemental, fundamental point, which is my hobby, if you will, in regard to education, and it applies not only to the training of the researcher, but also to the training of the engineer, or the training of the salesman, or the training of anybody, I do not care who he is.

We do not send our men out from our engineering schools with a good, broad, all round engineering training. We try, with all the best intentions in the world, to fill them full of information. We do not, however, put our principal emphasis on the thing that is the most important of all, to teach a man to use his mind in something more than memorizing things. We do not teach him, in other words, the habit of analysis and the habit of thinking for himself. We teach him superficial methods of computation in regard to electrical apparatus and electrical engineering, without letting him know that they are crowded as can be, from a scientific standpoint, with other approximations of the crudest nature. He assumes that they are correct, and when he goes on to work he uses those formulas, assuming they are based on sound rule, and the moment he goes outside the range of the most elementary or standard types of apparatus, he is all at sea, and his results do not check up, because he is not on a sound foundation at all. He requires some kind of fundamental training, training in thinking for himself, training, in the first place, in the fundamental principles to that end, so that he sees the phenomenon in his mind. He must have an imagination, if he is going to be an engineer of any kind whatsoever, research or otherwise, and he must have the power of visualizing these phenomena, he must see them in more than their crowded superficial aspects, he must be able to go into an analysis of them. It is not necessary for him to have all the mathematics in the world to be able to handle these problems mathematically in order to see that the flux density across the thickness of a lamination, particularly at high frequencies, is about the same at all parts—he can see that phenomenon physically without any difficulty whatsoever if he has any power of imagination, and it is his duty to do that, and his duty to recognize the fact that the calculations based on the assumption of constant density are only roughly approximate. That is the simplest possible explanation, but there are thousands of other instances, in the case of electrical machinery, where the crudeness is so gross, in some cases, that it is absolutely a crime to teach a man that these things are what they appear to be superficially.

So that the important thing from the educational standpoint, to my mind, is not the question as to whether the colleges are teaching a man research work, training him to be a researcher. That is not the important point at all, so much as whether he has been given a good foundation, has been taught to think and encouraged to think. We do not encourage the students to think. I speak advisedly. I have been teaching for twenty-six years this subject of electrical engineering, and have been in close contact with the products of other institutions as well as their instructors, and it is a great temptation to provide the student with this superficial, practical information, which will make him fairly useful and practical the moment he goes out of the college, but he would have been more useful had he been more thoroughly grounded in the fundamentals, so that he knows that he has something at his command, which he can use, and knows how to use.

I always feel a little bit ashamed of myself when I get started off on this hobby, but I have checked it up in various ways, and I am convinced, as I am of anything in the educational field, that I am right, and a man so trained will make a good researcher, and fill the requirements for manipulative skill. Frequently a perfectly good man has no manipulative skill, no resourcefulness in devising apparatus and things of that sort. My belief is that we should give him an opportunity, in other words, a chance, and train his mind in the broadest possible sense, and then let him go his way.

Just one other point in regard to this matter of coordinating research work. I am in absolute accord with that. It is just as unreasonable for us to go in this absolutely haphazard, hit or miss fashion, allowing a dozen men in one institution to carry on a lot of different researches, and another dozen of men in another institution, without any knowledge of what is going on in the first institution do some work of a similar character, none of them doing it thoroughly or satisfactorily or completing it, as it would be for each family to get along by itself and make its own clothes and shoes, and raise its own food supply, and provide for itself everything else that it required. We have learned to carry out the theory and practise of the division of labor, as society has developed, and there is no earthly reason why that principle should not be applied to this class of work.

Let me illustrate by a little plan of co-operation in research which I worked out for a particular purpose, for a particular field. There is the question, for example, of high-tension cable design, taking into consideration the stresses, etc. That is a question involving the most thorough knowledge of dielectrics and dielectric phenomena, a problem which is of the utmost practical importance, and our lack of knowledge of which stops our advance in the voltage which can be carried on high-tension cables at a certain point. We have not succeeded in making cables which will carry more than about 25,000 volts in practical operation, and we are having all kinds of trouble with Public Service Corporations with regard to the high-tension cables. There is a problem which should be tackled, not by one corporation or college, or one commercial industrial research laboratory—there is a problem which should be tackled in some such fashion as this—Have it financed, first, not by one corporation, but by a lot of big public service corporations and cable manufacturers. If the cable manufacturers are a little afraid of each other, and want to hang on each to his own little secrets, which do not amount to a hill of beans, let them do it. The Public Service Corporations alone could finance it. They could really handle it without touching their financial profits, because the sum which each would have to contribute would be very small, indeed. Then let us appoint a Committee of Research Experts, the heads of some of our big research laboratories, taking in our industrial research laboratories, our big university research laboratories, and independent research laboratories, such as that of which Dr. Sharp is the head. Appoint that committee and let that committee lay out, not a few superficial tests of cables in place, but a series of research operations, some dealing with the fundamental laws of dielectrics, some going a little further, and finally some dealing with the completed cables.

That work could be carried out at an insignificant expense to the individual corporation. The committee could meet occasionally, discuss the results obtained in these various laboratories, having the work distributed where it could be most easily handled. Then the plans could be laid for the next subject, and that to be followed by the next, and in the course of two or three years, because these things cannot be done in a month, you would have the combined, united and coordinated effort of all of these various agencies, and a result which really would amount to something, and might, in all probability, mean an advance greater than that which would ordinarily be reached in ten years.

Oberlin Smith: I am glad that Prof. Adams has spoken of the necessity of some coordination. Of course, the members of this Society are chiefly interested in electrical research, the ascertaining of various phenomena, which are not wholly known, etc.—but mechanical engineering is now so mixed up with electrical and mining and civil engineering that we have to go to each other in a good deal of our work. In my opinion there should be a United Engineering Committee, and the United Engineering Building in New York City would seem to be a world center for something of this kind. Cannot we, therefore, at some time in the near future have some such a place where all sorts of information can be obtained definitely, and if so, would not the United Engineering Society be the proper one to have various committees and sub-committees for the different lines of research proposed.

The next question is, can this Institute, at this meeting, or some other meeting, take some initial action in the matter and if the United Engineering Society is the proper body to carry on this work, send a suggestion or request to them, so that some action of the kind may be taken. Most of this information doubtless now exists in many laboratories that have made such experiments, and records thereof. Students in the various universities, having plenty of time for research work, will doubtless take up any class of such work if a request comes for it from a responsible headquarters.

William McClellan: Industrial research is organized primarily for the results obtained from it. Incidentally it is organized, of course, to develop the men in the research laboratory, but even there it is done for the same reason as you develop a man in the department of salesmanship, or in any other department, you undertake it to make the man more useful, to be a broader and more capable man. Primarily, industrial research is organized for the results we get from it. Research in colleges, if it is organized for anything else than the development of men, irrespective from what may come from it, is misplaced. The university is endowed primarily, and funds are given to it from the state primarily, for the purpose of advancing general and scientific knowledge. It is endowed and funds are given to it to develop men, and then, because men cannot possibly be developed without doing research work, the research work must go along with it.

When I read in the papers that such and such a university has had a million dollars given to it, and it has gone to London—they used to go to Paris and Berlin—to purchase a lot of magnificent apparatus with which to do research work, then I wonder, whether another university has quit some of the older subjects of instruction and taken up aviation, or some other modern subject. I presume that some of them have laboratories with eight or ten different sized airships, and are going to do research work. I think they have a misconception of what research work is. It seems to me that it is a mistaken idea that research work is something you can manufacture and make. Research, after all, is taking notice. That is what it is. Of course, work steadily and take notice. There was no difference between the research work of Maxwell and Ford, although you know one man's results are one kind, and another man's results are of another kind. Nevertheless, they were both taking notice, each man in his own way.

That is what I meant by saying originally that research work is a matter of method, and therefore, when we come to training for research work it is a question of stressing all the time the absolute necessity of whatever you are doing, whatever is taking place, whether an old or a new experiment, or idea, and trying to find out if the other fellow saw everything. It is not a question of trying to see what the other man saw, but an attitude of mind —did the other fellow see everything, or is there something else to see. Then, of course, this question of method comes in as a matter of training; the method of doing research and the method of keeping the record of it.

As to the training problem, what do you find? As a matter of fact, if you take a freshman engineering class, you will find that the men go to the courses of that class for one hundred or one hundred and fifty different reasons. One man has read a novel of some engineer in the West surmounting tremendous difficulties, in building a railroad or transmission line, and because the man has read that novel he wants to be an engineer; another man's father was an engineer, and so it goes. They have all grades of ability, and, therefore, some of them develop into one thing, and some into another. That is one view of it. On the other hand, you have a clear demand for operatives. You have got to make the universities or colleges more or less of a success, so far as numbers and finances are concerned. And, as I say, you have a clear demand for practical men who can go out and do something. You must amalgamate that heterogeneous mass that comes to you, and you are limited in what you can do by the question of expense. You cannot possibly run a universal course and give each man individual training—take this man and study his case and gradually discover that he ought to go off into the mathematical-physics field, and take this man and study his case and gradually discover that he will never be fit for anything at all but to record readings of ammeters, voltmeters or wattmeters. You cannot do that. So you are compelled to compromise. I have long ago given up the idea of trying to tell the men in the educational field just what they ought to do, but at the same time there is one fundamental thought, no matter what else you give them, the idea of research must be given. There is no reason whatever why a man should not be trained thoroughly as he goes along, and, of course, as he goes along if he discovers, as he naturally will, or others will, if he does not, that he has certain abilities which can be directed into certain activities, advantage can be taken of those abilities. It will likewise be discovered that certain men have no ability in that particular direction.

The opportunity, as Prof. Adams said a moment ago, must be provided for the development of each of these men after they become acquainted with any given subject. I know, as you know, that a man who cannot visualize magnetic induction, a man who cannot visualize the stresses and strains, a man to whom a mathematical formula is nothing but an arrangement over a line and something under it, had better leave certain fields of research alone, but that does not mean he cannot do some research work. Some of the best research work in agriculture has been done by the farmer who knew no chemistry whatever. It is an attitude of mind and method, and of course the colleges, say of them what you will, must, for a variety of reasons as I have tried to show, compromise on this problem to a certain extent, but the college which is most successful will not compromise in regard to its principles of instruction, it will only compromise, perhaps, on the material facilities which its building offers, in the spending of any limited amount of money which it has. But whatever it may be—whether it be as Sir. William Thomson crossing the ocean with nothing but his pencil and tablet, going to work at various problems about the balance wheel of a watch that happened to attract his attention, so any man, no matter what he is doing, if he is given that attitude of mind which leads to research, will take it up, some to a very great degree, and some to a very slight degree.

Alexander M. Gray: The college teacher has to handle material which has already been moulded by the underpaid teachers of the high schools and perhaps because of this we find many students who can follow instructions, but comparatively few of the research type. These men of research ability, probably not more than three out of every hundred, undoubtedly suffer by being put through a course designed to meet the needs of the other ninety-seven.

As to the use of the English language: my experience as a teacher has been that there is little difference in the type of report prepared by two students one of whom has had a course in English composition at the university, but that there is a great deal of difference between the type of work handed in by the accurate thinker and that which is prepared by the slovenly thinker.

W. L. R. Emmett: I was interested in what Mr. Behrend said, as to the undesirability of importing our scientific men, and I think that phase is a really important feature which we should consider. The actual doing of work may not require science, but most good work, in scientific or engineering lines, is inspired and enormously assisted by the higher developments of science. I should never go to Sir William Thomson, or to some great authority in a college to find out how to design, maybe, a turbine wheel, because I think that research is one thing, and scientific investigation another, in a sense; that is, the function of an engineer is to be just as deep as the occasion requires, and it is far easier to investigate a specific case than it is to investigate general cases. In engineering, I think that any recorded knowledge is seldom good enough to use, and I think that data generally has to be acquired newly in every case, but the possibilities would not be known if the required fundamentals of science were not recorded. Furthermore, there is a language of science, and there are methods of procedure which must be understood and must be familiar if a man is to pursue scientific methods. We are in urgent need of scientific men of the highest degree of training, and to supply that need we have drawn very largely upon Europe. We have done so to such a degree I have often felt ashamed, that we with all of our means for acquiring knowledge have depended so much upon Europe for all kinds of pure scientific work involving calculations and involving many forms of knowledge.

The reason for this, is, I think, apparent. In this country there has been a tremendous demand for men. There have been all sorts of activities, bringing rapid returns, whereas in Europe opportunities have been less. In Europe men have labored patiently through years of study in the hope that they may attain a higher level, and have been satisfied with small returns during that time. In Europe there has been an excess of men, whereas in America there has been a constant shortage of men, and for that reason we have drawn largely upon European research, but some of our very best scientists and some of our very best material are trained in American colleges, and I think that the conditions which have arisen now will lead to a realization of the importance of research, will show the students in colleges that there is big money in science, as compared with simple engineering education, and that more students will seek to be real scientists.

The impression which I have formed concerning knowledge of any kind is not that it is difficult to acquire, but that it is unfamiliar; that is, there is nothing complicated in the most advanced theory concerning the construction of an atom, the ideas in which people deal in such studies are no more complicated than the ideas of simple mechanical motions, but a man's imagination must form a picture, and he must be brought in contact with these methods, that he may know that such things exist and such means of procedure exist. An illustration was given this afternoon of some one who was made to go through the calculations relating to the revolutions of the sun—a boy who has seen that done, or who has done it, would know the way such problems have been solved, and having known it once, would never forget it, and if he had some problem of that kind to solve, he would seek the scientific and best way of doing it rather than flounder about in an uncertain way. There is not work enough in higher mathematics or in the higher ranges of science for a great number of men in any one industry, but there is a demand for them all over, and it has been shown, not only that they are very valuable, but that they are so valuable that they will be paid almost any money within reason, if they have the imagination and the scientific ability as well. I think of cases in point in our own research laboratory. I can think of several men, Americans, who have come out of colleges, as teachers or promising students who, through association with scientific men, have drifted to Schenectady. I remember one of these men talked to me when he first came. I was struck with the impression that he was a man who had for several years been simply devoting his thoughts to the abstract principles of physics and various scholastic ideals regarding a number of physical forces and chemical laws, and was peculiarly familiar with them, but out of touch with the uses which might be made of such knowledge. I thought of this man perhaps unjustly, as an accomplished amateur, as far as engineering was concerned, but saw that he was full of all sorts of knowledge which appealed to my imagination and struck me as wonderfully useful. Since that time he has produced and made commercial several very important inventions which have gone out into the world and become immensely useful.

I think the colleges should not try to be research institutions, unless they can pay men good money for valuable work which they may do there. I think that they should do research enough to give men an idea of the uses of science in connection with work, because if they do not see the uses to some extent, they will be less likely to intelligently study the laws. The important thing is to make them scientific, men who can really do the most difficult kinds of scientific work. We cannot make all of them such, but those who elect to be such should have the best possible opportunities.

C. E. Skinner: A number of those discussing this paper apparently assume that research men must be developed during the four-year college course. I intended to make it clear in my paper that it was my idea that specialization for research should begin after the four-year college course is finished.

For a number of years, I have been directing the work of a few graduate students in one or two schools, with the primary object of training research men. These men are paid enough to cover expenses and they give half their time to the problems we set, the remaining time being given to graduate work or to teaching. They spend their summers in the factory and in the laboratories and it is expected, of course, that they will come to us when their one or two years work is completed. This arrangement has given us some very excellent men.

Professor Karapetoff stated that the university must do research in order to teach research men. I agree with him absolutely and if the more practical problems are desired, there are many which could be outlined for use in this connection. For example, in line with Mr. Smith's suggestion, I would propose the problem of the determination of the hardness (or softness) of copper magnet wire. At the present time, we know of no really satisfactory measure of this property of copper wire—the tests and the judgment of the men doing the winding not infrequently being completely at variance.

Again, a number of speakers, while not specifically so stating, seem to refer to the research man as a man of genius. We need men of genius, but we also need many well-trained, thorough painstaking observers for each man of genius. There are few men with really creative minds and we could not run our research laboratories with only men of this kind. An excellent combination in the conducting of research is to assign a research problem to two men—one with the creative type of mind and the other with the mind of the painstaking observer who never loses sight of a point until he has accumulated every possible bit of information in connection with it.

I agree with Professor Adams that men should be taught principles and not filled up with information. Dynamo design taught in the college course usually has little in common with the dynamo design as practiced in the manufacturing companies.

The question of co-ordinating research has been the subject of very earnest discussion in the National Research Council. The members of the Council, while possibly not representing the entire field, are endeavoring to do exactly what has been suggested by one of the speakers, in that they are trying to co-ordinate research being carried on at the different universities and the different research laboratories. It is extremely desirable that there be not too much duplication of work and that the results of fundamental research done in any one place shall be available to other research workers.

Professor Gray states that the schools cannot change the training of 97 per cent of their men to accommodate the 3 per cent who aim to become research workers. I am entirely in agreement with this statement, but I see no reason why the men who have a leaning to research cannot be given some encouragement along that line and provision made for the ever increasing number of research men who are going to be necessary to the industry and to encourage them to take more than their regular four-year college course. If the need of trained research men were properly recognized, the money for such training would probably not be difficult to secure. Dr. Agnew brought out very strikingly the need of research men in Government work and we have information that the British Government and British engineers recognize at this time as never before the need for research men, not only at present, but after the war is over. During his recent visit here, Sir Ernest Rutherford made the statement that in the problem of submarine warfare alone there were very many promising leads which might be followed but for lack of the necessary research workers and facilities.

N. S. Diamant (communicated after adjournment): Both Mr. Skinner and Mr. Kennelly seem to agree that the "principal function of university research should be to train research men." Now, it will be conceded that universities cannot possibly do this properly unless they support, encourage and produce themselves high grade research. Thus we find Mr. Skinner stating that in training research men the universities will *naturally* become the custodians and the promoters of research.

In regard to the proper support of research men it is difficult to add anything to Mr. Skinner's statement. He describes the situation very vividly when he states that while writing his paper he received letters of inquiry for positions offering compensations not commensurable with the qualifications required of the men to fill the positions—teaching or teaching and research. I believe that the frank and salutory remarks made in this connection are very significant and worthy of special note: he says that it is generally understood, at least by those outside the teaching profession, that there are many compensations, such as short hours and long vacations. This calls for little comment except that the man who wants to work need not have either short hours or long vacations. Although financial support is not only necessary but fair, it is well not to lose sight of another element which money cannot buy—recognition. I can possibly illustrate best what I mean by a concrete case. Last May when so many college men answered the call to the Colors it was not uncommon to pick up a daily paper and find special mention of Mr. A or B, great athlete in such and such college; however, few if any graduate or undergraduates, Messrs. X. Y or Z were mentioned who had maintained an excellent standing throughout their college course, or men who had done some excellent piece of research, etc. This is intended merely as an illustration to show that research cannot be fostered and produced by money and equipment alone—though these are absolutely necessary, recognition and a sympathetic liberal atmosphere is also essential.

Some people seem to attach great importance to the relationship that should exist between the university research and the industry. However, it seems to me that the real problem is the introduction of high grade research itself, into as many schools of the country as possible, under the direction of capable and far-sighted men carrying instruction and investigation side by side. If this be accomplished, the rest will take care of themselves, for the simple reason that essentially industrial research and university research are fairly distinct and do not conflict. The principal distinction, as brought out by Past-President Carty over a year ago, is one of motive and ethics. The latter may of course change with a change of patent laws.

Because industrial laboratories have been so successful, universities should not attempt to imitate them blindly; they should as a rule attack fundamental problems and attempt general rather than particular solutions. For example, a manufacturer may study the ventilation and heating or commutation, or noise and humming, etc., of a new line of machines he is bringing out, with the view of remedying some definite troubles or improving certain definite characteristics. The university on the other hand should study the general laws of heat generation and heat transference or noise in machines. While the former is in a hurry for definite results and may well afford to use cut and try methods, the latter, should attempt to be systematic and attack the problem from a general and broad point of view. Of course, there will be some overlapping. Industrial laboratories have to do and are doing excellent general university research; however, it is probably best for colleges to do as little purely commercial and industrial research as possible, even though it offers the great inducement of immediate application and financial return.

Presented at the 334th meeting of the American Institute of Electrical Engineers, New York, November 9, 1917.

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AN EXPERIMENTAL METHOD OF OBTAINING THE SOLUTION OF ELECTROSTATIC PROBLEMS WITH NOTES ON HIGH-VOLTAGE BUSHING DESIGN

BY CHESTER W. RICE

ABSTRACT OF PAPER

The electrodynamic method for obtaining the solution of electrostatic and allied problems is developed to a high degree of accuracy. The method is then applied to the study of high-voltage bushings. An experimental high-air-efficiency bushing was built and tested with the result that the arc-over was very materially lower than had been anticipated.

A study was then made to ascertain the reason for this large discrepancy, which was found to be due to an unexpectedly large *surface effect* which varied greatly with different materials.

After obtaining the numerical value for the *surface effect* a reasonably accurate predetermination of the arc-over of structures, in which the stress distribution is known, can be made.

In order to determine the desirability of using artificial equipotential surfaces to increase the efficiency of the use of the supporting dielectric, diagrams were taken and a small bushing of this type constructed and tested.

A study was then made to find out whether the reduction in diameter of condenser bushings is principally due to equalization of potential or due the greater strength of insulation when barriers are used. As a result of this work, it is believed that the barrier effect greatly predominates.

A short discussion follows which shows the difficulties of obtaining a sufficiently exact theory of bushing design to enable us to predetermine the most efficient shape for a practical bushing.

A series of small bushings were made and tested with a view to determining the general shape and characteristics which go to make up a practical all around bushing.

The appendix gives an unexpurgated solution of the following two-flow problems.

I. The distribution of the electrostatic field when any two confocal hyperboloids of revolution of one sheet and of the same family are maintained at given potentials.

II. The distribution of the electrostatic field when any two confocal hyperboloids of revolution of two sheets and of the same family are maintained at given potentials.

INTRODUCTION

IN general, a bushing has to serve two purposes; first, it must support the leading-in wire or conductor and must act as a mechanical unit capable of being removed or replaced in case of damage; secondly, it must insulate the high-potential conductor from the tank which is usually at zero or ground potential. The leading-in wire, or what may be called the high-voltage electrode may have various shapes. The tank with hole in it may be called the grounded electrode and it may be shaped in various ways. A solid supporting dielectric is necessary to make the structure a mechanical unit, that is, to mechanically connect the high-voltage central electrode with the tank or grounded electrode. Between the grounded electrode and the high-voltage electrode, isolated metallic surfaces may be introduced which will form *artificial equipotential surfaces*. The condenser bushing is a familiar example.

The electrical part of the design consists in studying the electrostatic field distribution between the electrodes in order to use the various available insulating materials to their best advantage. It is obvious that there are large numbers of possible electrode shapes and arrangements, the electrostatic field distribution of which should be studied in order to determine the one which is best adapted for commercial use. The following investigation was undertaken with a view of determining the theoretical possibilities of some of the various possible arrangements. The work was started in July 1914, and extended with short interruptions to July 1916. Naturally an investigation of this nature is never complete but I hope that the work to be described will be of interest to engineers and also stimulate others to take up the work so that eventually we will have a better and more complete understanding of this and similar problems.

Preliminary Work

The preliminary work consisted in investigating the possibilities of obtaining a reasonably approximate solution of the bushing problem. The first question which presented itself was the following: Can two-dimensional fields be rotated and used as satisfactory approximations for three-dimensional problems? In order to test out this matter the literature was searched for a solution of the rod and torus problem, so as to be able to compare it with the solution obtained by rotating the field of two parallel wires. If the approximate solution, as obtained by the rotation of the plane figure, did not prove useful as a result of comparison with the exact solution, it was thought that possibly some simple law of distortion or stretching might be arbitrarily imposed upon the rotating-plane diagram which would change the lines of force and equipotential surfaces so as to closely conform with the accurate three-dimensional solution. If such a procedure was found possible, it was hoped that the same method could be used in transforming the many available two-dimensional problems into approximate solutions of the related three-dimensional figures of revolution.

The problem of insulating two parallel wires was studied in order to compare some of the various criterions which might be advanced for the correct and strongest surface along which to place the insulating material. For this preliminary work the question of different inductive capacities was neglected. The following surfaces were studied by graphical construction on a large sized diagram of the field between parallel wires drawn for equal tubes of electrostatic flux and equal differences of potential.

I. —Constant surface flux density.
  Equal areas between lines of flux on equipotential surfaces.
II. —Surface of constant potential gradient.
  Equal distances along lines of force between equipotential surfaces.
III.—Surface of constant volume energy density.
  Surface defined by unit cells of equal volume.
IV.—Surface such that the component of the potential gradient along the surface has a definite limited value, for example say, 50 per cent of the gradient. The object in studying such a surface is apparent if we assume that the surface of the insulation introduces a weakening effect. For in that case the component of the gradient tangent to the surface must not exceed the breakdown strength of the surface, whereas the actual gradient may be equal to the dielectric strength of the surrounding dielectric.
V. —Surfaces of constant creepage.
  Equal distances between equipotential surfaces.

The surfaces defined in I, II and III were seen to be identical as would be expected.

The corresponding surfaces were drawn for the solid figure resulting from the rotation of a right section of the plane figure. Of course, such a procedure does not give the solution of the torus problem, but as explained above it was done in order that the result might be compared with the true solution of that problem.

A search of the literature failed to yield a useful solution of the torus problem. C. L. Fortescue and S. W. Farnsworth[1] state that the smooth lines (in Fig. 4 of the paper) show theoretical equipotential surfaces of indicated potential for the given terminals. Inspection of the figure shows that the smooth surfaces are those cut out by revolving a family of circles about the axis of the rod which would yield a set of anchor rings, the orthogonal surfaces would then be a set of spheres. That this cannot be a solution of the torus problem is stated by W. E. Byerly.[2] "Indeed no possible distribution can make our anchor rings or our spheres a set of equipotential surfaces." It is therefore, evident that their solution and the calculations given in their curves Figs. 13, 14, 18 and 19 must have been the result of some sort of an approximation. It seems to me that a discussion would have added to the interest and clearness of their paper.

A study of the solution of the torus problem, as outlined by Byerly and also given by Hicks,[3] convinced me that the difficulties of calculating sufficient points for the construction of a diagram of the field would be very great, and even if the solution were available, it would not be of great assistance in solving the bushing problem because the rod and torus does not constitute a self-supporting structure resembling a bushing. It was realized, at this time, that if the solution of an infinite rod passing perpendicularly through a hole in a plane were available, it would be of considerable value as this would constitute the simplest form of bushing.

As a result of this preliminary work, the difficulty of obtaining even approximate mathematical solutions of such electrostatic problems was brought out.[4]

Various experimental methods were, therefore, looked into with the hope of obtaining a method which would enable us to obtain experimentally the solution of any desired electrostatic problem.

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[1] "Air as an Insulator when in the Presence of Insulating Bodies of Higher Specific Inductive Capacity." A. I. E. E. TRANS., 1913, Vol. XXXII, Part I, p. 893.

[2] Fourier's Series and Spherical Harmonics page 265.

[3] Toroidal Functions, Philosophical Transactions of the Royal Society, Part III, pages 608–562, 1881.

[4] In this connection it is interesting to read what Maxwell has to say about such problems; Electricity and Magnetism. Vol. I, page 177–178.

PLATE XLI.
A. I. E. E.
VOL. XXXVI, 1917

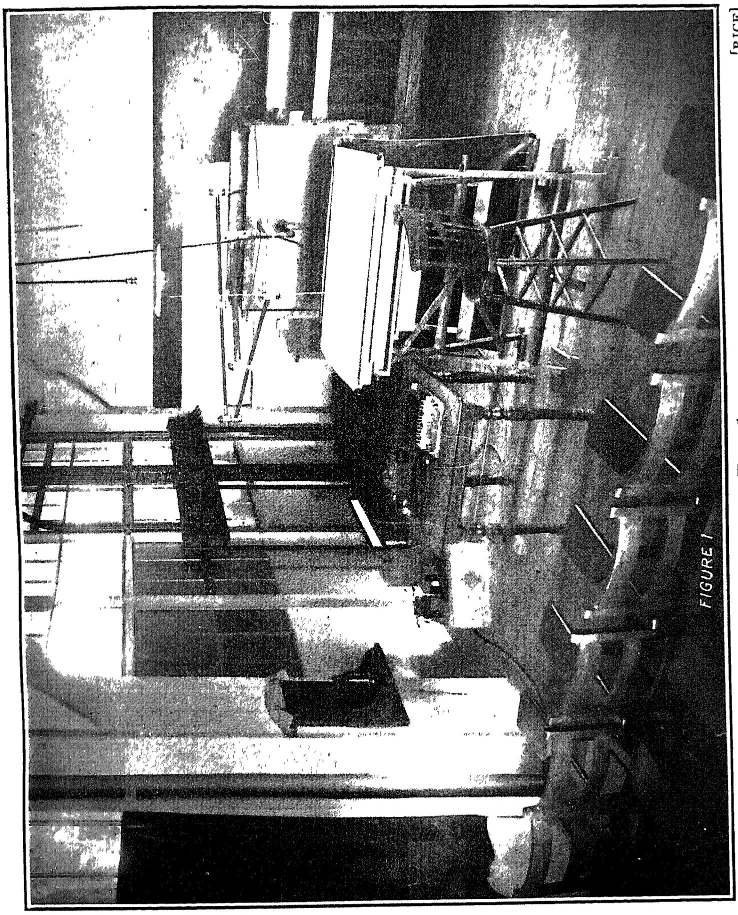

Fig. 1

[Rice]

The desired diagrams can be obtained in a variety of ways[5] some of which are enumerated below:

I. Directly calculated by the process of cut and try. This method is outlined by Karapetoff[6] from which I quote in part; "In order to calculate the permittance (capacity) of a given dielectric, or to find the flux densities and stresses in different parts of it, proceed as follows: The field is mapped out into small cells by lines of force and equipotential surfaces, drawing them to the best of ones judgment, the total permittance is calculated by properly combining the permittances of the cells in series and in parallel. Then the assumed directions are somewhat modified, the permittance is calculated again, and so on, until by successive trials the positions of the lines of force are found with which the permittance becomes a maximum." The method of successive approximations was systematized and used by Lord Rayleigh.[7]

While theoretically possible in all cases this method is very laborious even for problems in two dimensions and for three-dimensional problems it becomes still more exasperating, as will be readily discovered by anyone who tries it. A considerable assistance in the application of this method is rendered by experimentally obtaining the approximate direction of the lines of force by the well known method of using mica filings, or better fine needle-shaped pieces of glass which can be obtained by grinding up glass wool or fabric.

II. Obtain experimentally the isothermal surfaces in the related heat-flow problem. Experimental difficulties such as radiation and conduction, obviously make this method impractical.

III. Obtain the equipotential surfaces in the equivalent electrical conduction problem[8] or what I have termed the electrodynamic method.

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[5] For references to the numerous articles on this subject I will refer the reader to those contained in the excellent article on this same subject by John F. H. Douglas, A. I. E. E., TRANS. 1915, Vol. XXXIV, Part I, page 1067, "The Reluctance of Some Irregular Magnetic Fields."

[6] Electric Circuit, pages 160–163.

[7] See Phil. Trans. Royal Society, 1871, p. 77, "On the Theory of Resonance," also "Theory of Sound," Vol. II, p. 171.

[8] I understand from Mr. Douglas' paper "The Reluctance of Some Irregular Magnetic Fields", A. I. E. E., TRANS., 1915, Vol. XXXIV, Part I, p. 1081, that Kirchoff first proposed such a method in 1845, and was subsequently modified and employed by many investigators. Re-

The possibilities of this method appeared very attractive and it was, therefore, selected as the one best suited for the present purposes.

THE ELECTRODYNAMIC METHOD

The method consists in obtaining the equipotential surfaces for any desired shape of electrodes from the exactly analogous conduction problem in an electrolyte. Thus the chosen electrode shapes are placed in an electrolyte and alternating current passed between them. The equipotential surfaces are then obtained by an exploring point connected through a quadrant electrometer to a definite known potential with regards to that between the electrodes. The locus of the points of zero potential difference as thus read by the electrometer constitutes the desired equipotential surface.

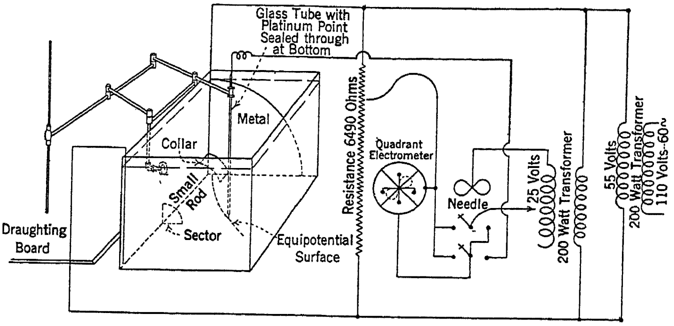

FIG. 2

The electrodes under investigation were placed in a large box made of paraffine treated wood and filled with an electrolyte—ordinary city water being found most convenient. The box was bolted together on the outside as will be seen in Fig. 1, thus eliminating all metal from contact with the electrolyte in the box. A small transformer (200 watt) was employed to step down the 110-volt, 60-cycle lighting circuit to 55 volts which was used as supply for the tests. A non-inductive resistance of 6490 ohms, divided into 50 equal parts, was shunted across the mains between the electrodes. The exploring pointer which was carried by the pantograph consisted of a slender glass tube

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cently C. L. Fortescue and S. W. Farnsworth, A. I. E. E., TRANS., 1913, Vol. XXXII, Part I, p. 893, "Air as an Insulator when in the Presence of Insulating Bodies of Higher Specific Inductive Capacity", as well as Mr. J. F. H. Douglas have employed this method.

with a small platinum wire sealed through at the end. The glass tube was slipped over a metal tube carried by the pantograph and the insulated wire brought out from the platinum point through the metal tube. The metal tube was used for mechanical stiffening and the glass tube with platinum point so as to avoid electrical leakage. Any defect in the insulation such as a crack in the glass tube being easily detected.

The pointer was connected to one pair of quadrants of the electrometer, the other pair being connected to a point of known potential on the resistance *R*, Fig. 2. The needle of the electrometer had a metallic suspension and was kept at a definite potential above that of the quadrants by the transformer as shown in the illustration. This method of excitation gives a constant sensitivity regardless of the point at which connection is made to the resistance *R*. It also makes it possible to change the sensitivity of the instrument by varying the potential applied to the needle. For example, in exploring the field between a given pair of electrodes we can apply a certain low potential to the needle when exploring the dense part of the field, and when exploring the weak part of the field we can raise the potential applied to the needle by selecting a tap on the exciting transformer. Thus, we can maintain equal accuracy in all portions of the field.

Some of the advantages of using the arrangement described above are as follows: The use of alternating current eliminates polarization, to a large extent, and automatically gives "reversed" readings. The use of a quadrant electrometer has the advantage that it takes practically no energy to operate, and it is a good zero instrument since when a point has been nearly located the difference in potential between the quadrants is small in comparison with that of the needle and under these conditions the deflection is very nearly proportional to the potential difference being measured.[9]

The fact that these experiments were carried out on a large scale combined with the small energy necessary to operate the electrometer made it possible to use ordinary city water as the electrolyte which is obviously a great convenience.

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9. For the theory of the quadrant electrometer reference may be made to Maxwell, "Electricity and Magnetism," Vol. I, page 338; Jeans, "Electricity and Magnetism," Vol. I, page 108; J. J. Thomson, "Electricity and Magnetism, Vol. I, p. 97–103.

Method of Procedure

In applying the electrodynamic method to problems having symmetry about an axis of revolution, a quadrant or octant of the actual figure was studied because in this manner the size of the electrodes used could be made larger and, therefore, the accuracy correspondingly improved. Skeleton electrodes were used because of the obvious ease with which they can be cut out of tin or other metal, and for the further reason that if it ever becomes possible to obtain a mathematical solution of such problems the boundary conditions would be of as simple geometry as possible. Of course in applying the resulting diagrams to the design of any piece of apparatus the electrode shape which would actually be built and used would conform to one of the experimentally obtained equipotential surfaces at some distance from the actual skeleton electrodes. Under some circumstances it would be desirable to employ actual models of the desired electrodes.

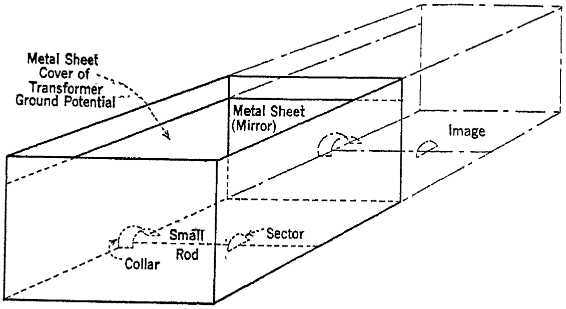

FIG. 3

The method of images is obviously applicable and is of considerable value in improving the accuracy and simplifying many problems. For example, if we wish to study the disturbing influence of one bushing in proximity to another, when they are at opposite potentials, we can set up the experiment as shown in Fig. 3. In this manner we can determine the maximum stress between the two bushings and the minimum stress on the outside of the bushing. Of course a diagram taken in this manner is no longer a plane section through a figure of rotation.

Theoretically there is no difficulty in obtaining the solution of electrostatic problems involving materials of different inductive capacities by this method, as we may employ electrolytes or materials having the proper relative conductances. Practically, however, there are obvious difficulties in obtaining materials having suitable characteristics especially for the case of three-dimensional problems.

In Fig. 4A is illustrated the use of a high-resistance substance such as carbon or a silicon clay composition, etc., as high inductive capacity material immersed in a suitable electrolyte to represent the surrounding air or low inductive capacity material.

Another method is to use an insulating diaphram to separate two electrolytes of different conductivities one inside and the other outside of the shell. In Fig. 4B a series of wire rings are used which are connected together metallically. In Fig. 4C, the same method is shown except that metal pins are substituted to connect the outside with the inside point for point. Still another and possibly the simplest method of carrying out the same principle is shown in Fig. 4D and was suggseted to me by Mr. G. B. Shanklin. It consists in building up the separating shell of alternate metallic and insulating washers. It will be observed that the methods employing the wire rings or washers are only applicable to problems having circular symmetry.

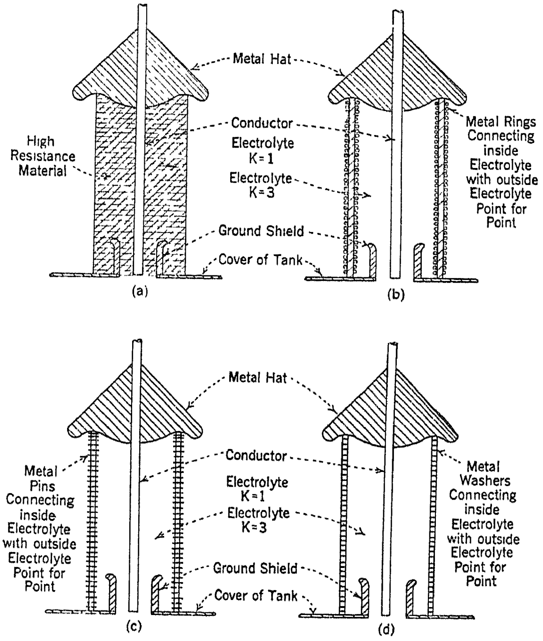
Metal Hat
High Resistance Material
Conductor
Electrolyte K=1
Electrolyte K=3
Ground Shield
Cover of Tank
Metal Rings Connecting inside Electrolyte with outside Electrolyte Point for Point
(a)
(b)
Metal Hat
Metal Pins Connecting inside Electrolyte with outside Electrolyte Point for Point
Conductor
Electrolyte K=1
Electrolyte K=3
Ground Shield
Cover of Tank
Metal Washers Connecting inside Electrolyte with outside Electrolyte Point for Point
(c)
(d)

FIG. 4

It is difficult in all of these methods to obtain the equipotential surfaces inside of the containing shell, although this can be accomplished in some instances. For example, in the case of a bushing, by placing the quarter section inverted at the top of the large wooden box somewhat as shown in Fig. 5, the exploring plane would then be at the surface of the electrolyte.

It will be seen that in this simple apparatus we have a most remarkable calculating machine which is able to obtain the solution of Laplace's equation and automatically calculate and plot the results for boundary conditions which as yet have not been obtained by analytical methods. The accuracy merely depends upon the scale and care with which the experiments are carried out. For example, in the experiments here described the potential of any point on the full sized diagram, 3 ft. by 4 ft. (90 cm. by 120 cm.) could be determined inside of a pin head. Of course, the diagrams are not accurate to this extent due to the fact that the electrodes used were large compared with the size of the containing box, and, therefore, the edges of the diagrams show large distortions, as will be readily seen by comparing the experimental diagram Fig. 9 with the mathematical solution for the similar case Fig. 10. The first two diagrams are also subject to slight pantograph errors arising from a lack of rigidity and accuracy of the original pantograph. The later diagrams are more accurate as a new pantograph was constructed of large thin walled steel tubing and provided with ball bearings throughout.

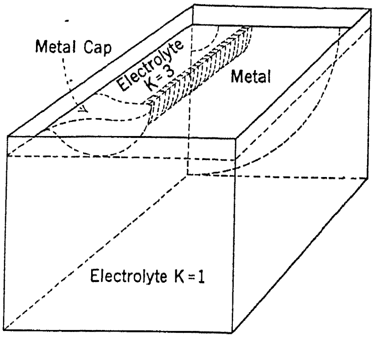

FIG. 5

Before taking a diagram the inside of the paraffine treated wooden box was painted over with a mixture of beeswax and rosin to eliminate or reduce to a minimum any conductivity of the wood.

Completing the Diagrams.

After obtaining an experimental diagram giving the equipotential surfaces for equal or unit differences of potential, in the manner described above, the diagram may be completed by drawing in the family of orthogonal surfaces which divide the field into unit tubes of electrostatic flux. Besides being a great help to the eye in using the diagrams this process furnishes a graphical check on the accuracy of the experimental work. If it is found possible to draw in an orthogonal family of surfaces so that the elementary condensers, cut out between the equipotential surfaces and tubes of electrostatic flux, have the same capacity throughout the field, we know that our diagram is the one and only solution of that particular electrostatic problem. For it is known that there is one and only one solution of any problem in electrostatics. In the second place, the following two conditions must be fulfilled when the field is mapped out in unit tubes of electrostatic flux, and in equipotential surfaces for every unit difference of potential.

I.—The tubes of flux must intersect the lines of force everywhere in the field at right angles, for if this were not true there would be a component of the potential gradient (or electric intensity) along an equipotential surface which is impossible in a static field.

II.—The unit cells, or elementary condensers, which are cut out by the intersection of unit tubes of flux (or force) and unit equipotential surfaces must have the same capacity everywhere in the field. The correctness of this condition can be seen by the following reasoning.

All the elementary condensers between two adjacent flux lines, that is, cut out by a unit tube of flux, are in series and if they were not of equal capacity they would not divide the potential difference equally between them. This is contrary to the assumption that the field is to be divided up into equipotential surfaces for every unit difference of potential.

Again we see that the elementary condensers between any two adjacent equipotential surfaces are in parallel and have unit impressed electromotive force across them. Therefore, if they were not of equal capacity they would not divide the flux equally between them ($\varphi = C e$), which would be contrary to the condition that the field is to be divided into equal tubes of flux.

The process of drawing in the lines of force was briefly as follows: An appropriate height for the unit cell was selected near the axis of symmetry of the diagram, a perpendicular[10] was erected from this point on the equipotential surface half way to the next one. The perpendicular was then drawn to the next equipotential surface which cut the first perpendicular half way between the two equipotential surfaces. The latter perpendicular was extended half way to the next equipotential surface and the process continued,—in this follow your nose fashion,—until an entire flux line was completed. A smooth curve was then drawn through the intersection of the perpendiculars. After completing a pair of lines in this manner the mean heights of the elementary condensers or unit cells were calculated assuming a constant value of capacity. These mean heights were then plotted on the diagram and it was inspected to see whether the flux line, obtained by perpendiculars, was a good mean curve through all these points. At the beginning of a diagram these points were generally high and low in an erratic manner as would be expected. Another flux line was then obtained by the method of perpendiculars, the smooth curve drawn through the intersection and the mean heights calculated as before and compared with the drawn in curve. In this manner the flux lines were built up one upon another. After completing six or seven flux lines it was generally found that the calculated heights of the cells were showing a slight consistent deviation from the curve as obtained by perpendiculars in some part of the field. The constant error was then distributed throughout all of the flux lines so as to make the two criterions, namely, perpendicularity and equal capacities, come into as harmonious agreement as possible. The work of drawing in more flux lines was then continued and the various processes repeated until the entire diagram was completed.

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[10] A convenient method of erecting the perpendiculars is to employ a small plane mirror held, for example by a block of wood at right angles to the drawing board. When the part of the equipotential line which is seen in the mirror forms a continuous smooth curve with the part of the equipotential line which is viewed directly, perpendicularity exists and a line can be drawn using the mirror as a straight edge.

The fact that it was possible to satisfactorily complete the above process is considered a good check on the accuracy of the experimental work.

Determination of Capacities from the Diagrams

After completing the diagrams by dividing up the field into elementary cells, we can calculate the capacity of any desired part of the field by properly combining the elementary cylindrical condensers in series and in parallel. For example, in the diagram, Fig. 12, we may desire to calculate the capacity assuming a single dielectric between equipotential surfaces 0 to 36 and bounded by flux lines 0 to 54. Here we have 54 of our elementary condensers in parallel and 36 in series. Now since the small cylindrical condensers are all of the same capacity C_u we can write as the total capacity of the part of the circuit under consideration

$$C = \frac{54}{36} C_u$$

or,

$$C = \frac{m}{n} C_u$$

where m = number of elementary cells in parallel.
n = number of elementary cells in series.
C_u = the capacity of the unit cell or the capacity of the elementary cylindrical condenser assumed for the particular diagram.

C_u will obviously depend upon the assumed inductive capacity and size of the diagram.

In selecting the mean height of the first cell which determines the capacity of the unit cells no particular value was chosen, the spacing for the flux lines being selected only for convenience in construction. They are not therefore, strictly speaking, unit cells.

It is obviously necessary to assume some limit to the extent of the field in making any calculation on a field of this type for, if we assumed the central rod with cap infinite in extent as well as an infinite plane with hole and collar, the capacity of the entire circuit would be infinite.

A check on the accuracy of the capacity as calculated from any of the diagrams could be obtained from a measurement of the resistance of the circuit as set up in the large box combined with a determination of the specific resistance of the electrolyte used. I am sorry that I neglected to accurately record this data when taking the various diagrams.

In the case of the diagram of Fig. 28, the following resistance measurements were retained.

Artificial Equipotential Surfaces

Voltage applied, 60-cycle	54.2	volts.
Current	1.39	amperes.
Resistance	39.0	ohms.

Artificial Equipotential Surfaces Removed

Voltage applied, 60-cycle	54.2	volts
Current	1.30	amperes.
Resistance	41.7	ohms.

The resistivity of the electrolyte used (Schenectady city water) was determined from a sample taken from the box during the experiment and was found to be approximately 3000 ohm-cm. at 25 deg. cent.

A comparison of the measured resistance of the circuit with and without artificial equipotential surfaces shows that by their introduction we have lowered the resistance of the circuit or, if considering the capacity, we have increased the capacity by the addition of artificial equipotential surfaces.

It will be readily seen that in those cases where we are merely interested in determining the resistance to flow for any of the problems which obey the Fourier-Ohm law, such as the flow of an ideal incompressible fluid, heat, or the so-called electrostatic and magnetic fluxes and current, we merely have to set up the desired electrodes in an electrolyte of known specific resistance and make a determination of resistance. The model used in the tests need not be the same size as the actual piece of apparatus which it is desired to study but may be either a magnified or reduced image. If the linear dimensions are all n times as large as the original then the conductance will be n times larger than it would be for the original model and inversely. A simple and also general method of obtaining the relation between the resistance in ohms as determined from the conduction experiments and the electrostatic capacity of the equivalent electrostatic problem is to compare the two cases for parallel plane electrodes at close spacing when neglecting all edge effect.

The well known expression for the capacity of a parallel plate condenser expressed in practical units is

$$C = \frac{A}{4\pi d} \; k \, \frac{1}{9 \times 10^{11}} \text{ farads}$$

and the resistance between the same electrodes is

$$R = \frac{d}{A} \rho \text{ ohms}$$

where A = area of the dielectric or electrolyte, or what is the same thing, the area of one plane.
d = spacing between the planes.
k = specific inductance capacity (permittivity) of the dielectric.
ρ = specific resistance (resistivity) of the electrolyte in ohms cm.[3]

If we now substitute the value of $\frac{A}{d}$ in terms of R and ρ in the above equation for the capacity we obtain

$$C = \frac{k\,\rho}{4\,\pi\,R} \quad \frac{1}{9 \times 10^{11}} \text{ farads}$$

as the expression giving the capacity of our electrodes in terms of the resistance measurements[11] R and ρ.

As an illustration, we may calculate the capacity of a bushing built from the diagram of Fig. 28 full size, assuming permittivity unity. The resistance measurement of a quadrant of the bushing gave approximately 40 ohms, and therefore, the resistance for the complete structure would be 10 ohms. Assuming $\rho = 3{,}000$ ohm-cm., we have

$$C = \frac{\rho}{4\,\pi R} \quad \frac{1}{9 \times 10^{5}} \text{ microfarads}$$

$$= \frac{3000}{4\,\pi \times 10} \quad \frac{1}{9 \times 10^{5}} = 0.000027 \text{ microfarad}$$

Study of Electrode Shapes

The following are some of the questions which presented themselves at the outset, and which it was hoped could be answered by taking various diagrams of the electrostatic field by the electrodynamic method:

I. What is the best form of equipotential surface for the rod or high-potential electrode?

II. What is the best form of surface to select for the cover of the tank with a hole in it?

III. Is there a proper best ratio of rod to hole diameter under various conditions?

IV. Are hats desirable? Of how great an assistance are they in screening the bushing from surrounding influence? Are they useful in acting as a rain shed and thereby reducing the field distortion under rain conditions, etc.?

V. Can artificial equipotential surfaces be of material assistance in bringing about a better distribution of stress in the solid or supporting dielectric; and, at the same time, what are their effects upon the distribution of stress in the air and under oil part of the dielectric? Are their effects conflicting?

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[11] See A. E. Kennelly, *Electrical World*, December 29, 1906, Vol. XLVIII, page 1239, who has used this method for determining the capacity of wireless antenna.

APPLICATION OF THE ELECTROSTATIC-FIELD DIAGRAMS TO BUSHING DESIGN

The problem of high-voltage bushing design is in reality a problem involving two or more dielectrics of different inductive capacities. Therefore, in general the diagrams obtained by using a single electrolyte only offer approximate solutions of our problem.

There are two conditions, however, in which the introduction of high inductive capacity material does not result in a distortion of the field by flux refraction.

Case I. High inductive capacity material in parallel with the air dielectric, that is, when the supporting dielectric conforms to a line of flow. In this case the form and distribution of the flux and equipotential surfaces are neither altered in form nor distribution—nothing is changed under these conditions when considering perfect dielectrics.

Case II. High inductive capacity material in series with the air dielectric, that is, when the additional material conforms to an equipotential surface. In this case the form of the equipotential and flux surfaces is not distorted by refraction. There is, however, an increase in the total amount of flux at constant applied potential difference which causes a change in the distribution of the stresses. The stress in the part of the field where the high inductive capacity (permittivity) material has been inserted is reduced, and that in the low capacity (permittivity) material increased. Therefore, in this case allowance has to be made for this effect.

In the above cases it is not necessary to have the surface of discontinuity rigorously conform to the flux or the equipotential surface since a small variation will not greatly disturb the field. This can be easily seen by making simple calculations by the law of flux refraction, which states that the tangent of the angle of incidence θ_1 is to the tangent of the angle of refraction θ_2 as permittivity of the first medium k_1 is to the permittivity of the second k_2.[12]

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[12] For example see Jeans, "Electricity and Magnetism," p. 335; Webster, "Electricity and Magnetism," p. 327; Karapetoff, "The Electric Circuit," p. 28 or 163.

Thus

$$\frac{\tan \theta_1}{\tan \theta_2} = \frac{k_1}{k_2} \qquad \text{(See Fig. 6)}$$

As an example consider the case in which

$$k_1 = 1 \text{ (air)}$$
$$k_2 = 3 \text{ (oil)}$$
$$\theta_1 = 70 \text{ deg. (the assumed angle of incidence)}$$

Then,

$$\tan \theta_2 = \tan \theta_1 \frac{k_2}{k_1}$$
$$\tan \theta_2 = 2.747 \times 3 = 8.25$$
$$\theta_2 = 83 \text{ deg. (the angle of refraction)}$$

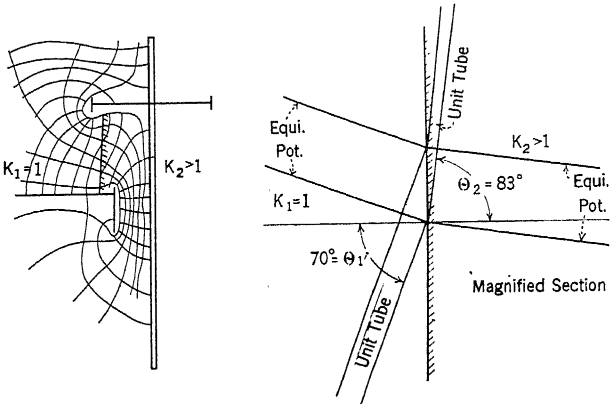

FIG. 6

As θ_1 approaches 90 deg., that is coincidence with the line of force, θ_2 also approaches 90 deg., and in the limit we have the case of two dielectrics of different permittivities in parallel. Under these conditions the form of the electrostatic field is unaltered by the presence of the two dissimilar materials, and the flux density in the air is not increased thereby. If θ_1 approaches zero degree then, in the limit, we have the case of two dissimilar dielectrics in series, and for this case the flux density in the two materials is the same, but due to the increase in total amount of flux the density in the air is increased by the presence of the higher permittivity material.

The above considerations show that there are two cases in which we can apply the diagrams obtained by using a single electrolyte to bushing design without fear of large errors being introduced due to flux refraction between the dissimilar materials which are in practise necessarily used for bushing.

Uniform-Field Type of Bushing

Thus applying Case I we see that it may be possible to so shape the electrode that the *strongest surface of discontinuity* between the supporting or high inductive capacity material and the air conforms closely to a line of flux. In that case the introduction of the solid material will not alter the field distribution and we would thus obtain one of the infinite number of theoretically correct designs. If this design can be obtained without making the electrodes of an impractical shape or size, this particular solution of the problem would be all that we require. For the case of ideal or perfect dielectrics, which assumes that the surface of discontinuity between the air and solid dielectric does not introduce a weakening influence, *the strongest surface would be an equigradient surface which coincides with a line of flux.*[13]

In this type of bushing it will be observed that the surface of discontinuity receives the full value of the potential gradient and therefore any weakening influence due to the surface of discontinuity between dissimilar dielectrics will exert its maximum ill effect. In the following a bushing built along these lines has been referred to as the "High-Air-Efficiency Bushing." It may also be called the "Uniform-Field Type," since the supporting dielectric and air are in parallel in an essentially uniform field.

By the use of artificial equipotential surfaces placed so as to bring about uniform gradient in the supporting dielectric, this type of bushing would, assuming perfect dielectrics, be as small as it is possible to construct, that is, it would use the air and any available supporting dielectric to their maximum efficiencies.

Radial-Field Type of Bushing

If we apply Case II to bushing design we would obtain a condition in which the potential gradient is zero along the surface of discontinuity between the air and supporting dielectric, and, therefore, has its full value normal to the surface. A bushing of this type would not be affected by surface conditions. It is obviously impossible to have the supporting dielectric connecting the grounded and high-potential electrodes actually coincide with an equipotential surface since the two electrodes must be at different potentials, nevertheless the condition can be approximately obtained in practise. This type of bushing has been referred to as the radial-field type. In this case the supporting and air dielectrics are essentially in series in a radial field. If the *surface effect* is very great, that is, if the component of the potential gradient along the surface has to be very small, this type of bushing when provided with artificial equipotential surfaces to obtain uniform stress in the supporting dielectric would result in as small a bushing as it is possible to construct. In this case the potential gradient would be approximately

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[13] It may be of interest to compare this suggested criterion which assumes ideal dielectrics with that suggested by C. L. Fortescue and S. W. Farnsworth, and J. M. Weed's discussion, A. I. E. E., TRANS., 1913, Vol. XXXII, Part I, p. 893, "Air as an Insulator when in the Presence of Insulating Bodies of Higher Specific Inductive Capacity."

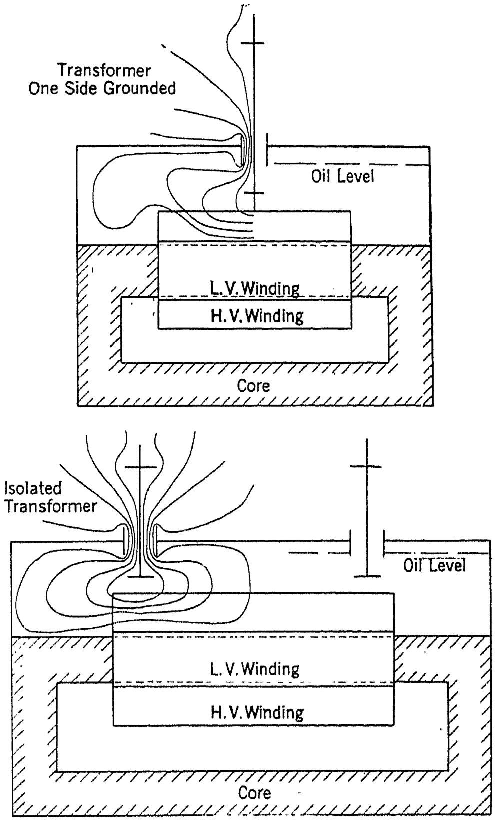

FIG. 7

normal to the surface and is, of course, only limited by the strength of air.

Under-Oil End. In the above discussion we have considered the top part of the bushing where two greatly dissimilar dielectrics must generally be employed. The under-oil end of the bushing can, as a fair approximation, be treated as a single dielectric problem since the dielectrics usually employed do not differ very greatly in inductive capacities.

The under-oil end requires careful study due to the proximity of disturbing influences such as the core and windings of the transformer. It may be found desirable to use a rather large cap on the under-oil end to shield the bushing from these disturbing influences.

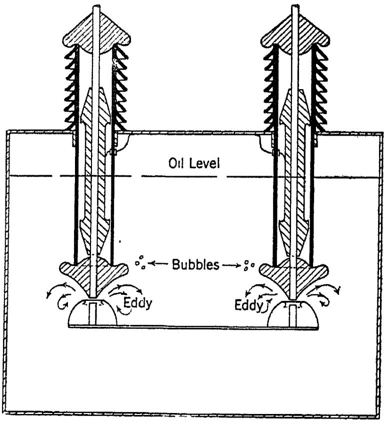

Fig. 8

This part of the problem will have to be carefully considered in conjunction with the whole design of the transformer or switch, etc. Fig. 7 is an illustrative suggestion of transformer design which would assist in an efficient under-oil-bushing design. Fig. 8 shows a switch design in which a large cap is used to reduce the tendency for an arc to re-strike, and, at the same time, provides a sort of deflector which throws the gas bubbles, etc., away from the surface of the dielectric. This could be given a sort of curved or bucket shape so as to produce eddies which might reduce the height of oil necessary over the contacts, etc.

Explanation of Diagrams

The diagrams are drawn for equal tubes of electrostatic flux or force and equal differences of potential between equipotentials.

That is, they are plane sections through the solid figure of revolution. Thus, if we imagine this plane section rotated about the axis of symmetry, the lines in the diagrams will cut out the correct surfaces—the equipotential lines and flux lines will generate equal tubes of flux and equipotential surfaces for equal differences of potential. The elementary cells, or elementary cylindrical condensers, thus cut out will have the same capacity throughout the field.

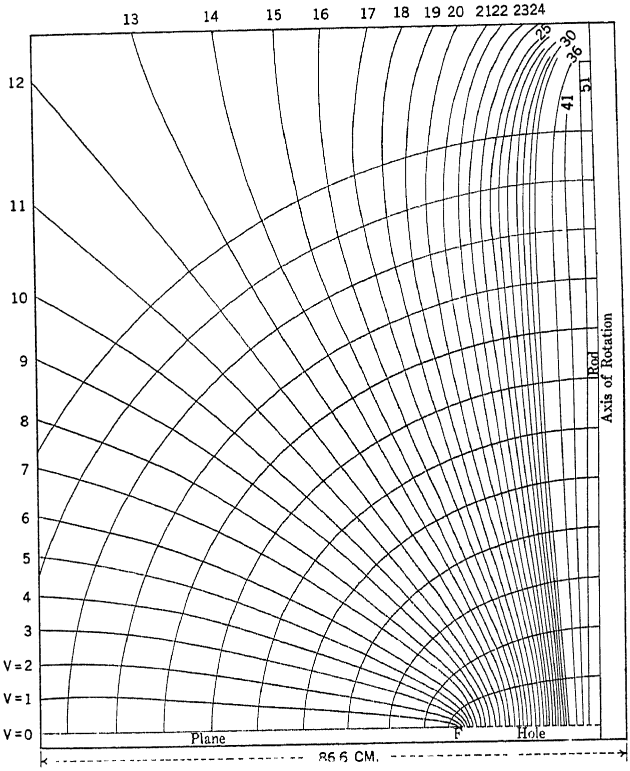

FIG. 9

In the case of diagrams, Figs. 27 and 28 dealing with the use of artificial equipotential surfaces the tubes of flux were not drawn in since they would be discontinuous at the metal surfaces. For this reason it was thought that they would confuse rather than add to the diagrams.

Discussion of Diagrams

Fig. 9. (*Mathematical Solution Fig.* 10). It will be readily seen that if we assume our transformer or switch tank sufficiently large, we can consider the top of the tank as an infinite plane with a circular hole in the middle. As our desired means of metallic connection the simplest would be merely a wire passing perpendicularly through the hole in the tank connecting the inside with the outside. As these electrodes constitute the simplest imaginable form from which to design a bushing their electrostatic field was the first studied.

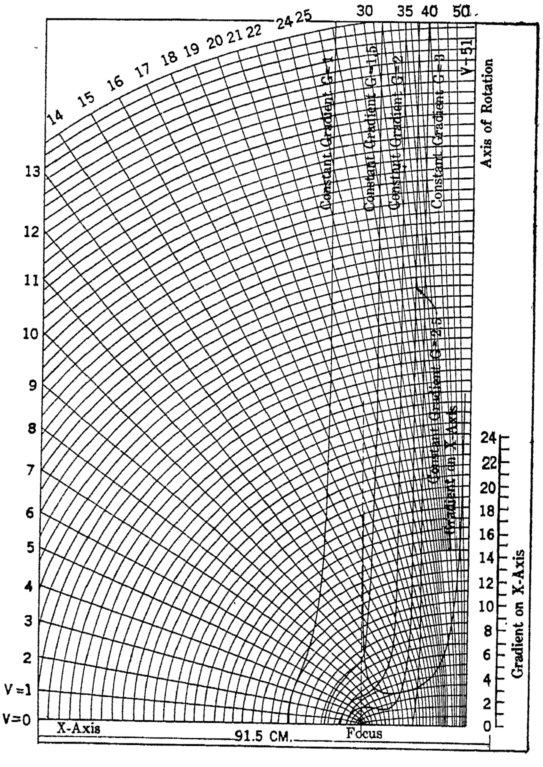

Fig. 10

This diagram shows the characteristics of what has for convenience been termed the radial-field type.

In designing a bushing from this diagram we would probably select equipotential surface 1 or 2 to represent the cover of the tank with hole and rounded edge which makes the stress on the supporting dielectric about three times as high as allowable for the air part of the dielectric. In order to stress the supporting dielectric near the rod to approximately the same value as that at the edge of the hole we would select as our rod an equipotential surface somewhere between No. 25 and No. 30. Under these conditions the ratio of rod diameter to hole diameter would be about 5 to 1. There are, however, many factors (such as the relative strengths of the dielectrics employed and whether the strength is equal in all directions, etc.) which will influence the above ratio and it should, therefore, not be considered as a fixed quantity.

The edge of the hole in the tank must clearly be embedded in the supporting, stronger than air, dielectric to eliminate over stressed air near the edge. That is, the value of a ground shield is clearly shown. This type of bushing will necessarily be quite high in order not to over stress the air at the top.

A study of this diagram suggested that a mathematical solution of this case should be quite easily obtained, due to the apparent simplicity of the equipotential surfaces and their mutual resemblance. Some of the lines of force were drawn in and were seen to conform fairly closely to confocal ellipses, having the edge of the plane as focus (see Fig. 9). This suggested that if there were no disturbing influences, that is, if the rod were very small in diameter and infinitely long and the plane also infinite in extent, the solution of the problem would consist in a confocal system of hyperboloids of one sheet as the equipotential surfaces, and confocal oblate spheroids as the surfaces of force. A plane section through the axis of symmetry of this figure is then a family of confocal hyperbolas and ellipses. The minor axis of the ellipses is the axis of symmetry or the axis about which the plane figure is rotated in order to cut out the solid figure.

In order to test this assumed solution of the problem, a diagram was constructed as follows:

Two large confocal ellipses were constructed having the minor axis along the rod and the radius of the hole in the plane as semi-major axis. The space between these two ellipses was then divided up into cells (see Fig. 11) which obeyed the required geometrical law:

$$\frac{2\pi r \times h}{d} = C \text{ (a constant)}$$

where r = the mean radius of the cell

h = distance between the two confocal ellipses

d = the other dimension of the small rectangular cell (or the distance between the concentric cylinders).

By cut and try, the space between the two ellipses was divided up into small cells of constant C. This value, C, is proportional to the capacity of the small concentric cylinder condensers which would be cut out by rotation around the minor axis. The family of hyperbolas which has the same focus as the ellipses was then drawn through the cells. The figure was then completed by drawing in the complete family of confocal ellipses which divided the whole field into cells having the same capacity or constant C. The fact that it was possible to complete this construction is a graphical proof of the correctness of the assumed solution; namely, that confocal hyperboloids of one sheet are the equipotential surfaces, and confocal oblate spheroids the boundary surfaces of the tubes of flux.

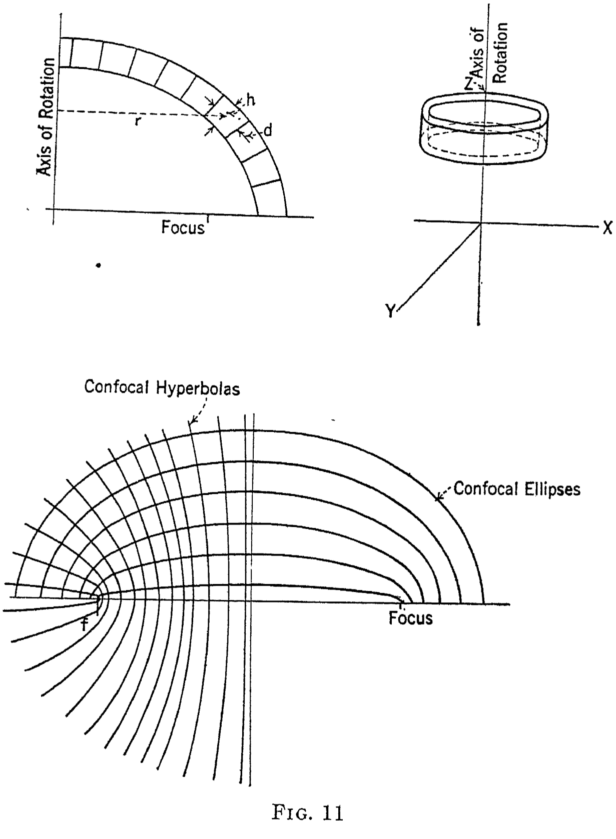

FIG. 11

After obtaining the solution of the problem by this method, I found that it was well known; for example Maxwell[14] discusses the mathematical solution of this and similar problems. Other treatments will also be found in many books.[15]

Notwithstanding the numerous treatments of the subject it was not until a lengthy study of these sources, besides correspondence and later the great privilege of an interview with Professor W. E. Byerly, that I was able to obtain what appeared to me to be a clear and useful solution of the problem.

I believe that there are other engineers who find it difficult to readily understand, and therefore use, to the best advantage, the results of the great deal of extremely valuable mathematical work which is available in the various treatises. It seems to me a great pity that the mathematicians do not more frequently reduce their results to a readily utilizable form, and wherever possible sketch out, with examples, some of the applications which must occur to them while working on the subject. In spite of the drudgery which necessarily accompanies any numerical calculations, I believe that a writer would be amply repaid for his trouble by the greatly increased number of people who would study and be able to use his results.

Another difficulty, which I have frequently encountered, is the fact that the writer assumes too great a familiarity with existing mathematical works on the part of his readers. No one is better able to supply page references to what he considers a good and clear treatment of his statements " it has been proved ". I also believe that no work would suffer from the inclusion of an appendix of "it can be easily shown" and in some cases the "hences". All of these additions could be skipped by the fluent mathematical readers but would be available as wonderful time savers and often life savers to the ordinary engineer. For these reasons I have included an unexpurgated edition of two electrostatic problems in the appendix to this paper.

Problem I. The distribution of the electrostatic field when any two confocal hyperboloids of revolution of one sheet, and of the same family, are maintained at given potentials.

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[14] See Maxwell, Electricity and Magnetism, Vol. I, p. 235 and following.

[15] See Byerly, Fourier Series and Spherical Harmonics, p. 238–247. J. H. Jeans, Electricity and Magnetism, p. 238–244; A. G. Webster, Electricity and Magnetism, p. 203–242, 273, etc.; I. Todhunter, The Functions of Laplace, Lamé and Bessel, Chapter XXI p. 211.

This problem is useful as an approximation in studying high-voltage bushing design; it also gives us an interesting variety of possible electrode shapes for use in testing insulating materials, where it is desirable to be able to calculate the gradients. When testing a dielectric of given inductive capacity immersed in a dielectric having a different inductive capacity (*i.e.*, hard rubber in air or oil) the edge effect can be eliminated by shaping the test piece so as to follow a line of force or flux.

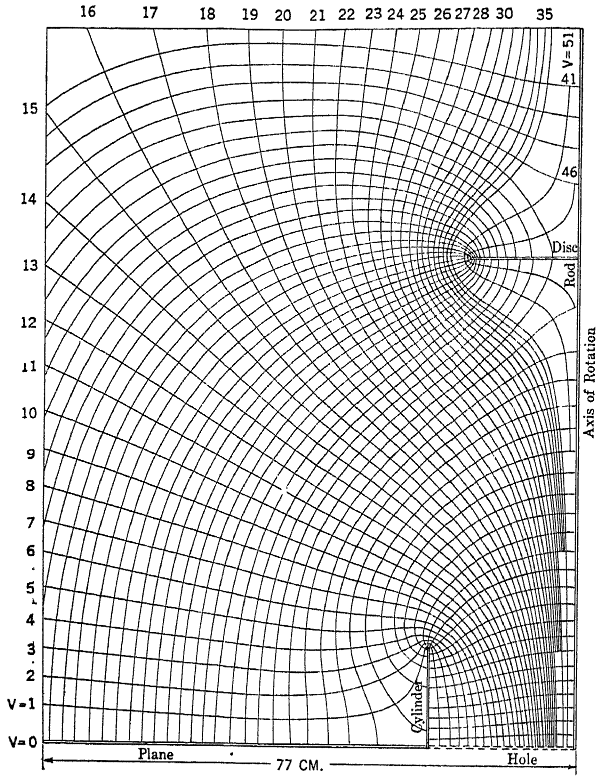

FIG. 12

Problem II. The distribution of the electrostatic field when any two confocal hyperboloids of revolution of two sheets and of the same family are maintained at given potentials. This problem is of interest as an approximation to a group of problems which are of frequent occurrence in engineering. For example, it may be applied to switch electrodes at various spacings; also, as an approximation in vacuum tube designs, such as X-ray tubes, kenotrons, etc. That is, it is an approximation to the problems of two elongated electrodes at various spacings, (*i.e.*, two needles or a needle and a plane) etc.

Fig. 12. This diagram[16] shows a form of field intermediate between the radial and uniform-field types. The effect produced by adding a collar to the edge of the hole in the plane is shown. The addition of a collar or extended ground shield is generally used on the air end to shield the bolts necessary in clamping the bushing to the tank. It may also be used to shield switch mechanism, etc. A ground shield extending below the surface of the oil on the under-oil end is usually necessary in order to remove the stress on the air above the oil.

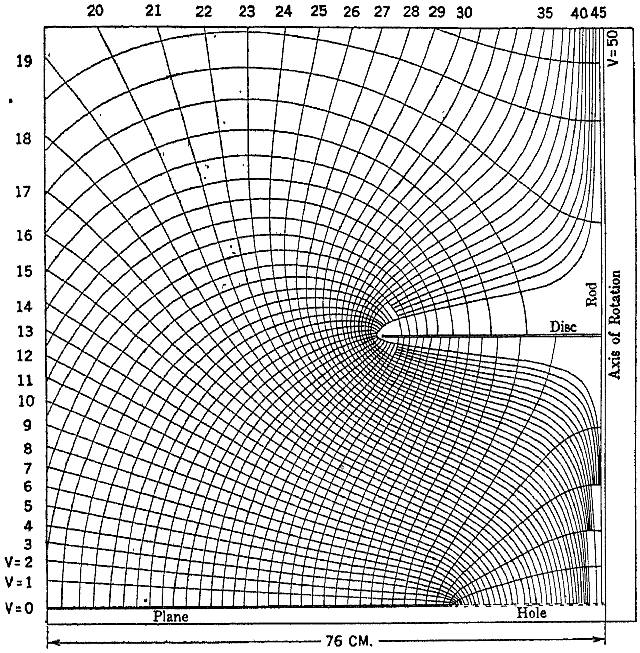

FIG. 13

In designing a bushing from this diagram, we would probably select equipotential surface No. 2 or No. 3 as cover of the tank with hole and collar or ground shield and an equipotential surface resembling No. 20 as high potential electrode. In practise we would probably not build the electrode of the shape of surface No. 20, but would make up the electrode of a straight rod and cap which would be roughly equivalent to surface No. 20. The addition of the cap and collar does not greatly alter the conclusions regarding the ratio of rod to hole diameters as outlined above when discussing this point for the simplest case, that of a rod passing through a hole in a plane. That is, the edge effect of the collar or cylinder is about the same as that for the edge of a plane.

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[16] Dr. C. P. Steinmetz has published this diagram in the fourth edition of his Electrical Engineering, page 116, which has recently appeared.

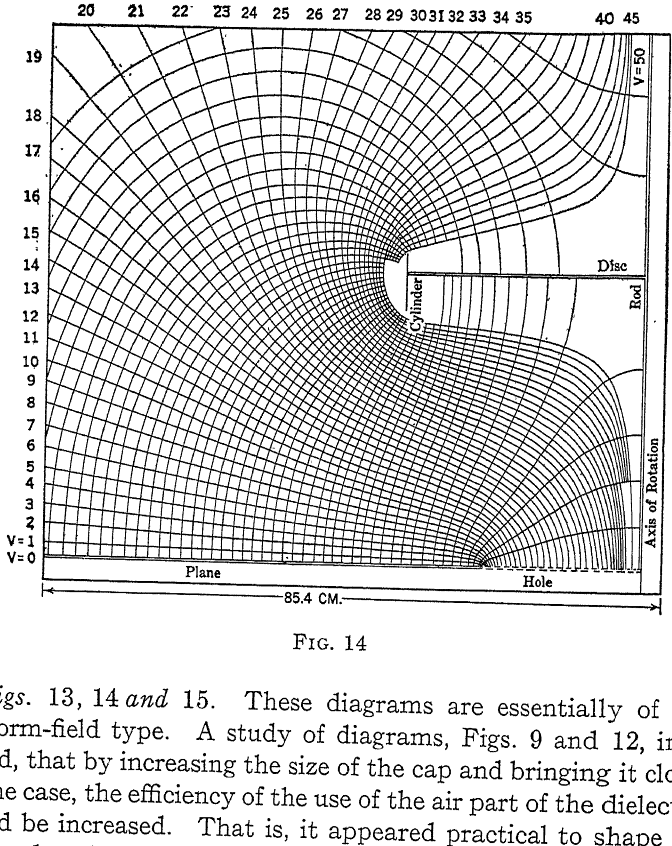

FIG. 14

Figs. 13, 14 *and* 15. These diagrams are essentially of the uniform-field type. A study of diagrams, Figs. 9 and 12, indicated, that by increasing the size of the cap and bringing it closer to the case, the efficiency of the use of the air part of the dielectric could be increased. That is, it appeared practical to shape the electrodes of the bushing in such a way that the distribution of the electrostatic field would approach the condition in which the strongest surface assuming ideal dielectric (constant gradient or flux density) coincides with a line of force in the part of the field where it is desired to introduce the dielectric of higher specific inductive capacity than the rest, which may be air or oil. As previously stated the resulting bushing would use the air at its maximum efficiency and hence the resulting bushing would be as efficient, with respect to the air path, as any of the infinite other solutions which could be obtained by using flux refraction combined with various electrode configurations.

In order to experimentally test out this conclusion, Figs. 13, 14 and 15 were taken. Skeleton electrodes were used, as in the previous experimental work, in order to simplify matters. For instance, one electrode may consist in a thin piece of sheet iron, or tin, with a hole in it; the other electrode a small wire with an attached metal disk. The proper actual shape of the rod with cap and the tank will then be selected from the equipotential surfaces which are obtained by the experiment.

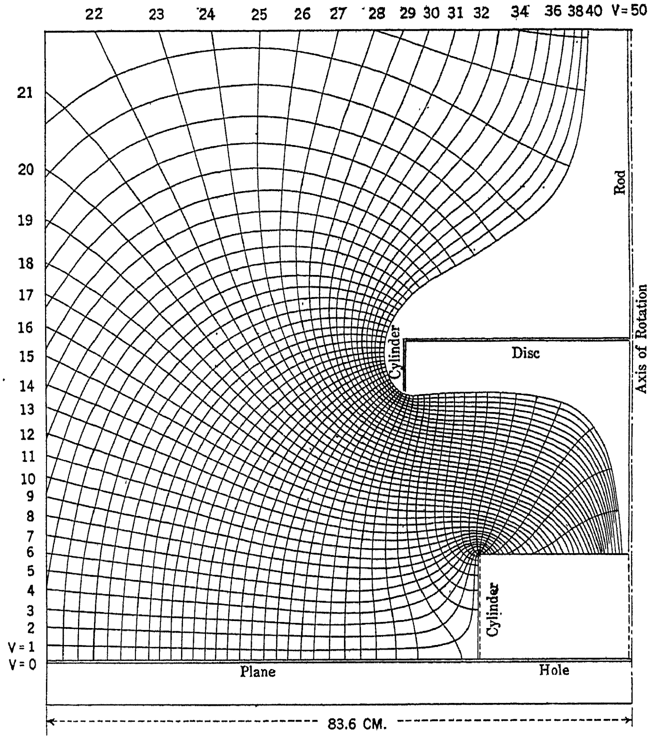

FIG. 15

For the new diagrams the size of the cap was increased so as to have a diameter of about $1\frac{1}{2}$ times that of the hole in the plane, and was placed at about one diameter from the plane. The short cylinder was added to the sheet-iron disk for Fig. 14, to form a skeleton electrode which would flatten out the equipotential surfaces near the edge of the disk, and also to bring the lines of force into a more nearly cylindrical form. Briefly, to give the effect of a cap having a greater radius of curvature at its edge.

In applying these diagrams to bushing design we would probably select the equipotential surface 2 as the top of the tank, and an equipotential surface between 28 to 32 as our rod with cap. This selection gives approximately equal gradient at the edge of the hole in the tank and at the rod. Also, the maximum gradient on the cap is not too different from that near the tank. The supporting insulation, stronger than air and of higher specific inductive capacity, would probably be put in somewhere between flux lines number 4 to 7.

It is apparent from the diagram that this bushing would have its maximum air stress near the edge of the cap, and therefore, would probably arc-over clear of the solid insulation, unless the surface introduces appreciable weakening influences. This matter will have to be settled by experiment and then given consideration according to its magnitude. Assuming a surface which does not appreciably affect the arc-over of the bushing, we obtain from the full size diagram, Fig. 14, the following result. Selecting equipotential surface No. 30 as our rod with cap, we see that the maximum potential gradient in the air occurs near the edge of the cap. We must, therefore, select this gradient to be that at which air breaks down or approximately 21 kv. effective per cm. The distance between equipotential surfaces 30 and 29 is about 0.6 cm. (on the full size diagram).
Hence

$$\frac{e\Big|_{30}^{29}}{0.6} = 21 \text{ kv. effective per cm. (assumed strength of air.)}$$

$$e\Big|_{30}^{29} = 12.6 \text{ kv. effective, the potential at which breakdown will occur between these two surfaces.}$$

Between the cap and the tank (equipotential surface No. 2) there are 28 equipotential surfaces drawn for equal differences of potential. Therefore, the arc-over voltage of this terminal would be obtained by multiplying the number of equipotential surfaces between that selected as the rod with cap and the tank, by the voltage which would cause breakdown if applied between the two consecutive surfaces in the densest part of the air field. In this case the number of surfaces is 28, and the voltage at which breakdown first occurs between two surfaces is 12.6 kv. effective.

Hence

$$28 \times 12.6 = 350 \text{ kv. effective arc-over voltage.}$$

The operating voltage, assuming a safety factor of 3, would be 116 kv. effective. This calculation assumes that as soon as the gradient at the edge of the cap reaches the breakdown value, arc-over will occur (unstable condition).

With regards to the stresses on the solid or liquid dielectric which is used in this design, it may be of interest to observe that the maximum gradient, which occurs along the path between rod and edge of tank, is approximately 63 kv. effective per cm. or three times that at which air breaks down.

The efficiency with which this bushing uses the air part of the dielectric may be estimated as follows:

Assuming arc-over to occur from the point of maximum stress on the cap, that is, near the outer edge, to the tank a distance of approximately 30 cm., we see that this air path under uniform gradient, should arc-over at

$$30 \times 21 = 630 \text{ kv. effective.}$$

Hence, taking the efficiency to be the ratio of the arc-over voltage as estimated above from the diagram, to this uniform gradient condition, we have

$$\text{Efficiency} = \frac{350 \text{ kv.}}{630 \text{ kv.}} = 55 \text{ per cent.}$$

Fig. 15, shows the effect of the addition of a collar or cylindrical ground shield to a bushing of this type. The conclusions are similar to those which may be drawn from the previous diagrams.

TESTS ON EXPERIMENTAL HIGH-AIR-EFFICIENCY BUSHING

An experimental bushing was constructed from a photographic reduction of Fig. 14, as outlined in Fig. 16. Two supporting dielectrics were tried. In the first case a smooth porcelain piece conforming as closely as possible to the flux line No. 7 was used. In the second case the porcelain surface was corrugated as shown in the figure between flux lines No. 4 and No. 7.

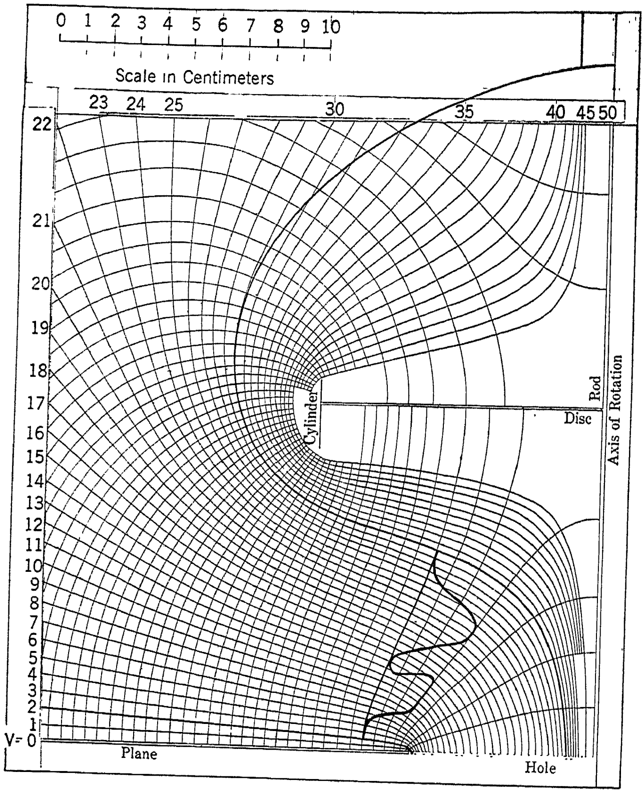

FIG. 16

The electrodes and porcelain pieces were cemented together with a sort of sealing wax compound, considerable effort being made to make good joints at the electrodes. Equipotential surface No. 30 was chosen as the central electrode shape but was modified at the top as shown in the figure as it was not thought important to conform to the diagram at the top part of the electrode where the stress on the air is very low. Equipotential surface

PLATE XLII.
A. I. E. E.
VOL. XXXVI, 1917

[RICE]

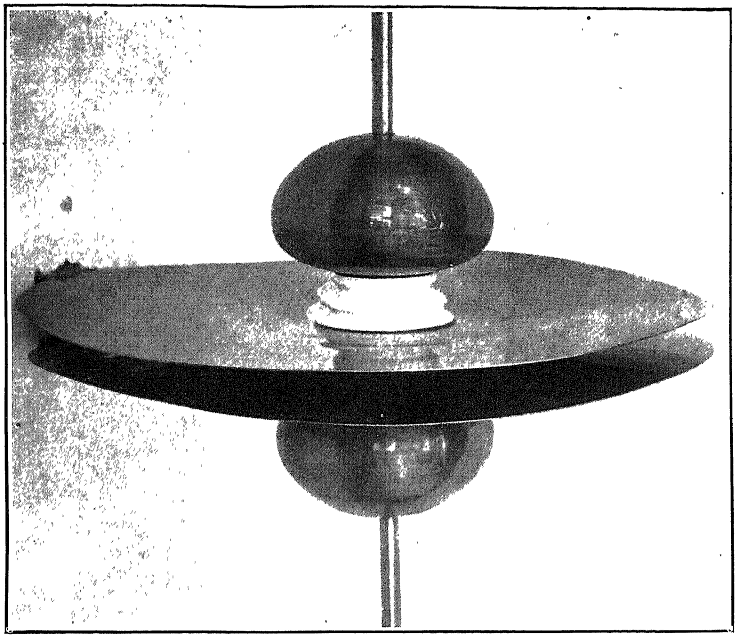

FIG. 18

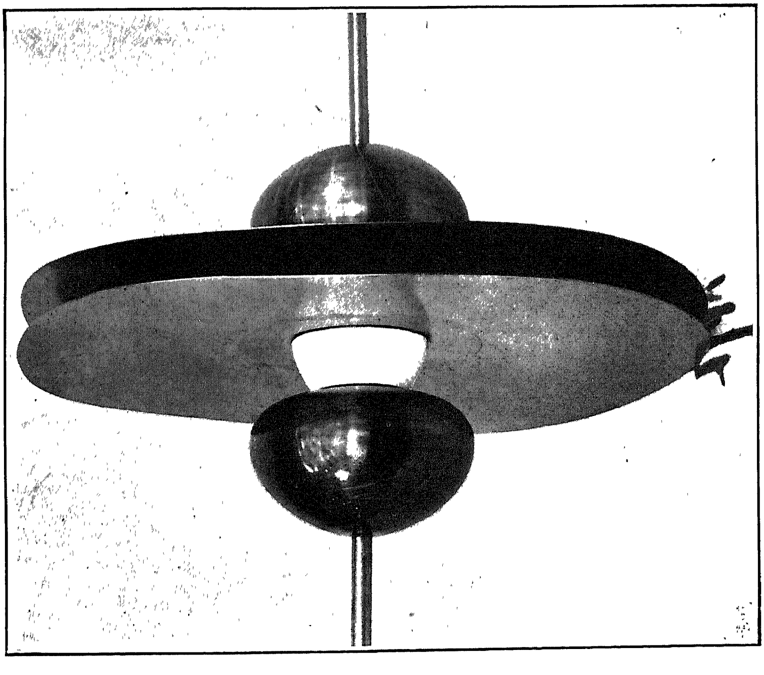

FIG. 17

No. 2 was selected as the ground plane and extended 43 cm. from the axis. The upper and lower parts of the bushing were made as nearly alike as possible; Fig. 17 and 18 show the general appearance and construction of the bushings.

The principal electrical characteristics, as obtained from the experimental diagram, Fig. 14, are as follows: The maximum gradient on the air part of the dielectric occurs on the cap of the central electrodes and well away from the surface of the supporting dielectric. The gradient at this point reaches 21 kv. effective per cm. (or the assumed breakdown strength of air) at about 117 kv. effective. The gradient along flux line No. 7 (7.3 cm. long) is practically uniform and will reach 21 kv. effective per cm. at about 153 kv. The gradient at the edge of the ground electrode and the central rod are approximately equal and are about three times the maximum gradient on the air.

The arc-over voltages of the corrugated and smooth bushings may be estimated as follows:

(1) *Smooth Surface*

If we assume that the surface and joints of the porcelain do not introduce disturbing influences, arc-over should occur clear of the surface at 117 kv. effective. For this potential difference the gradient along the surface of the porcelain would be 16 kv. effective per cm.

(2) *Corrugated Surface.*

If we neglect refraction effects and make the very rough assumption that the corrugations introduce half air and half porcelain in series in a uniform field we would have for the arc-over voltage

$$e = G_1 \left(x_1 + \frac{x_2}{k} \right) \text{ arc-over voltage}$$

Where

$$G_1 = 21 \text{ kv. per cm.}$$

$$x_1 = x_2 = \frac{7.3}{2} \text{ cm.}$$

$$k = 5 \text{ porcelain}$$

Hence,

$$e = 21 \left(3.65 + \frac{3.65}{5} \right)$$

$$e = 92 \text{ kv. effective,}$$

and arc-over would occur by unstable corona formation in the corrugations at 92 kv. effective.

At this voltage the average potential gradient along the arc-over path would be $\frac{92}{7.3} = 12.6$ kv. effective per cm.

Summary of Tests

Sixty-Cycle Arc-over Voltage. (average of ten readings)

Surfaces cleaned with absolute alcohol and wiped dry

I	Smooth Surface	80 kv. ± 10 kv.
	Average gradient	11 kv. per cm.
II	Corrugated Surface	72 kv. ± 10 kv.
	Average gradient	9.9 kv. per cm.

Surfaces cleaned as above and then covered with an oil film.

I	Smooth Surface	75 kv. ± 10 kv.
	Average gradient	10.3 kv. per cm.
II	Corrugated Surface	82 kv. ± 5 kv.
	Average gradient	11.2 kv. per cm.

The arc-over voltages after visible carbonization points have been formed at the sealing wax joints and on the porcelain is given below. This condition is reached after about 15 arc-overs, when dry clean surfaces are used and in 5 to 10 arc-overs when the surface is oiled. Handling the surface results in about the same arc-over voltages.

I	Smooth Surface	50 kv. ± 2 kv.
	Average gradient	6.85 kv. per cm.
II.	Corrugated Surface	5 kv. ± 2 kv.
	Average gradient	7.25 kv. per cm.

Surface of electrodes and porcelain covered with city water, that is, the surfaces were sprayed after all the oil had been removed.

I	Smooth Surface	23 kv. ± 3 kv.
	Average gradient	3.15 kv. per cm.
II	Corrugated Surface	27 kv. ± 3 kv.
	Average gradient	3.7 kv. per cm.

Surface oiled and a needle point ¼ in. long placed upright at the corrugated surface on the ground plate.

First arc-over	50 kv.
Average gradient	6.85 kv. per cm.

After about three arc-overs a carbonized point could be seen to form at the arcing point on the upper electrode above the needle, also considerable carbonization took place around the needle.

Under these conditions the arc-over was............................35 kv.
An average gradient of............. 4.8 kv. per cm.

60 *Cycle Under-Oil Tests.* After the above mentioned tests were completed the bushing was immersed in oil (40 kv. oil as measured by standard gap of 0.2 in. (0.5 cm.) between 0.5 in. (1.27 cm.) terminals) in order to determine the under-oil characteristics.

In both cases arc-over took place over the porcelain surfaces and in the corrugated bushing followed in and out of the corrugations. The results were

I	Smooth Surface	122 kv.
	Average gradient	16.7 kv. per cm.
II	Corrugated Surface	144 kv.
	Average gradient	19.7 kv. per cm.

Impulse Arc-over Tests. Impulse tests were made using F. W. Peek's impulse generator.[17] Impulses having various frequencies and wave shapes were used but as the results were in good agreement in all cases only a typical set of readings will be given. The calculated frequency of the impulse wave is given as 500 kilocycles with a maximum voltage corresponding to 188 kv. effective.

The arc-over voltages as measured by 12.5-cm. spheres placed in parallel with the bushing were as follows:

I	Smooth Surface	111.5	spheres only
		117.	half and half
		121.	bushing only
	Average gradient	16.	kv. per cm.
II	Corrugated Surface	104.	spheres only
		111.	half and half
		117.	bushing only
	Average gradient	15.2	kv. per cm.

The results were practically identical with clean dry or oiled surfaces.

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[17] "The Effect of Transient Voltages on Dielectrics." A. I. E. E., TRANS., 1915, Vol. XXXIV, Part II, p. 1857.

Observations and Discussion

In all of the tests on the smooth bushing the arc-over appeared to follow close to the surface. In the corrugated case arc-over appeared to take place along flux line No. 7, that is, did not follow in and out of the corrugations. (The only exception was in the case mentioned of the under-oil arc-over of the corrugated bushing where the path followed the corrugations in and out).

The previous calculations indicated that the smooth porcelain bushing should arc-over clear of the surface at about 117 kv. whereas in test it arced-over along the surface at about 80 kv. ± 10 kv. (oily and dry surfaces do not seem to be greatly different) or say approximately 70 per cent of the calculated arc-over voltage. However, this comparison is not a truly correct one since the arc-over occurred at the surface and not at the point of maximum gradient. A better comparison of calculated and observed arc-overs is obtained by comparing the calculated surface arc-over with the actual surface arc-over.

Sixty-Cycle Tests. (A) Averaging the clean and oily surfaces we have

$$\frac{\text{calc. surface arc-over}}{\text{measured surface arc-over}} \cdots\cdots = \frac{153}{77} = 2.0$$

(B) Carbonization points formed after a number of arc-overs

$$\frac{\text{calc. surface arc-over}}{\text{measured surface arc-over}} \cdots\cdots = \frac{153}{50} = 3.1$$

(C) Surfaces and electrodes sprayed with water. . $= \frac{153}{23} = 6.6$

Impulse Tests. (D) Clean or oily surfaces. . . . . $= \frac{153}{117} = 1.3$

Many explanations may be advanced for the above results; at first I was inclined to account for them in the following manner.

Case A. Here breakdown occurred over the surface of the bushing at approximately one-half the calculated voltage. To account for this we might assume that a conducting corona ring forms prematurely due to an imperfect joint between the porcelain and electrodes, since at the joint a high stress might exist on the air due to the high inductive capacity material being in series with the air at this point. If such a conducting corona ring was formed the maximum stress upon it would be approximately twice that existing in the uniform part of the field. For if we insert a cylindrical conductor in a uniform field the maximum gradient on the cylinder will be twice the gradient of the uniform field in which it is placed or if we have two infinite plane electrodes having hemicylindrical bosses upon them, then for spacings between the planes such that the field between is essentially uniform in the middle the gradient in the vicinity of the cylindrical bosses will be a maximum on the crest of the boss and will be twice the gradient existing in the uniform part of the field.

The solution of this problem is obtained by the method of images; by superimposing a uniform field upon the field of a linear doublet.[18] A complete diagram of the electrostatic field is given by Maxwell in Plate XV at the end of Vol. II of "Electricity and Magnetism."

If we make the further assumption that the corona ring is unstable then we see that arc-over should occur at approximately one-half the calculated arc-over voltage assuming perfect joints and surfaces, which is about the value observed from the tests. It is of course probable that the gradient at the surface of the boss will have to be somewhat above 21 kv. effective per cm. at breakdown, or what is the same thing, the breakdown strength of air (assumed 21 kv. eff. per cm.) must be reached at a finite though small distance from the surface of the boss. This same effect is well known in the case of wires or concentric cylinders, etc., for example, it has been discussed by F. W. Peek, Jr.[19]

Case B. Carbonization points formed after a number of arc-overs. In this case arc-over took place over the surface at about one third the calculated value. A method of approximately explaining this case may be based upon the assumption that a small hemispherical corona boss forms on the surface of the electrodes. The solution of the electrostatic problem of two infinite planes having hemispherical bosses is obtained by superimposing a uniform field upon the field of a spherical doublet. The gradient is a maximum on the crest of the boss and is three times that in the uniform part of the field.[20]

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[18] See "Electricity and Magnetism". A. G. Webster, p. 202, also p. 88. Notes on Electric Field Distribution. Journal of the Franklin Institute, July, 1913. W. S. Franklin, p. 72–75.

[19] A. I. E. E., TRANS., 1911, Vol. XXX, Part III, p. 1889, "The Law of Corona and the Dielectric Strength of Air."

[20] J. H. Jeans, "Electricity and Magnetism," p. 188–191; A. G. Webster, "Electricity and Magnetism", p. 371–373; Lord Kelvin Papers on "Electrostatics and Magnetism," p. 492; J. J. Thomson, "Electricity and Magnetism," p. 158–160.

Diagrams of the lines of force or flux will be found in the various references given below and a somewhat similar and instructive case is shown in Plate III at the end of Vol. I—Maxwell, "Electricity and Magnetism."

Case C. Water particles on the surface. In this case arc-over occurs at about one-half the needle-gap arc-over at this spacing. This low arc-over under these conditions may be a sort of progressive breakdown between adjacent water particles or distortion of the field by erratic conduction, etc.

Case D. The impulse test shows an arc-over voltage which is very close to the calculated arc-over assuming that it takes place clear of the surface. To the eye, however, the arc-over seems to take place along or very close to the surface and therefore there seems to be some disturbing influence at work which the above explanations do not take into account.

The previous calculations for the corrugated porcelain bushing indicated that the arc-over should take place along the edge of the corrugations due to unstable corona formation in the valleys of the corrugations. The calculation is necessarily only approximate as the flux refraction produced at the corrugations is neglected.

A comparison of the various cases may, however, be of interest.

Sixty-cycle Tests.

Case A. Oily and clean surfaces ratio of calculated to test arc-over........................ $\frac{92}{77} = 1.2$

Case B. Carbonization points formed after a number of arc-overs........................ $\frac{92}{53} = 1.74$

Case C. Surfaces sprayed with water............. $\frac{92}{27} = 3.4$

Impulse Test

Case D. Clean and Oily Surface................ $\frac{92}{111} = 0.83$

Explanations similar to those offered for the smooth surface might be advanced but cannot be quantitatively applied as the field conditions are not as definitely known.

Under-Oil Tests (60 *Cycles*). The under-oil arc-over tests also show a lower value than would be expected if we assume perfect joints and ideal dielectrics.

The strength of oil at these large spacings (7.3 cm.) is about 33 kv. effective per cm., and therefore the surface arc-over voltage should be 7.3 × 33 = 240 kv. effective for the smooth porcelain surface

$$\frac{\text{calculated surface arc-over}}{\text{measured surface arc-over}} = \frac{240}{122} = 2.0$$

It will be observed that the explanation above (corona ring or joint effect) could be applied to these tests and give what would appear to be a satisfactory explanation of the observed discrepancy between calculated surface arc-over voltage and that obtained from the tests. The corrugated porcelain surface could be discussed qualitatively in a similar manner.

ADDITIONAL TESTS ON "HIGH-AIR-EFFICIENCY BUSHINGS.

The object of the following tests was to determine the disturbing influence of the walls and floor of the room or ground upon the arc-over voltage of this type of bushing. The following tests were made using the *corrugated supporting dielectric* as it was still assembled with the electrodes. The previous under-oil tests had damaged the porcelain surface on one side, and therefore, necessitated submerging the injured end under oil. For this purpose, a wooden oil tank was used, the tests being made on the uninjured end. The surface of the porcelain was oiled for all tests.

Summary of Tests

Sixty-Cycle Arc-Over Voltage. (average of ten readings). Corrugated Surface.

(A) Plane grounded
    Arc-over voltage.................67 kv. ± 5 kv. eff.
    Average gradient................. 9.2 kv. per cm.

(B) Plane isolated, central electrode grounded
    Arc-over voltage.................67 kv. ± 4 kv.
    Average gradient................. 9.2 kv. per cm.

(C) Bushing entirely isolated.
    Arc-over voltage.................80 kv. ± 10 kv.
    Average gradient.................11 kv. per cm.

A comparison of these tests with the previous ones show a reduction of the arc-over voltage for the grounded cases to about 80 per cent of the previous values. The isolated case agrees closely with the previous tests where the plane was grounded.

The results might be expected from the following consideration: In the previous tests the bushing was suspended about 10 ft. above the floor of the room, and therefore was not appreciably influenced by the floor and walls, whereas the present tests were made with the bushing about 2 to 3 ft. from the floor of the room. The conditions of the test had to be changed, due to the fact that it was necessary to have the damaged end of the bushing under oil. Under these conditions the proximity of the floor of the room or ground would have a greater effect upon the grounded cases, *A* and *B*, than for case *C* where both plane and central electrode were isolated.

It may be interesting to note, at this point, that the type of electrostatic field, which we are here concerned with, for example, an infinite rod passing through a hole in an infinite plane is essentially different from the case of two spheres at different potentials in space. In our case the rod and the plane both are considered as going to infinity and are at different potentials. Thus, it is evident that the potential at infinity is indeterminate and the electrostatic field distribution merely depends upon the relative potential between the two electrodes. This can be seen analytically from the mathematical solution for this simplest case (see appendix). The case of two spheres in infinite space is quite different. For example, assume them to have equal and opposite potentials, in that case, the equipotential surfaces are approximately spherical at great distances from the spheres and at an infinite distance the equipotential surfaces around each sphere are two infinite spheres made up of the infinite plane which passes midway between them. By reason of symmetry this infinite plane is seen to be at zero potential since it forms the equipotential surface which lies midway between the two spheres and since this plane goes to infinity the potential at infinity is zero.

If we now assume one sphere at zero potential and the other at a positive potential of such a value as to make the relative potential between the two spheres the same as before, the electrostatic field distribution is no longer symmetrical about the plane midway between the spheres, that is, the field has been changed though the relative potential between the spheres is the same. A diagram of the electrostatic field, under these conditions, is shown in Fig. II at the end of Vol. I, Electricity and Magnetism, by Clerk Maxwell. In this diagram, *Q*, is the spherical surface at zero potential and, *A*, the other sphere at a positive potential. The concentration of the field about, A, will be observed. This is quite a different matter from the effect of floor, walls, etc., of the room. I have taken the liberty of calling attention to this fact because it has puzzled me considerably and may have bothered others also. The above remarks, of course, do not apply to the case of two bushings in proximity, in which case the fields of the two bushings would have to be treated as one.

Conclusions

In general, the results of tests on the above high-air-efficiency bushings were not very encouraging from the point of view of building a satisfactory commercial bushing of this type. However, if the above suggested explanation of the discrepancy between theory and test were found to be correct, that is, if the disturbing influence is due to corona at the joints between the porcelain and electrodes, it should be possible to eliminate it by electrostatic shielding of these joints. This might eliminate all the lowering of arc-over except that due to rain or sprayed surfaces, and, therefore, might give us a useful bushing for indoor use, especially on testing transformers, where the *impulse safety factor* is not important.

Investigation of the Discrepancy Between Calculated and Test Arc-Over Voltage of Experimental High-Air-Efficiency Bushing

Joint Effect

The following tests were carried out in order to obtain an experimental check upon the explanation offered above which was based upon the assumption that a corona ring formed prematurely at the joint between the supporting dielectric and the electrodes.

Large plane electrodes with smoothly rounded edges were spun up of brass of the form shown in Fig. 19. The object of the design was to obtain as nearly as possible, a uniform field between the electrodes without disturbing edge effects. The sixty-cycle arc-over voltage was then obtained at various spacings, with planes alone, and when provided with small rings, hemispherical bosses and points. The results are given in the form of curves in Fig. 19.

It will be observed from the data given on the planes alone that the field between them is not accurately uniform since the breakdown gradient changes with the spacing and is in general lower than 21 kv. effective per cm. the approximate strength of air at these large spacings. The arc-overs were, however, well distributed and did not show a tendency to take place at the rounded edges. The 200 kilocycle impulses arc-over of the planes is given on the curve, and approximately represents the true strength of air under uniform field conditions, as the edge effect does not seem to greatly lower the impulse arc-over.

To show roughly the effect of hemicylindrical bosses small copper rings ⅛ in. (0.318 cm.) cross-section were used. Two diameters for the rings were tried first, one-inch (0.254 cm.) diameter and secondly, four-inch (10.15 cm.). The results were, however, not appreciably different and, therefore, the averages are shown, first for the case of a single ring on the upper plane, and secondly for a ring on each plane. The rings were not made half-sections because it was not thought necessary for such rough tests. Of course the effect produced by the ring is not rigorously equivalent to a straight hemicylindrical boss except where the cross section of the ring is exceedingly small and the diameter of the ring extremely large in comparison.

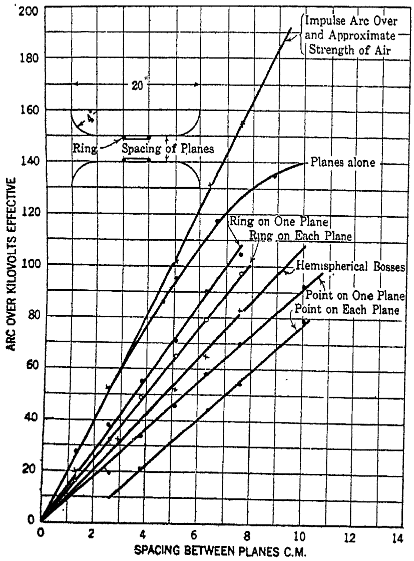

FIG. 19—SIXTY-CYCLE TESTS IN AIR

The hemispherical bosses used had a radius of ⅛ in. (0.318 cm.) No appreciable difference was observed between the case in which a boss was placed on one or on both planes, and, therefore, a single curve is shown.

The points or cones which were employed were approximately 30 deg. and ¼ in. (0.635 cm.) high and had rather dull points.

From the previous discussion we saw that the apparent arc-over gradient when hemicylindrical bosses were placed upon the planes should be, assuming any corona formation to be unstable somewhat above 10.5 kv. eff. per cm. due to the so-called "energy distance effect" which requires that the surface gradient exceed the breakdown strength of air. The rough tests given above indicate an apparent surface gradient of about 13 kv. eff. per cm. Observation in the dark indicated that arc-over takes place without previously showing corona.

A similar explanation probably suffices to explain the difference between the apparent arc-over gradient, as discussed above, for the case of a hemispherical boss which indicated that on the assumption of unstable corona formation the arc-over would take place at an apparent gradient somewhat above 7 kv. eff. per cm. In test the apparent gradient was found to be about 10 kv. eff. per cm. No corona could be observed in the dark before arc-over.

The case of the small points on the planes is not so easily calculated but is interesting in showing the extreme condition. Observation in the dark showed streamers from the point to the plane just below the arc-over voltage, sometimes resulting in arc-over and sometimes going out, the appearance being like very fine so-called "static sparks" and not a corona like glow.

Surface Effect

Tests in Air and Oil. We have seen that the explanation of the discrepancy between calculated and test arc-over voltages of the high-air-efficiency bushing can be fairly satisfactorily explained on the assumption that the joint between the porcelain and metal of the electrodes was the disturbing influence which brought about the reduction in arc-over voltage. It has also been pointed out that, if this were the true explanation, it should be possible to eliminate the effect by proper electrostatic shielding of the joints. The following preliminary tests were, therefore, carried out in order to roughly check the assumption experimentally.

For this purpose the two brass electrodes used in the previous tests were available, and in addition a second pair of planes were spun up on exactly the same form, except that the annular grooves were added as shown in Fig. 20. The object of the grooves was to supply an electrostatic seal for the edge of the test piece (a glass cylinder in these tests). It is believed that in this manner we have shielded the joint between the test piece and the electrodes, and, therefore, should have eliminated any disturbing influence which might arise from the joints. An imaginary sketch of the electrostatic field about the grooves has been drawn which is also a detail drawing of the groove. Reference may also be made to Figs. XI and XIII, at the back of Vol. I, Electricity and Magnetism, by Clerk Maxwell, which show the screening effect of plates and gratings, which are somewhat analogous cases. Under these conditions the introduction of the glass cylinder of high specific inductive capacity in parallel with the air dielectric should have no effect upon the arc-over voltage, providing the strength of air is not different when in contact with the test piece.

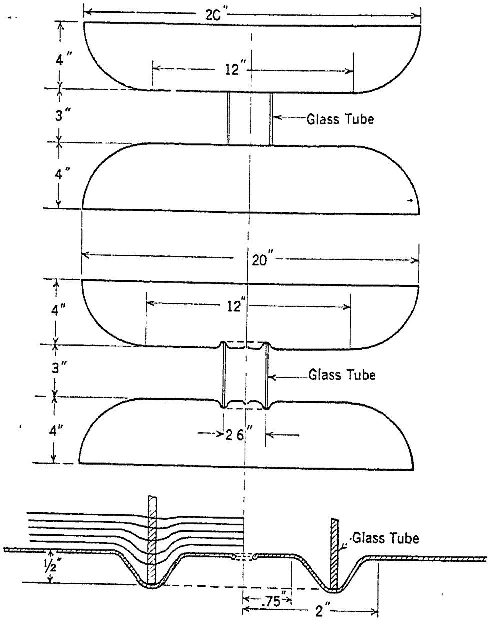

Fig. 20

The data obtained using various test pieces between the smooth and grooved planes in air and oil are summarized by the curves, Figs. 21 to 25. During the tests the barometer varied between 74.5—76. cm. Hg.; the temperature between 23 deg. cent. to 27 deg. cent. and humidity between 34 per cent to

53 per cent. The effect of these variations did not show themselves above the experimental errors involved in the tests and therefore all the data were averaged together. The glass cylinders used as test pieces were ordinary soda glass approximately 2.5 inches (6.25 cm.) diameter with a wall thickness of about 0.07 inches (0.18 cm). In all cases minute longitudinal flaws were visible along the grain of the glass. The edges of the cylinders were roughly ground but the process left appreciable irregularities which probably would be sufficient to introduce corona disturbances, if they are appreciable.

Three conditions of the glass surface were investigated: I—Glass cylinders cleaned with absolute alcohol and dried with a cloth. This case is represented on the curves as dry glass (I).

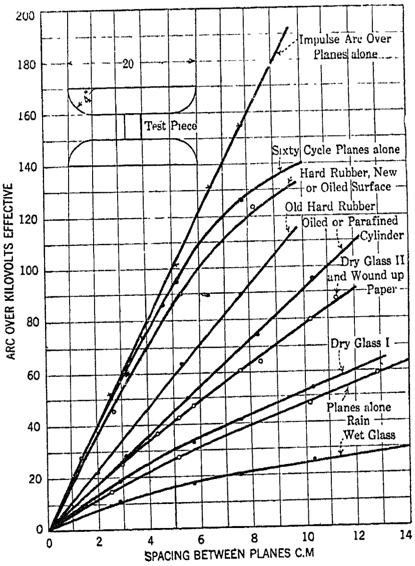

FIG. 21—SMOOTH PLANES—SIXTY CYCLE TESTS IN AIR

II—Glass cylinders cleaned as in (I) and then heated for about 3 hours at 180 deg. cent. The cylinders were then removed from the oven, handling being done by means of dried fibre strips, and set up between the planes for test. Arc-overs were taken while the cylinders were still hot and also after they had reached room temperature. The later condition is that referred to on the curves as dry glass (II). The hot arc-overs differed from the room temperature ones merely in proportion to the change in air density on the supposition of a heated film of air in the vicinity of the hot cylinders.

III—Wet glass cylinders. For these tests the cylinders were held under the water faucet just before tests were made.

For the tests in which the cylinders were coated with paraffine, or oil, etc., the test piece was soaked in the hot material and then tested after reaching room temperature.

The tests with hard rubber varied considerably as shown by the two curves. The upper curve is for hard rubber cylinders which had been re-surfaced with fine emery just before test. The lower curve shows the arc-over after the test piece had stood around the laboratory for a week or so. The tests with oiled hard rubber refer to cylinders which had been soaked in No. 6 transil oil for a few days.

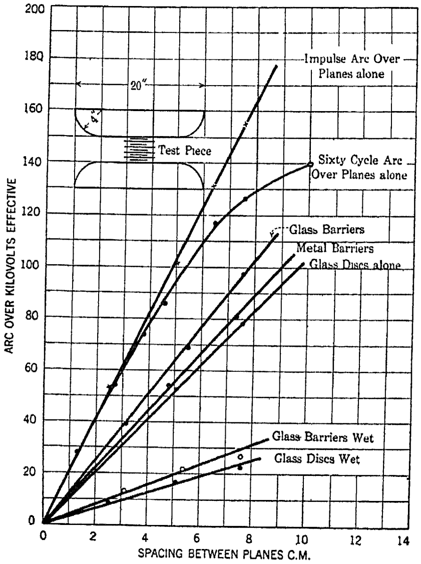

FIG. 22—SMOOTH PLANES—SIXTY-CYCLE TESTS IN AIR

Tests were also made on wound-up paper cylinders impregnated with compound.

The tests using glass disks given on curve sheet Fig. 22 were intended to show the effect of breaking up the surface first by the natural irregularity of the piled up disks (see Fig. 26), secondly with larger glass disks inserted at intervals, and thirdly with metal disks inserted at intervals. The glass disks were 0.075 in. (0.19 cm.) thick and 2 in. (5 cm.) and 4 in. (10 cm.) diameter and had roughly ground edges. The metal barriers were 5 in. (12.7 cm.) in diameter and 0.019 in. (0.048 cm.) thick.

The under-oil tests were carried out in a large wooden oil tank containing No. 6 transil oil, testing 40 kv. between 0.5-in. (1.27 cm.) terminals at 0.2 inches (0.51 cm.) spacing.

The impulse tests were made using F. W. Peek's impulse generator [21] with an impulse resembling half a cycle of a 200 kilocycle sine wave. The voltages given are on the assumption that arc-over occurs at the top of the sine wave impulse.

A few tests were made using glass tubes and hard rubber cylinders having different diameters, so as to see whether the arc-over was affected by changes in the surface resistance between the planes. The tests did not, however, show any variation from the previous ones.

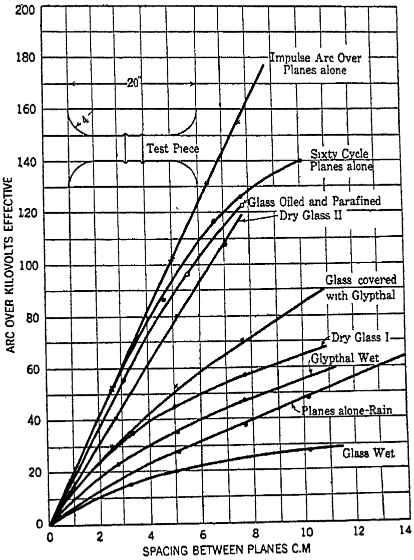

FIG. 23—GROOVED PLANES—SIXTY-CYCLE TESTS IN AIR

The incompleteness of the data and the irregularities which occurred between different samples of the same material as well as the rather crude methods of test make any conclusions rather questionable. I am, however, inclined at present to the following conclusions.

If all precautions were taken to absolutely free the test samples from moisture, and furthermore, if practically perfect joints were made between the test samples and the planes, either by electrostatistically shielding the joints or by mechanical fit, I believe that the presence of insulation in parallel with air or oil, etc., in a uniform field would not result in a breakdown lower than that of the weakest material. When, however, the joint is not carefully made, I believe that premature breakdown occurs at the joint when a high inductive capacity material is placed in parallel with a lower inductive capacity material.

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[21] See footnote 17.

When dealing with dielectrics, which have been cleaned and dried, with what would be considered great care in the household sense of the word, there appears to be a very large reduction in arc-over under the conditions here discussed, namely, under uniform field conditions, depending upon the character of the material which cannot be accounted for by joint effect. In general, it may be further pointed out that materials such as clean new hard rubber, oiled, or paraffined surfaces, which Mr. Harvey L. Curtis has shown have a high surface resistivity under varying humidity conditions, also show a high breakdown under the test conditions here described. From a purely electrostatic point of view it is, of course, obvious that if the surface conduction took place in a perfectly regular and uniform manner it should not result in any reduction in the arc-over of the test pieces as the equipotential surfaces of conduction would coincide

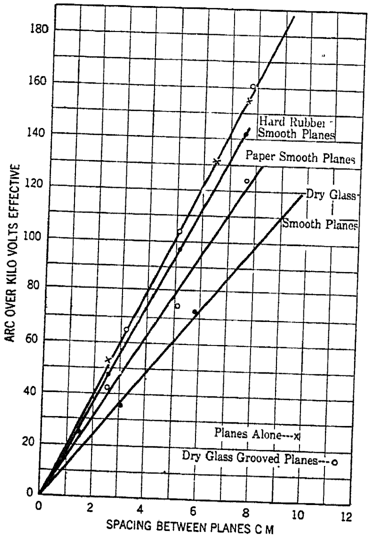

FIG. 24—IMPULSE TESTS IN AIR—200 KILO-CYCLES

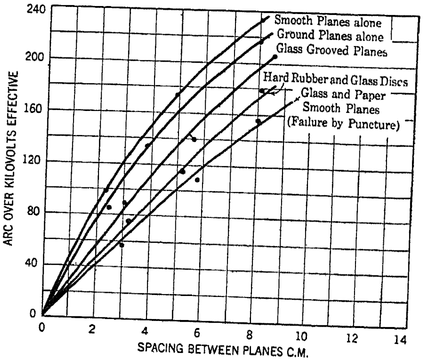

FIG. 25—SIXTY-CYCLE TESTS IN OIL

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22. The Volume Resistivity and Surface Resistivity of Insulating Materials—*General Electric Review*, October 1915, p. 996.

with electrostatic ones and no distortion or increase in stress would thereby result. If, therefore, the reduction is to be attributed to surface leakage or conduction, this effect must be of an erratic and discontinuous nature.

It would be of considerable theoretical interest to test this conclusion experimentally. For example, I believe that a suitably high resistance and homogeneous metallic film could be obtained on a glass cylinder by subjecting it either to a cathode spray or by volatilization in a high vacuum. Some tests were tried in which ground-glass cylinders were coated with graphite, etc., but without satisfactory results.

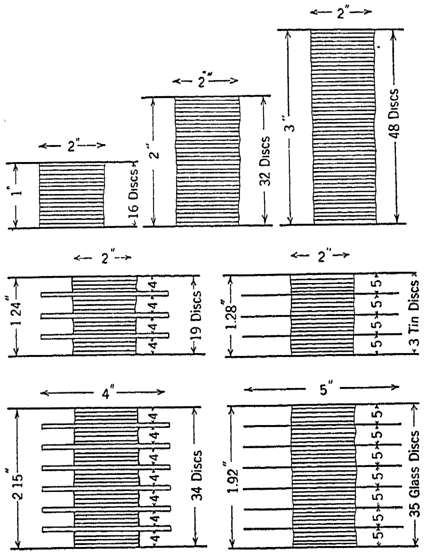

FIG. 26

I believe that a thorough and accurate investigation of this phenomena would be of considerable practical as well as theoretical interest.

REVISED ARC-OVER CALCULATION OF HIGH-AIR-EFFICIENCY BUSHING

A revised calculation of the arc-over voltage of the high-air-efficiency bushing may now be made which takes into account the surface and joint effects. We have seen previously that the surface arc-over of the bushing in air should take place under ideal conditions at about 150 kv. effective. Now, since the electrostatic conditions along the surface are practically identical for the bushing and for the test pieces between the smooth planes, a direct comparison is legitimate except for the fact that a glass surface is probably not identical with a glazed porcelain surface. Nevertheless the comparison is of interest.

Arc-over of High-Air-Efficiency Bushing	Arc-over of Smooth Planes 7.3 cm. Spacing
Tests in Air	
Average of dry and oiled smooth porcelain surfaces	Average of dry and oiled glass
60 cycle, 73 kv. effective	60 cycle, 67 kv. effective
Impulse, 117 kv. effective	Impulse, 90 kv. effective
Surface wet with city water	Surface wet with city water
60 cycle, 23 kv. effective	60 cycle, 21 kv. effective
Tests in Oil.	
Smooth porcelain surface	Glass cylinder,
60 cycle, 122 kv. effective	140 kv. effective

In conclusion it should be observed that if we are forced to use supporting dielectrics, which show large surface effects, we must consider this fact when we form any criterion for the best use of air or oil in combination with the supporting dielectric. If we assume that our surface of discontinuity between the air part of the dielectric and the solid or supporting dielectric, for instance glass, is greatly weaker than air alone, for example—only stands a potential gradient of 7.3 kv. effective per cm. instead of 21 kv. effective as would probably be the case if perfectly dry and clean; then considering this feature alone, we see that in order to use both materials most efficiently, the component of the potential gradient along the surface should not exceed 7.3 kv. effective per cm., whereas the air itself should be used to its full strength of 21 kv. effective per cm. Thus, the component of the potential gradient normal to the surface should be $G = \sqrt{21.^2 - 7.3^2} =$ 19.7 kv. effective per cm. or the flux lines should make an angle of approximately 70 deg. from the surface. This is practically the condition existing in the so-called "Radial Type of Field" in which the supporting dielectric and air are approximately in series in a radial field. It will be seen that when considering ordinary materials from which bushings are usually made that this surface effect practically requires us to go to the radial type of field in constructing a bushing.

CALCULATION OF THE ARC-OVER OF THE TEST PIECE, DESCRIBED AND TESTED BY C. L. FORTESCUE AND S. W. FARNSWORTH.[23]

The electrodes consisted of confocal hyperboloids of revolution of two sheets. The equation of the hyperbolas which constitute the generating curves were obtained from the description and Fig. 20 of the paper as follows:[24]

$$\frac{x^2}{a^2} - \frac{z^2}{c^2} = 1$$

Where

$f = \sqrt{a^2 + c^2} = 1$ in. the semi-focal distance.

and $a = 0.875$ in.

$c = 0.485$ in.

from which we have

$$\frac{x^2}{(0.875)^2} - \frac{z^2}{(0.485)^2} = 1$$

The ellipsoidal hard-rubber supporting dielectric is cut out by rotating the ellipse whose equation was determined in a similar manner to be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Where $f = \sqrt{a^2 - b^2} = 1$ in.

and $b = 3.1$ in.

$a = 3.26$ in.

and

$$\frac{x^2}{(3.26)^2} + \frac{y^2}{(3.1)^2} = 1$$

Reference to Fig. 18 or 19 of the appendix shows that the hypobola No. 27 is very approximately the one corresponding to that used for the electrodes. The ellipse bounding the hard rubber is No. 22 counting out from the focal ellipse as 0.

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[23] See footnote 1.

[24] See p. 863 and Figs. 1 and 2 of the appendix.

Assuming the breakdown strength of air as 21 kv. effective per cm. and perfect dielectrics, corona would start at the junction between the hard rubber and electrodes at 205 kv. eff. If we further assume that the corona formation is unstable arc-over should occur assuming perfect dielectrics without surface or joint effects at about this value. The factor which takes into account the reduction in arc-over, due to what has been called surface and joint effect, for the case of hard rubber in parallel with air in a uniform field, in which the full value of the gradient is along the surface, is applicable in this case. The tests have shown that a factor of about 73 per cent should be applied to the value as calculated neglecting this effect. Thus, the calculated arc-over of this structure is about 150 kv. effective, the test results reported by Fortescue and Farnsworth were given as 160 kv. effective.

OUTLINE OF SOME OF THE POSSIBLE METHODS FOR INCREASING THE EFFICIENCY OF THE USE OF THE SUPPORTING DIELECTRIC

In the previous discussion on bushing design attention has been principally directed towards studying the possibilities of an efficient use of the air dielectric. Theoretically, the ideal bushing would be that in which the whole dielectric is used to its maximum efficiency, that is, it would be on the point of breakdown simultaneously at all points. Therefore, it is interesting to study the possibilities of increasing the efficiency of the use of the solid or supporting dielectric. For example, inspection of the diagram, Fig. 14 shows that the supporting dielectric in a bushing built along these lines is not used to its maximum strength except right near the rod and edge of the hole in the tank. Therefore, if it is possible by some means to redistribute the stress in the solid dielectric so as to bring about a more uniform condition, it would be possible to greatly reduce the diameter of the bushing as a whole. The following are some of the possible methods which might be employed.

I—Theoretically we could bring about the desired condition provided we had suitable dielectrics of various inductive capacities and dielectric strengths. We could then place the high inductive capacity materials near the rod and edge of the hole in the tank putting the lower inductive capacity materials in the less dense parts of the field and in some such manner as this bring about the condition in which all the dielectrics used were stressed to their maximum strengths. This is the well known principle of graded insulation. Unfortunately there is not a very wide variation in the inductive capacities of the materials which are available in practise.

II—If there were available extremely high resistance materials, or dielectrics, we could construct a bushing which would maintain a proper potential distribution by conduction through the dielectric. This might be called a resistance bushing. A graded resistance bushing would also be possible.

III—Another method is to insert artificial equipotential surfaces and thereby control the stresses in the manner which we desire. In order to effect the desired distribution of stresses the artificial equipotential surfaces must be maintained at the proper potentials. Some of the methods which can theoretically be used for this purpose are as follows:

A. Metallic connection to a proper source of potential. For example, to transformer taps or external balancing resistances, inductances or capacities, etc. A balancing resistance might even be embedded in the dielectric.

B. By conduction through the supporting dielectric. This would be similar to the resistance bushing mentioned above except that the distribution of the conduction current could be modified, in various ways, by the insertion of artificial equipotential surfaces.

C. Electrostatically (or the well known condenser principle)[25] in this case we have the balancing condensers embedded in the supporting dielectric, and, at the same time, forming the artificial equipotential surfaces. Of course graded insulation is also applicable to this type of bushing.

Theoretically where artificial equipotential surfaces are used they could be extended through the surface of the solid or supporting dielectric into the air part of the field where they could be used to assist in bringing about the desired field distribution, and at the same time, even be made to act as petticoats to shield the surface from rain, etc.

Obviously the possibility exists of making various combinations of the above principles.

When the practical difficulties of utilizing the above methods are considered it appears that case III alone, or possibly in combination with grading by inductive capacities, is the most feasible at the present time. Therefore, a brief study of this case was undertaken making use of the electrodynamic method.

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[25] R. Nagel *Elektrische Bahnen und Betriebe*, 1906, p. 278; A. B. Reynders, A. I. E. E., TRANS., 1909, Vol. XXVIII, Part I, p. 209; C. L. Fortescue, *Electrical Journal*, August 1913, p. 718; W. S. Franklin, *Journal of the Franklin Institute*, July 1913.

PRELIMINARY STUDY OF THE USE OF ARTIFICIAL EQUIPOTENTIAL SURFACES OR POTENTIAL EQUALIZERS

Diagrams of Figs. 27 and 28, were taken as a preliminary study of the effects produced by forcing a uniform distribution of potential gradient in the solid or supporting dielectric by the use of cylindrical artificial equipotential surfaces. Diagram, Fig. 27, shows the effect when the radial-field type is used on the air end and diagram, Fig. 28, for the uniform-field type. The under-oil ends are what might be termed semi-radial.

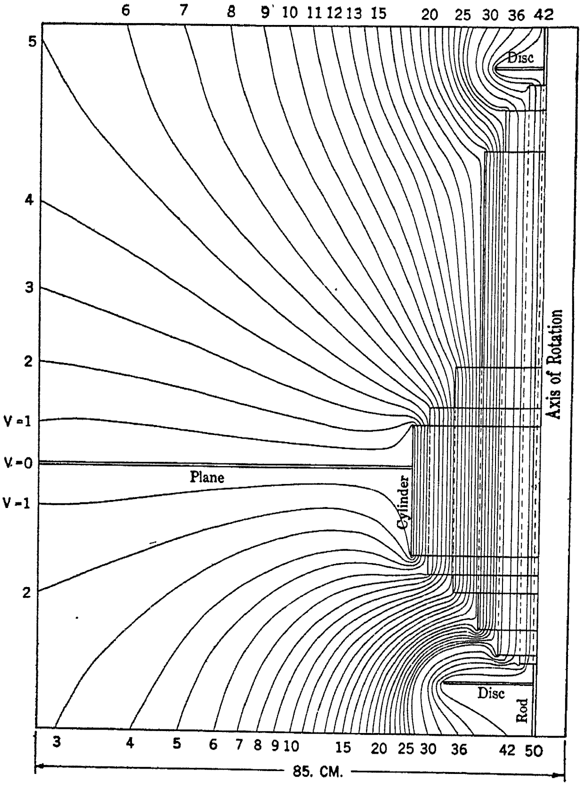

FIG. 27

At the outset, it should be noted that it is necessary to study both the air and oil ends simultaneously if they are not symmetrical, otherwise the problem is indeterminate. Briefly the method of experiment was as follows: The skeleton electrodes, for Fig. 27 consisted of a plane with cylinder (ground shield) and a central rod provided with a tin disk at both ends. The disk on the under-oil end was put in so as to make a definite field in which to end the artificial equipotential surfaces. If this were not done the bushing would be greatly affected by the piece of apparatus in

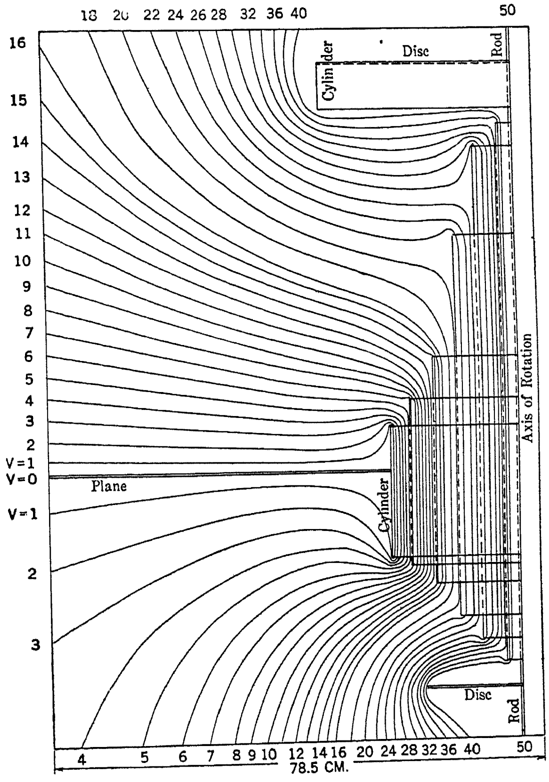

FIG. 28

which it was afterwards used. I believe that this feature is quite important. The ground shield was used in order to relieve the stress on the air above the oil level, as usual, and was projected into the air end so as to shield the joint which would be necessary in construction. Five tin quarter-cylinders were then added as the artificial equipotential surfaces. These were constructed of two pieces so that they could be telescoped, thereby making it possible to change their length and the position of their ends both in the air and oil parts of the field. The adjustment was accomplished by a system of strings so that different positions could be tried while experimenting with water in the box. The apparatus was set up in this manner and the position of the cylinders in the air and oil ends adjusted until the potential gradient between the cylinders was as nearly equal as could be easily predetermined. It was during the course of this work that it was found necessary to add the tin disk at the top of the bushing, (See Fig. 27) in order to have a dense field in which to end the two inner cylinders. If this were not done they would have had to be either too close to the under-oil disk which would mean too great a stress at the bottom of the under-oil end, or greatly extended in the air end. In the latter case the resulting bushing would have been much higher than seemed desirable for the preliminary work where an exaggeration of effects was desirable. Naturally, the position of the potential equalizing cylinders may be almost anything, that is, the potential of any of the cylinders could be brought to the desired value either by adjustment of the ends of the cylinder in the air or oil end. Such an arbitrary arrangement, without regard to the stresses brought about on other parts of the bushing, would obviously result in an impractical design. For instance, it might result in bad stresses at the ends of the cylinders and outside of them at the expense of the uniform gradient thus obtained within. In both diagrams the object was to obtain, as nearly as possible, equal gradient within the cylinders, and at the same time, obtain the best external field consistent with this requirement. Naturally, in practise it would probably be best to compromise between the internal and external parts of the field but for a preliminary study it seemed best to take one object to start with and study the consequences.

When the diagram was completed, it was found that the preliminary adjustment had not actually obtained the uniform gradient in the solid material as had been attempted. For example, the distributions between the various artificial equipotential surfaces starting from the ground shield were as follows: (See Figure 27).

I —Number of lines.................. 6.
Average gradient.............. $\frac{6 \text{ lines}}{2.92 \text{ cm.}} = 2.05$

II —Number of lines.................. 6.4
Average gradient.............. $\frac{6.4 \text{ lines}}{3.94 \text{ cm.}} = 1.62$

III —Number of lines.................. 7.
Average gradient.............. $\frac{7\text{ lines}}{4.2\text{ cm.}} = 1.66$

IV —Number of lines.................. 6.5
Average gradient.............. $\frac{6.5\text{ lines}}{3.3\text{ cm.}} = 1.97$

V —Number of lines.................. 10
Average gradient.............. $\frac{10\text{ lines}}{3.8\text{ cm.}} = 2.64$

VI —Number of lines.................. 14
Average gradient.............. $\frac{14\text{ lines}}{2.16\text{ cm.}} = 6.5$

No attempt was made to obtain a low gradient in the last case, as it was contemplated to select the equipotential surface of the inner cylinder as our rod with cap.

Inspection of Fig. 27 will show that a concentration of the field at the top and bottom ends of the bushing has resulted from forcing the nearly uniform gradient between the cylinders. It should also be observed that the capacities of the concentric cylinder condensers formed by the artificial equipotential surfaces have no definite relation, under these conditions, where the effect of the caps and assymmetry of the problem introduce large effects. For example, assuming as the length of the condensers the mean height of the two adjacent cylinders we have for the capacities starting from the ground shield.

I $C = \frac{\text{mean height}}{\log_\epsilon \frac{R}{r}} = \frac{25.2\text{ cm.}}{\log_\epsilon 1.16} = \frac{25.2}{0.148} = 170.$

II C proportional to............ $\frac{33.3\text{ cm.}}{\log_\epsilon 1.29} = \frac{33.3}{0.255} = 131.$

III C " " .... $\frac{59.4\text{ cm.}}{\log_\epsilon 1.43} = \frac{59.4}{0.358} = 166.$

IV C " " .... $\frac{86.0\text{ cm.}}{\log_\epsilon 1.52} = \frac{86}{0.419} = 205.$

V C " " .... $\frac{94.2\text{ cm.}}{\log_\epsilon 2.5} = \frac{94.2}{0.916} = 103.$

VI C " " .... $\frac{100\text{ cm.}}{\log_\epsilon 6.7} = \frac{100}{1.90} = 53.5$

I wish to emphasize caution against any conclusions which might be drawn from the above values for the capacities; for an entirely different adjustment of the lengths of the cylinders could obviously be obtained which would still give equal potential gradient between the successive cylinders and in which the capacities calculated as above would have entirely different values and interrelations.

Before leaving the discussion of this diagram it may be interesting to note that if we built a bushing along these lines; for example, if we selected equipotential surface No. 36 as our central rod with cap and equipotential surface No. 0 as plane with ground shield, corona would start at the top of the bushing at about 238 kv. effective and would probably arc-over slightly above this value. The average voltage per cm. of height of this bushing would be $\frac{238 \text{ kv.}}{70 \text{ cm.}} = 3.4$ kv. effective per cm.

Diagram, Fig. 28 was taken using the same electrode configuration except that the large skeleton cap was put in place of the small one used for Fig. 27. The position of the cylinders was adjusted with the same objects in view as for the previous diagram.

The distribution of potential between the various artificial equipotential surfaces starting from the ground shield will be seen to be as follows:
(See Diagram, Fig. 28).

I. —Number of lines.................. 6.5

Average gradient.............. $\frac{6.5 \text{ lines}}{3.05 \text{ cm.}} = 2.13$

II. —Number of lines..................7.

Average gradient.............. $\frac{7. \text{ lines}}{3.94 \text{ cm.}} = 1.77$

III.—Number of lines.................. 7.5

Average gradient.............. $\frac{7.5 \text{ lines}}{3.94 \text{ cm.}} = 1.90$

IV.—Number of lines.................. 6.

Average gradient.............. $\frac{6 \text{ lines}}{3.55 \text{ cm.}} = 1.69$

V. —Number of lines.................. 10

Average gradient.............. $\frac{10 \text{ lines}}{3.82 \text{ cm.}} = 2.62$

VI.—Number of lines.................. 13

Average gradient.............. $\dfrac{13 \text{ lines}}{2.16 \text{ cm.}} = 6.0$

As before, no attempt was made to obtain a low gradient inside of the small last cylinder as it was contemplated using this equipotential surface for the rod with cap, as in the previous case.

The capacities of the adjacent cylinders considered as concentric cylinder condensers, starting from the ground shield, are as follows:

I.— $C =$ .... $\dfrac{\text{mean height}}{\log_\epsilon \dfrac{R}{r}} = \dfrac{24.9 \text{ cm.}}{\log_\epsilon 1.17} = \dfrac{24.9}{0.157} = 158$

II.— C proportional to .... $\dfrac{32.8 \text{ cm.}}{\log_\epsilon 1.27} = \dfrac{32.8}{0.239} = 138$

III.— " " " .... $\dfrac{51 \text{ cm.}}{\log_\epsilon 1.41} = \dfrac{51.}{0.344} = 148$

IV.— " " " .... $\dfrac{73 \text{ cm.}}{\log_\epsilon 1.56} = \dfrac{73}{0.445} = 164$

V.— " " " .... $\dfrac{86 \text{ cm.}}{\log_\epsilon 2.5} = \dfrac{86}{0.916} = 94$

VI.— " " " .... $\dfrac{99 \text{ cm.}}{\log_\epsilon 6.7} = \dfrac{99}{1.90} = 52$

As in the previous case, caution should be urged against hasty conclusions drawn from the above capacity values, for in this case also an entirely different adjustment would be possible in which the magnitudes and interrelations between the capacities would have very different values.

In designing a bushing from this diagram we might select equipotential surface No. 38 for the rod with cap and equipotential surface No. 0 for the plane with ground shield. A straight cylindrical shell would be added enclosing the central core and the ground shield. The resulting distribution of the potential on the air end of such a bushing would be approximately uniform. Under these conditions corona would start at the cap on the air end at about 440 kv. effective which gives an average gradient along the surface of 7.75 kv. effective per cm.

The tests made on glass cylinders in a uniform field in which the surface breakdown gradient was found to be about 9.2 kv.

per cm. indicate that the air end of this bushing is about as short as practical assuming good surface conditions. A shell would also be added to the under-oil end of the bushing enclosing the ground shield. Inspection of the under-oil end shows a concentration of the field about the cap. Also in view of the under oil arc-over tests on glass cylinders and hard rubber it looks as if the under-oil end is about as short as possible. The diameter of this bushing is evidently larger than necessary to withstand puncture as will be readily seen by comparing the thickness of insulation used with that found in practise to be necessary.[26] It will also be observed that the ratio of external diameter to rod diameter as used in practise is considerably less than that used for these preliminary experiments where it was about 8, if we take the inner equalizer as our rod with caps. For example, the ratio between outside diameter to rod diameter varies in practise between 2.3 and 3.7. With these relatively small diameter ratios the gradient without artificial equipotential surfaces would not be very far from uniform to start with (see diagram, Fig. 10). There is, therefore, very little potential equalizing to do. In our tests, however, we have taken a ratio of about 8, which means that we have an exaggerated condition and considerable equalizing to do.

This immediately brings up the question as to whether the small diameters of the condenser bushings is due to the equalizing effect of the tin-foil layers in bringing about uniform potential gradient, or whether the effect is not largely due to the subdivision of the solid material by metallic barriers as well as the laminated structure of the dielectric itself, thereby increasing the apparent strength of the structure,(an effect analogous to the use of solid barriers, pressboard, etc.) in liquid dielectrics. This matter will be briefly discussed before leaving the subject.

It should be observed that there is no definite proper ratio between the diameters of the central rod and the hole when artificial equipotential surfaces are used in this manner. It is, however, undesirable to use too small a central electrode because this means that more redistribution of stress has to be brought about by the artificial equipotential surfaces. This in general necessitates either too long a bushing or an excessive stress at the ends. If we select a ratio of rod to hole diameter which gives more nearly equal gradient at the edge of the hole and rod without artificial equipotential surfaces, we will have less potential equalizing to do which will probably be found advantageous.

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[26] See "Construction and Application of the Condenser Terminal", by J. E. Mateer, *Electrical Journal*, August, 1913.

EXPERIMENTAL BUSHING WITH ARTIFICIAL EQUIPOTENTIAL SURFACES, UNIFORM-FIELD TYPE

A bushing of the uniform-field type was built from a photographic reduction of Fig. 28. It has been pointed out that in taking the diagram the artificial equipotential surfaces were adjusted so as to give an approximately uniform distribution of stress in the solid or supporting dielectric, while at the same time we attempted to produce as good an external field distribution as could be foreseen before the diagram was actually completed.

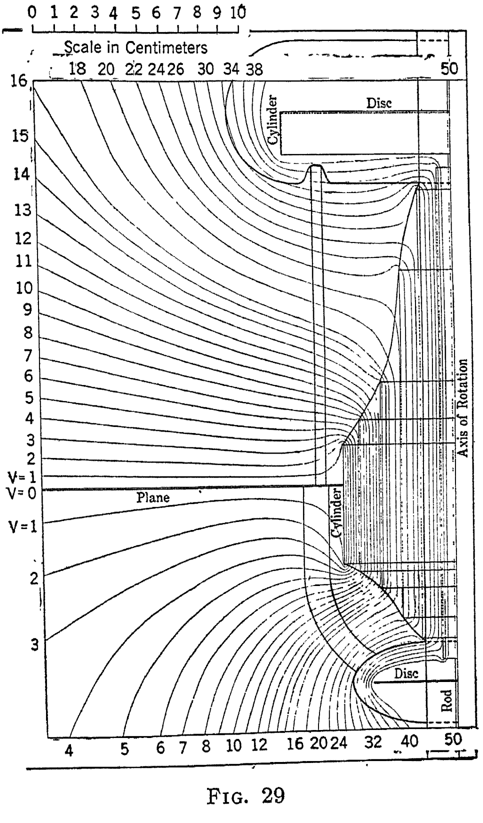

FIG. 29

Fig. 29 shows the outline of the bushing drawn on the diagram of Fig. 28. The electrodes were not made to rigorously conform to any of the equipotential surfaces in the diagram as it did not seem necessary to do so, since the field will probably not be greatly changed by the deviations made. A groove was put in the cap on the air end to electrostatically shield the joint between the supporting dielectric and the air. The condenser core of the bushing was made of wound-up shellac paper with tin-foil layers at intervals of 1/16 in. It was turned down to the shape shown in Fig. 29 and then stepped off as will be seen from the photograph Fig. 30.

A shellac paper cylinder enclosed the air end of the bushing and the shell for the under-oil end was built up of hard-rubber rings. Sarco, an asphaltic compound, was used as filler.

Arc-Over Voltage as Calculated from the Diagram

An inspection of Fig. 29 will show that the gradient on the air end is very nearly parallel to the surface of the containing cylinder and is approximately uniformly distributed being somewhat higher near the top. The maximum gradient on the air end is seen to be on the cap away from the surface. Under these conditions corona should start on the cap at about 90 kv. effective as calculated from the diagram making allowance for the fact that the cap used did not rigorously conform to any single equipotential surface of the diagram. The bushing is 15 cm. high and therefore the average gradient along the surface would be 6.7 kv. eff. per cm.

Inspection of the under-oil end shows that a concentration of stress has resulted on the cap from forcing the uniform internal stress by the artificial equipotential surfaces. The distance between lines No. 22 and 28 (6 lines) where the under-oil shell meets the cap is 0.8 cm. thus we have 7.5 lines per cm. Tests on the under-oil arc-over of rubber cylinders between parallel planes gave a surface arc-over gradient of about 23 kv. effective per cm. Assuming this data to apply in this case, arc-over of the under-oil end of the bushing should take place at

about $\frac{30}{7.5} \times 23 = 92$ kv. effective.

It will be observed from these calculations that the air and under-oil ends are about equally strong. Before the tests were made it was therefore doubtful which end would arc-over first as the calculations are necessarily only approximate, since we have not actually followed the equipotential surfaces of the diagram.

Sixty-Cycle Dry Arc-Over Tests

These tests were made with the plane grounded:

Arc-over voltage.....................86 kv. effective
Average gradient along surface........ 5.7 kv. per cm.

Arc-over took place clear of the surface from the cap to the plane. Upon applying the voltage for the third time the bushing apparently punctured either inside of the shell or under oil at 76 kv. effective. Inspection did not indicate an under-oil arc-over and therefore the bushing had to be taken to pieces in order to determine the cause of failure. This process necessitated destroying the bushing as it is difficult to get the compound out in any other way. The investigation showed that the result of failure was due to an arc-over of the bushing on the air end along the inside surface of the containing shell. A large blister about 1½ in. (4 cm.) long was found at this point which had held the compound away from the surface and thus left an air pocket which weakened the inside surface and resulted in the failure.

I was disappointed not to be able to obtain wet arc-over, impulse arc-over and under-oil arc-over, but did not believe that these results would warrant rebuilding a bushing of this sort for it is quite certain that the results would be low, for the same reasons which were met with in the tests on the high-air-efficiency bushing and from which it was concluded that a more nearly radial type of field is generally desirable in practise.

A reference to Fig. 10, will show that in the radial-field type of bushing the natural equipotential surfaces are approximately cylindrical in form and therefore it will be impossible to produce any considerable change in the potential distribution by means of cylindrical artificial equipotential surfaces, unless they extend to a great distance in the air end, or unless a cap or hat, or the equivalent, is provided on the under-oil end, which changes the natural equipotential surfaces from the cylindrical form, and therefore, allows us to bring about a change in potential distribution by inserting cylindrical artificial equipotential surfaces. That is, if our inserted metal equipotential surfaces do not differ in form from the naturally existing equipotential surfaces no effect in the potential distribution results from their insertion, regardless of number, spacing, etc. It is, therefore, useless to employ them unless they produce an effective increase in the apparent strength of the solid or supporting dielectric by virtue of a subdivision of the dielectric into elements. The following preliminary tests were, therefore, undertaken to check up this point.

EFFECT OF BARRIERS ON STRENGTH OF INSULATION

It is well known that the strength of dielectrics decrease with increase in thickness. I believe that Lord Kelvin was the first to observe this effect for the case of air[27] in 1860.

For example, assume that we have two parallel plane electrodes with such a gradual curvature at the edges that the gradient is always less at the edges than that in the uniform part of the field. If we then take measurements of the voltage required to breakdown various thicknesses of air, we find that the apparent strength of air as determined by the potential gradient necessary to cause breakdown decreases as we increase the spacing of the planes. At exceedingly small spacings the potential gradient required to produce breakdown may be very great, for example, tests recently made by F. W. Peek, Jr.,[28] using 2.54 cm. diam. spheres at 0.0035 cm. spacing gave as the disruptive gradient 150 kv. effective per cm. As we increase the spacing the breakdown gradient for air apparently approaches a constant value of approximately 21 kv. effective per cm. at normal temperature and pressure.

De La Rue and Müller investigated this effect[29] for various spacings of parallel plane electrodes at constant pressure and temperature as well as the variation under constant spacing and varying pressure and concluded that, "The law of the hyperbola holds equally well for a constant pressure and varying distance as it does for a constant distance and varying pressure; the obstacle in the way of a discharge being as the number of molecules intervening between the terminals up to a certain point." Harris[30] had previously found that a change in air density pro-

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[24] Measurement of the electromotive force required to produce a spark in air between parallel metal plates at different distances. Proceedings Royal Society, Feb. 23 and April 12, 1860, or *Philosophical Magazine*, 1860, or Papers on Electrostatics and Magnetism, p. 247–259 Lord Kelvin. An account of these tests will also be found in, Electricity in Gases, J. S. Townsend, p. 346.

[28] Law of Corona and Dielectric Strength of Air, III., F. W. Peek, Jr., A. I. E. E., TRANS., 1913, Vol. XXXII, Part II, p. 1767.

[29] Experimental Researches on the Electric Discharge with the Chloride of Silver Battery. Phil. Trans. Royal Society, Part I, 1880, p. 79–83.

[30] W. S. Harris, Phil. Trans. Royal Society, 1834, p. 230.

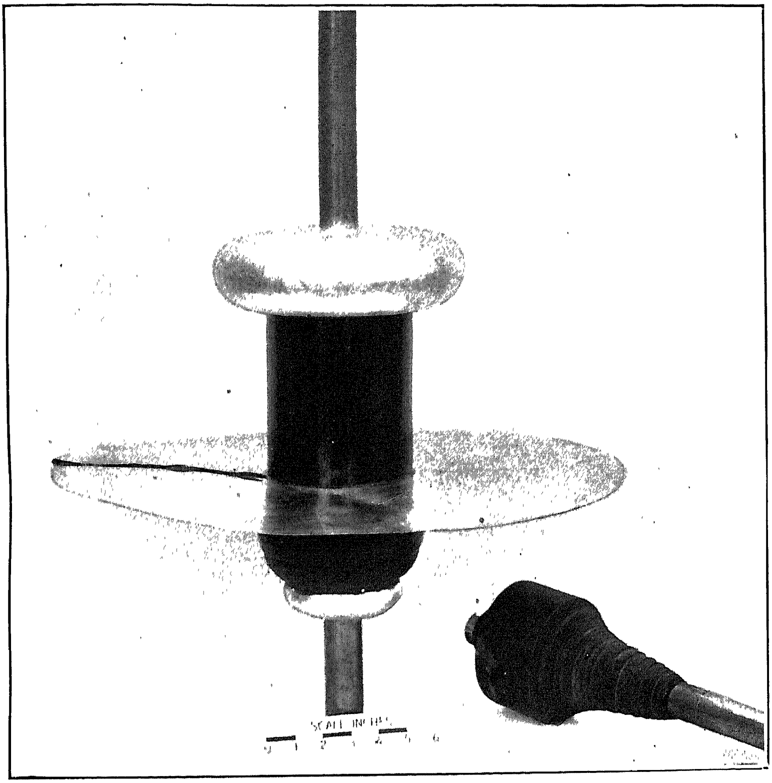

FIG. 30

[RICE]

duced the same effect whether due to temperature or pressure, that is, when air is contained in an air-tight receiver, so that the density remains constant the potential difference between two electrodes required to produce a discharge was unaltered when the temperature varied from 50 deg. fahr. to 300 deg. fahr.

If we immersed our electrodes in oil we would obtain a similarly shaped curve showing the relation between the disruptive gradient and spacing. The constants in the equations for air and oil would probably, however, be different. Again if we imagine our parallel plane electrodes embedded in some solid dielectric, for example glass, we would obtain a similarly shaped curve. For very great and very small spacings it is quite possible that all materials have the same apparent strength.

Now, if we assume that our dielectric is homogeneous and isotropic and is a perfect dielectric in every respect (no conduction, dielectric losses, etc.,) the equipotential surfaces for equal differences of potential would be equally spaced between our parallel planes. If we desired we could put in infinitely thin conducting sheets at any desired intervals without in any way changing the problem from a purely electrostatic point of view. For convenience we could space them at equal intervals so that the dielectric between any two adjacent metal equipotential surfaces between the planes was subjected to the same potential gradient. If we wished we could also metallically connect the inserted metal sheets to equal intervals on a balancing resistance shunted across our parallel plane electrodes, that is, connect the equipotential surfaces to a source of potential having the value naturally existing. Obviously nothing has been changed electrostatically by any of these processes and therefore it is evident that we can confine our attention to the dielectric included between any two equipotential surfaces, and if we now raise the potential between our parallel plane electrodes the potential gradient on the dielectric between the two equipotential surfaces which we have selected to watch will increase until finally breakdown will occur when the gradient exceeds a certain value called the strength of the material. It is observed that nothing has been said about the actual thickness of the section of the dielectric between our two metal equipotential surfaces under observation. In other words, we might in one case insert a great number of equipotential surfaces dividing the dielectric into exceedingly thin layers and in another case use fewer metallic surfaces and have thicker insulation under observation. When considering ideal dielectrics from a purely electrostatic point of view, as here described, I think that it is clear that the actual thickness of the dielectric should have no influence upon the breakdown strength. For it should not make any difference whether we inserted isolated thin metal equipotential surfaces or left them out, or inserted them and connected them to a proper source of potential. Nothing that we could detect from a purely electrostatic view point has been changed. It is also interesting to observe that a uniform leakage current should not affect these conclusions provided the heating effects were taken care of, since the equipotential surfaces for current flow would coincide with the electrostatic equipotential surfaces and no potential distortion would result.

The fact that we observe a difference in the apparent strength of dielectrics with different thicknesses, which is contrary to the above electrostatic reasoning, based on the assumption that the metal equipotential surfaces could be inserted without changing the breakdown strength, makes it interesting to determine what does take place in practise when a dielectric is divided up in this manner. Obviously when we consider the case of the metallic equipotential surfaces connected to the proper sources of potential, the breakdown strength must be the same for a given spacing between the surfaces or thickness of dielectric, no matter how many are connected in series, or what is the same thing, regardless of the total thickness of insulation under test, for each element forms a separate test piece isolated between the two metal surfaces and connected to a definite potential. The interesting thing therefore is a comparison of the dielectric strength of a given thickness of insulation with and without isolated metallic barriers placed at various spacings.

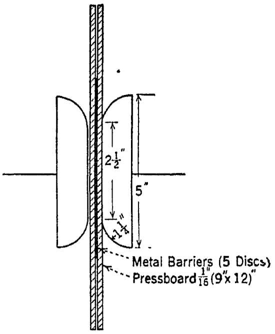

FIG. 31—ARRANGEMENT OF PRESSBOARD AND BARRIERS

I am sorry that the experimental results, which I have to offer, are so crude and incomplete, but if they will serve the purpose of starting some one on a more complete investigation of this subject I will regard them as having served a good purpose. I believe that a complete investigation of this subject would lead to a great deal better understanding of the true mechanism of the breakdown of insulations, or the theory of ionization by collision which of course offers a qualitative explanation of the phenomena.

TESTS TO DETERMINE THE EFFECT OF BARRIERS ON SOLID AND GASEOUS INSULATION UNDER UNIFORM-FIELD CONDITIONS

The first set of tests recorded below in Table I were made on sheets of oiled pressboard 1/16 in. (0.16 cm.) thick. The barriers and pressboard were sealed together under No. 6 transil oil and then placed between the parallel plane electrodes as shown in Fig. 31, the whole then being immersed under No. 6 transil oil at 25 deg. cent. for test.

The object of using solid metal barriers and comparing their effect with wire-gauze barriers was to see whether the mean velocity of the particles or ions in the solid dielectric, which must precede rupture, was sufficiently high to allow an appreciable number, to pass through the openings in the gauze, whereas they should all be stopped by a metal sheet. Electrostatically the gauze should not be essentially different from the solid metal sheet.

TABLE I

Instantaneous 60-cycle puncture tests in No. 6 transil oil at 25 deg. cent. on dried 1/16 in. oiled pressboard. The average of 5 readings are given. A maximum variation of ± 2 kv. existed between individual raedings.

No. of sheets of pressboard each 0.16 cm. thick	60-cycle puncture kv. effective	Average gradient kv. eff. per cm.	Number of isolated barriers.
1	70.5	445.0	no barriers
2	113.0	355.0	" "
3	164.5	345.0	" "
2	117.0	368.0	1 tin barrier
2	124.0	390.0	Fine copper gauze
2	121.0	380.0	Coarse copper gauze

The points of puncture were well distributed over the surface of the insulation and in no cases occurred at the edge of the barriers. Contrary to the expectation the gauze barrier seems to be more effective than the tin barrier but the tests are so fragmentary that conclusions are not safely drawn. If, we attempted to use larger thicknesses arc-over occurred around the edge of the pressboard before puncture. A more satisfactory method of test was, therefore, sought. The method decided upon consisted in setting up a series of parallel plane electrodes as shown in Fig. 32, the sheet insulation being placed between the several electrodes in series. Thus, each pair of electrodes which are connected together may be considered as a barrier. It is also possible, under these conditions, to have the barriers isolated or connected across a proper balancing resistance. An additional barrier can be placed midway between each pair of electrodes without fear of distorting the field.

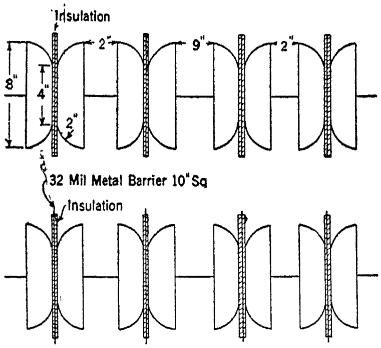

FIG. 32

A great difficulty with this method is the unbalancing of stress which the capacity or electrostatic flux to ground of the various electrodes will introduce. It was hoped that possibly this effect would not be appreciable when using fairly high inductive capacity material for the tests, as the capacity between the electrodes proper would then be large in comparison with that to ground. A few tests made with one end of the series grounded and compared with the case in which the electrodes were isolated, in which case the neutral or ground potential would be in the middle of the series, shows a rather large effect which means that

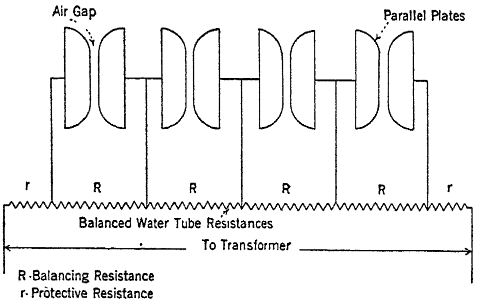

FIG. 33

the isolated tests are subject to voltage distortion from this cause though to a less extent than when one end was grounded.

Fig. 32 shows the arrangement of barriers and insulation for four and eight isolated barriers respectively. When it was desired to metallically connect the barriers to the proper potential, water tube resistances were used of such a value as to allow 7 to 10 times the capacity current to flow (see Fig. 33). The maximum variation of the individual resistance tubes was about 2 per cent. The apparatus was placed at about 42 in. above the floor of the room and the gap line parallel to the ground. The tests were made in No. 6 transil oil (40 kv. flat electrodes 0.2 in. (0.51 cm.) gap, 0.5 in. (1.27 cm.) diameter) at 25 deg. cent.

The insulation used was 12-mil black varnished cambric the individual sheets being sealed to each other under oil. When the balancing or shunted resistance was connected the voltage readings were corrected for the drop in the protecting resistances. The results are tabulated in Table II.

TABLE II

Instantaneous 60-cycle tests on 12-mil black varnished cambric immersed in No. 6 transil oil at 25 deg. cent. Terminals 4 in. plane with 2 in. radius at edge. Individual readings differed by ± 1 kv.

Variation of Insulation Strength with thickness or number of layers.

Average of five readings

Using one pair of planes

No. of sheets	Total thickness insulation cm.	60-cycle kv. effective	Kv. per cm. effective
1	0.031	16.5	540.0
2	0.061	29.7	480.0
4	0.122	51.5	422.0
8	0.244	92.5	378.0
16	0.488	138.3	284.0

Data repeated using four pair of planes connected in parallel to obtain the effective strength when larger areas are used and therefore greater chances for weak spots.

Average of two readings.

1	0.031	14.7	482.0
2	0.061	28.5	451.0
4	0.122	51.3	421.0
8	0.244	91.7	376.0
16	0.488	136.5	280.0

Effect of Isolated Barriers on Puncture Strength of Insulation.

Condition of test	Total thickness insulation under test cm.	Thickness between barriers cm.	Non-Grounded			One End Grounded	
			Puncture kv. eff.	Kv./cm.	Kv./cm. roughly corrected	Puncture kv. eff.	Kv./cm.
2 sheets between each of 4 planes in series	0 244	0.061	106.8	438.0	521.0	86.8	355.0
1 sheet between each of 8 barriers in series	0.244	0.031	105.8	434.0	516.0	....	....
4 sheets between each of 4 barriers in series.	0.488	0.122	168.5	345.0	411.0	....	....

Effect of Barriers when Shunted by High Resistance on the Puncture Strength of Sheet Insulation

2 sheets between each of 4 barriers in series. Each unit shunted by 8 × 10^5 ohms.	0.244	0.061	112.6	462.0	433.0	98.1	403.0
1 sheet between each of 8 barriers in series. Each unit shunted 4 × 10^5 ohms	0.244	0.031	110.0	452.0		....	....
4 sheets between each of 4 barriers in series. Each unit shunted 8 × 10^5 ohms	0.488	0.122	195.8	402 0		....	....

In order to obtain an estimate of the unbalancing due to the capacity of the parallel planes to ground in this method of determining the effect of barriers on the strength of insulation the following tests were carried out using air as the dielectric. Two sets of tests were tried, the first using ½-in. (1.27 cm.) spacing between the individual planes and for the second a ¼-in. (0.635 cm.) gap was used. The gaps between the individual pairs of planes were set as accurately as feasible and individual spark-over readings taken non-grounded and with one end grounded. The gaps were then all connected in multiple to check the strength of this combination as compared with the weakest gap. After doing this they were connected in series and arc-over taken both grounded and isolated with and without shunted balancing resistance. A summary of the data is given in Table III.

From other sources we know that the strength of air for large spacings (*i. e.*, 2 in. or 5.1 cm.) between parallel planes is about 21 kv. eff. per cm. Whereas these data show an average of 18.1 kv. eff. per cm. when the tests were made non-grounded and 14.2 kv. eff. per cm. for the grounded case. That is, when using air as our dielectric with this arrangement we have an increase in stress of $1 - \frac{18.1}{21.} = 14$ per cent brought about by unbalance of voltage due to capacity to ground, etc. When operating with one side grounded we have $1 - \frac{14.2}{21} = 32$ per cent or approximately twice the ill effect.

This immediately gives us a rough method of correcting the previous tests for this unbalancing effect where no balancing resistance was used. We may write the following from the data given in the table for isolated barriers.

$1 - \frac{438}{x}$ = per cent apparent decrease in strength of insulation assumed due to capacity to ground for the non-grounded tests.

$1 - \frac{355}{x}$ = per cent apparent decrease in strength of insulation for grounded tests.

Where

x = assumed true strength of the material.

If we further assume that the apparent decrease for the grounded tests is twice the decrease for the non-grounded tests as shown to be approximately the case in air, we can write

$$2\left(1 - \frac{438}{x}\right) = \left(1 - \frac{355}{x}\right)$$

$x = 521$ assumed true strength of the material.

TABLE III

½-INCH GAP

Relative Humidity 40 per cent. Bar. 76 cm. Temp. 25 deg. cent.

Condition of test	Non-Grounded		Grounded	
	60-cycle kv. eff.	Kv./cm. eff.	60-cycle kv. eff.	Kv./cm. eff.
Gap 1 ½ in.	26.3		26.3	
2	26.3		26.3	
3	26.5		26.4	
4	26 4		26.4	
Ave.	26.4	20.8	26.4	20.8
Four gaps in multiple..	26.3	20.7	26.3	20.7
Four gaps in series.	87.5	17.2	69.5	13.5
Four gaps in series each shunted by 7 × 10^5 ohms......	106.0	20.8	106.0	20.8

¼-INCH GAP

Temp. 20 deg. cent. Bar. 76 cm. = 1.00

Condition of test	Non-Grounded		Grounded	
	60-cycle kv. eff.	Kv./cm. eff.	60-cycle kv. eff.	Kv./cm. eff.
Gap 1 ¼ in.	13.9	22.0		
2	14.1	22.2		
3	13.8	21.8		
4	13.9	22.0	14.0	22.1
Average	13.9	22.0		
Four gaps in multiple (arc-over No. 3)...	13.8	21.8		
Four gaps in series.	48.5	19.0	38.0	15.0
Four gaps in series each shunted by 8.5 × 10^5 ohms with 1.4 × 10^5 protective resistance.....	52.0	20.5	51.0	20.1

Averaging the data for ½-in. and ¼-in. spacings.

Four gaps in multiple		21.2		
Four gaps in series.		18.1		14.2
Four gaps in series each shunted by resistance.........		20.7		20.4

The strength as indicated from the tests was 438 kv. effective per cm. or only 84 per cent of the value obtained after applying this rough method of correction. I have also applied the same correction factor to the rest of the values in this part of the table. We might consider the corrected column as an upper limit and the data as observed from test as a lower limit to the true value. In that case the average of these two columns would give another estimate of the strength as affected by barriers.

When considering the tests using air dielectric in which balancing resistances were used, we see that the stress due to capacity to ground is practically eliminated. It should also be noted from these tests that there is no appreciable increase in stress analogous to that which has been discussed for the case of two spheres in space where it was seen that even though there is no potential unbalance between the spheres, there is nevertheless considerable increase in stress when one sphere is at zero potential and the other at a given high potential, over the case in which they are both isolated and have the same difference of potential between them.

Therefore, it would seem that the best method of estimating the true strength of the insulation when balancing resistances are used is to average directly the non-grounded tests with the grounded tests. The data obtained are not sufficiently extensive to enable us to see whether there is a real difference between these two cases or not.

TABLE IV

AVERAGE OF DATA

Thickness of insulation No barriers. cm.	Thickness of insulation between barriers. cm.	Average strength of material No barriers. Kv. eff./cm.	Strength of material with barriers average isolated and shunted kv. eff./cm.
0.031	0.031	511	467
0.061	0.061	466	464
0.122	0.122	422	386

In conclusion it appears probable from a theoretical point of view that the strength of insulation under uniform-field conditions is the same whether the barriers are isolated or connected metallically to the proper sources of potential and also that the strength merely depends upon the thickness of the material between the barriers. The averages, as given in the above table, do not warrant the above conclusions, but I am inclined to believe that the discrepancy is due to insufficient data and errors in the method. I hope, therefore, that someone will take up the subject and make a careful study of this important theoretical and also practical problem.

Another interesting question would be to determine the limiting value of the electrostatic flux density which would result in excessive heating, and therefore, breakdown irrespective of the so-called "instantaneous strength" of the structure. The effect of metal barriers in distributing the heating, eddy-current losses in the metal barriers due to electromagnetic flux accompanying the charging current, etc., would also be of interest.

GENERAL DISCUSSION OF THE PROBLEM OF HIGH-VOLTAGE-BUSHING DESIGN

So far we have considered the problem of high-voltage-bushing design mainly from an ideal and purely electrostatic point of view, that is, we have greatly simplified the problem by assuming ideal or perfect dielectrics and a constant or static condition of the equipotential surfaces. These conditions do not however, actually exist in practise, and therefore, the effects brought about by rain, dirt, snow, steep wave front impulses must be considered in a complete theory of the correct and most effcient design.

For example, under rain conditions the equipotential surfaces are in a general constantly changing and will be different from the dry condition. There is probably also a certain space or volume charge effect due to the rain drops becoming charged from contact with the high potential electrodes and then falling in the vicinity of the bushing to the grounded electrode. Furthermore, a considerable change in the electrostatic field may be brought about by an effective increase in the permittivity of the air part of the dielectric due to the presence of small rain drops. The problem is further complicated by the presence of conduction over wet or dirty surfaces with the consequent change in the potential distribution as well as the possibility of an arc-over resulting from what might be called fuse action, that is, the conducting material may become overheated by conduction and blow like a fuse and thus precipitate an arc-over along the path of the hot gases.

There are a great many other disturbing factors and possibilities which have not been mentioned, but I believe that the above are sufficient to show the multitude of factors over which the designer can have no control and which, therefore, must be included in the theoretically correct and most efficient bushing design.

We have already observed the large surface effect which exists even with relatively clean dry surfaces, also the change in the dielectric strength of materials when subdivided into thin layers by metal barriers and probably, to some extent, merely by laminating the material.

It might appear from what has been said about the complexity of the problem that it is beyond the power of analysis to try to obtain a complete theory of bushing design. Strictly speaking, I believe that this is the case, nevertheless, I believe that the incomplete theory which we have is of great value in determining the proper lines along which to experiment.

We will now briefly describe the construction and tests on a series of small experimental bushings which were built for the purpose of determing the rain and dry characteristics of various types.

The object was to determine the general type of bushing which would best meet the following conditions.

I. Rain and dry arc-over should have as nearly as possible the same value.

II. The arc-over of the under-oil end should be considerably above either the rain or dry-air end arc-over.

III. Puncture should occur considerably above the under-oil arc-over.

IV. The impulse safety factor or ratio should be as high as possible, both under wet and dry conditions and application of a large number of impulses should not result in puncture.

The materials used in the construction of these bushings were selected from the point of view of ease of construction and alterations. The caps used on the air and oil ends were turned up of wood and metal covered by the Schoop metal spraying process. After completing the tests with a certain shape of cap it could be put in a lathe and altered in the desired manner and then re-metallized. Hard rubber was selected as the material for the core since it can be readily machined to any desired form; for example, the necessary protection of the edges of the ground shield was easily obtained (see Fig. 34). This material also has other desirable qualities such as homogeneity, high dielectric strength, small surface effect.

A single glass tube was used for the containing shell in order to eliminate the difficulty of making oil tight joints. The only joints being that at the lower cap which was satisfactorily sealed with ceresine. A glass seal was provided by which connection was easily made between the ground shield and the tin disk which represents the transformer tank cover. For convenience, a transparent Russian mineral oil was used as filler. During tests the under-oil end was placed in a 5-gallon tin can filled with oil, having dimensions 9 in. by 9 in. by 14 in. high. All tests were made at 60 cycles and with the tin can and tin disk connected together and grounded.

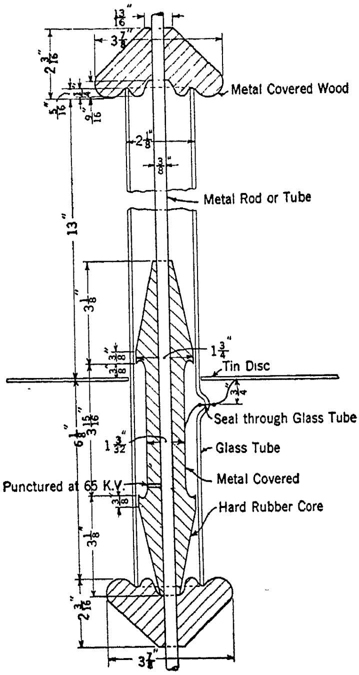

FIG. 34

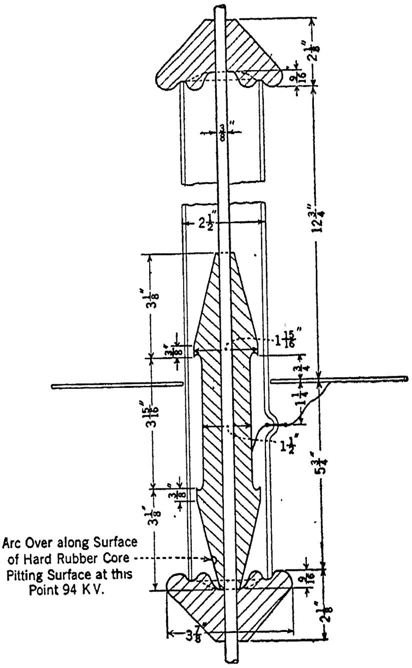
Arc Over along Surface of Hard Rubber Core
Pitting Surface at this Point 94 K.V.

FIG. 35

Determinatoin of Puncture and Under-Oil Arc-Over

This series of tests were made to determine the under-oil and puncture characteristics of the bushings.

Bushing No. 1. A bushing was built up as shown to scale in Fig. 34. The joints between the glass containing cylinder and the metal caps were shielded by the grooves as shown. The joint between the hard-rubber core and cap on the under-oil end was likewise protected. The electrostatic field of the under-oil end is practically of the uniform-field type (Fig. 14), the desirability of which has been pointed out. The air end is essentially the radial-field type.

In test this bushing punctured at 65 kv. effective at the point shown in Fig. 34. After sawing the core open longitudinally, inspection showed that the hole had not been smoothly drilled which may have accounted for the puncture, as the space thus left between the rod and core may not have been properly filled with oil.

Bushing No. 2. (See Fig. 35). The core was increased to 1½ in. (3.8 cm.) external diameter so as to insure a safe margin against puncture. As an additional precaution a brass tube closed at the bottom end and having small holes drilled radially at intervals inside the core was used in place of the brass rod used in Bushing No. 1. The object being to enable oil poured in at the top of the tube to fill up any irregularities resulting from imperfect fit between the core and central rod.

In test this bushing arced over along the surface of the hard rubber and jumped to the inner guard ring on the cap at 94 kv. effective. Small chips were taken out of the rubber at the point shown in Fig. No. 35, apparently where the surface arc turned to jump to the guard ring.

Bushing No. 2-*A*. Bushing No. 2 was disassembled and the inner guard ring cut down to the shape shown by the dotted lines in Fig. 35. In assembly the core was inverted putting the damaged part toward the air end of the bushing, bringing the new end under test.

Arc-over occurred over the surface of the hard rubber core on the under-oil end without pitting the surface at 93.5 kv. effective. The voltage was then brought up again and arc-over occurred over the surface of the hard-rubber core on the air end to the previously damaged spots and there punctured to the rod at 82.5 kv.

Bushing No. 3. (See Fig. 36). The hard-rubber core, which had been damaged by the previous tests, was turned down to the form shown in Fig. 36, thereby removing the damaged material. The under-oil end cap was also altered to a form approximately as shown. Practically the only electrostatic change made by this alteration was to somewhat reduce the shielding effect of the cap on the hard-rubber core. The air end was also materially shortened.

This bushing arced-over the air end at 81 kv. Upon bringing up the voltage a second time arc-over occurred over the surface of the hard-rubber core on the under-oil end without damaging the core at 86.5 kv. effective.

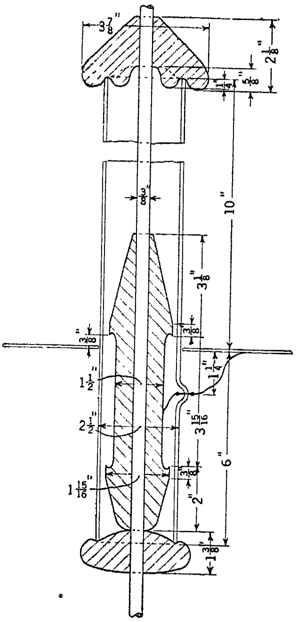

FIG. 36

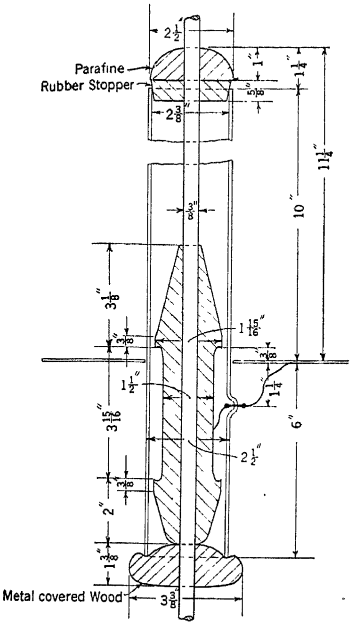

FIG. 37

Determination of the Air-End Characteristics

Having secured a fairly efficient and satisfactory under-oil end the work was then directed towards a study of the rain and dry characteristics of various air-end combinations.

Further Tests on Bushing No. 3. (See Fig. 36.) Temp. 24 deg. cent. Barometer 76 cm. Humidity 57 per cent.

Bushing Dry

Corona visible air end near ground shield..........	41 kv.
Corona visible on cap of air end..................	59 kv.
Bushing outlined air end.........................	87 kv.

Arc-over was not taken.

Rain about 0.2 in. per minute at 45 deg.

Corona starts top and bottom	10 kv.
First streamers	16 kv.
Whole tube outlined	20 kv.
Arc-over	52 kv.

This bushing is obviously unsatisfactory because of the low corona starting points both wet and dry and the very low rain arc-over.

Bushing No. 4. (See Fig. 37). This bushing was similar to No. 3 except that the metal cap at the top was replaced by a rubber stopper covered with paraffine. The object being to make the field on the air end as nearly radial as possible.

Summary of Tests on Bushing No. 4

Temp. 23.5 deg. cent. Barometer 76 cm Hum. 50 per cent.

Bushing Dry

Corona starts near ground shield	35 kv.
Corona at top and bottom	60 kv.
Voltage raised without arc-over to	85 kv.

Rain Test about 0.2 in. per minute 45 deg.

Corona starts top and bottom	15 kv.
Tube outlined	17 kv.
Vicious streamers over surface	34 kv.
Arc-over	57 kv.
Arc-over (paraffine removed)	55 kv.

This bushing is unsatisfactory for the same reasons as stated for No. 3.

Bushing No. 5. (See Fig. 38). This bushing was exactly similar to No. 4 except that the internal brass shield was added as shown in Fig. 38. The object of adding this element was to somewhat relieve the stress on the rubber stopper and paraffine and to throw it away from the rod at the top of the bushing.

Summary of Tests on Bushing No. 5

Temp. 23 5 deg. cent. Barometer 76 cm. Hum. 50 per cent

Bushing Dry

Corona starts at ground shield	32 kv.
Corona top and bottom	68 kv.
Voltage raised without arc-over to	80 kv.

Rain Test about 0.2 in per minute 45 deg.

Corona starts top and bottom	18 kv.
Arc-over	42 kv.

This bushing is obviously unsatisfactory.

Bushing No. 6. (See Fig. 39). This bushing was constructed as shown in Fig. 39. The air end was shortened and the glass tube drawn down so as to get rid of the rubber stopper and paraffine, which gave trouble in the previous two bushings by carbonization and burning. The internal shield shown was used for the same reasons as in Bushing No. 5.

Summary of Tests on Bushing No. 6.

Bushing Dry

Corona starts at ground shield and extends up the surface of glass tube	50 kv.
Corona appears opposite guard ring on the air end	77 kv.
Arc-over	79 kv.

Rain Test about 0.2 in. per minute 45 deg.

Corona starts at top near tin foil	5 kv.
Corona streamers run out from tin foil over the glass surface	10 kv.
Bushing outlined	12 kv.
Arc-over	36 kv.

Bushing No. 6-A. A metal cone similar to that shown in Fig. 41 was slipped over the top of bushing No. 6 and the following results obtained.

Bushing Dry

Corona on surface of glass opposite guard ring	30 kv.
Bushing outlined	60 kv.
Arc-over	78 kv.

Rain Tests approximately 0.2 in. per minute 45 deg.

Corona starts at edge of metal cone	14 kv.
Bushing outlined	20 kv.
Arc-over	29 kv.

These bushings No. 6 and No. 6-A, are seen to be unsatisfactory from the point of view of corona starting point and wet arc-over.

Bushing No. 7. (See Fig. 40). The previous tests showed the necessity of breaking up the flow of water which adheres to the smooth glass surface and thereby produces great field distortion, or a sort of short circuit, or conducting sheet over the bushing. In order to accomplish this result the usual method of adding petticoats was resorted to. These were made by belling out glass tubing as will be seen from the figures. The petticoats were then slipped over the glass shell and the intervening space filled in with ceresine. In other respects, the bushing was exactly similar to No. 6.

From an electrostatic point of view, it will be observed that in the radial-field type of bushing the corrugations approximately coincide with the direction of the flux lines. Therefore, the potential gradient is along the surface of the corrugations and will tend to break down any surface layer of dust or dirt. For this reason it would appear that their effectiveness is somewhat limited. We may further observe from Fig. 9 that when dry the presence of the corrugations will not appreciably affect the field distribution in this type of bushing, except those which are close to the tank. In this latter case they are seen to introduce an appreciable percentage of high inductive capacity material in series with the air. Therefore, if petticoats of this kind are used it may be best not to extend them too near the tank.

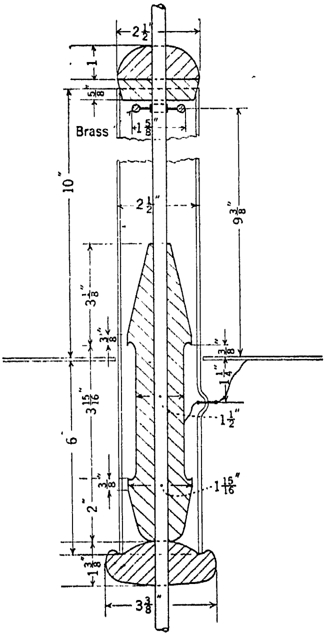

Fig. 38

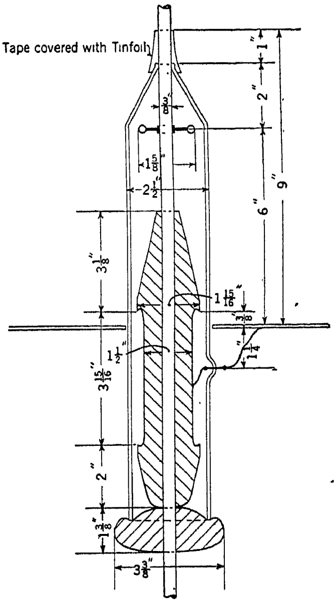

Fig. 39

In the uniform-field type of bushing it will be observed that the petticoats introduce high inductive capacity material in series with the air (except in case of a vacuum bushing). This means that when dry the stress on the air will be increased by their addition. The surfaces of the corrugations will lie practically along the equipotential surfaces; hence, an accumulation of dust, water, etc., would not result in so great a field distortion.

Summary of Tests on Bushing No. 7.

Humidity 60 per cent. Barometer 76 cm. Temp. 24 deg. cent.

Bushing Dry

Corona on tin-foil at top of bushing	24 kv.
Corona near ground shield	38 kv.
Bushing outlined	60 kv.
Streamers over the entire surface of glass	70 kv.

Rain Tests approximately 0.2 in. per minute 45 deg.

Corona on tin-foil at top	26 kv.
Corona on edge of upper petticoat	34 kv.

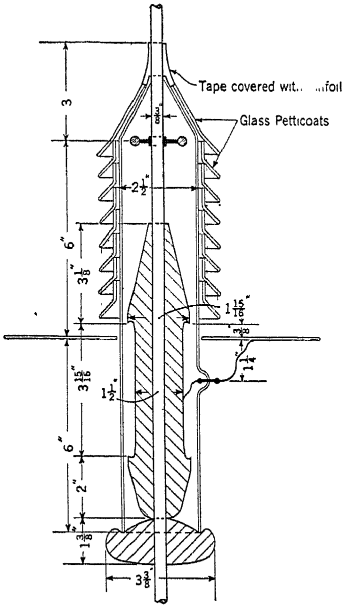

FIG. 40

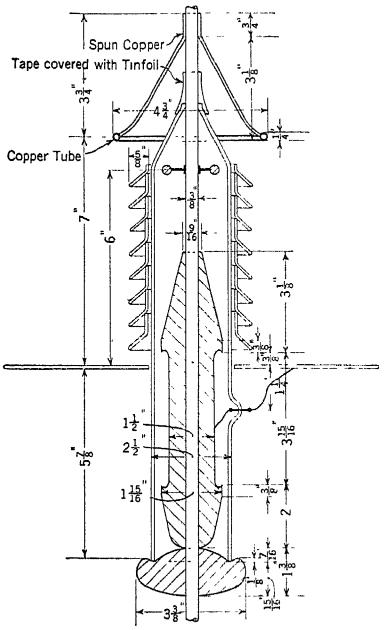

FIG. 41

Corona near ground shield	50 kv.
Corona streamers over surface	70 kv.
Arc-over	75 kv.

It will be observed that the addition of the petticoats has greatly improved the wet arc-over of the bushing.

Bushing No. 8. (See Fig. 41 and Fig. 42). This bushing is identical to No. 7 except that the spun copper cone was added in order to act as rain shed and thereby eliminate the formation of streamers which went out over the top surface of the glass in the previous bushing No. 7.

Summary of Tests on Bushing No. 8.

Humidity 60 per cent. Barometer 76 cm. Temp. 24 deg. cent.

Dry Tests

Corona near ground shield	46 kv.
Arc-over	69 kv.

Rain Test approximately 0.2 in. per minute at 45 deg.

Corona on edge of cone and near ground shield	36 kv.
Bushing outlined	43 kv.
Arc-over	69 kv.

This bushing is seen to be quite satisfactory from the point of view of wet and dry arc.

Bushing No. 9. (See Figs. 43 and 44). For this bushing a new core and caps were turned up as shown in Fig. 43; also an idea of the appearance can be obtained from Fig. 44 which is of a similar bushing. The air and oil ends were made identical except for the addition of petticoats to the air end. The method of assembly of the petticoats previously used namely to fill the space between the petticoats and glass shell with ceresine was given up. Instead the individual petticoats were cemented together with a sealing wax composition. The resulting petticoat shell was slipped over the inner containing glass cylinder and the space filled with a viscous compound. This made a more satisfactory scheme for assembly and disassembly.

Summary of Tests on Bushing No. 9

Temp. 24 deg. cent. Barometer 76 cm. Humidity 60 per cent.

60-cycle Tests

Bushing Dry

Corona on edge of upper cap	51 ± 2 kv.
Arc-over	59 ± 2 kv.

Rain Test approximately 0.2 in. per minute 45 deg.

Corona on edge of upper cap	24 ± 2 kv.
Bushing outlined	42 ± 4 kv.
Arc-over (clear of surface)	48± 5 kv.

$$\frac{\text{wet arc-over}}{\text{dry arc-over}} = 0.813$$

Impulse Tests

200-kilocycle "B" wave

Bushing Dry

Arc-over (1 out of 10 impulses)	99 kv. eff.
Arc-over (5 " " ")	103 kv. eff.

Rain Test 0.2 in. per minute 45 deg.

Arc-over (1 out of 10 impulses).............. 93 kv. eff.
Arc-over (5 " " " ")..............95.3 kv. eff.

Impulse ratio or safety factor

$$\text{Dry} = \frac{99}{59} = 1.68$$

$$\text{Rain} = \frac{93}{48} = 1.93$$

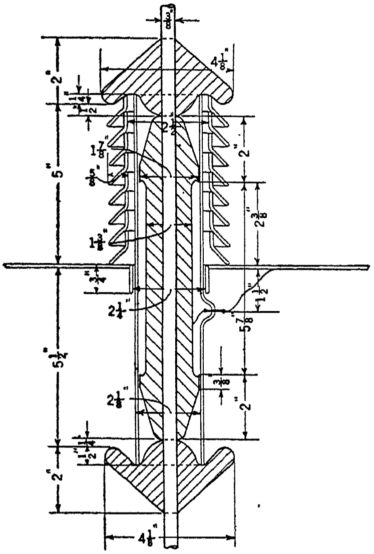

FIG. 43

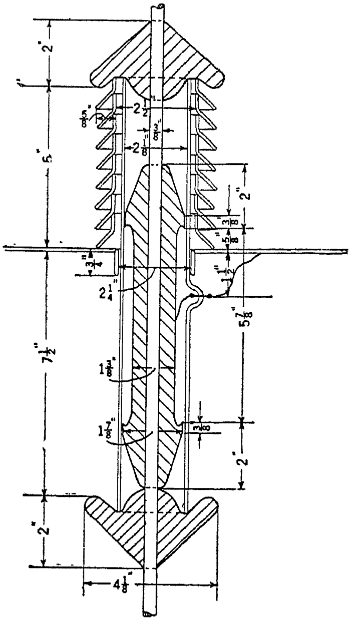

FIG. 45

Bushing No. 10. (See Figs. 45 and 46). This bushing was identical to No. 9 except that the hard-rubber core was moved down to the position shown in Fig. 45, thereby changing the type of field used on the upper part of the air end from the parallel field type to a more radial type. This bushing is shown in the photograph Fig. 46.

Summary of Tests on Bushing No. 10
60-cycle Tests

Bushing Dry

Corona on edge of upper cap................. 54 ± 4 kv.
Arc-over (clear of surface)................... 64 ± 1 kv.

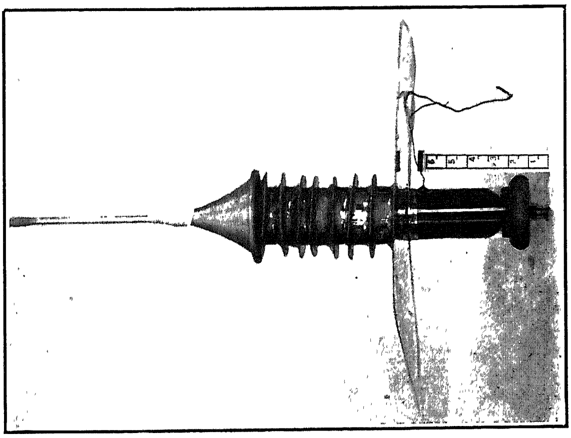

FIG. 42

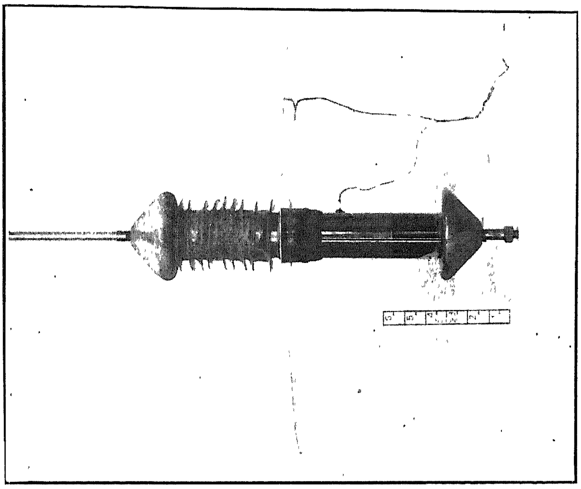

FIG. 44

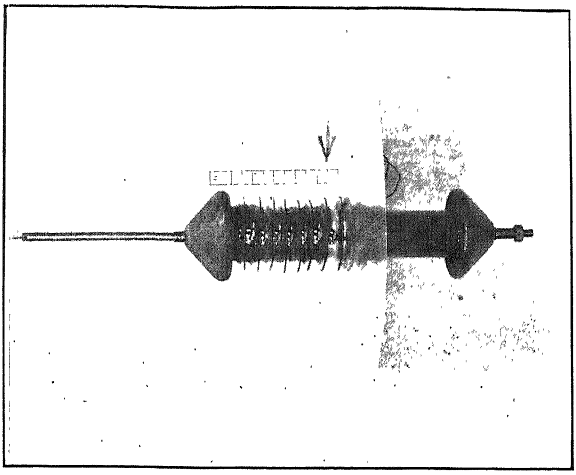

FIG. 16

[RICE]

Rain Test approximately 0.2 in. per minute 45 deg.
  Bushing outlined.......................... 40 ± 3 kv.
  Arc-over (clear of surface).................. 46 ± 2 kv.

$$\frac{\text{wet arc-over}}{\text{dry arc-over}} = 0.72$$

Impulse Tests
200-kilocycle "B" Wave

Bushing Dry
  Arc-over (1 out of 10 impulses)............ 111 kv. eff.
  Arc-over (5 " " " ")............ 114 kv. eff.
Bushing Rain approximately 0.2 in. per minute at 45 deg.
  Arc-over (1 out of 10 impulses)............ 104 kv. eff.
  Arc-over (5 " " " ")............. 107.5 kv. eff.

Impulse ratio or safety factor

$$\text{Dry} = \frac{111}{64} = 1.73$$

$$\text{Rain} = \frac{104}{46} = 2.26$$

Of this series of bushings it appears that the general form shown in Figs. 43 to 46 best fulfills the various requirements which an all around bushing has to meet.

Acknowledgments

In conclusion I wish to express my appreciation of the interest which was shown by Dr. C. P. Steinmetz, Mr. G. Faccioli, and Dr. E. J. Berg in connection with the experimental part of the work and to Professor W. E. Byerly for the great assistance which he so kindly gave me in the course of the mathematical work contained in the appendix. I wish, furthermore, to acknowledge the help which Mr. B. L. Stemmons has given in making the various tests.

APPENDIX

MATHEMATICAL SOLUTION OF TWO ELECTROSTATIC PROBLEMS

It is not in general possible, by known mathematical methods, to obtain the solution of Laplace's equation so as to fulfil arbitrarily given boundary conditions.[31] If, however, we take the simplest conceivable electrode arrangement which would form the skeleton of a bushing, we can obtain a mathematical solution for this case. The most simple electrode arrangement will be seen to be a fine wire passing perpendicularly through a hole in an infinite plane. The mathematical formulation of these skeleton electrodes can be obtained. Thus, for instance, we can formulate the equation of an infinite plane with a hole in its centre as the focal or limiting hyperboloid of revolution of one sheet. The edge of the hole will be at the focus of the hyperboloid. We can now formulate the equation of a fine wire passing perpendicularly through the centre of the hole in our plane. It will be represented by the other limiting confocal hyperboloid, that is, the one which degenerates into the axis of revolution of the confocal family. You will readily see that we have thus reduced the problem to finding the distribution of the electrostatic field between two confocal surfaces of the same family maintained at given potentials. In this case the equipotential surfaces will be hyperboloids of revolution.

We have thus reduced our problem to a form which can be treated quite simply mathematically and for which the form of the solution is known.[32] Here Maxwell states that the equipotential surfaces will be confocal hyperboloids of one sheet and the surfaces of flow will be the confocal oblate spheroids.

The simplicity of the mathematical solution rests upon a wise selection of the coordinates as it does in many other problems of this type. For example, in dealing with cylindrical distributions we would employ cylindrical co-ordinates; in spherical distributions we would use spherical co-ordinates.

In this case where we have to deal with surfaces formed by the revolution of hyperbolas about their conjugate axis thus forming hyperboloids of one sheet; a system of curvilinear co-ordinates of a proper form are made use of. Since most engineers are not very familiar with their meaning and use I have started from the beginning and gone through the development of the system of curvilinear co-ordinates which are used in solving our problem. This makes the work appear rather long and clumsy, but will, I believe, make the whole subject quite simple and clear, and is therefore justified. Assuming a familiarity with the system of co-ordinates, the solution is exceedingly simple, in fact it is practically as simple as the familiar solution for parallel wires at large spacing where the diameter of the wires is neglected.

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[31] An extremely interesting discussion of this subject will be found in Maxwell's Electricity and Magnetism, Vol. I Chap. VII, page 175.

[32] See—Maxwell's Electricity and Magnetism, Vol. I, page 235.

CURVILINEAR CO-ORDINATES; CONICOIDS OR QUADRIC SURFACES

A surface whose equation is of the second degree in x, y and z, is called a quadric surface or conicoid. The sphere is a special case of such a surface.

It is possible, by a suitable transformation of co-ordinates, to reduce the general equation of the second degree in x, y, and z, namely;

$$\begin{aligned} &Ax^2 + By^2 + Cz^2 + 2\,Dyz + 2\,Ezx + 2\,Fxy. \\ &\quad + 2\,Gx + 2\,Hy + 2\,Iz + K = 0 \end{aligned} \tag{1}$$

to the following form in which the terms yz, zx and xy are all absent. That is, the equation can be put into the following simpler form;

$$A'x^2 + B'y^2 + C'z^2 + 2\,G'x + 2\,H'y + 2\,I'z + K' = 0 \tag{2}$$

If in this equation the constants A', B', C' are all finite, we can further simplify it by making a change in the origin of co-ordinates and obtain an equation which when referred to its new axis is of the form

$$A'x^2 + B'y^2 + C'z^2 = D' \tag{3}$$

The locus of this equation is evidently symmetrical with respect to each of the co-ordinate planes, and hence with respect to the origin. Such surfaces are therefore called central *quadric surfaces.*

We may now divide equation (**3**) through by D' and obtain

$$\frac{x^2}{\frac{D'}{A'}} + \frac{y^2}{\frac{D'}{B'}} + \frac{z^2}{\frac{D'}{C'}} = 1 \tag{4}$$

If in (**4**) we substitute

$$a_1 = \frac{D'}{A'},\ a_2 = \frac{D'}{B'},\ a_3 = \frac{D'}{C'}$$

We have the familiar equation of a central quadric surface referred to, its principal axes, namely;[33]

$$\frac{x^2}{a_1} + \frac{y^2}{a_2} + \frac{z^2}{a_3} = 1 \qquad (\mathbf{5})$$

where a_1, a_2 and a_3 may be positive or negative. If they are all negative, the surface is imaginary. We will now consider the nature of the surfaces under the various other conditions.

First. Suppose one of the constants a_1, a_2 or a_3 is negative, while the remaining two are positive.

$$\begin{aligned} \text{Thus, let } a_3 &= -c^2 \\ \text{while } a_1 &= a^2 \\ a_2 &= b^2 \end{aligned}$$

Also assume that, a, is numerically greater than b, and, b, greater than c,

or $$a > b > c$$

Substituting these values in (**5**) we obtain

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1 \qquad (\mathbf{6})$$

This particular central quadric surface will be recognized to be that of a hyperboloid of one sheet with conjugate axis along the z axis.

This can be seen by considering the shape of the plane curves which results as the intersection of this solid figure with the reference planes, or what are generally called the principal sections of the surface. First take the X-Y plane, that is, the plane whose equation is $z = 0$. Substituting this value in (**6**) we obtain.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

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[33] Proofs of the statements made above may be found in any good book on Solid Analytic Geometry, for example; Solid Geometry by Charles Smith,—Macmillan and Company.

That is, the surface is cut by the X-Y plane in the ellipse whose semi-axes are a and b. The foci are at a distance from the origin.

$$f = \sqrt{a^2 - b^2} = \sqrt{a_1 - a_2}$$

which is seen by inspection from Fig. 1.

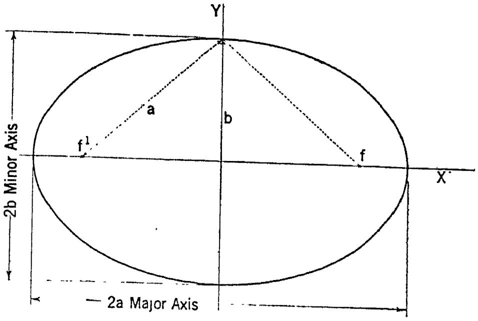

Fig. 1—Ellipse

The section cut out by the Z-X plane ($y = 0$) is seen to be the hyperbola

$$\frac{x^2}{a^2} - \frac{z^2}{c^2} = 1$$

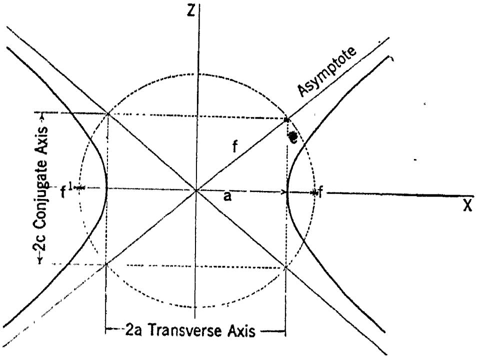

Fig. 2—Hyperbola

with semi-axes a and c. The foci are at the distance

$$f = \sqrt{a^2 + c^2} = \sqrt{a_1 - a_3}$$

on the X axis which can be seen by inspection from the values of the various quantities as given in Fig. 2.

The section by the Y-Z plane ($x = 0$) is the hyperbola

$$\frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

with semi-axes b and c and foci at the distance

$$f = \sqrt{b^2 + c^2} = \sqrt{a_2 - a_3}$$

on the Y axis.

Thus the surface defined by equation (**6**) is seen to be a hyperboloid of one sheet with the conjugate axis on the Z axis.

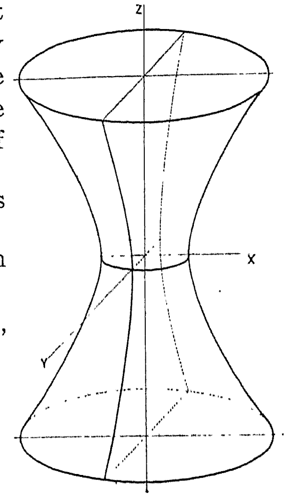

FIG. 3—HYPERBOLOID OF ONE SHEET

If in the above equation (**6**) we let $a = b$ the section of the surface by any plane parallel to the X-Y plane (plane $z = K$) is a circle. Hence, the surface would be formed by the revolution of the hyperbola $\frac{x^2}{a^2} - \frac{z^2}{c^2} = 1$ about its conjugate axis. Fig. 3 shows the form of the surface.

Second. Let two of the constants a_1, a_2, a_3 in equation (**5**) be negative.

For example, let

$$\begin{aligned} a_2 &= -b^2 \\ a_3 &= -c^2 \\ a_1 &= a^2 \end{aligned}$$

Also assume as before that a is numerically greater than b and b greater than c,

$$\text{or} \quad a > b > c$$

Substituting these values in (**5**) we obtain,

$$\frac{x^2}{a^2} - \frac{y_2}{b^2} - \frac{z^2}{c^2} = 1 \qquad (\mathbf{7})$$

and the surface is called a hyperboloid of two sheets.

The sections by the co-ordinate planes and their focal distances are found as before.

The section by the X-Y plane ($z = 0$) is the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

with semi-axes a and b and focal distance

$$\sqrt{a^2 + b^2} = \sqrt{a_1 - a_2}$$

The section by the ZX plane ($y = 0$) is the hyperbola

$$\frac{x^2}{a^2} - \frac{z^2}{c^2} = 1$$

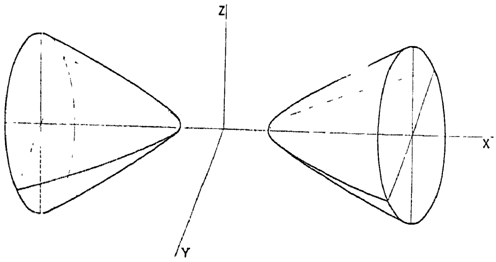

FIG. 4—HYPERBOLOID OF REVOLUTION OF TWO SHEETS

with semi-axes a and c and focal distance

$$\sqrt{a^2 + c^2} = \sqrt{a_1 - a_3}$$

The section by the YZ plane ($x = 0$) is the imaginary ellipses $\frac{y^2}{b^2} + \frac{z^2}{c^2} = -1$. With semi-axes b and c and focal distance

$$\sqrt{-(b^2 - c^2)} = \sqrt{a_2 - a_3}$$

If in equation (**7**) we let $b = c$ the section by any plane parallel to the plane $x = 0$ is a circle. Hence the surface is in this case formed by the revolution of the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

about its tranverse axis. The form of the surface is shown in Fig. 4.

Third. Let all of the constants a_1, a_2 and a_3 be positive.

Thus, let

$$a_1 = a^2$$
$$a_2 = b^2$$
$$a_3 = c^2$$

Also let us assume that the relative magnitude of the constants are as indicated by the equation

$$a > b > c$$

Upon substituting these values in equation (**5**) we obtain

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \tag{8}$$

which is seen to be the equation of an ellipsoid.

The sections by the co-ordinate planes and their focal distances are given below.

The section by the X-Y plane ($z = 0$) is the ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with semi-axes a and b and focal distance

$$f = \sqrt{a^2 - b^2} = \sqrt{a_1 - a_2}$$

The section by the Z-X plane ($y = 0$) is the ellipse

$\frac{x^2}{a^2} + \frac{z^2}{c^2} = 1$ with semi-axes a and c and focal distance

$$f = \sqrt{a^2 - c^2} = \sqrt{a_1 - a_3}$$

The section by the Y-X plane ($x = 0$) is the ellipse

$\frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ with semi-axes b and c and focal distance

$$f = \sqrt{b^2 - c^2} = \sqrt{a_2 - a_3}$$

If in equation (**8**) we let $a = b$ the sections by the planes parallel to the X-Y plane will be circles whose centres lie along the Z axis and the surface is cut out by revolving the ellipse

$$\frac{x^2}{a^2} + \frac{z^2}{c^2} = 1$$

about the Z axis. If a is the major axis and c the minor axis of this ellipse as has been assumed and stated in the assumption

$a > c$, then the figure is cut out by revolving an ellipse about its minor axis. Such a figure is called an oblate spheroid. If the rotation occurs around the major axis the figure is called a prolade spheroid.

If $a = b = c$ the ellipsoid reduces to a sphere.

The form of the oblate spheroid is shown in Fig. 5.

CONFOCAL QUADRIC SURFACES

In all three cases investigated above it will be observed that the squares of the focal distances of the principal sections (sections cut out on the X-Y, Y-Z, Z-X planes) are the square roots of the differences of the three constants a_1, a_2 and a_3. Therefore, if we add a constant to each of the three constants a_1, a_2 and a_3 we obtain a surface whose principal sections have the same foci as before or what is called a surface confocal with respect to the original.

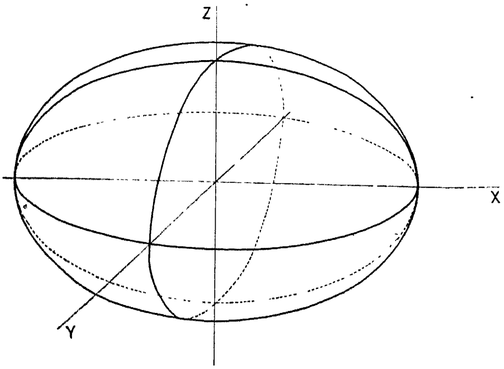

FIG. 5—OBLATE SPHEROID
Axis of revolution about Z axis.

For example, if we take the equation of an ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \qquad (9)$$

the focal distance of the principal section made by the X-Y plane has been seen to be

$$f = \sqrt{a^2 - b^2} = \sqrt{a_1 - a_2}$$

If we now add the constant quantity ρ to a_1 and a_2 we have as the focal distance of this principal section.

$$\sqrt{(a_1 - \rho) - (a_2 - \rho)} = \sqrt{a_1 - \rho - a_2 + \rho} = \sqrt{a_1 - a_2}$$

and thus the focal distance has remained the same. Similarly for the other principal sections, and we therefore see that

$$\frac{x^2}{a^2 + \rho} + \frac{y^2}{b^2 + \rho} + \frac{z^2}{c^2 + \rho} = 1 \tag{10}$$

represents a surface confocal with the original for any real value positive or negative of ρ.

If as we have assumed throughout, a is greater than b and b is greater than c or,

$$a > b > c$$

then the character of the surface represented by equation (**10**) will be seen to be determined as follows:

(1) If $\rho > - c^2$ the surface is an ellipsoid.
(2) If $- c^2 > \rho > - b^2$ the surface is a hyperboloid of one sheet.
(3) If $- b^2 > \rho > - a^2$ the surface is a hyperboloid of two sheets.
(4) If $- a^2 > \rho$ the surface is imaginary.

Suppose we now wish to pass through a given point in space x, y, z a quadric surface confocal with the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \tag{11}$$

Where $a > b > c.$

We have seen that its equation will be

$$\frac{x^2}{a^2 + \rho} + \frac{y^2}{b^2 + \rho} + \frac{z^2}{c^2 + \rho} = 1 \tag{12}$$

from which the value of ρ can be determined when the values of x, y, z of the desired point are given.

It will be seen that this equation (**12**) is a cubic equation in ρ and therefore will have three roots say ρ_1, ρ_2, ρ_3 and if it is found that these roots are all real there will be three confocal surfaces which will pass through any desired point x, y, z in space. Since an equation of odd degree always has at least one real root we see that the other two roots will be real or both imaginary since imaginary roots always enter an equation in pairs.

In order to investigate the roots of our equation (**12**) we can write it when cleared of fractions as follows:

$$(a^2+\rho)(b^2+\rho)(c^2+\rho)-x^2(b^2+\rho)(c^2+\rho)-y^2(a^2+\rho)(c^2+\rho)$$
$$-z^2(a^2+\rho)(b^2+\rho)=f(\rho)=0 \qquad (\mathbf{13})$$

This is our cubic equation in ρ and $a, b; c, x, y, z$ are the constants.

We have just seen that the character of the confocal surface depends upon the magnitude of ρ with respect to the three constants a, b and c. To recapitulate

(1) If $\rho > -c^2$ the surface is an ellipsoid.
(2) If $-c^2 > \rho > -b^2$ the surface is a hyperboloid of one sheet.
(3) If $-b^2 > \rho > -a^2$ the surface is a hyperboloid of two sheets.
(4) If $-a^2 > \rho$ the surface is imaginary.

We will now substitute for ρ in equation (**13**) the values

(1) $\rho = +\infty$
(2) $\rho = -c^2$
(3) $\rho = -b^2$
(4) $\rho = -a^2$

which will be seen to be the extreme values as outlined above. It should be observed that $\rho = -\infty$ would result in an imaginary surface and therefore is not considered.

Upon making these substitutions for ρ in equation (**13**) and remembering that $a > b > c$ we will obtain the following as regards to changes of sign of our cubic equation in ρ which is represented for brevity by $f(\rho)$.

When $\qquad \rho = +\infty$
sign $\qquad f(\rho) = +\infty$

and it is observed that $f(\rho)$ is a positive quantity for all values of x, y, z, a, b, c constant under the assumed relation $a > b > c$.

When $\qquad \rho = -c^2$

$$f(\rho) = -z^2(a^2-c^2)(b^2-c^2)$$

sign $\qquad f(\rho) = (-)(+)\quad(+) = (-)$

in this case we observe that $f(\rho)$ is negative and therefore $f(\rho)$ has changed from a positive value in the previous case to a negative value in this case. Therefore, $f(\rho)$ has passed through zero somewhere between $\rho = +\infty$ and $\rho = -c^2$, that is, there is a real root between these two values. Call this root λ and observe that it occurs in the region which specified that the confocal surface will be an ellipsoid.

When $\rho = -b^2$

$f(\rho) = -y^2(c^2 - b^2)(a^2 - b^2)$

sign $f(\rho) = (-)(-)\quad(+) = (+)$

Here again there has been a change in sign and therefore there is another real root somewhere between $\rho = -c^2$ and $\rho = -b^2$. Call this root μ and observe that it occurs in the region which specifies that the confocal surface defined by it will be a hyperboloid of one sheet.

When $\rho = -a^2$

$f(\rho) = -x^2(b^2 - a^2)(c^2 - a^2)$

sign $f(\rho) = (-)\quad(-)\quad(-)\quad = (-)$

and we see that there is another change in sign and therefore another real root in the interval between $\rho = -b^2$ and $\rho = -a^2$. Call this root ν and observe that it lies in the interval which specifies that the confocal shall be a hyperboloid of two sheets.

We have thus accounted for the three roots of the cubic and found their general location also that they are all real.[34] Thus we have seen that through any given point in space, say x', y', z' there will pass one surface of each of the three kinds and therefore we may represent the given point in space either by its rectangular co-ordinates x', y', z' or by the three values λ, μ, ν which are the three values of ρ as determined by the cubic equation in

$$\frac{x^2}{a^2+\rho} + \frac{y^2}{b^2+\rho} + \frac{z^2}{c^2+\rho} = 1$$

in which the values of x, y, z are taken as x' y' z'.

If we write for the sake of brevity.

$$F(\lambda, x, y, z) = \frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} + \frac{z^2}{c^2+\lambda} - 1 = 0$$

$$F(\lambda, x, y, z) = 0 \tag{14}$$

We see that equation (**14**) defines λ as a function of x, y, z which is a point in space and therefore λ is said to be defined as a point function. Similarly

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[34] A complete treatment of the cubic equations will be found in "Todhunter's Theory of Equations". Page 109.

$$F(\mu, x, y, z) = 0 \tag{15}$$

$$F(\nu, x, y, z) = 0 \tag{16}$$

define μ and ν as point functions.

If we have given the values of λ, μ, ν in these three equations we determine completely the ellipsoid λ = constant, and the hyperboloid of one sheet μ = constant, and the hyperboloid of two sheets ν = constant, and hence their point of intersection x, y, z and its seven symmetrical points in the other quadrants. It can be further shown that the three surfaces defined by

$$\begin{aligned}\lambda &= \text{constant}\\ \mu &= \text{constant}\\ \nu &= \text{constant}\end{aligned}$$

are mutually perpendicular at every point of intersection.[35] λ, μ and ν are called the *ellipsoidal* or *elliptic co-ordinates* of the point. They form a system of Orthogonal Curvilinear Co-ordinates.[36]

NORMAL OBLATE SPHEROIDAL CO-ORDINATES

When any two semi-axes of our standard ellipsoid become equal the above described system of curvilinear co-ordinates breaks down. For in that case the equation

$$\frac{x^2}{a^2+\rho} + \frac{y^2}{b^2+\rho} + \frac{z^2}{c^2+\rho} = 1$$

reduces to a quadratic equation in ρ and therefore has only two roots which we may call λ and μ. The surfaces λ = constant and μ = constant are now confocal ellipsoids and hyperboloids of revolution. Obviously a third family of surfaces is required before the position of a point in space can be fixed by their intersection. Such a family of surfaces, orthogonal to the two present families, is supplied by the system of diametral planes through the axis of revolution of the standard spheroid. The two cases in which the standard ellipsoid is a prolate spheroid and an oblate spheroid require separate treatment. We are here principally interested in the later case which will now be described.

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[35] A good discussion will be found in "Solid Geometry", Charles Smith, Macmillan and Co., Page 147.

[36] The above discussion has mainly been taken from A. G. Webster, "Electricity and Magnetism", pages 27-31.

Let us assume as the standard oblate spheroid

$$\frac{x^2 + y^2}{a^2} + \frac{z^2}{c^2} = 1 \tag{17}$$

where $a > c$ that is, we have taken the case where the figure is rotated about the z axis.

The equation of the confocal family of surfaces is then

$$\frac{x^2 + y^2}{a^2 + \rho} + \frac{z^2}{c^2 + \rho} = 1 \tag{18}$$

which is seen to be a quadratic ρ regarding x, y, z, a, c as constants.

As we have seen before the character of the surface will depend upon the magnitude of ρ with respect to the constants a and c. Remembering $a > c$

(1) If $\rho > -c^2$ the surface is an oblate spheroid.
(2) If $-c^2 > \rho > -a^2$ the surface is a hyperboloid of revolution of one sheet.
(3) If $-a^2 > \rho$ the surface is imaginary.

In order to investigate the roots of (**18**) we may write it

$$(a^2 + \rho)(c^2 + \rho) - (x^2 + y^2)(c^2 + \rho) - z^2(a^2 + \rho) = f(\rho) = 0 \tag{19}$$

and observe the changes of sign when the following values are substituted for ρ in equation (**19**).

(1) $\rho = +\infty$
(2) $\rho = -c^2$
(3) $\rho = -a^2$

When $\rho = +\infty$
sign $f(\rho) = +\infty$

and it is observed that $f(\rho)$ is a positive quantity for all values of x, y, z, a, c consistent with the assumed relation $a > c$.

When $\rho = -c^2$
$f(\rho) = -z^2(a^2 - c^2)$
sign $f(\rho) = (-)\ (+) = (-)$

in this case we see that $f(\rho)$ is negative and has therefore changed from a positive value in the previous case to a negative value in this case. Therefore, $f(\rho)$ has passed through zero somewhere between $\rho = +\infty$ and $\rho = -c^2$, that is, there is a real root between these two values. Call this root λ and observe that it occurs in the region which specifies that the confocal surface will be an oblate spheroid.

When $\rho = -a^2$

$$f(\rho) = -(x^2 + y^2)(c^2 - a^2)$$

sign $f(\rho) = (-) \quad (+) \quad (-) \quad = (+)$

Here again there has been a change in sign and therefore there is another real root somewhere between $\rho = -c^2$ and $\rho = -a^2$ Call this root μ and observe that it is in the region which specifies that the confocal surface will be an hyperboloid of revolution of one sheet. We have thus accounted for the two roots of our quadratic and also found their approximate location; also that they are real.

We may now write

$$F(\lambda, x, y, z) = \frac{x^2 + y^2}{a^2 + \lambda} + \frac{z^2}{c^2 + \lambda} - 1 = 0$$

or,

$$F(\lambda, x, y, z) = 0 \tag{20}$$

Similarly,

$$F(\mu, x, y, z) = 0 \tag{21}$$

Now, as we have seen before, in order to determine completely a point in space we must have another equation which defines a surface which will intersect the two present surfaces λ = constant, and, μ = constant, at all real points. As has already been said such a system of surfaces is that of the diametral planes through the axis of revolution of our standard oblate spheroid.

In this case, we have taken the z-axis as the axis of revolution. We, therefore, wish the equation of the plane which passes through the intersection of the planes $x = 0$ and $y = 0$. The equation of this plane may be written.[37]

$$y - \nu x = 0$$

where ν may have any value.

---

[37] See—Charles Smith "Solid Geometry," Macmillan and Co., page11.

Thus we may write

$$F\,(\nu, x, y, z) = y - \nu\, x = 0 \tag{22}$$

If we are given the values of λ, μ, ν these three equations determine completely the oblate spheroid, λ = constant, the hyperboloid of revolution of one sheet, μ = constant, and the diametral plane through the axis of revolution, ν = constant. Hence their point of intersection x, y, z and its seven symmetrical points in the other quadrants is completely determined. It can further be shown that the three surfaces defined by

$$\begin{aligned} \lambda &= \text{constant} \\ \mu &= \text{constant} \\ \nu &= \text{constant} \end{aligned}$$

are mutually perpendicular at every point of intersection. We may thus take λ, μ, ν as a set of normal curvilinear co-ordinates.

In order to simplify the work which will follow we will make certain simplifications in the form of the equations of the three surfaces, the intersection of which represent our three co-ordinates of a point. Rewriting equations (**20**), (**21**), (**22**) of the three orthogonal surfaces,

$$F\,(\lambda, x, y, z) = \frac{x^2}{a^2 + \lambda} + \frac{y^2}{a^2 + \lambda} + \frac{z^2}{c^2 + \lambda} - 1 = 0 \tag{23}$$

$$F\,(\mu, x, y, z) = \frac{x^2}{a^2 + \mu} + \frac{y^2}{a^2 + \mu} + \frac{z^2}{c^2 + \mu} - 1 = 0 \tag{24}$$

$$F\,(\nu, x, y, z) = y - \nu\, x = 0 \tag{25}$$

Where

$$\begin{gathered} a > c \\ \lambda > -c^2 \\ -c^2 > \mu > -a^2 \end{gathered}$$

ν may have any real value positive or negative.

Let us now make the following substitutions in equations (**23**) and (**24**).

Let

$$a^2 - c^2 = E^2 \tag{26}$$

and

$$a^2 + \lambda = E^2\, D \tag{27}$$

subtracting we obtain

$$c^2 + \lambda = E^2 (D - 1) \tag{28}$$

also let

$$a^2 + \mu = E^2 F \tag{29}$$

subtracting (**26**) from (**29**) we obtain

$$c^2 + \mu = E^2 (F - 1) \tag{30}$$

If we now substitute (**27**) and (**28**) in (**23**), and (**29**) and (**30**) in (**24**) we have

$$\frac{x^2}{E^2 D} + \frac{y^2}{E^2 D} + \frac{z^2}{E^2 (D - 1)} - 1 = 0 \tag{31}$$

$$\frac{x^2}{E^2 F} + \frac{y^2}{E^2 F} + \frac{z^2}{E^2 (F - 1)} - 1 = 0 \tag{32}$$

We may now make the further substitutions,

Putting
$$E^2 = c'^2$$
$$E^2 D = \lambda'^2$$
$$E^2 F = \mu'^2$$

We then may write (**31**) and (**32**)

$$\frac{x^2}{\lambda'^2} + \frac{y^2}{\lambda'^2} + \frac{z^2}{\lambda'^2 - c'^2} - 1 = 0 \tag{33}$$

$$\frac{x^2}{\mu'^2} + \frac{y^2}{\mu'^2} + \frac{z^2}{\mu'^2 - c'^2} - 1 = 0 \tag{34}$$

If in these equations λ'^2 is greater than c'^2 and c'^2 is greater than μ'^2, that is

$$\lambda'^2 > c'^2 > \mu'^2$$

then equation (**33**) is the equation of a confocal family of oblate spheroids in a simplified form and equation (**34**) the confocal family of hyperboloids of revolution. The Z-axis is the axis of revolution and $2\,c'$ is the focal distance. In other words, if we take an ellipse and a hyperbola having the same foci (focal distance $2\,c'$) and revolve them about the minor axis of the ellipse (in this case the z-axis) we shall get a pair of surfaces which are mutually perpendicular; also a plane through the axis of revolution will cut both the spheroid and hyperboloid orthogonally. We may now drop the primes and write the equations of the three surfaces:

$$F_1(x, y, z, \lambda) = \frac{x^2}{\lambda^2} + \frac{y^2}{\lambda^2} + \frac{z^2}{\lambda^2 - c^2} - 1 = 0 \qquad (35)$$

$$F_2(x, y, z, \mu) = \frac{x^2}{\mu^2} + \frac{y^2}{\mu^2} + \frac{z^2}{\mu^2 - c^2} - 1 = 0 \qquad (36)$$

$$F_3(x, y, z, \nu) = y - \nu x = 0 \qquad (37)$$

where $\lambda^2 > c^2 > \mu^2$, $2c$ being the distance between foci.

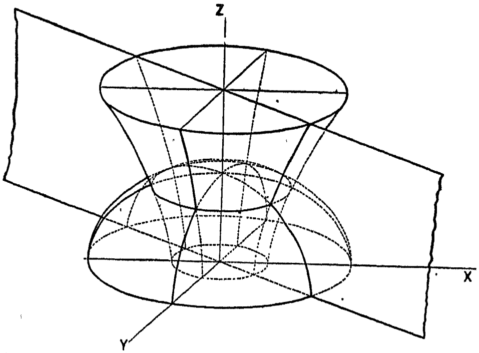


Fig. 6—Intersection of an Oblate Spheroid, Confocal Hyperboloid and a Plane through the Axis of Rotation

For all values of λ, μ and ν consistent with the inequality written above the surfaces (**35**), (**36**) and (**37**) intersect in real points and cut orthogonally. We can, therefore, represent any given point in space say x, y, z by the intersection of the three surfaces provided we assign the proper values to λ, μ and ν. Therefore, λ, μ and ν are a set of orthogonal curvilinear co-ordinates.

The form of the three surfaces is shown in Fig. 6.

Laplace's Equation in Normal Oblate Spheroidal Co-ordinates

If we solve equations (**35**), (**36**) and (**37**) simultaneously we will obtain the values of x, y and z in terms of our co-ordinates λ, μ and ν.

To do this we may divide equation (**35**) through by μ^2 and equation (**36**) through by λ^2 and then subtract. We obtain upon simplification.

$$z^2 = \frac{(\lambda^2 - c^2)\,(c^2 - \mu^2)}{c^2} \tag{38}$$

Substituting this value of z^2 in equation (**35**) we obtain

$$x^2 + y^2 = \frac{\lambda^2\,\mu^2}{c^2}$$

and upon substituting the value of y from equation (**37**) we obtain

$$x^2 = \frac{\lambda^2\,\mu^2}{c^2\,(1 + \nu^2)} \tag{39}$$

similarly we obtain

$$y^2 = \frac{\lambda^2\,\mu^2\,\nu^2}{c^2\,(1 + \nu^2)} \tag{40}$$

Equations (**38**), (**39**) and (**40**) enable us to express the position of a point in space x, y, z when the values of λ, μ and ν are given.

Let $x = x_0$, $y = y_0$, $z = z_0$ be the rectangular co-ordinates of the point in space determined by the intersection of the three surfaces $\lambda = \lambda_0$, $\mu = \mu_0$, $\nu = \nu_0$.

The rectangular co-ordinates of the point

$$\lambda = \lambda_0 + d\,\lambda$$

$$\mu = \mu_0$$

$$\nu = \nu_0$$

will be

$$x = x_0 + \frac{\partial\,x}{\partial\,\lambda}\,d\,\lambda$$

$$y = y_0 + \frac{\partial\,y}{\partial\,\lambda}\,d\,\lambda$$

$$z = z_0 + \frac{\partial\,z}{\partial\,\lambda}\,d\,\lambda$$

That is, it will be the original position plus the rate of change of x with respect to a change in λ multiplied by the increment in λ. This will be readily seen from Fig. 7.

The square of the distance between the points λ_0 and $\lambda_0 + d\,\lambda$ will be

$$dn_1^2 = \left(\frac{\partial\, x}{\partial\, \lambda}\right)^2 d\,\lambda^2 + \left(\frac{\partial\, y}{\partial\, \lambda}\right)^2 d\,\lambda^2 + \left(\frac{\partial\, z}{\partial\, \lambda}\right)^2 d\,\lambda^2$$

$$dn_1^2 = \left[\left(\frac{\partial\, x}{\partial\, \lambda}\right)^2 + \left(\frac{\partial\, y}{\partial\, \lambda}\right)^2 + \left(\frac{\partial\, z}{\partial\, \lambda}\right)^2\right] d\,\lambda^2 \qquad \textbf{(41)}$$

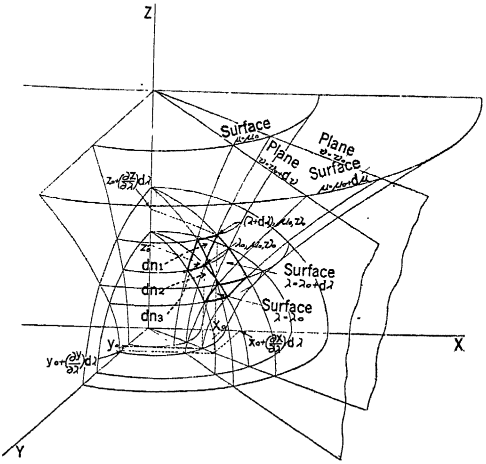

Z
Surface $\mu=\mu_0$
Plane $\nu=\nu_0$
Plane $\nu=\nu_0+d\nu$
Surface $\mu=\mu_0+d\mu$
$z_0+\left(\frac{\partial z}{\partial \lambda}\right)d\lambda$
$(\lambda+d\lambda), \mu_0, \nu_0$
λ_0, μ_0, ν_0
z_0
dn_1
dn_2
dn_3
Surface $\lambda=\lambda_0+d\lambda$
Surface $\lambda=\lambda_0$
x_0
$x_0+\left(\frac{\partial x}{\partial \lambda}\right)d\lambda$
y_0
$y_0+\left(\frac{\partial y}{\partial \lambda}\right)d\lambda$
X
Y

FIG. 7—ELEMENTARY SOLID CUT OUT BY THE INTERSECTION OF A PAIR OF CONFOCAL HYPERBOLOIDS OF REVOLUTION, A PAIR OF ELLIPSOIDS OF REVOLUTION AND A PAIR OF PLANES THROUGH THE AXIS OF REVOLUTION (*Z* AXIS)

In the limit the elementary curvilinear rectangular parallelopiped is simply a rectangular parallelopiped, that is, the effect of the curvative will introduce an effect which will be an infinitesimal of higher order than $d\,\lambda$ and dn_1 and, therefore, may be neglected. From this consideration it will be observed that dn_1 is the differential unit of length normal to the surface $\lambda = \lambda_0$. Thus, a considerable simplification has resulted from the fact that our system of curvilinear co-ordinates are orthogonal.

In like manner, we can write down the expressions for the differential unit normals to the other two surfaces

$\mu = \mu_0$ and $\nu = \nu_0$.

Thus,

$$dn_2^2 = \left[\left(\frac{\partial x}{\partial \mu}\right)^2 + \left(\frac{\partial y}{\partial \mu}\right)^2 + \left(\frac{\partial z}{\partial \mu}\right)^2\right] d\,\mu^2 \qquad (42)$$

which is the elementary normal to the surface $\mu = \mu_0$
Similarly

$$dn_3^2 = \left[\left(\frac{\partial x}{\partial \nu}\right)^2 + \left(\frac{\partial y}{\partial \nu}\right)^2 + \left(\frac{\partial z}{\partial \nu}\right)^2\right] d\,\nu^2 \qquad (43)$$

is the elementary normal to the surface $\nu = \nu_0$
If we now let

$$\frac{1}{h_1^2} = \left(\frac{\partial x}{\partial \lambda}\right)^2 + \left(\frac{\partial y}{\partial \lambda}\right)^2 + \left(\frac{\partial z}{\partial \lambda}\right)^2 \qquad (44)$$

$$\frac{1}{h_2^2} = \left(\frac{\partial x}{\partial \mu}\right)^2 + \left(\frac{\partial y}{\partial \mu}\right)^2 + \left(\frac{\partial z}{\partial \mu}\right)^2 \qquad (45)$$

$$\frac{1}{h_3^2} = \left(\frac{\partial x}{\partial \nu}\right)^2 + \left(\frac{\partial y}{\partial \nu}\right)^2 + \left(\frac{\partial z}{\partial \nu}\right)^2 \qquad (46)$$

we can write the three equations for the differential normals in the more compact form.

$$dn_1 = \frac{d\,\lambda}{h_1} \qquad (47)$$

$$dn_2 = \frac{d\,\mu}{h_2} \qquad (48)$$

$$dn_3 = \frac{d\,\nu}{h_3} \qquad (49)$$

The elementary surfaces which form the sides of the differential volume may now be written (see Fig. 7)

$$d\,S_1 = dn_2\,dn_3 = \left(\frac{d\,\mu\,d\,\nu}{h_2\,h_3}\right) \qquad (50)$$

for the surface $\lambda = \lambda_0$

$$d\,S_2 = dn_1\,dn_3 = \frac{d\,\lambda\,d\,\nu}{h_1\,h_3} \tag{51}$$

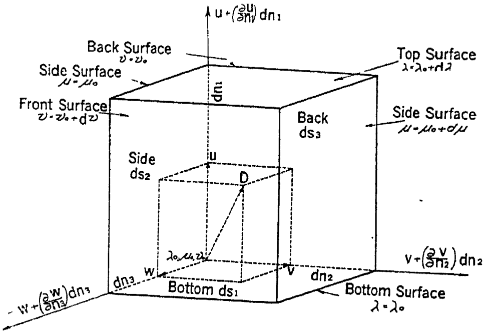
for the surface $\mu = \mu_0$

$$d\,S_3 = dn_1\,dn_2 = \frac{d\,\lambda\,d\,\mu}{h_1\,h_2} \tag{52}$$

for the surface $\nu = \nu_0$

We may now write the differential volume $d\,\tau$ of the rectangular parallelopiped

$$d\,\tau = dn_1\,dn_2\,dn_3 = \frac{d\,\lambda\,d\,\mu\,d\,\nu}{h_1\,h_2\,h_3} \tag{53}$$



$u + \left(\frac{\partial u}{\partial n_1}\right) dn_1$
Back Surface
$\nu = \nu_0$
Side Surface
$\mu = \mu_0$
Front Surface
$\nu = \nu_0 + d\nu$
Top Surface
$\lambda = \lambda_0 + d\lambda$
dn_1
Back
ds_3
Side Surface
$\mu = \mu_0 + d\mu$
Side
ds_2
u
D
λ_0, μ_0, ν_0
w
v
$v + \left(\frac{\partial v}{\partial n_2}\right) dn_2$
dn_3
Bottom ds_1
dn_2
Bottom Surface
$\lambda = \lambda_0$
$w + \left(\frac{\partial w}{\partial n_3}\right) dn_3$

FIG. 8

We can now proceed to write down Laplace's equation in our present system of orthogonal curvilinear co-ordinates. We will here consider the case of a homogeneous and isotropic medium. The clearest method of obtaining Laplace's equation is to apply the well known laws of flow of an incompressible fluid to the infinitesimal unit of volume bounded by the intersection of the six surfaces,

$$\lambda = \lambda_0 \qquad \mu = \mu_0 \qquad \nu = \nu_0$$

$$\lambda = \lambda_0 + d\,\lambda \qquad \mu = \mu_0 + d\,\mu \qquad \nu = \nu_0 + d\,\nu$$

Now let (see Fig. 8)

u equal the component of the flow density or flux density, D, at the point λ_0, μ_0, ν_0, parallel to dn_1

Similarly

v = component of D parallel to dn_2.

w = component of D parallel to dn_3.

Now the total amount of flux (or flow) leaving the infinitesimal parallelopiped is the sum of the six amounts leaving it through the six faces; each of these is expressed as; flow density × surface.

Therefore, they are,

$$\left.\begin{aligned} &- u\,d\,S_1 + \left(u + \frac{\partial u}{\partial n_1} dn_1\right)\left(d\,S_1 + \frac{\partial\,(d\,S_1)}{\partial n_1} dn_1\right) \\ &- v\,d\,S_2 + \left(v + \frac{\partial v}{\partial n_2} dn_2\right)\left(d\,S_2 + \frac{\partial\,(d\,S_2)}{\partial n_1} dn_2\right) \\ &- w\,d\,S_3 + \left(w + \frac{\partial w}{\partial n_3} dn_3\right)\left(d\,S_3 + \frac{\partial\,(d\,S_3)}{\partial n_3} d\,n_3\right) \end{aligned}\right\} \qquad (54)$$

In order to see the exact meaning of these three equations we will put the first one in word form. The first term says that the total flux (or flow) entering the lower surface of the infinitesimal parallelopiped is equal to the component of the flux density in the direction perpendicular to the surface × the area of the lower surface. Since we desire to express the outward flow we designate this inward flow by the minus sign. The second term expresses the total flux outward through the top surface. For this case the component of the flux density will be the original component plus the rate of increase of the component as we move towards the top of the cube multiplied by the height of the cube. This gives the component of the flux density at the top of the cube and if we multiply this by the area of the top we obtain the total outward flow at the top surface.

It will be observed that the area of the top surface is the area of the bottom plus the rate of increase of the area as we go from the bottom towards the top multiplied by the distance.

Multiplying out the second term of the first line of equation (**54**) we obtain:

$$u\,d\,S_1 + \frac{\partial u}{\partial n_1}\,d\,n_1\,d\,S_1 + u\,\frac{\partial\,(d\,S_1)}{\partial n_1}\,d\,n_1$$
$$+ \frac{\partial u}{\partial n_1}\,\frac{\partial\,(d\,S_1)}{\partial n_1}\,d\,n_1^2 \qquad (55)$$

the fourth term is of higher order and therefore may be neglected

Factoring out the $d\,n_1$, we can write (**55**) as:

$$\left[\frac{\partial u}{\partial n_1}\,d\,S_1 + u\,\frac{\partial\,(d\,S_1)}{\partial n_1}\right] d\,n_1 \qquad (\mathbf{56})$$

It will now be evident that this is merely the differential of the product of the two functions u and $d\,S_1$. (I am indebted to Professor J. N. Vedder, who called my attention to this relation while discussing the problem with him.) That is

$$d\,[u\,d\,S_1] = u\,d\,(d\,S_1) + d\,S_1\,d\,u$$

We can therefore write (**56**)

$$\frac{\partial}{\partial n_1}\,[u\,d\,S_1]\,d\,n_1$$

Applying the same reasoning to the other similar equations and adding them all together, we obtain the following equation, which represents the total amount of flux leaving the infinitesimal volume.

$$\frac{\partial}{\partial n_1}\,(u\,d\,S_1)\,d\,n_1 + \frac{\partial}{\partial n_2}\,(v\,d\,S_2)\,d\,n_2 + \frac{\partial}{\partial n_3}(w\,d\,S_3)\,d\,n_3 \qquad (\mathbf{57})$$

If we substitute the values of $d\,n_1$, $d\,n_2$ and $d\,n_3$ as given by equations (**47**), (**48**) and (**49**); as well as $d\,S_1$, $d\,S_2$, $d\,S_3$, as given by equations (**50**), (**51**) and (**52**) in equation (**57**) we obtain

$$h_1\,\frac{\partial}{\partial \lambda}\left(u\,\frac{d\,\mu\,d\,\nu}{h_2\,h_3}\right)\frac{d\,\lambda}{h_1} + h_2\,\frac{\partial}{\partial \mu}\left(v\,\frac{d\,\lambda\,d\,\nu}{h_1\,h_3}\right)\frac{d\,\mu}{h_2}$$

$$+ h_3\,\frac{\partial}{\partial \nu}\left(w\,\frac{d\,\lambda\,d\,\mu}{h_1\,h_2}\right)\frac{d\,\nu}{h_3} \qquad (\mathbf{58})$$

Simplifying and dividing by (**53**) to express the equation for unit of volume, we get:

$$\left[\frac{\partial}{\partial \lambda}\left(\frac{u}{h_2\,h_3}\right) + \frac{\partial}{\partial \mu}\left(\frac{v}{h_1\,h_3}\right) + \frac{\partial}{\partial \nu}\left(\frac{w}{h_1\,h_2}\right)\right] h_1\,h_2\,h_3 \qquad (\mathbf{59})$$

Now for any point in space which is neither a source nor a sink of flux, we see that the same quantity of flux must leave as enters any closed surface in order that there may be no accumulation within. Therefore, we may set this equation equal to zero. This now constitutes the familiar equation of continuity in the λ, μ, ν system of coordinates.

To obtain the same physical statement in terms of potential we merely have to substitute the relation between flux density and potential gradient as expressed by the Fourier-Ohm law namely,

$$\left.\begin{aligned} u &= -\kappa \frac{\partial V}{\partial n_1} \\ v &= -\kappa \frac{\partial V}{\partial n_2} \\ w &= -\kappa \frac{\partial V}{\partial n_3} \end{aligned}\right\} \qquad (60)$$

In other words these equations state that the flow density in a given direction is equal to the potential gradient in that direction multiplied by the specific conductivity. The minus sign indicating that the potential is decreasing in the direction of the flow. We will here take the specific conductivity to be unity and equal in all directions, that is, the material has unit specific resistance and is homogeneous and isotropic.

We may, therefore, rewrite equation (**56**) for this case.

$$\left.\begin{aligned} u &= -\frac{\partial V}{\partial n_1} \\ v &= -\frac{\partial V}{\partial n_2} \\ w &= -\frac{\partial V}{\partial n_3} \end{aligned}\right\} \qquad (61)$$

Substituting the values of dn_1, dn_2 and dn_3 as given by equations (**47**), (**48**) and (**49**) in equation (**61**) we obtain,

$$\left.\begin{aligned} u &= -h_1 \frac{\partial V}{\partial \lambda} \\ v &= -h_2 \frac{\partial V}{\partial \mu} \\ w &= -h_3 \frac{\partial V}{\partial \nu} \end{aligned}\right\} \qquad (62)$$

Therefore, if we substitute the values given in equation (**62**) in equation (**59**), we obtain after changing signs:

$$\frac{\partial}{\partial \lambda}\left(\frac{h_1}{h_2\, h_3}\,\frac{\partial V}{\partial \lambda}\right) + \frac{\partial}{\partial \mu}\left(\frac{h_2}{h_1\, h_3}\,\frac{\partial \nu}{\partial \mu}\right)$$

$$+\frac{\partial}{\partial \nu}\left(\frac{h_3}{h_1\, h_2}\,\frac{\partial v}{\partial \nu}\right) h_1\, h_2\, h_3 = 0 \qquad (\mathbf{63})$$

which is Laplace's equation in orthogonal curvilinear coordinates. This simple method of deducing the above equation was first given by Sir William Thomson (see Mathematical and Physical Papers Vol. I, p. 25).

We may now proceed to obtain the values of h_1, h_2 and h_3 for our case of normal oblate spheroidal co-ordinates from equations (**38**), (**39**) and (**40**) when combined with equations (**44**), (**45**) and (**46**).

Thus differentiating equations (**38**), (**39**) and (**40**) with respect to λ we obtain

$$\left.\begin{aligned} \frac{\partial x}{\partial \lambda} &= \frac{\mu}{c\sqrt{1+\nu^2}} \\ \frac{\partial y}{\partial \lambda} &= \frac{\mu\, \nu}{c\sqrt{1+\nu^2}} \\ \frac{\partial z}{\partial \lambda} &= \frac{\lambda}{c}\sqrt{\frac{c^2-\mu^2}{\lambda^2-c^2}} \end{aligned}\right\} \qquad (\mathbf{64})$$

Substituting these values in equation (**44**) we obtain

$$\frac{1}{h_1^2} = \left(\frac{\mu}{c\sqrt{1+\nu^2}}\right)^2 + \left(\frac{\mu\, \nu}{c\sqrt{1+\nu^2}}\right)^2 + \left(\frac{\lambda}{c}\sqrt{\frac{c^2-\mu^2}{\lambda^2-c^2}}\right)^2$$

Simplifying we get

$$\frac{1}{h_1^2} = \frac{\lambda^2-\mu^2}{\lambda^2-c^2} \qquad (\mathbf{65})$$

Similarly we obtain

$$\left.\begin{aligned} \frac{\partial x}{\partial \mu} &= \frac{\lambda}{c\sqrt{1+\nu^2}} \\ \frac{\partial y}{\partial \mu} &= \frac{\lambda \nu}{c\sqrt{1+\nu^2}} \\ \frac{\partial z}{\partial \mu} &= -\frac{\mu}{c}\sqrt{\frac{\lambda^2-c^2}{c^2-\mu^2}} \end{aligned}\right\} \tag{66}$$

From which

$$\frac{1}{h_2{}^2} = \frac{\lambda^2-\mu^2}{c^2-\mu^2} \tag{67}$$

Again

$$\left.\begin{aligned} \frac{\partial x}{\partial \nu} &= -\frac{\lambda \mu \nu}{c(1+\nu^2)^{3/2}} \\ \frac{\partial y}{\partial \nu} &= \frac{\lambda \mu}{c(1+\nu^2)^{3/2}} \\ \frac{\partial z}{\partial \nu} &= 0 \end{aligned}\right\} \tag{68}$$

and

$$\frac{1}{h_3{}^2} = \frac{\lambda^2 \mu^2}{c^2(1+\nu^2)^2} \tag{69}$$

If we now substitute these values of $\frac{1}{h_1}\ \frac{1}{h_2},\ \frac{1}{h_3},$ in equation (**63**) we get

$$\frac{\mu}{c(1+\nu^2)\sqrt{c^2-\mu^2}}\frac{\partial}{\partial \lambda}\left[\lambda\sqrt{\lambda^2-c^2}\frac{\partial V}{\partial \lambda}\right] +$$

$$\frac{\lambda}{c(1+\nu^2)\sqrt{\lambda^2-c^2}}\frac{\partial}{\partial \mu}\left[\mu\sqrt{c^2-\mu^2}\frac{\partial V}{\partial \mu}\right] +$$

$$\frac{c(\lambda^2-\mu^2)}{\lambda\mu\sqrt{(\lambda^2-c^2)(c^2-\mu^2)}}\frac{\partial}{\partial \nu}\left[(1+\nu^2)\frac{\partial V}{\partial \nu}\right] = 0 \tag{70}$$

which is Laplace's equation in our spheroidal coordinates λ, μ and ν.

If now in place of λ, μ and ν we can find some function of λ, some function of μ, and some function of ν which, therefore, will represent the same set of orthogonal surfaces, and if we can choose these functions which we shall designate by α, β and γ, which of course are also functions of x, y and z, so that they are solutions of Laplace's equation when expressed in rectangular co-ordinates, and, therefore, satisfy it when substituted therein. That is, we would have,

$$\left.\begin{aligned} \frac{\partial^2 \alpha}{\partial x^2} + \frac{\partial^2 \alpha}{\partial y^2} + \frac{\partial^2 \alpha}{\partial z^2} &= 0 \\ \frac{\partial^2 \beta}{\partial x^2} + \frac{\partial^2 \beta}{\partial y^2} + \frac{\partial^2 \beta}{\partial z^2} &= 0 \\ \frac{\partial^2 \gamma}{\partial x^2} + \frac{\partial^2 \gamma}{\partial y^2} + \frac{\partial^2 \gamma}{\partial z^2} &= 0 \end{aligned}\right\}$$

under these circumstances equation (**70**) will be reduced to a more simple and symmetrical form. It must be remembered that the existence of these functions α, β, γ, is merely tentative. The criterion for their existence will be given later.

Equation (**70**) is Laplace's equation expressed in terms of λ, μ and ν.

Assuming now that in equation (**70**) V is a function of λ only; in that case

$$\frac{\partial V}{\partial \mu} = 0 \text{ and } \frac{\partial V}{\partial \nu} = 0$$

and equation (**70**) would then reduce to

$$\frac{\partial}{\partial \lambda}\left[\lambda \sqrt{\lambda^2 - c^2}\, \frac{\partial V}{\partial \lambda}\right] = 0 \qquad \textbf{(71)}$$

Integrating with respect to λ we get

$$\lambda \sqrt{\lambda^2 - c^2}\, \frac{d V}{d \lambda} = c_1 \qquad \textbf{(72)}$$

whence

$$d\,V = \frac{c_1\,d\,\lambda}{\lambda\sqrt{\lambda^2 - c^2}} \tag{73}$$

and integrating again

$$V = \frac{c_1}{c}\sec^{-1}\frac{\lambda}{c} + K_1 \tag{74}$$

Here V is a function of λ which satisfies Laplace's equation. Call this value of V which, is a function of λ and which satisfies Laplace's equation, α leaving the constant c_1 undertermined thus

$$d\,\alpha = \frac{c_1\,d\,\lambda}{\lambda\sqrt{\lambda^2 - c^2}} \tag{75}$$

and

$$\alpha = \frac{c_1}{c}\sec^{-1}\frac{\lambda}{c} + K_1 \tag{76}$$

The constant K_1 is zero since,

$$\alpha = 0$$

when

$$\lambda = c$$

and

$$\sec^{-1} 1 = 0$$

As above assumed now that V in equation (70) is a function of μ only; in this case $\frac{\partial V}{\partial \lambda} = 0$ and $\frac{\partial V}{\partial \nu} = 0$ and

$$\frac{\partial}{\partial \mu}\left[\mu\sqrt{c^2 - \mu^2}\,\frac{\partial V}{\partial \mu}\right] = 0 \tag{77}$$

Integrating

$$\mu\sqrt{c^2 - \mu^2}\,\frac{d\,V}{d\,\mu} = c_2 \tag{78}$$

$$d\,V = \frac{c_2\,d\,\mu}{\mu\sqrt{c^2 - \mu^2}} \tag{79}$$

$$V = -\frac{c_2}{c}\operatorname{sech}^{-1}\frac{\mu}{c} + K_2 \tag{80}$$

Call this value of V which is a function of μ only, and which satisfies Laplace's equation, β, leaving the constant c_2 undetermined we have

$$d\,\beta = \frac{c_2\,d\,\mu}{\mu\sqrt{c^2-\mu^2}} \tag{81}$$

$$\beta = -\frac{c_2}{c}\operatorname{sech}^{-1}\frac{\mu}{c} + K_2 \tag{82}$$

The constant of integrating K_2 is zero since

$$\beta = 0$$

when

$$\mu = c$$

and

$$\operatorname{sech}^{-1} 1 = 0$$

Similarly

$$\frac{\partial}{\partial\,\nu}\left[(1+\nu^2)\frac{\partial\,V}{\partial\,\nu}\right] = 0 \tag{83}$$

$$(1+\nu^2)\frac{d\,V}{d\,\nu} = c_3 \tag{84}$$

$$d\,V = \frac{c_3\,d\,\nu}{1+\nu^2} \tag{85}$$

$$V = c_3\tan^{-1}\nu + K_3 \tag{86}$$

If, as before, we call this value of V, which is a function of ν only and which satisfies Laplace's equation, γ, leaving the constant c_3 undetermined we have

$$d\,\gamma = \frac{c_3\,d\,\nu}{1+\nu^2} \tag{87}$$

$$\gamma = c_3\tan^{-1}\nu + K_3 \tag{88}$$

The constant of integration K_3 vanishes since

$$\gamma = 0$$

when

$$\nu = 0$$

and

$$\tan^{-1} 0 = 0$$

If we now substitute these values in equation (**70**) we can express Laplace's equation in terms of our new functions α, β and γ and thereby obtain a simpler equation. To do this let us take equation (**70**) term by term.

The first term is

$$\frac{\mu}{c\,(1+\nu^2)\sqrt{c^2-\mu^2}}\,\frac{\partial}{\partial\,\lambda}\left[\lambda\sqrt{\lambda^2-c^2}\,\frac{\partial\,V}{\partial\,\lambda}\right]$$

expanding the indicated differentiation we get

$$\frac{\mu}{c\,(1+\nu^2)\sqrt{c^2-\mu^2}}\left[\frac{\partial}{\partial\,\lambda}\left(\lambda\sqrt{\lambda^2-c^2}\,\frac{\partial\,V}{\partial\,\lambda}\right.\right.$$
$$\left.\left.+\,\lambda\sqrt{\lambda^2-c^2}\,\frac{\partial^2\,V}{\partial\,\lambda^2}\right)\right] \qquad \text{(89)}$$

From equation (**71**) the first term within the bracket is equal to zero.

If we now assign the following values to the constants, c_1, c_2 and c_3

$$c_1 = -\,c_2 = c \text{ and } c_3 = 1$$

We have from equation (**73**)

$$\frac{d\,V}{d\,\lambda} = \frac{c}{\lambda\sqrt{\lambda^2-c^2}}$$

$$\frac{d^2\,V}{d\,\lambda^2} = \frac{d}{d\,\lambda}\left(\frac{c}{\lambda\sqrt{\lambda^2-c^2}}\right)$$

$$\frac{d^2\,V}{d\,\lambda^2} = \frac{-\,c\,[\,(\lambda^2-c^2)+\lambda^2]}{\lambda^2\,(\lambda^2-c^2)\sqrt{\lambda^2-c^2}} \qquad \text{(90)}$$

Now from equation (**75**) we have

$$\frac{d\,\lambda}{d\,\alpha} = \frac{\lambda\sqrt{\lambda^2-c^2}}{c}$$

$$\left(\frac{d\,\lambda}{d\,\alpha}\right)^2 = \frac{\lambda^2\,(\lambda^2-c^2)}{c^2} \qquad \text{(91)}$$

Combining (**90**) and (**91**) we obtain

$$\frac{d^2\, V}{d\ \alpha^2} = \frac{d^2\, V}{d\ \lambda^2}\frac{d\ \lambda^2}{d\ \alpha^2} = -\ \frac{(\lambda^2 - c^2) + \lambda^2}{c\sqrt{\lambda^2 - c^2}} \tag{92}$$

Now comparing (**90**) and (**92**) it will be seen that

$$\frac{d^2\, V}{d\ \lambda^2} = \frac{c^2}{\lambda^2\,(\lambda^2 - c^2)}\frac{d^2\, V}{d\ \alpha^2} \tag{93}$$

If we now substitute this value of $\frac{d^2\, V}{d\ \lambda^2}$ in the second term of equation (**89**) we get

$$\frac{\mu}{c\,(1+\nu^2)\sqrt{c^2-\mu^2}}\left\{\lambda\sqrt{\lambda^2 - c^2}\left[\frac{c^2}{\lambda^2\,(\lambda^2-c^2)}\frac{\partial^2\, V}{\partial\ \alpha^2}\right]\right\}$$

or simplifying the first term of equation (**70**) when expressed in terms of $\frac{\partial^2\, V}{\partial\ \alpha^2}$ we get

$$\frac{c}{\lambda\,(1+\nu^2)\ \sqrt{c^2-\mu^2}\ \sqrt{\lambda^2-c^2}}\frac{\partial^2\, V}{\partial\ \alpha^2} \tag{94}$$

If we now perform the same operation upon the second term of equation (**70**) which is

$$\frac{\lambda}{c\,(1+\nu^2)\ \sqrt{\lambda^2 - c^2}}\frac{\partial}{\partial\,\mu}\left[\mu\sqrt{c^2 - \mu^2}\,\frac{\partial\, V}{\partial\,\mu}\right] \tag{95}$$

expanding the indicated differentiation we have

$$\frac{\lambda}{c\,(1+\nu^2)\ \sqrt{\lambda^2 - c^2}}\left[\frac{\partial}{\partial\,\mu}\left(\mu\sqrt{c^2 - \mu^2}\right)\frac{\partial\, V}{\partial\,\mu} + \mu\sqrt{c^2 - \mu^2}\,\frac{\partial^2\, V}{\partial\ \mu^2}\right] \tag{96}$$

Equation (**77**) tells us that the first term within the bracket is zero.

Now from equation (**79**) putting $c_2 = -c$ we have

$$\frac{d\,V}{d\,\mu} = \frac{-c}{\mu\ \sqrt{c^2-\mu^2}}$$

and then

$$\frac{d^2\,V}{d\,\mu^2} = \frac{c\,[\,(c^2-\mu^2)-\mu^2]}{\mu^2\,(c^2-\mu^2)\ \sqrt{c^2-\mu^2}} \tag{97}$$

From equation (**81**) we have

$$\frac{d\,\mu}{d\,\beta} = \frac{\mu\,\sqrt{c^2-\mu^2}}{-c}$$

$$\frac{d\ \mu^2}{d\ \beta^2} = \frac{\mu^2(c^2-\mu^2)}{c^2} \tag{98}$$

From (**97**) and (**98**) we obtain

$$\frac{d^2\ V}{d\ \beta^2} = \frac{d^2\ V}{d\ \mu^2}\frac{d\ \mu^2}{d\ \beta^2} = \frac{(c^2-\mu^2)-\mu^2}{c\,\sqrt{c^2-\mu^2}} \tag{99}$$

Now from (**97**) and (**99**) it will be seen that the following relation exists

$$\frac{d^2\ V}{d\ \mu^2} = \frac{c^2}{\mu^2\ (c^2-\mu^2)}\frac{d^2\ V}{d\ \beta^2} \tag{100}$$

Hence substituting this value in equation (**96**) we get for the second term of equation (**70**) when expressed in terms of $\frac{\partial^2\ V}{\partial\ \beta^2}$ the following.

$$\frac{\lambda\,c}{\mu\,(1+\nu^2)\ \sqrt{c^2-\mu^2}\ \sqrt{\lambda^2-c^2}}\,\frac{\partial^2\ V}{\partial\ \beta^2} \tag{101}$$

Now as before expand the third term of equation (**70**)

$$\frac{c\,(\lambda^2-\mu^2)}{\lambda\,\mu\ \sqrt{\lambda^2-c^2}\ \sqrt{c^2-\mu^2}}\left[\frac{\partial}{\partial\,\nu}\,(1+\nu^2)\,\frac{\partial\,V}{\partial\,\nu} + (1+\nu^2)\,\frac{\partial^2\ V}{\partial\,\nu^2}\right] \tag{102}$$

and observe that the first term within the brackets vanishes by equation (**83**).

Now from equation (**85**) putting $c_3 = 1$, we get,

$$\frac{d V}{d \nu} = \frac{1}{1 + \nu^2}$$

$$\frac{d^2 V}{d \nu^2} = \frac{-2 \nu}{(1 + \nu^2)^2} \qquad (\mathbf{103})$$

From equation (**87**)

$$\frac{d \nu}{d \gamma} = 1 + \nu^2$$

$$(\mathbf{104})$$

$$\frac{d \nu^2}{d \gamma^2} = (1 + \nu^2)^2$$

From (**103**) and (**104**)

$$\frac{d^2 V}{d \gamma^2} = \frac{d^2 V}{d \nu^2} \frac{d \nu^2}{d \gamma^2} = -2 \nu \qquad (\mathbf{105})$$

Now from (**103**) and (**105**) we get

$$\frac{d^2 V}{d \nu^2} = \frac{1}{(1 + \nu^2)^2} \frac{d^2 V}{d \gamma^2} \qquad (\mathbf{106})$$

Hence substituting this value in equation (**102**) we get for the third term of equation (**70**) when expressed in terms of $\frac{\partial^2 V}{\partial \gamma^2}$ the following.

$$\frac{c (\lambda^2 - \mu^2)}{\lambda \mu (1 + \nu^2) \sqrt{c^2 - \mu^2} \sqrt{\lambda^2 - c^2}} \frac{\partial^2 V}{\partial \gamma^2} \qquad (\mathbf{107})$$

If we now combine the three terms as given by equations (**94**), (**101**) and (**107**) we get

$$\frac{1}{\lambda^2} \frac{\partial^2 V}{\partial \alpha^2} + \frac{1}{\mu^2} \frac{\partial^2 V}{\partial \beta^2} + \frac{\lambda^2 - \mu^2}{\lambda^2 - \mu^2} \frac{\partial^2 V}{\partial \gamma^2} = 0 \qquad (\mathbf{108})$$

or since from equation (**76**), (**82**) and (**88**) and remembering that we put $c_1 = -c_2 = c_3 = 1$ we have

$$\left.\begin{aligned} \lambda &= c \sec \alpha \\ \mu &= c \operatorname{sech} \beta \\ \nu &= \tan \gamma \end{aligned}\right\} \qquad (\mathbf{109})$$

we can now write equation (**108**)

$$\cos^2 \alpha \frac{\partial^2 V}{\partial \alpha^2} + \cosh^2 \beta \frac{\partial^2 V}{\partial \beta^2} + (\cosh^2 \beta - \cos^2 \alpha) \frac{\partial^2 V}{\partial \gamma^2} = 0 \quad (\mathbf{110})$$

which is Laplace's equation expressed in terms of what have been called normal oblate spheroidal co-ordinates.

When using this equation (**110**) it is to be noted that the point whose co-ordinates are (α, β, γ) is the point of intersection of an oblate spheroid whose semi-axes are

$c \sec \alpha$ (major axis)
$c \tan \alpha$ (minor axis and the axis of revolution.)

and a hyperboloid of revolution of one sheet whose semi-axes are

$c \operatorname{sech} \beta$ (transverse axis)
$c \tanh \beta$ (conjugate axis, the axis of revolution.)

and a plane containing the axis of revolution of the system and making the angle γ with a fixed plane.

If the axis of revolution is the z axis and the fixed plane of reference is the XZ plane; the rectangular co-ordinates of α, β, γ, will be obtained by substituting the values of λ, μ and ν given by equation (**109**) into equations (**38**), (**39**) and (**40**). This substitution gives

$$\left.\begin{aligned} x &= c \sec \alpha \operatorname{sech} \beta \cos \gamma \\ y &= c \sec \alpha \operatorname{sech} \beta \sin \gamma \\ z &= c \tan \alpha \tanh \beta \end{aligned}\right\} \qquad (\mathbf{111})$$

If now we let α range from 0 to $\frac{\pi}{2}$, β from $-\infty$ to $+\infty$, and γ from 0 to 2π, we shall be able to represent all points in space;

and if we agree that negative values of β shall belong to points below a plane through the origin and perpendicular to the axis of revolution and positive values of β to points above that plane. Then not only shall we have no ambiguity, but also the rectangular co-ordinates of any point in space as given by equations (**111**) will have their proper signs. (See Fig. 9)

The above transformation of Laplace's equation, as expressed in terms of the orthogonal curvilinear co-ordinates λ, μ and ν which represent an orthogonal system of surfaces, to the simplified form in which it was expressed in terms of the new co-ordinates α, β, γ was made possible by the assumption that certain functions α, β, γ of λ, μ and ν exist. Thus we determined the value of α by solving equation (**70**) on the assumption that V is a function of λ only; μ and ν not entering. This means that we assumed that the curvilinear co-ordinate λ corresponds to a possible equipotential or isothermal surface, for in that case there would be a function of λ which would satisfy Laplace's equation.

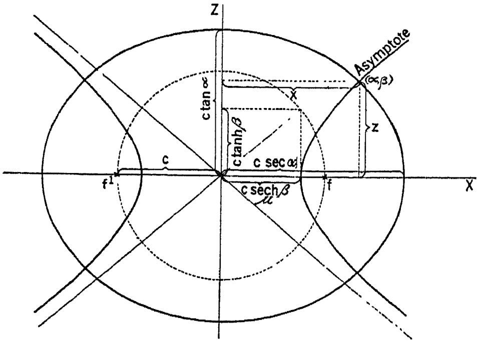


FIG. 9—CONFOCAL ELLIPSE AND HYPERBOLA

We may write it symbolically as follows:

Starting with Laplace's equation in terms of rectangular coordinates namely $\Delta^2 V = 0$ and remembering that $\lambda = f(x, y, z)$. Then if it happens that $V = f(\lambda)$ only, is a solution of Laplace's equation, then we may write it

$$V = f(\lambda) = \alpha$$

and upon substituting back in Laplace's equation we would get the condition that

$$\Delta^2 \alpha = 0$$

In other words, this means that α as found by this process is a value of the potential V corresponding to such a distribution that the surface obtained by giving particular values to λ are equipotential surfaces. The particular value of the function of λ, α is given in equation (**76**).

The condition that such a function α should exist, for a given system of surfaces, that is, that the distribution described above should be possible will be obtained analytically below.

We have seen that the required condition is merely that V in Laplace's equation may be a function of λ alone.

If V is a function of λ alone and we also remember that λ is a function of x, y and z, we may start from this information to write out Laplace's equation.

We have

$$V = f_1 (\lambda)$$

$$\lambda = f_2 (x, y, z)$$

Hence

$$\left.\begin{aligned}
\frac{\partial V}{\partial x} &= \frac{d V}{d \lambda}\frac{\partial \lambda}{\partial x} \\
\frac{\partial V}{\partial y} &= \frac{d V}{d \lambda}\frac{\partial \lambda}{\partial y} \\
\frac{\partial V}{\partial z} &= \frac{d V}{d \lambda}\frac{\partial \lambda}{\partial z}
\end{aligned}\right\}$$

Differentiating again we obtain

$$\left.\begin{aligned}
\frac{\partial^2 V}{\partial x^2} &= \frac{d^2 V}{d \lambda^2}\frac{\partial \lambda^2}{\partial x^2} + \frac{d V}{d \lambda}\frac{\partial^2 \lambda}{\partial x^2} \\
\frac{\partial^2 V}{\partial y^2} &= \frac{d^2 V}{d \lambda^2}\frac{\partial \lambda^2}{\partial y^2} + \frac{d V}{d \lambda}\frac{\partial^2 \lambda}{\partial y^2} \\
\frac{\partial^2 V}{\partial z^2} &= \frac{d^2 V}{d \lambda^2}\frac{\partial \lambda^2}{\partial z^2} + \frac{d V}{d \lambda}\frac{\partial^2 \lambda}{\partial z^2}
\end{aligned}\right\}$$

Adding these three equations and equating the result to zero in order to obtain Laplace's equation we get

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0.$$

also

$$\left[\left(\frac{\partial \lambda}{\partial x}\right)^2 + \left(\frac{\partial \lambda}{\partial y}\right)^2 + \left(\frac{\partial \lambda}{\partial z}\right)^2\right]\frac{d^2 V}{d \lambda^2} +$$

$$\left[\frac{\partial^2 \lambda}{\partial x^2} + \frac{\partial^2 \lambda}{\partial y^2} + \frac{\partial^2 \lambda}{\partial z^2}\right]\frac{d V}{d \lambda} = 0$$

We thus have obtained an expression for Laplace's equation based upon the assumption that the potential V shall be a function of λ alone. If we now write it

$$\frac{\dfrac{\partial^2 \lambda}{\partial x^2} + \dfrac{\partial^2 \lambda}{\partial y^2} + \dfrac{\partial^2 \lambda}{\partial z^2}}{\left(\dfrac{\partial \lambda}{\partial x}\right)^2 + \left(\dfrac{\partial \lambda}{\partial y}\right)^2 + \left(\dfrac{\partial \lambda}{\partial z}\right)^2} = - \frac{\dfrac{d^2 V}{d \lambda^2}}{\dfrac{d V}{d \lambda}}$$

This may be written in symbolic form

$$\frac{\Delta^2 \lambda}{h_1^2} = F_1 (\lambda)$$

where $F_1 (\lambda)$ may be any function of λ alone. Thus our required condition is that

$$\frac{\Delta^2 \lambda}{h_1^2} = F_1 (\lambda) \qquad \textbf{(112)}$$

and when this is fulfilled the original curvilinear co-ordinate λ corresponds to a possible equipotential or isothermal surface and in this case a function α exists. If similar relations are found to hold for β and γ the reduction of Laplace's equation to the so-called symmetrical form is possible.

Similarly we determine the values of β and γ on the assumption that V is a function of μ only in the case of β and a function of ν only in the case of γ. In that case it is evident that

$\Delta^2 \beta = 0$ and in the second case
$\Delta^2 \gamma = 0$

It is therefore evident that β is a value of V corresponding to such a distribution that the surfaces obtained by giving particular values to μ are equipotential surfaces; and that γ is a value of V corresponding to such a distribution that the surfaces obtained by giving particular value to ν are equipotential surfaces.

From what has been said above it is evident why Lamé, the inventor of this system of co-ordinates called the functions α, β and γ thermometric parameters.[38]

The analytical method of obtaining the criterion for the existence of these functions β and γ is exactly the same as that for the case of α which has been worked out. It is in the case of λ

$$\frac{\Delta^2 \lambda}{h_1^2} = F_1(\lambda)$$

and it is for μ

$$\left.\begin{aligned} \frac{\Delta^2 \mu}{h_2^2} &= F_2(\mu) \\ \text{and for } \nu \qquad & \\ \frac{\Delta^2 \nu}{h_3^2} &= F_3(\nu) \end{aligned}\right\} \qquad \textbf{(113)}$$

When these three conditions are fulfilled for λ, μ and ν, the original curvilinear co-ordinates λ, μ and ν correspond to possible equipotential or isothermal surfaces and thermometric parameters α, β and γ exist and the reduction of Laplace's equation to the so-called symmetrical form given by equation (**108**) is possible.

In all of the above work I have followed the development of the subject of orthogonal curvilinear co-ordinates as given in Byerly's Fourier Series and Spherical Harmonics pages 238 to 245 and merely tried to give an unexpurgated description of the process. That is, I have tried to fill in some of the steps in the process which are necessary in order that the average engineer may follow the development with ease.

The solution of the problems given in the following pages is due to Professor W. E. Byerly. I had been struggling with the problem for some time without success and finally took the liberty of writing Professor Byerly concerning my troubles. His reply of January 9th, 1915 to my letter exceeded my expectations for it gave the desired solution as given in the following pages. On February 17th, 1915, I had the further privilege of talking over the problems with him, at which time he cleared up the remaining troublesome points.

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[38] Numerous references to Lamé's work will be found in the following works.

Maxwell, Electricity and Magnetism, page 232; Webster, Electricity and Magnetism, page 22, 64,173; Byerly, Fourier's Series and Spherical Harmonics, page 244, 274; J. H. Jeans, Electricty and Magnetism, page 247; Goursat-Hedrick, Mathematical Analysis, Vol. I, page 80; I. Todhunter, The Functions of Laplace, Lamé and Bessel, page 210.

POTENTIAL AT ANY POINT IN SPACE

Laplace's equation (**110**) expressed in the symmetrical form was seen to be

$$\cos^2 \alpha \frac{\partial^2 V}{\partial \alpha^2} + \cosh^2 \beta \frac{\partial^2 V}{\partial \beta^2} + (\cosh^2 \beta - \cos^2 \alpha) \frac{\partial^2 V}{\partial \gamma^2} = 0 \quad \textbf{(114)}$$

In the present problem we wish to determine the potential at any point in space between two confocal hyperboloids; one of which is chosen to represent the tank with a hole in it; the other representing the rod or connection which passes through the hole in the cover of the tank.

The potential V must satisfy the above equation and is a function of β only. This follows from the fact that the equipotential surfaces will be confocal hyperboloids of revolution.

Therefore, we have,

$$\frac{\partial^2 V}{\partial \alpha^2} = 0 \text{ and } \frac{\partial^2 V}{\partial \gamma^2} = 0$$

and Laplace's equation reduces in our case to

$$\cosh^2 \beta \frac{\partial^2 V}{\partial \beta^2} = 0$$

This equation must be true for all values of β and V and since $\cosh^2 \beta$ is not zero for all values of β, $\frac{\partial^2 V}{\partial \beta^2}$ must be equal to zero. Thus our equation will be

$$\frac{\partial^2 V}{\partial \beta^2} = 0 \quad \textbf{(115)}$$

The solution of this differential equation is seen to be obtained as follows:

Integrating,

$$\frac{d\,V}{d\,\beta} = A$$

and

$$d\,V = A\,d\,\beta$$

$$V = A\,\beta + C \tag{116}$$

which is the solution for the potential at any point in space. The values of the constants of integration A and C will be found from the conditions of the problem.

Thus if $\beta = \beta_0$ for the hyperboloid selected as the tank with hole in it, the potential is

$$V = 0$$

Assuming the tank to be grounded,

Hence $$0 = A\,\beta_0 + C$$

Also when $\beta = \beta_1$ for the hyperboloid selected as the electrode which passes through the hole in the tank

$$V = V_1$$

Hence

$$V_1 = A\,\beta_1 + C$$

Solving these equations simultaneously we have

$$\left.\begin{aligned} C &= \frac{V_1}{1 - \dfrac{\beta_1}{\beta_0}} \\ A &= \frac{V_1}{\beta_1 - \beta_0} \end{aligned}\right\} \tag{117}$$

Substituting the values of A and C as thus determined into equation (**116**) we have.

$$V = \frac{V_1\,\beta}{\beta_1 - \beta_0} + \frac{V_1}{1 - \dfrac{\beta_1}{\beta_0}}$$

or

$$V = \frac{V_1}{\beta_1 - \beta_0}\,(\beta - \beta_0) \tag{118}$$

which is the complete expression for the potential at any point in space under the assumed terminal conditions.

From considerations of symmetry it is obvious that the potential has circular symmetry about the axis of revolution; and, in this case, the z axis has been chosen.

We may now determine the potential at any point in space in rectangular co-ordinates as follows. Suppose we desire the potential at the point x, z for the case in which the semifocal distance is c.

We have for the value of μ which is the semi-transverse axis of the hyperbola

$$\mu = \frac{1}{2}\,(P\,F_2 - P\,F_1)$$

(see Fig. 10)

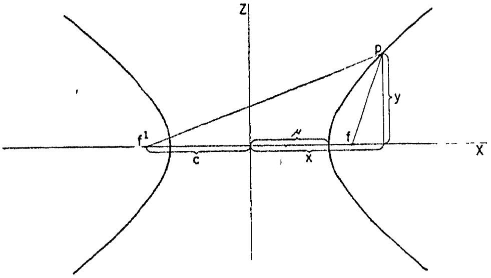

FIG. 10—HYPERBOLA

Hence,

$$\mu = \frac{1}{2}\left[\sqrt{(x+c)^2 + z^2} - \sqrt{(x-c)^2 + z^2}\right] \tag{119}$$

Also from equation (**109**) we have,

$$\mu = c \operatorname{sech} \beta$$

$$\cosh \beta = \frac{c}{\mu}$$

$$\beta = \cosh^{-1} \frac{c}{\mu} \tag{120}$$

Thus we can determine the potential at any point P in space under any desired conditions of c, β_1, β_0, V_1.

POTENTIAL DISTRIBUTION ON THE X-AXIS

For this case we have $z = 0$ in equation (**119**) and hence it reduces to

$$\mu = c \tag{121}$$

Now under these conditions μ and x are identical in value for points along the x axis between the origin and the focus. We can, therefore, use equation (**120**) directly without reference to equation (**119**) or (**121**).

Thus we can determine the values of β_1 and β_0 for any value of μ from equation (**120**) or rewriting it here

$$\beta = \cosh^{-1} \frac{c}{\mu} \tag{122}$$

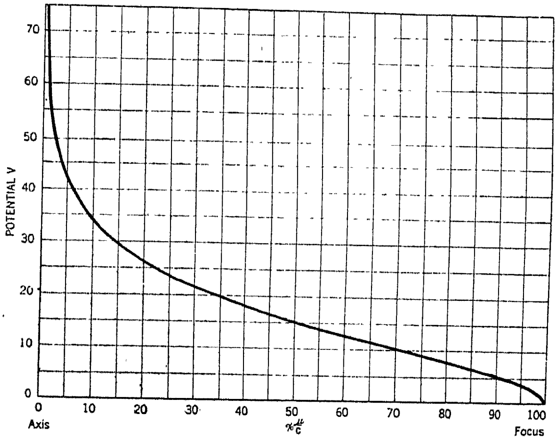

FIG. 11—CALCULATED POTENTIAL DISTRIBUTION ALONG THE X-AXIS

If we select for the hyperboloid β_0 the limiting case that of the infinite plane with a hole in the centre $\beta_0 = 0$ for

$$\beta = \cosh^{-1} \frac{c}{\mu}$$

where $\mu = c$

$$\beta = \beta_0 = \cosh^{-1} 1 = 0$$

We may then write equation (**118**) as follows:—

$$V = \left(\frac{V_1}{\beta_1}\right) \beta \tag{123}$$

The potential distribution for this case is shown in the curve, (see Fig. 11). From this curve we may find the proper spacing for the hyperbolas which when rotated about their z axis cut out the equipotential surfaces for unit differences of potential. All we have to do is to divide the potential into equal increments and note the corresponding value of μ or x. In this manner we can construct a diagram similar to Fig. 12. In the case before us the potential has been divided into 51 equal increments.

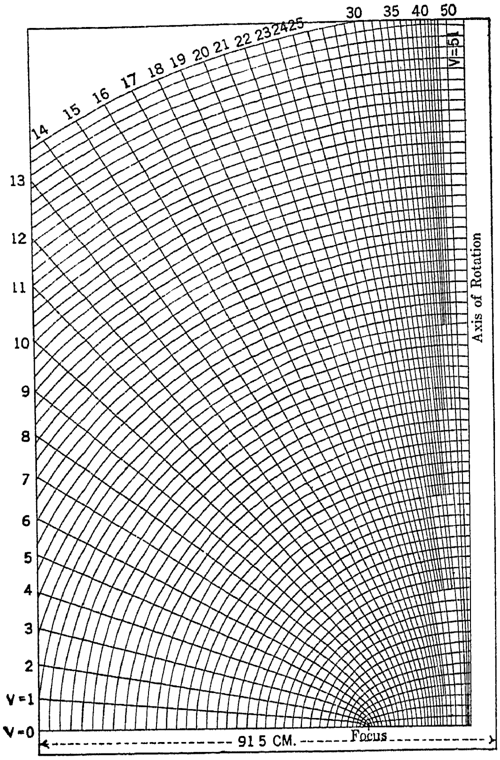

FIG. 12

POTENTIAL GRADIENT AT ANY POINT IN SPACE

The potential gradient is defined as the rate of change of potential in the direction of the greatest decrease in potential, that is, along the normal to the equipotential surface in the direction of decreasing potential. The sign will depend upon whether our unit normal is taken in the direction of increasing or decreasing potential. For the former case the expression for the potential gradient will be positive and for the latter negative. By reference to Figs. 7 and 8 and remembering that we have assumed the tank to be at zero potential it will be seen that we have selected the differential normal $d\,n_2$ in the direction of decreasing potential and, therefore, our expression for the potential gradient will be $-\dfrac{d\,V}{d\,n_2}$.

We have seen that the potential at any point in space is given by equation (**116**) and is

$$V = A\,\beta + C$$

Thus the potential gradient will be

$$-\frac{d\,V}{d\,n_2} = A\,\frac{d\,\beta}{d\,n_2} \qquad \textbf{(124)}$$

where $d\,n_2$ is the normal to the surface $\mu = \mu_0$, (a constant). and in the direction of decreasing potential.

In order to determine β we have the relation

$$\beta = \operatorname{sech}^{-1}\frac{\mu}{c}$$

Hence,

$$\frac{d\,\beta}{d\,\mu} = -\frac{c}{\mu\sqrt{c^2-\mu^2}} \qquad \textbf{(125)}$$

Now we have seen from equation (**48**) that

$$d\,n_2 = \frac{d\,\mu}{h_2} \qquad \textbf{(126)}$$

Equation (**67**) gives us the value of $\dfrac{1}{h_2{}^2}$

$$\frac{1}{h_2{}^2} = \frac{\lambda^2-\mu^2}{c^2-\mu^2}$$

$$\frac{1}{h_2} = \sqrt{\frac{\lambda^2-\mu^2}{c^2-\mu^2}} \qquad \textbf{(127)}$$

We may now substitute this value in equation (**126**) and we obtain

$$d\, n_2 = \sqrt{\frac{\lambda^2 - \mu^2}{c^2 - \mu^2}}\, d\, \mu \tag{128}$$

Thus,

$$\frac{d\, \mu}{d\, n_2} = \sqrt{\frac{c^2 - \mu^2}{\lambda^2 - \mu^2}} \tag{129}$$

Combining now equation (**125**) and (**129**) we have,

$$\frac{d\, V}{d\, n_2} = A\, \frac{d\, \beta}{d\, \mu} \frac{d\, \mu}{d\, n_2} \tag{130}$$

or,

$$\frac{d\, V}{d\, n_2} = - \frac{A\, c}{\mu \sqrt{c^2 - \mu^2}} \sqrt{\frac{c^2 - \mu^2}{\lambda^2 - \mu^2}} \tag{131}$$

and

$$- \frac{d\, V}{d\, n_2} = \frac{A\, c}{\mu \sqrt{\lambda^2 - \mu^2}} \tag{132}$$

The value of the potential gradient at any point in space.

The value of the constant A is that given in equation (**117**) and in order to determine the values of μ and λ in rectangular co-ordinates we have the relations given by equations (**109**) and (**111**). It will here be observed that the potential gradient is independent γ which is obvious from the symmetrical arrangement, that is, the potential gradient will be similar for any of the diametral planes γ = constant. Thus, we may write

$$\left.\begin{aligned} x &= c \sec \alpha \operatorname{sech} \beta \\ y &= c \tan \alpha \tanh \beta \end{aligned}\right\} \tag{133}$$

also,

$$\left.\begin{aligned} \lambda &= c \sec \alpha \\ \mu &= c \operatorname{sech} \beta \end{aligned}\right\} \tag{134}$$

These equations in combination with (**132**) enable us to calculate the potential gradient at any point in space in rectangular co-ordinates. (See Fig. 9.)

CURVES OF CONSTANT POTENTIAL GRADIENT

The curves of constant potential gradient may now be obtained by putting

$$-\frac{d\,V}{d\ n_2} = G = \frac{A\,c}{\mu\sqrt{\lambda^2-\mu^2}} \qquad \textbf{(135)}$$

Then

$$\mu\sqrt{\lambda^2-\mu^2} = \frac{A\,c}{G} \qquad \textbf{(136)}$$

Thus for any given condition the right hand side of equation (**136**) is a constant. If we now substitute the values of λ and μ as given in equation (**134**) we may write

$$\sec\alpha = \pm\sqrt{\operatorname{sech}^2\beta + \frac{\left(\frac{A}{c\,G}\right)^2}{\operatorname{sech}^2\beta}}$$

or for purposes of calculation we may put for

$$\operatorname{sech}^2\beta = 1 - \tanh^2\beta$$

and we have

$$\sec\alpha = \pm\sqrt{(1-\tanh^2\beta) + \frac{\left(\frac{A}{c\,G}\right)^2}{(1-\tanh^2\beta)}} \qquad \textbf{(137)}$$

where the values in terms of x and y are

$$\left.\begin{aligned} x &= c\sec\alpha\operatorname{sech}\beta \\ y &= c\tan\alpha\tanh\beta \end{aligned}\right\}$$

The curves of constant potential gradient as calculated from these equations are shown in Fig. 13.

POTENTIAL GRADIENT ALONG THE X AXIS

The value of the potential gradient at any point along the x axis between the origin and the focus may be obtained from equation (**132**) as follows:

From equation (**111**) we have

$$z = c\tan\alpha\tanh\beta$$

Now when $z = 0$

$$c \tan \alpha \tanh \beta = 0$$

and since $c \tanh \beta$ is not zero we have

$$\tan \alpha = 0$$
$$\alpha = 0$$

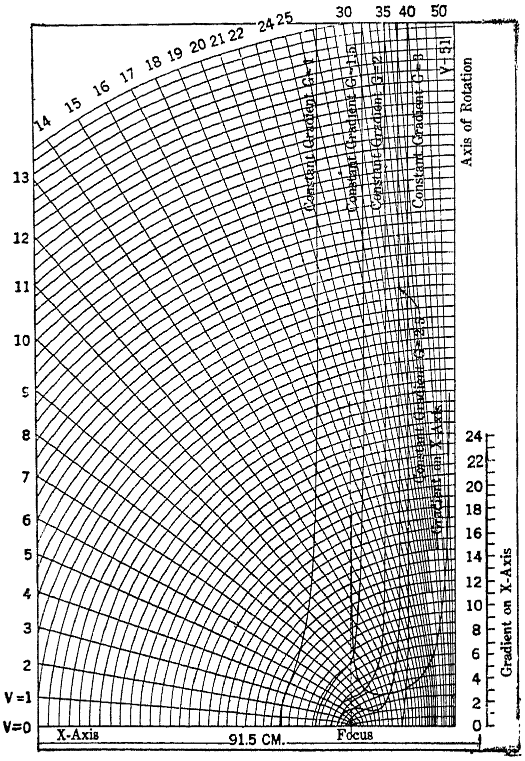

FIG. 13

Substituting this value of α in equation (**134**)

$$\lambda = c$$

and further upon substituting this value of λ in equation (**132**) we have

$$-\frac{d\,V}{d\,n_2} = \frac{A\,c}{\mu \sqrt{c^2 - \mu^2}} \tag{138}$$

The potential gradient along the x axis.

Another method of obtaining the same result is to observe that the potential gradient along the x axis is merely $-\frac{d\,V}{d\,\mu}$ since the hyperbolas cut the x axis normally. Thus as we have seen

$$V = A\,\beta + C$$

Differentiating with respect to μ

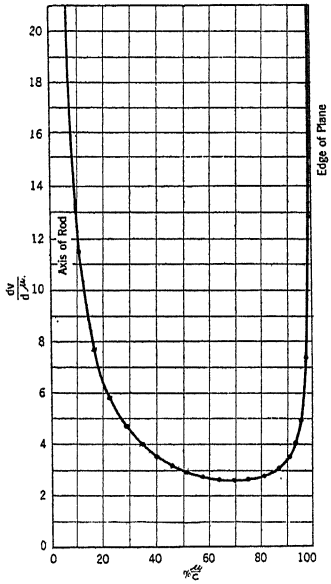

FIG. 14—CALCULATED POTENTIAL GRADIENT BETWEEN ROD AND EDGE OF PLANE

$$\frac{d\,V}{d\,\mu} = A\,\frac{d\,\beta}{d\,\mu}$$

Also,

$$\beta = \operatorname{sech}^{-1}\frac{\mu}{c}$$

Hence differentiating

$$\frac{d\,V}{d\,\mu} = A\,\frac{d\,\beta}{d\,\mu} = \frac{-A\,c}{\mu\,\sqrt{c^2 - \mu^2}}$$

Thus,

$$-\frac{d\,V}{d\,\mu} = \frac{A\,c}{\mu\,\sqrt{c^2 - \mu^2}} \qquad \textbf{(139)}$$

which is identical with equation (**138**) and is the potential gradient along the x axis between the origin and the focus. The potential gradient curve for this case is calculated and plotted in the curve Fig. 14.

Testing this equation for the values $\mu = 0$ and $\mu = c$ it will be observed that the potential gradient is infinite at the edge of the hole in the plane or focus and at the origin or along the z axis. A minimum value of the gradient will exist somewhere between these two limits. Its value will be found in the usual manner as

$$\frac{d}{d\,\mu}\left(\frac{d\,V}{d\,\mu}\right) = 0$$

Thus differentiating equation (**139**) with respect to μ and equating the result to zero we have

$$\frac{d^2\ V}{d\ \mu^2} = \frac{A\ c\,[\,(c^2 - \mu^2) - \mu^2]}{\mu^2\ (b^2 - \mu^2)^{3/2}}$$

$$= \frac{A\ c\,[\,(c^2 - 2\,\mu^2)\,]}{\mu^2\ (b^2 - \mu^2)^{3/2}}$$

Equating to zero and observing that $A\ c$ is not zero we have

$$c^2 = 2\,\mu^2$$

$$\mu = \frac{c}{\sqrt{2}} \tag{140}$$

Thus the minimum value of the potential gradient occurs at this point which is about 71 per cent of the distance from the axis towards the edge of the hole in the infinite plane.

The potential gradient curve enables us to determine the proper size of central electrode so as not to over stress the dielectric. Or by selecting the hyperboloid which we choose to represent our tank with hole we can select a central electrode which has the same value of the potential gradient or any other value which we may desire for other reasons. Knowing the characteristics of our dielectric used we can determine the puncture voltage of the apparatus, etc.

Solution for the Tubes of Flow or Flux

The flux or flow across any area may be defined as the surface integral of the normal component of the flux density, and as we have seen from equations (**56**) and (**57**) this in turn is equal to the normal component of the potential gradient.

Thus,

$$\left.\begin{aligned} &\int\!\!\int - \frac{d\ V}{d\ n_2}\, d\,n_1\, d\,n_3 \\ \text{or}\quad &\int\!\!\int_s - \frac{d\ V}{d\ n_2}\, d\,S_2 \end{aligned}\right\} \tag{141}$$

We have from equation (**132**)

$$-\frac{d\ V}{d\ n_2} = \frac{A\ c}{\mu\,\sqrt{\lambda^2 - \mu^2}}$$

We also have for the values of $d\,n_1$ and $d\,n_3$ or $d\,S_2$, equation (**51**)

$$d\,S_2 = d\,n_1\,d\,n_3 = \frac{d\,\lambda\,d\,\nu}{h_1\,h_3} \qquad (\mathbf{142})$$

and from equations (**65**) and (**69**)we have

$$\frac{1}{h_1} = \sqrt{\frac{\lambda^2 - \mu^2}{\lambda^2 - c^2}}$$

$$\frac{1}{h_3} = \frac{\lambda\,\mu}{c\,(1 + \nu^2)}$$

Substituting these values in (**142**) we get

$$d\,S_2 = \sqrt{\frac{\lambda^2 - \mu^2}{\lambda^2 - c^2}}\left(\frac{\lambda\,\mu}{c\,(1 + \nu^2)}\right) d\,\lambda\,d\,\nu$$

and combining in equation (**141**) and simplifying we get

$$\int\!\!\int_{S_2} - \frac{d\,V}{d\;n_2}\,d\,S_2 = \frac{A\,\lambda}{\sqrt{\lambda^2 - c^2}\,(1 + \nu^2)}\,d\,\lambda\,d\,\nu$$

$$= A \int \frac{d\,\nu}{(1 + \nu^2)} \int \frac{\lambda\,d\,\lambda}{\sqrt{\lambda^2 - c^2}}$$

Integrating the first part between the limits $\nu = 0$ and $\nu = \nu_0$ we have,

$$\Big[\tan^{-1}\nu + K\Big]_{\nu=0}^{\nu=\nu_0} = -\tan^{-1}\nu_0$$

and since ν_0 may have any value we can neglect it as far as the spacing of the surfaces of flow are concerned. Integrating the second part between the limits $\lambda = \lambda_0$ and $\lambda = \lambda_1$ we have

$$\int_{\lambda=\lambda_0}^{\lambda=\lambda_1} \frac{\lambda\,d\,\lambda}{\sqrt{\lambda^2 - c^2}} = \Big[\sqrt{\lambda^2 - c^2} + K\Big]_{\lambda=\lambda_0}^{\lambda=\lambda_1}$$

That is we have

$$\int_{S_2}\int - \frac{d\ V}{d\ n_2}\, d\, S_2 = A\ \left(\sqrt{\lambda_0{}^2 - c^2} - \sqrt{\lambda_1 - c^2}\right) \tag{143}$$

If we now remember that from equation (**109**)

$$\lambda = c \sec \alpha$$

we may write

$$\int_{S_2}\int - \frac{d\ V}{d\ n_2}\, d\, S_2 = A\ \ c\ \left(\sqrt{\sec^2 \alpha_0 - 1} - \sqrt{\sec^2 \alpha_1 - 1}\right) \tag{144}$$

Also, $\sec^2 \alpha - 1 = \tan^2 \alpha$

Substituting we get

$$\int_{S_2}\int - \frac{d\ V}{d\ n_2}\, d\, S_2 = A\ (c \tan \alpha_0 - c \tan \alpha_1) \tag{145}$$

But it will be observed from Fig. 9 that $c \tan \alpha$ is the minor axis of an oblate spheroid, and, therefore, for equal tubes of flow the surfaces will have any constant spacing on the minor axis of the spheroid. (See Figs. 12 and 13.)

In the above work we have obtained the solution of the electrostatic problem in which any two confocal hyperboloids of revolution of one sheet and of the same family are maintained at definite potentials. The solution of this problem should be of interest to engineers as it furnishes us with a solution which may be applied as an approximation to many engineering problems. For example, it is a useful guide in studying the high-voltage bushing problem; it also gives us an interesting variety of possible electrode shapes for use in testing insulating materials, where it is very desirable to be able to calculate the gradients, etc., etc.

Another problem which should also be of considerable interest to engineers is the case when any two confocal hyperboloids of revolution of two sheets and of the same family are maintained at definite potentials. The solution of this problem may be applied as an approximation to another group of problems which are of frequent occurrence in engineering. For example, it may be applied to switch electrodes at various spacings; also as an approximation in vacuum-tube designs, such as X-ray tubes, kenotrons, etc. That is, it is an approximation to the group of problems of two elongated electrodes at various spacings, or such as an electrode and a plane (*i.e.*, two needle points or a needle and a plane).

The solution for this case is obtained in exactly the same manner as that described above except that in this case we select as our standard ellipsoid the prolate spheroid and express Laplace's equation in normal prolate spheroidal co-ordinates.

Two other problems namely, the distribution of the electrostatic field about the charged oblate or prolate spheroids are obtained in exactly the same manner.

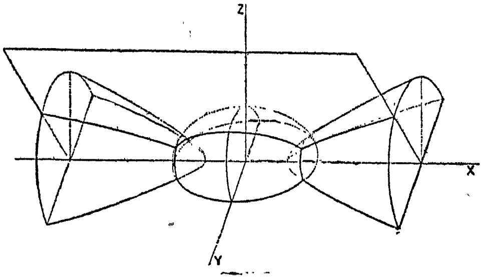

FIG. 15—INTERSECTION OF PROLATE SPHEROID, CONFOCAL HYPERBOLOID OF TWO SHEETS AND A PLANE THROUGH THE AXIS OF ROTATION

NORMAL PROLATE SPHEROIDAL CO-ORDINATES[39]

In this case we take for our standard ellipsoid the prolate spheroid, the axis of rotation is then the major axis and our plane must contain that axis. Taking the x axis as axis of rotation and the XY plane as plane of reference we have for the equation of our three orthogonal surfaces the following. (See Fig. 15).

$$\left.\begin{aligned} \frac{x^2}{\lambda^2} + \frac{y^2}{\lambda^2 - b^2} + \frac{z^2}{\lambda^2 - b^2} &= 0 \\ \frac{x^2}{\mu^2} + \frac{y^2}{\mu^2 - b^2} + \frac{z^2}{\mu^2 - b^2} &= 0 \\ z - \nu\, y &= 0 \end{aligned}\right\} \qquad (146)$$

---

[39] See Byerly Fourier's Series and Spherical Harmonics, page 243, from which this discussion has been taken.

where $\lambda^2 > b^2 > \mu^2$
Here, b, is the semi-major axis of the prolate spheroid.
Solving the three equations simultaneously for the values of x, y, z as functions of λ, μ and ν as we did in the oblate spheroidal problem, and then differentiating these partially with respect to λ, μ and ν we will have functional equations as before, namely;

$$\frac{\partial x}{\partial \lambda} = f_1 (\lambda, \mu, \nu)$$

$$\frac{\partial y}{\partial \lambda} = f_2 (\lambda, \mu, \nu)$$

$$\frac{\partial z}{\partial \lambda} = f_3 (\lambda, \mu, \nu)$$

and similar relations for x, y, z with respect to μ and ν. We also have as before

$$\left.\begin{aligned} \frac{1}{h_1^2} &= \left(\frac{\partial x}{\partial \lambda}\right)^2 + \left(\frac{\partial y}{\partial \lambda}\right)^2 + \left(\frac{\partial z}{\partial \lambda}\right)^2 \\ \frac{1}{h_2^2} &= \left(\frac{\partial x}{\partial \mu}\right)^2 + \left(\frac{\partial y}{\partial \mu}\right)^2 + \left(\frac{\partial z}{\partial \mu}\right)^2 \\ \frac{1}{h_3^2} &= \left(\frac{\partial x}{\partial \nu}\right)^2 + \left(\frac{\partial y}{\partial \nu}\right)^2 + \left(\frac{\partial z}{\partial \nu}\right)^2 \end{aligned}\right\}$$

and obtain the following values

$$\left.\begin{aligned} h_1^2 &= \frac{\lambda^2 - b^2}{\lambda^2 - \mu^2} \\ h_2^2 &= \frac{b^2 - \mu^2}{\lambda^2 - \mu^2} \\ h_3^2 &= \frac{b^2 (1 + \nu)^2}{(\lambda^2 - b^2)(b^2 - \mu^2)} \end{aligned}\right\} \qquad \textbf{(147)}$$

From this we have

$$\left.\begin{aligned} d\,n_1 &= \frac{d\,\lambda}{h_1} = \sqrt{\frac{\lambda^2-\mu^2}{\lambda^2-b^2}}\,d\,\lambda \\ d\,n_2 &= \frac{d\,\mu}{h_2} = \sqrt{\frac{\lambda^2-\mu^2}{b^2-\mu^2}}\,d\,\mu \\ d\,n_3 &= \frac{d\,\nu}{h_3} = \sqrt{\frac{(\lambda^2-b^2)\,(b^2-\mu^2)}{b^2\,(1+\nu^2)^2}}\,d\,\nu \end{aligned}\right\} \quad \textbf{(148)}$$

and Laplace's equation in terms of our spheroidal co-ordinates λ, μ and ν becomes.

$$\frac{1}{b^2\,(1+\nu^2)}\,\frac{\partial}{\partial\,\lambda}\left[(\lambda^2-b^2)\,\frac{\partial\,V}{\partial\,\lambda}\right]+\frac{1}{b^2\,(1+\nu^2)}\,\frac{\partial}{\partial\,\mu}\left[(b^2-\mu^2)\,\frac{\partial\,V}{\partial\,\mu}\right]+\frac{\lambda^2-\mu^2}{(\lambda^2-b^2)\,(b^2-\mu^2)}\,\frac{\partial}{\partial\,\nu}\left[(1+\nu^2)\,\frac{\partial\,V}{\partial\,\nu}\right]=0 \quad \textbf{(149)}$$

and reduces in exactly the same manner as the oblate spheroid problem to

$$\frac{1}{\lambda^2-b^2}\,\frac{\partial^2\,V}{\partial\,\alpha^2}+\frac{1}{b^2-\mu^2}\,\frac{\partial^2\,V}{\partial\,\beta^2}+\frac{\lambda^2-\mu^2}{(\lambda^2-b^2)\,(b^2-\mu^2)}\,\frac{\partial^2\,V}{\partial\,\gamma^2}=0 \quad \textbf{(150)}$$

Where

$$\left.\begin{aligned} d\,\alpha &= -\frac{b}{\lambda^2-b^2}\,d\,\lambda \\ d\,\beta &= \frac{b}{b^2-\mu^2}\,d\,\mu \\ d\,\gamma &= \frac{1}{1+\nu^2}\,d\,\nu \end{aligned}\right\} \quad \textbf{(151)}$$

$$\left.\begin{aligned} \alpha &= \coth^{-1}\frac{\lambda}{b} \\ \beta &= \tanh^{-1}\frac{\mu}{b} \\ \gamma &= \tan^{-1}\nu \end{aligned}\right\} \qquad (\mathbf{152})$$

or,

$$\left.\begin{aligned} \lambda &= b\coth\alpha \\ \mu &= b\tanh\beta \\ \nu &= \tan\gamma \end{aligned}\right\} \qquad (\mathbf{153})$$

Substituting these values in equation (**150**) we obtain Laplace's equation in the form

$$\sinh^2\alpha\,\frac{\partial^2 V}{\partial\,\alpha^2} + \cosh^2\beta\,\frac{\partial^2 V}{\partial\,\beta^2} + (\sinh^2\alpha + \cosh^2\beta)\,\frac{\partial^2 V}{\partial\,\gamma^2} = 0 \qquad (\mathbf{154})$$

In using this equation it is to be noted that the point $(\alpha,\ \beta,\ \gamma)$ is the point of intersection of a prolate spheroid whose semi-axes are

$$\left.\begin{aligned} &b\coth\alpha \text{ major} \\ &b\,\text{csch}\,\alpha \text{ minor} \end{aligned}\right\} \qquad (\mathbf{155})$$

a biparted hyperboloid of revolution whose semi-axes are

$$\left.\begin{aligned} &b\tanh\beta \text{ transverse axis} \\ &b\,\text{sech}\,\beta \text{ conjugate axis} \end{aligned}\right\}$$

and a plane containing the axis of revolution and making the angle γ with a fixed plane. If the fixed plane of reference is that of XY the rectangular co-ordinates of any point in space $(\alpha,\ \beta,\ \gamma)$ are

$$\left.\begin{aligned} x &= b\coth\alpha\tanh\beta \\ y &= b\,\text{csch}\,\alpha\,\text{sech}\,\beta\cos\gamma \\ z &= b\,\text{csch}\,\alpha\,\text{sech}\,\beta\sin\gamma \end{aligned}\right\} \qquad (\mathbf{156})$$

and α may range from $+\infty$ to 0, β from $-\infty$ to $+\infty$, and γ from 0 to 2π. Negative values of β are to be taken for points lying to the left of a plane through the origin perpendicular to the axis of revolution; in this case, to the left of the *YZ* plane. (See Fig. 16).

POTENTIAL AT ANY POINT IN SPACE

If we are given the potentials of any two confocal hyperboloids of two sheets, that is, for any two values of β we can obtain the potential at any point in space between them. For in that case we have seen exactly as in the oblate spheroid problem that V is a function of β only, hence in Laplace's equation

$$\frac{\partial^2 V}{\partial \alpha^2} = 0 \text{ and } \frac{\partial^2 V}{\partial \gamma^2} = 0$$

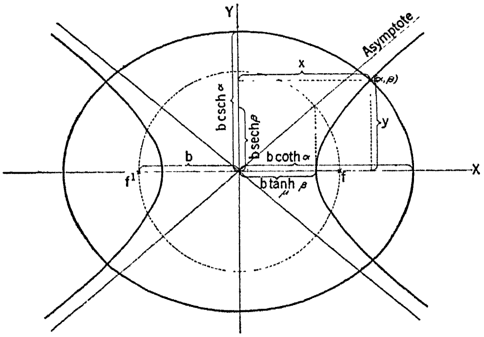

FIG. 16—CONFOCAL ELIPSE AND HYPERBOLA

Since $\cosh^2 \beta$ is not equal to zero for all values of β Laplace's equation reduces in our case to

$$\frac{\partial^2 V}{\partial \beta^2} = 0 \tag{157}$$

The solution of which is seen to be

$$V = A\,\beta + C \tag{158}$$

The constants A and C will be determined from the boundary conditions of the problem as before.
Assuming the case

$$\left.\begin{aligned} V &= 0 \text{ when } \beta = 0 \\ V &= V_1 \text{ when } \beta = \beta_1 \end{aligned}\right\} \tag{159}$$

Substituting these values in equation (**158**) we get

$$\left.\begin{aligned} C &= 0 \\ A &= \frac{V_1}{\beta_1} \end{aligned}\right\}$$

Thus we may write our solution under these boundary conditions simply as

$$V = A \cdot \beta \tag{160}$$

or since $\beta = \tanh^{-1} \frac{\mu}{b}$

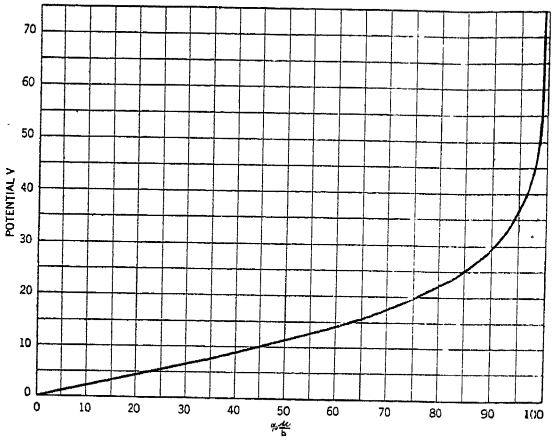

FIG. 17—CALCULATION OF POTENTIAL DISTRIBUTION ALONG THE X-AXIS

we may write it in terms of μ and b as follows.

$$V = \frac{V_1}{\tanh^{-1} \frac{\mu_1}{b}} \tanh^{-1} \frac{\mu}{b} \tag{161}$$

This equation enables us to calculate the potential at any point along the x axis between the foci and has been plotted in Fig. 17.

A diagram of the field has been constructed in the same manner as described above for the oblate problem. (See Fig. 18)

Potential Gradient at any Point in Space

The potential gradient will be

$$\frac{d\,V}{d\,n_2} = A\,\frac{d\,\beta}{d\,n_2} \tag{162}$$

where $d\,n_2$ is the elementary normal to the surface of the hyperboloid (say for $\mu = \mu_0$) under consideration. The sign $\frac{d\,V}{d\,n_2}$ which expresses the potential gradient is here taken as positive that is, $d\,n_2$ is assumed to be in the direction of increasing potential.

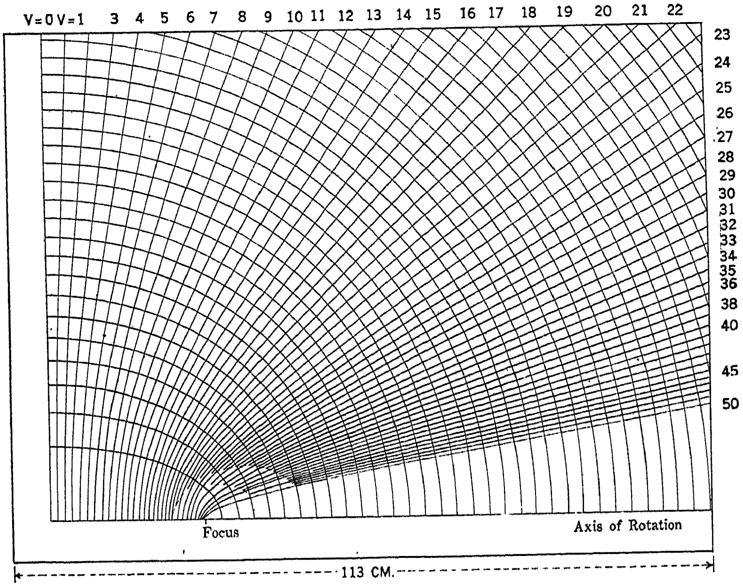

Fig. 18

In order to determine β we have the relation

$$\beta = \tanh^{-1}\frac{\mu}{b}$$

Hence,

$$\frac{d\,\beta}{d\,\mu} = \frac{b}{b^2 - \mu^2} \tag{163}$$

We also have

$$d\,n_2 = \frac{d\,\mu}{h_2}$$

$$d\,n_2 = \sqrt{\frac{\lambda^2 - \mu^2}{b^2 - \mu^2}}\,d\,\mu$$

and

$$\frac{d\,\mu}{d\,n_2} = \sqrt{\frac{b^2 - \mu^2}{\lambda^2 - \mu^2}} \tag{164}$$

Thus for the gradient we have from equations (**163**) and (**164**)

$$\frac{d\,V}{d\,n_2} = A\,\frac{d\,\beta}{d\,\mu}\,\frac{d\,\mu}{d\,n_2} = A\,\frac{b}{b^2 - \mu^2}\sqrt{\frac{b^2 - \mu^2}{\lambda^2 - \mu^2}}$$

Simplifying

$$\frac{d\,V}{d\,n_2} = \frac{A\,b}{\sqrt{(b^2 - \mu^2)\,(\lambda^2 - \mu^2)}} \tag{165}$$

which is the potential gradient at any point in space. It is observed that the potential gradient is independent of γ as is obvious from the symmetry of the problem. We may, therefore, express the potential gradient at any point in space in rectangular co-ordinates with the help of the following relations (See Fig. 16).

$$\left.\begin{aligned} \lambda &= b \coth \alpha \\ \mu &= b \tanh \beta \end{aligned}\right\}$$

and

$$\left.\begin{aligned} x &= b \coth \alpha \tanh \beta \\ y &= b \operatorname{csch} \alpha \operatorname{sech} \beta \end{aligned}\right\}$$

Curves of Constant Potential Gradient

The curves of constant potential gradient may now be obtained by putting equation equal to the various constant values desired.

Thus,

$$\frac{d\,V}{d\,n_2} = G = \frac{A\,b}{\sqrt{(b^2 - \mu^2)\,(\lambda^2 - \mu^2)}}$$

$$\sqrt{(b^2 - \mu^2)\,(\lambda^2 - \mu^2)} = \frac{A\,b}{G} \tag{166}$$

Thus for any given condition and value of potential gradient G the right hand member of this equation is a constant.

If we now substitute the values of λ and μ we may write it for the purpose of calculation.

$$\coth \alpha = \pm \sqrt{\tanh^2 \beta + \frac{\left(\frac{A}{G\,b}\right)^2}{\operatorname{sech}^2 \beta}}$$

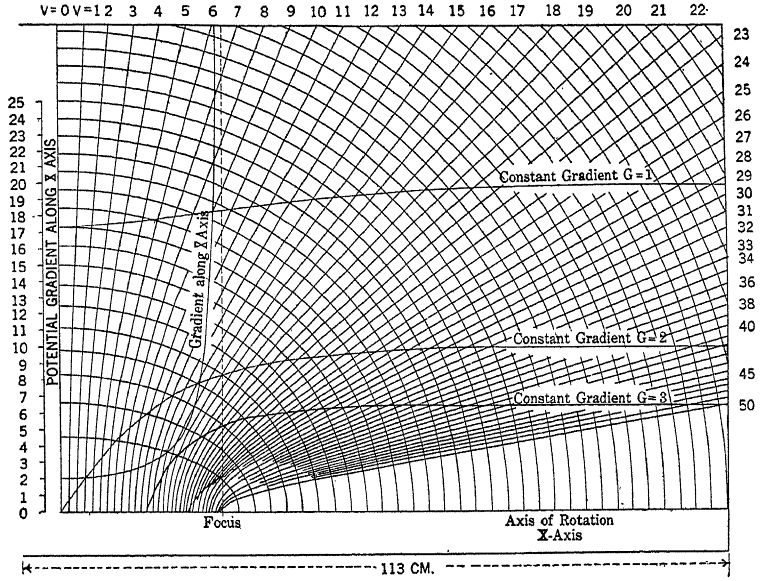

FIG. 19

or if tables of $\operatorname{sech}^2 \beta$ are not available we can write it

$$\coth \alpha = \pm \sqrt{\tanh^2 \beta + \frac{\left(\frac{A}{G\,b}\right)^2}{1-\tanh^2 \beta}} \qquad \textbf{(167)}$$

The values of x and y are

$$\left.\begin{aligned} x &= b \coth \alpha \tanh \beta \\ y &= b \operatorname{csch} \alpha \operatorname{sech} \beta \end{aligned}\right\}$$

The curves of constant potential gradient as calculated from these equations are shown in Fig. 19.

POTENTIAL GRADIENT ALONG THE X AXIS

The value of the potential gradient at any point along the x axis between the origin and the foci may be found directly from the general equation (**165**) or independently as $\frac{d\,V}{d\,\mu}$.

It is the value obtained when $y = 0$

$$y = 0 = b \operatorname{coch} \alpha \operatorname{sech} \alpha$$

and since b sech β is not zero we have

$$\operatorname{csch} \alpha = 0$$

$$\frac{1}{\sinh \alpha} = 0$$

$$\alpha = \infty$$

Substituting this value in the equation

$$\lambda = b \coth \alpha$$

we have $\coth \infty = 1$

and $\lambda = b$

Hence substituting $\lambda = b$ in our equation (**165**) for the potential gradient we get

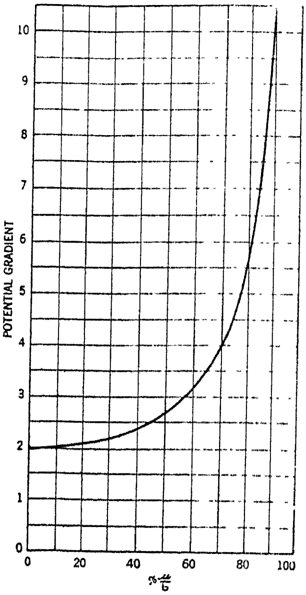

FIG. 20—CALCULATED POTENTIAL GRADIENT ALONG THE X-AXIS

$$\frac{d\,V}{d\,n_2} = \frac{A\,b}{\sqrt{(b^2-\mu^2)\,(b^2-\mu^2)}} = \frac{A\,b}{b^2-\mu^2} \qquad \textbf{(168)}$$

for the value of the potential gradient along the x axis between the foci. The curve is shown in Fig. 20.

SOLUTION FOR THE TUBES OF FLOW OF FLUX

The total flux across any area may be defined as the surface integral of the normal component of the flux density; in the case of air dielectric, flux density and gradient are equivalent. We may, therefore, write it

$$\int\!\!\int \frac{d\,V}{d\,n_2}\, d\,n_1\, d\,n_3 = \int_{S_1}\!\!\int \frac{d\,V}{d\,n_2}\, d\,S_2$$

we have seen that

$$\frac{d\ V}{d\ n_2} = \frac{A\ b}{\sqrt{(b^2 - \mu^2)\ (\lambda^2 - \mu^2)}}$$

$$d\ n_1 = \sqrt{\frac{\lambda^2 - \mu^2}{\lambda^2 - b^2}}\ d\ \lambda$$

$$d\ n_3 = \sqrt{\frac{(\lambda^2 - b^2)\ (b^2 - \mu^2)}{b^2\ (1 + \nu^2)^2}}\ d\ \nu$$

Substituting these values and simplifying we get

$$\int\!\!\int \frac{A}{1 + \nu^2}\ d\ \lambda\ d\ \nu$$

As far as the spacing of the surfaces of flow is concerned we may neglect the integration with respect to ν by reason of the symmetry of the tubes of flow about the axis of rotation.
We thus have,

$$\int\!\!\int_{S_2} \frac{d\ V}{d\ n_2}\ d\ S_2 = A\ (\lambda_0 - \lambda_1)$$

But we have seen that

$$\lambda = b \coth \alpha$$

also that $b \coth \alpha$ is the major semi-axis of the prolate spheroids. Therefore, equal tubes of flux will be obtained by taking equal spaces along the major axis of the spheroids. (See Figs. 18 and 19).

DISCUSSION ON "AN EXPERIMENTAL METHOD OF OBTAINING THE SOLUTION OF ELECTROSTATIC PROBLEMS WITH NOTES ON HIGH-VOLTAGE BUSHING DESIGN," (RICE), NEW YORK, N. Y., NOVEMBER 9, 1917.

Ralph Mershon: There is one matter on which I shall be glad if Mr. Rice can give us some light. That is, as to what will happen in case the bushing has an arc-over backed up by considerable power; and as to what work, if any, has been done in devising means of protecting bushings from the effect of power arcs. This is a subject which is not perhaps immediately germane to Mr. Rice's paper, but which certainly is germane to the matter of bushings and insulators. It is a subject which has interested many of us for a long time.

Some time ago I conceived the idea that if we could build an insulator in the form of a framed structure the members of which were made up of units of massive porcelain we would have an insulator which it would be impossible to puncture and that if the members were so arranged that in case of a flash-over the resulting power arc could rise freely from them, we would have an insulator which would also be free from damage by power arcs.

After a great deal of difficulty I succeeded in having made out of porcelain the units for constructing the members of the structural insulator. These units were made up of solid porcelain, with comparatively small petticoats; they each had a net length of about nine inches. They were connected together end to end to form the structural member by cementing the ends into metal collars. The shape of the completed member was a good deal that of the outside of the bushing shown in Fig. 3 of Mr. Rice's paper. The manufacture of the units involved the production of massive porcelain absolutely free from flaws and having a high tensile strength. The units as constructed could be flashed over indefinitely under oil without puncture. They had a strength in tension of from 13,000 to 16,000 pounds, which meant, as I remember it, a unit stress of about 2000 pounds per square inch.

These insulators tested out very nicely with a testing transformer. It was absolutely impossible to puncture them, and the arc following a flash-over immediately rose from the surface of the insulator, as contemplated. But when the insulators were subject to a flash-over backed up by large power capacity the story was an entirely different one.

Through the kindness of the Pennsylvania Water & Power Company a portion of their generating plant was put at my disposal for power arc tests. A test was made on several of the porcelain units above described, assembled as they would be in one member of a structural insulator. This structural member was placed in a horizontal position and arcs started across its surface by means of fine wire fuses. In every case the arc instead of rising from the porcelain units as it had done in the case of testing transformers, and as it seemed to me it ought to, would not rise at all. It hugged in close against the petticoats and peeled them off, one by one. If the arc was allowed to continue long enough, it took pieces from the porcelain core, after having denuded it of the petticoats. The endeavor was made to protect the units from the power arc by arcing-rings, or similar devices, but without success. The power arc in every case stubbornly persisted in clinging close to the surface of the porcelain units. I have never been able to satisfactorily account for the peculiar behavior of the power arcs in this experiment, or to explain why the behavior should be so different in the case of heavy power arcs and the light arcs obtained from a source of limited capacity.

In the case of the bushing shown in this paper, which will presumably be used on transformers, switches, etc., to go out of doors, and which may arc over at times, no matter how carefully they may be designed or how great their factor of safety, I wonder what, if any, provision is contemplated to protect them from the effects of an arc backed up by a lot of power. If it is intended that they shall not arc over at all, we might recall some of the experiences we have had in the past with apparatus which was not going to do certain things.

C. O. Mailloux: The method described by the author, aside from its practical value as a means of graphical representation of the distribution of points of equal stress and of the course of the lines of action of force and energy between points having different degrees of stress, is of the highest theoretical interest.

The author has made a very important contribution to our working tools in physics, and to our methods of dealing with fields of force and their distribution in space. We find, in this paper, what is perhaps the first instance where engineers have been called upon to make personal acquaintance with the well known and time-honored "Equation of Laplace." I do not remember ever having seen, before, a paper not on a distinctly and purely physical subject in which that equation made its appearance. Anybody who has studied the potential function and who has made a passing acquaintance with Laplace's equation has acquired a great respect for it. He has learned that it is, indeed, a most wonderful means of analysis and a most potent tool for the physicist. It is agreeable to find that it will no longer be monopolized by the physicist, but that the electrical engineer is now going to make use of it. If, fifteen or twenty years ago, anyone had said that the electrical engineer would some day be able to handle the potential function and to use Laplace's equation as beautifully and as effectively as it is done in this paper, he would not have been believed.

It is especially in the study of fields of force in tri-dimensional space,—the most interesting, but also the most difficult field of physical research—that the equation of Laplace has proved a most wonderful instrument to the mathematical physicist. It has done this by virtue of certain remarkable properties which enable it to render valuable aid in the specification and the determination of the physical conditions obtaining in fields of force at the stage of action of the forces involved, at which a state of balance or equilibrium, either transitional or permanent, occurs. Its range of adaptability to fields of force problems is very great, being, indeed limited only by the mathematical knowledge and skill employed in its use. It can, theoretically, be applied to fields of force in which the lines of force, or the lines of flow of energy, follow paths of the most diversified character from the simplest, like straight parallel lines, to the most complex, like some of the cases discussed in the paper.

What interested me very much is the very ingenious development of methods by which the author, starting from the consideration of Laplace's equation in ordinary rectangular coordinates, develops and applies it to a very complex system of coordinates, designated as "Normal prolate spheroidal coordinates." That may be an awe-inspiring name to many of you, but it is a name well worth becoming familiar with. Even though it may take some time and study to learn all of the mathematics which precede it, it is well worth while. Laplace's equation has, as I just said, some remarkable properties; and one of its most remarkable properties, is that it enables physicists to reason about the phenomena incidental to the action of physical forces, and to reach absolutely logical, rigorous, scientific and true conclusions without the need of any postulates or any assumptions as to the intervening media, in other words, as to the nature and properties of the ether or of any medium in space through which force acts, and in which fields of force are produced by physical forces. It was, in my opinion, one of the great achievements of Laplace, that he was able to devise an instrument of thought, an instrument of mathematical analysis, the use of which was independent of any such complicated, worrisome and perplexing postulates. With Laplace's equation, used in each case in manner suitable for the purpose in view, it is possible to deal with the distribution of force under a great variety of circumstances, to determine exactly what is going to happen, and to obtain very interesting, useful and beautiful results.

The interesting feature of the mathematical part of this paper is that by the modification of the system of co-ordinates which the author has worked out, he is able to apply and utilize Laplace's equation in the study of the distribution, flow, and apportionment of forces under conditions where the lines of flow are no longer as you would find them in space which is free from all constraints or boundary conditions, but such as they are in certain special conditions, where the field is distorted by constraints and barriers, causing various reactions, as described in the paper and as shown in the diagrams.

The author's initial statement in the Appendix is almost contradicted by the fact that he himself has seemingly succeeded in finding solutions of Laplace's equation, which do fulfill at least some, if not all, of the arbitrary conditions involved in the problems. Further success and a nearer approach to the goal will result, presumably, from further modifications of coordinate systems still better adapted than those thus far developed and used for expressing the points of freedom, and the constraints necessary in the given case.

In conclusion, I consider this one of the most interesting mathematical papers that has been presented before the American Institute of Electrical Engineers in a long time. It is interesting, not only on account of the useful application made to an immediate practical purpose, but also because, it shows a new development and an extension of a method of using the equation of Laplace, and at the same time contains ideas which may be followed by others, and may lead to still further extensions of the method, so that, in presenting a new method of study of phenomena occurring in fields of force, the author has incidentally made a very interesting contribution to applied mathematics.

John B. Taylor: I am rather curious to know how you get such beautiful curves in a method where the devices introduced for observing the quantity you want to know, change the conditions under which you are working; that is, how is it possible to get this exploring point all through the medium without making apparent a change in conditions. It is trite to say when you put a voltmeter in the circuit you shall not draw so much current that the voltage is made different, or that an ammeter shall not have so much resistance that the current is changed. In acoustics we have difficulty in determining the form of vibration in the air, because the diaphragm usually changes the vibration at that particular point, and doubtless similar difficulties exist here, and I will be glad, if it is not covered in the paper, to hear a word in reply as to how that has been regarded.

A. M. Gray: I have had considerable success with the following photographic method of obtaining the dielectric flux lines in two-dimensional problems. To obtain the stress distribution, in slot insulation, for example, a model of part of the slot and conductors was made of metal as shown in Fig. 1, and placed on a photographic plate, the two pieces were connected to the high-voltage terminals of a transformer, the low-tension side of which was excited by direct current, and an impulse of high voltage was applied across the insulation by opening the direct-current circuit. No flash was visible but on developing the plate the flux lines showed up beautifully.

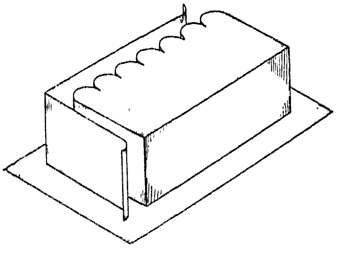

FIG. 1

To test whether or not this picture is a true representation of the stress distribution, two or three simpler cases were tried which could be solved mathematically, and the agreement was such as to justify the use of this experimental method for more complicated cases.

The application of the method to the three-dimensional problem of a rod passing through a plate would be more difficult. A small sector parallel to the lines of flux would have to be placed in the field and made of material which had the same permittivity as the film, the stress distribution due to a transient voltage, as well as that due to the contant potential from a static machine, could then be determined.

Charles L. Fortescue: In his text the author has several times referred to the paper by Mr. Farnsworth and myself on "Air as an Insulator in the Presence of Insulating Bodies of Higher Specific Inductive Capacity." The object of that paper was to show that dielectrics even though far from ideal do not substantially weaken the air path in contact with them, provided these dielectrics are introduced into the field in such a manner that no distortion is produced. Several errors crept into the paper, chiefly due to the limited time available for carrying out the work. Among those errors is that one pointed out by Mr. Rice in the present paper, namely, the use of circles for cross-sections of equipotential surfaces for the rod and ring problem. It was the original intention to work out the proper solution by the trial and error method, which Mr. Rice has described in his paper and the system of circles was taken as a first approximation, but the work proved so laborious that it was finally given up on account of lack of time.

While it would be extremely interesting from a theoretical point of view to obtain a solution of Laplace's equation which could be applied to any boundary conditions, it is more important from an engineering point of view to obtain a simple and accurate experimental method for determining the equipotential surfaces and flow lines for any system of insulated bodies. Mr. Rice's work is a valuable contribution in this direction.

The principles outlined in our original paper have been a great aid to me in determining the best form for insulating structures, but the lack of facilities for plotting the field form experimentally have been a great drawback. I hope to be able to set up a permanent outfit for carrying out such work in the future.

Referring to Mr. Rice's criterion for maximum efficiency in an insulator combining air and other materials, I wish to remark that it is the same as that given by us in our paper, although Mr. Weed among others, seems to find in the paper a general statement to the effect that any surface following a line of force was the strongest surface. What was actually intended, and I think brought out clearly in that paper, was the principle that the surface of the insulating body should conform to some system of flow lines and the electrodes should be so designed as to make the intensities along the surface as nearly uniform as possible. It appears probable that when substantial uniformity of stress is not present, it may be obtained by slightly deforming the insulation from the shape of the flow line.

Referring to Mr. Rice's calculation for the arcing over of the rubber test piece described in our paper, I regret to have to inform him that his conclusions do not agree with results of tests and since perhaps the original statement in our paper for the breakdown of this test piece led him astray, we owe him some explanation. The original test was made on a very jumpy circuit, as Mr. Chubb and I found to our great discomfort when calibrating the sphere gaps. The value quoted was the lowest value obtained, whereas, as I have pointed out in connection with the sphere-gap calibrations, on such a circuit, with apparatus on tests of such a character, the high values are the correct ones to use, as the low ratios are due to surges which the measuring devices do not record.

We found on this test piece that it did not make any appreciable difference in breakdown voltage if it were dirty or not. We let it lie on the floor for three months to accumulate a heavy coating of dust, and it tested just as high as when clean.

Later on, the electrodes were carefully re-cemented in such a way as to do away with the possibility of air pockets, etc., and when tested again under better conditions the breakdown value by ratio lay between 180 kv. and 190 kv. With proper corrections for crest value, this would represent an effective value of somewhere between 197,000 and 218,000 volts. Tests on this test piece were witnessed by a number of people, among them Prof. H. J. Ryan and Mr. Lieb.

I wish to take this opportunity to correct some wrong conceptions that have formed in connection with the condenser terminal. In some cases wrong descriptions have appeared in books on electric theory. I consider it rather regrettable that authors do not take the trouble to inform themselves as to the correctness of the information before presenting it. The condenser terminal is made with equal increments of length of the metallic cylinders for equal increments in their potential. Thus the adjacent pairs of cylinders have all the same capacity and decrease in length by equal steps as their diameter increases.

In my paper on "The Application of a Theorem of Electrostatics to Insulation Problems," I called attention to a theorem of electrostatics which is of much broader scope than that outlined by the first paper.* (Mr. Fortescue then read an extract from his paper referred to, followed by an extract from Maxwell's work.†

Engineers would do well to give this paragraph careful consideration, as it presents great possibilities in the design of insulating structures. Among the most noteworthy of the applications of this principle are the suspension type insulator and the condenser terminal. The principle of design of the latter may be presented as follows: Consider a cylinder passing through a zero potential plate whose thickness can be varied at will, and let us suppose the two ends of the cylinder to have conductors of a given form, for simplicity let us take infinite planes parallel to the plate, at each end. We shall first consider the region between the plates as having zero specific inductive capacity and let us consider an imaginary field of uniform strength mapped out in this region, conforming to that which would exist if the cylinder were not present. In order to present a surface around this cylinder having at all points the same potential as the assumed field, the dielectric must have the following characteristics:

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* A. I. E. E. TRANS. 1913, Vol. XXXII, Part I, p. 907.

† 1904 Edition, Chapter VII, Art. 117.

(1) Radially it must be a perfect dielectric;

(2) Axially, it must be a perfect conductor.

With a dielectric of these characteristics, the bounding surface may be so formed that the equipotential surfaces of the same value in the region between the rod and bounding surfaces will coincide with those in the region between the two plates, external to the dielectric. If the external region has a finite specific inductive capacity there will be a displacement current necessary in an axial direction from the external region into the dielectric body. This will require a slight modification of the surface of the dielectric to take care of this additional displacement, which must eventually pass radially through the outer cylinders and plate. It is, of course, impracticable to obtain such a dielectric, although there are many ways in which it may be approximated, one of them being the use of a system of coaxial conductors cylindrically imbedded in the insulation. The external surfaces should, of course, be infinite, but in most cases they extend far enough to give satisfactory results. The dielectric surrounding the rod is not uniformly stressed, nor is it desirable that it should be; on the contrary the inner portion should be stressed more lightly than the outer, as the heat set up by the dielectric losses must be conducted out of the dielectric.

If the writer of the paper had made use of this theorem, he would naturally have been led to design his dielectric and system of conducting bodies so as to give the most favorable distribution in the external region, and he would have reached a different conclusion.

Some tests were made a few years ago on glass rods between plates by students at Worcester Polytechnic Institute, and, as I recall them, these tests were quite satisfactory in establishing the non-interference of the surface of high specific inductive capacity material on the breakdown value between plates.

I have recently obtained excellent results with porcelain, though not so good as with moulded or machined materials. The less satisfactory results with this material seem to be due to the irregular form of the contact surfaces between the conductors and the dielectric.

E. DeWitt Eby: With reference to the matter in the general statement for an ideal bushing, I wish to comment, perhaps take exception to Mr. Rice's statement, that in an ideal bushing all of the dielectric is stressed uniformly with respect to its strength, that is, that it will all begin to break down at the same time. This would mean there would be little energy consumption up to the time of rupture or breakdown. It would mean, secondarily, that under impulse conditions the arc-over would be very little different from that under low-frequency stresses. In actual operation or in practise, the low-frequency stresses to which the bushing is subjected are as a rule definitely related to the operating potential, whereas the impulse or lightning stresses are not necessarily related to the line potential, outside of their limitation by line insulation. Therefore, it appears to me that a better relation of the stresses would prevail, if instead of all the insulation breaking down uniformly or at the same time, a part of it should begin to break down, preferably so that it would not destroy or damage the bushing. For instance, the surrounding air should break down earlier than the rest of the structure. In that way energy consumption would take place which would precede the total breakdown, and increase the ratio between the impulse and the low-frequency arc-over.

I wish to emphasize, or add my approval, to the conditions for the design of bushing which Mr. Rice sets forth, that it should arc-over without puncture, at both low frequencies and impulses, and that it should do this without damage to the insulation; also that it should have a high wet arc-over with respect to its dry arc-over value, and, repeating what I said, that its impulse arc-over should be high with respect to its low-frequency arc-over.

I think it was Mr. Mershon who made inquiry as to whether any attempt had been made to control the performance of the bushing under power conditions, that is, with power behind the test circuit or the arc-over, and while not attempting to answer his question, I will simply remark that I know of a number of instances in which successful operation or performance of bushing has prevailed during arc-over with power behind the arc, without doing any damage to the bushing. I also know of one isolated case where the bushing was damaged by the power arc.

I wish to call attention briefly to the effect of the specific resistance of the testing water upon the wet arc-over voltage of a bushing or insulator, a thing which apparently has been overlooked a great deal in making rain or wet tests, and recording the data. The specific resistance of the water has a very great effect upon the wet arc-over voltage, so that, for instance, with such water as Mr. Rice probably used in his tests, the arc-over of the bushing was probably 75 per cent of the voltage it would have been had water been used which was of a higher resistance, such as is available in most natural water systems, that is mountain or reservoir water supplies. I assume that the tests which Mr. Rice has made, were made with water having the resistance of about 4000 ohms per cu. cm., whereas a resistance of 15,000 ohms per cu. cm. would have increased the results about one-third.

We are all aware of the effect of altitude upon the dry and wet arc-over voltage of bushings, both at low frequency and under impulse conditions, and these things have to be taken into account in the application of bushings in actual operation.

It is interesting also to know that there is a temperature effect, and that this effect of temperature is of the order that one degree centigrade difference in temperature is equivalent to about 100 ft. (30.4 m.) difference in altitude.

There is one other point I wish to bring to your attention, and that is, that there should be a relation between the arc-over voltage of the bushing and the voltage of the system to which it is applied. This can best be related to the protection of the system, that is, to the lightning arrester gap settings, since if the system is properly designed the bushing will have a high impulse ratio. It is necessary to consider principally the low-frequency arc over of the bushing, and this should be—theoretically anyway—at least twice the low-frequency arc-over of the lightning arrester gap which protects the system. This is because, as we are all aware, a reflected wave which is lower than the lightning arrester will discharge, will return with double its initial value, and the bushing should not arc-over at that voltage.

It is interesting to note that the factor of safety which the Institute has set down as a testing value for assembled apparatus such as lightning arresters, switches, etc., seems to be prompted by experience, so far as bushings are concerned; that is, that a test value of two and one-quarter times the operating line voltage gives satisfactory results in services, and very few arc-overs take place at that voltage.

F. W. Peek, Jr.: I will give a physical conception or picture, of the meaning of the dielectric flux diagrams in practise.

A bushing or insulator is made up of metal parts or electrodes, oil, necessary supporting solid dielectric, and the air in which it is immersed. When voltage is applied between electrodes, stresses are caused in the solid insulation and the air. If these stresses anywhere exceed the strength of the insulation, breakdown will occur. The strength of air is much less than porcelain. A bushing requiring the minimum amount of material would be one in which the stresses in the different insulations were uniform and in proportion to the respective strengths of the dielectrics. Such a bushing would break down everywhere at once. However, since the air part of the bushing automatically replaces itself when it punctures or flashes over, it is best to make the designs so that flash-over takes place before puncture of the porcelain. As the air is the weakest dielectric, it plays a great part in determining size. Mr. Rice's diagrams are maps of the stress everywhere around the electrodes; the importance of these diagrams in design is at once apparent.

The strength of air is 54 kv. per in. If the full strength of the air could be utilized, a bushing with a 500,000-volt arc-over need not be over 9.5 in. long. There are a number of good reasons why such bushings should, in practise, be perhaps six times that length:

(1) It is not practicable, and for certain reasons not desirable, to design for an absolutely uniform field.

(2) Even if the bushing is designed for a uniform field, the field is not uniform because of apparent conduction along the surface of the solid insulation, which greatly lowers the arc-over voltage. Mr. Rice's views on this are confirmed by experiments that I have made.* The effect takes place on clean porcelain rubber, or glass, and is not appreciably reduced if the surfaces are polished. I believe this is the reason why Mr. Fortescue discovered very little difference in surface effect over the range of surface condition that he investigated in his very interesting and important paper of several years ago. The effect is greatly reduced if the surfaces are oiled.

(3) The third reason that a bushing must be made long is that it must be used out of doors in rain, snow, and dirt. Petticoats must be used. Under such conditions a uniform distribution is not for a moment possible. The designs must include all of these practicable variables.

(4) When flash-over occurs, it is generally caused by lightning. The bushing or insulator should be designed for a "lightning arc-over voltage" much higher than the 60-cycle arc-over voltage, just as the lightning arrester gap should be designed for a low "lightning arc-over voltage."† I have seen apparatus designed in just the opposite way.

You have seen in Mr. Rice's paper many beautiful diagrams determined experimentally; mathematically, it is very difficult to draw any but the simplest diagrams. The meaning of these plots is, however, quite simple. They map out the stresses in the space surrounding the electrodes.

Look at Fig. 9. There is a metal plane, and a rod perpendicular to the plane, passing through a hole in the center of it. You will see certain lines starting at right angles from the plane, and ending perpendicularly, on the central rod. These are the lines of force. They are so drawn that the flux between any two lines is the same. In other words, it means that the capacity between any two lines is equal. You will see at right angles to these lines another set of lines or curves; these represent the equipotential surfaces. Each point of a given curve is at the same potential. They are so drawn that the potential differences between any two adjacent equipotential surfaces is equal. For instance, No. 1 starts at a plane at zero potential, and the next line has a potential of 2, the next 3, and so on up to 51, which is the potential of the rod. The potential difference between each adjacent surface is one. The capacities of all of the little cells, cut out by the lines of force and equipotential surfaces, are equal. If the diagram is revolved about the rod as axis, these cells cut out condensers of equal capacity in space. You will note that the equipotential surfaces are crowded together in places. The unit stress or gradient in volts per inch or per cm. at these points is high, since the voltage differences between two adjacent surfaces is the same as it is where the lines are much farther apart.

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* "Dielectric Phenomena in High-Voltage Engineering", pp. 190 and 220-225.

† "The Effect of Transient Voltages on Dielectrics." F. W. Peek, Jr, A. I. E. E., TRANS., 1915, Vol. XXXIV, Part II, p. 1857.

"Lightning", F. W. Peek, Jr., *G. E. Review*, June, 1916.

It can be seen that these fields may be easily changed, not only by changing the dielectric, but also by changing the metal parts. Obviously, insulation strength may often be increased by adding metal, and some times decreased by adding insulation.

Mr. Rice has been able to draw these wonderful diagrams by the use of the analogy between the electric and dielectric circuits. In the electric circuit there is voltage, and in the dielectric circuit voltage, in the dielectric circuit flux, in the electric circuit current, in the dielectric circuit permittivity and in the electric circuit conductivity, etc.

There is one point which Mr. Rice has made to which I wish to call attention. Mr. Rice states: "It seems to me a great pity that the mathematicians do not more frequently reduce their results to a readily utilizable form, and whenever possible sketch out, with examples, some of the applications which must occur to them while working on the subject." He also says: "Another difficulty, which I have frequently encountered, is the fact that the writer assume too great a familiarity with existing mathematical works on the part of his readers." I wish heartily to endorse these statements. Any one who has ever tried to use some of the mathematical solutions, and has had to give up in despair, because of the great time it required to puzzle out certain steps and the meaning of certain symbols, even in cases where the work is really quite simple, will appreciate this. It generally requires very little extra work to define terms and symbols. I think Mr. Rice has made a good example here of how mathematical work should be presented.

M. E. Tressler: Some very interesting and enlightening individual tests made to determine the cause for certain results, such as "the apparent decrease in disruptive voltage kilovolts per cm. with the increasing thickness of insulation"; "lowered arc-over voltage between terminals along the junction point of two dielectrics of different permittivity, in parallel"; etc.

Mr. Rice draws the conclusion that if the dielectric in parallel with the air is absolutely free from moisture and if perfect joints are made between the dielectric and the terminal, that the presence of insulation in parallel with air or oil in a uniform field would not result in a breakdown lower than that of the weakest material; this conclusion may be correct, but part of the tests would seem to prove that the dielectric strength of air was different when in contact with the test piece of solid dielectric and depending on the properties of this dielectric, as was suggested in the paper. For instance, the difference in arc-over voltage between an oiled hard-rubber surface and an oiled glass surface, Fig. 21, at 8 cm. spacing is 47 kv. In this particular case where a good comparison can be made we should be justified in assuming that the joint between the oiled glass and metal planes was equally as good as the joint between the oiled hard rubber and the metal planes, yet the arc-over voltage on the oiled hard rubber at the same arcing distance was about 65 per cent higher than on the oiled glass surface.

In examining the curves, Fig. 23, with grooved planes and comparing with those in Fig. 21, it will be noted that with the grooved planes or electrostatically shielded joints the arc-over voltages are much higher, but if the arc over occurs the full length of the dielectric cylinders to the bottom of the grooves, the arcing distance is greater than the distance between planes by the depth of the grooves, or if it occurs from the bend of the plane into the groove the air must be broken down here first due to non-uniform field and consequently a condition is obtained similar to the so-called joint effect.

One other reason for assuming that the introduction of a solid dielectric into the uniform field in air causes a change in the dielectric strength of air or else that the curves in Fig. 23 disprove the assumption that the joint effect is eliminated by electrostatic shielding of the ends of the solid dielectric, is that, if the arc-over curves on oiled glass and dry glass II, Fig. 23, are extrapolated a very short distance they cross the curve for arc-over between planes alone, indicating that arc-over voltage with the dielectric inserted is greater than the air alone between the planes, which is a result that would not be expected if the lowering of the arc-over voltages below certain spacings is caused by the joint-effect at the ends or moisture on the surface of the solid dielectric.

H. O. Stevens: The point I think we should all get in this paper is the fact that we have brought before us here in fairly simple mathematical manner a few fundamental principles. We have an indefinitely extended wire passing through a hole in an indefinitely extended plane. That is a fundamental theory worked out beautifully. In other words, we have an ideal there to work to. We can take that fundamental principle and add certain things to it, which may improve the design and work up a satisfactory bushing. Unfortunately, many of the so-called practical engineers when they start out to attack a problem do not have the time, or perhaps the proper mathematical and physical knowledge of the subject to study the problem in this manner, so the thing is done by a cut and try method. When we are designing a bushing, we put a certain amount of insulation on it, and bring it out through a transformer tank and the voltage is increased a little bit more, and we add more insulation, until ultimately we get in trouble. Then some one like Mr. Rice takes the problem and studies it, from a mathematical and scientific standpoint, and while oftentimes the results obtained do not correspond with what we would expect, yet as I said before, it gives us an ideal to work to, and that is the point we want to bear in mind in this paper. If we can work out the fundamental mathematical conception of a given problem, although our theory will not be strictly borne out in practise, yet if we apply to that our mathematical knowledge, the solution of the problem is made a great deal easier to us.

Sidney W. Farnsworth: Mr. Rice speaks very frankly of the difficulties which he has met and brings vividly to mind the early days of our work on this problem, in which we encountered similar difficulties—we would work to get the bushing completed, and then have it puncture on the inside and be obliged to tear it down, and we would feel that the work of first construction had been so difficult it would not warrant us in replacing it.

As engineers we are interested in the application we can make of the information which is given by the author, and the last speaker voiced, I think, the sentiments of those here—I would say that as a result of our work we were able to embody the principles in commercial apparatus and to save money for the company, and also to extend the upper limit of voltage range. Mr. Rice has worked at very low voltages, and perhaps it is not appreciated that the principles involved here permit of multiplication or expansion any number of times. For instance, if you just multiply his dimensions by two, you should get just twice the breakdown voltage, multiply by three, three times the breakdown voltage, etc. That is the principle upon which he has worked. Therefore, he has not solved the problem simply for the low voltages which are mentioned here, but for higher voltages as well.

There is one striking example of the consideration of the principles set forth in this paper, and those which we encounter in our work, and that is of the addition on a high-voltage condenser terminal of the hat, so-called. We had a terminal for a very high-voltage transformer which showed distress at something over 300,000 volts. It was equipped originally with a disk of some 18 in. diameter, with a rim of perhaps 0.75 in. (19 mm.) radius and 1.5 in. (38.1 mm.) diameter. The distress occurred on the terminal prior to distress anywhere else, and the terminal itself probably represented an outlay of some $600 or $700, and remembering the principles we encountered in the course of the design of the transformer and the work connected therewith, we had a wooden hat about six ft. (182.8 cm.) in diameter, and one ft. (30.4 cm.) thick, as I recall it, made, the hat being covered with tinfoil, and this was used as a temporary expedient. A thing that cost perhaps $15 or $20 and made almost over night. We put it on top of the terminal, and distress did not show in that terminal up to 571,000 volts; in other words, there had been an increase of about 150,000 volts in the usefulness of that terminal. By making a gain of 150,000 volts in this way we are doing well. It seems to me that is how we should make use of the principles which are so well set forth in this paper.

I would like to ask, why the charts were made with sheet metal and with wires. In our work we made them with rods of appreciable diameter and with sheets of appreciable thickness, with rounded edges. Fig. 9, giving the equipotential surfaces, shows only the location of the surfaces from 1 to about 30, up to about 24 with what an engineer would call a reasonable degree of accuracy of ease of reading. Now, that is only about half of the total number that should be there, that is, there is 50 volts between the rod and plate. I understand that having the plates and rod conform to line 30, and the plate conform to line 2, that the distribution of the field between the two does not change; that is, the principle is the same. But suppose we did do that, plates and rod conforming to line 30, and an irregular rod and irregular shaped piece conforming to line 2, then we could put that back into the path, and each of these do have the voltage distribution between these two surfaces, the surfaces which we intend to use, and in that way greatly increase the area of the disruption.

Selby Haar: I wish to ask Mr. Rice if he made any tests on his experimental insulators with high direct potentials?

B. A. Behrend: The parallel which exists between the electrostatic case, in which we are interested, and which Mr. Rice has treated with such skill, and the hydrodynamic case, is well known. The hydrodynamic case, so far as its consideration in two dimensions is concerned, has been beautifully worked out experimentally by Prof. Hele Shaw in some admirable papers which were published in the Royal Society Transactions. Now, then, in the latest work I have had an opportunity to consult in connection with hydrodynamics, the treatment is limited to two-dimensional problems altogether. I want to ask Mr. Rice, in replying to the many statements that have been made here tonight, to what extent a correct scientific solution has been given for three-dimensional problems. Prof. Horace Lamb states that three-dimensional cases have not been treated successfully, and a similar statement is made by Sir George Greenhill. The treatment of three-dimensional cases is, of course, fundamental in connection with the treatment of the subject laid before us by Mr. Rice. I trust that Mr. Rice, in his reply, will be kind enough to comment on this phase of the subject, and tell us how far it has been possible to obtain complete solutions of the three-dimensional cases.

Joseph B. Morrill (communicated after adjournment): I should like to discuss a few points that occurred to me in going through Mr. Rice's excellent paper.

First, in connection with the introduction of the diametral planes on page 1001 one might wonder where these planes come from and whether or not they are a new family. If, indeed, this were a new family we would have the impossible condition of *four* mutually orthogonal families, which our three-dimensional space will not permit. I shall endeavor to show that these planes are in reality a degeneration of the hyperboloids of two sheets.

Referring to the middle of page 1000, we find the root that of the cubic in the parameter, ρ, which gives us the hyperboloids of two sheets, lies between $-b^2$ and $-a^2$. Let us assume that the distance between $-b^2$ and $-a^2$ on the $\rho = 0$ axis is divided up into m equal parts each of which has a value or length p. We may assign any positive value to m. Since the root which gives us the hyperboloid of two sheets must lie between $-b^2$ and $-a^2$ we can call this root

$$\rho = -a^2 + n\,p$$

where n may have any positive value less than m. (It must be positive for $a > b$). Substituting this value of ρ in the equation

$$\frac{x^2}{a^2+\rho} + \frac{y^2}{b^2+\rho} + \frac{z^2}{c^2+\rho} = 1$$

and remembering that $a > b > c$, we have

$$\frac{x^2}{n\,p} - \frac{y^2}{a^2-b^2-n\,p} - \frac{z^2}{a^2-c^2-n\,p} = 1$$

which is an hyperboloid of two sheets.

Since $$a^2 - b^2 = m\,p$$

we can rewrite the foregoing equation as follows:

$$\frac{x^2}{n\,p} - \frac{y^2}{(m-n)\,p} - \frac{z^2}{a^2-c^2-n\,p} = 1$$

Multiplying this equation through by p it becomes

$$\frac{x^2}{n} - \frac{y^2}{m-n} - \frac{p\,z^2}{a^2-c^2-n\,p} = p$$

For the oblate spheriodal co-ordinates the z axis becomes an axis of revolution and a^2 becomes equal to b^2, which is the same as letting p approach zero. As p approaches zero, both m and n remain finite for as a^2 and b^2 come closer together we can still continue to divide the distance into m parts, each part smaller in the same ratio as the distance between a^2 and b^2 becomes smaller. If p approaches zero our last equation becomes

$$\frac{x^2}{n} - \frac{y^2}{m-n} = 0$$

or

$$y = \pm\sqrt{\frac{m-n}{n}}\,x$$

which gives us the diametral planes of equation (**22**). Since m and n are always positive and $m > n$ the radical

$$\pm\sqrt{\frac{m-n}{n}}$$

can take any value and we may substitute ν for it and obtain

$$y - \nu x = 0$$

where ν may have any value.

The same method may be employed to show that the third equation of (**146**) is a degenerate hyperboloid of one sheet.

A point in connection with the introduction of the thermometric parameters α, β and γ, occurs to me, which might possibly aid one who likes to have a picture of such a step in the analysis. In introducing the parameter β, for instance, it is assumed, and the assumption later justified analytically, that the potential, under certain conditions is a function of μ only. The conditions under which this is true is later shown to be that two of the hyperboloids be kept at definite potentials. Referring to Fig. 10, it is seen that μ is the intercept of the hyperboloids on the X Y plane, that is, μ determines the position of any hyperboloid. From a graphical standpoint, then, it is justifiable to assume that if two of the hyperboloids are maintained at a definite potential, made equipotential surfaces, the potential of the other hyperboloids will depend only on their position with respect to these two, in other words, tha t V is a function of μ only.

On page 1028 in solving the problem of the infinite rod through the hole in an infinite plane, the three following equations are given:

$$\frac{\partial^2 V}{\partial \alpha^2} = 0, \quad \frac{\partial^2 V}{\partial \gamma^2} = 0, \text{ and } \frac{\partial^2 V}{\partial \beta^2} = 0$$

the last one being equation (**115**) and is solved. One might well wonder why the other two equations are not solved if they are true equations. Of course the first two equations are *not* true for, if V is neither a function of α, nor γ, how can $\frac{\partial^2 V}{\partial \alpha^2}$ or $\frac{\partial^2 V}{\partial \gamma^2}$ be equal to zero unless it were purely an accident? Equation (**115**) is the natural consequence of equation (**114**) for neither the first nor the third term of (**114**) can have a meaning if we assume that V is a function of β *only.* Is not this point similar to the following case in algebra? If we have the equation $10\,X = 0$, we know that $X = 0$ and do not have to make the untrue statement that $10 = 1$ in order to get it. I believe this same mis-statement is made in Professor Byerly's, "Fouriers Series and Spherical Harmonics," to which Mr. Rice refers.

The two problems which Mr. Rice has solved by Laplace's equation in spheroidal coordinates can only be checked by experiment, as is done in his paper, and then only approximately, due to the impossibility of using either an infinite rod or an infinite plane. It might be interesting to work a simple problem using Laplace's equation in this form which could be checked by some other method. One such problem obviously is to calculate the electrostatic capacity of an isolated thin circular disk. Such a disk is obtained by allowing the oblate spheroids to degenerate into a circular disk of radius c (see Fig. 9). The oblate spheroids will be equipotential surfaces and the tubes of flux will be bounded by the hyperboloids of one sheet and the diametral planes. Under these conditions the potential V is a function of α only and equation (**115**) becomes

$$\cos^2 \alpha \frac{\partial^2 V}{\partial \alpha^2} = 0$$

and since this must be true for all values of α, we have

$$\frac{\partial^2 V}{\partial \alpha^2} = 0$$

Solving this differential equation we obtain

$$V = A\,\alpha + C$$

To evaluate the two constants of integration, A and C, we shall let the disk have a potential of V_0 and since the disk is isolated, $V = 0$ at infinity. We have therefore the two boundary conditions.

$$V = V_0 \text{ when } \alpha = 0 \text{ and } V = 0 \text{ when } \alpha = \frac{\pi}{2}.$$

Substituting these conditions in the last equation we obtain

$$V = V_0\left(1 - \frac{2\,\alpha}{\pi}\right)$$

The electrostatic flux, ψ, is given by the equation,

$$\psi = \int\!\!\int - \frac{d\,V}{d\,n_1}\, d\,n_2\, d\,n_3$$

The potential gradient is

$$\frac{d\,V}{d\,n_1} = -\frac{2\,V_0}{\pi} \cdot \frac{d\,\alpha}{d\,n_1} = -\frac{2\,V_0}{\pi} \cdot \frac{d\,\alpha}{d\,\lambda} \cdot \frac{d\,\lambda}{d\,n_1},$$

which is obtained by differentiating the equation for potential given above with respect to the normal, n_1.

Since

$$\lambda = c \sec \alpha,$$

$$\frac{d\,\alpha}{d\,\lambda} = \frac{c}{\lambda\sqrt{\lambda^2 - c^2}} \qquad \text{(Equation 75)}$$

From equation (**47**) and equation (**65**) we get

$$\frac{d\,\lambda}{d\,n_1} = h_1 = \frac{\sqrt{\lambda^2 - c^2}}{\sqrt{\lambda^2 - \mu^2}}$$

$$\therefore\ \frac{d\,V}{d\,n_1} = -\,\frac{2\,V_0}{\pi}\cdot\frac{c}{\lambda\sqrt{\lambda^2 - \mu^2}}$$

From equation (**48**) and equation (**67**) we obtain

$$d\,n_2 = \frac{d\,\mu}{h_2} = \frac{\sqrt{\lambda^2 - \mu^2}}{\sqrt{c^2 - \mu^2}}\,d\,\mu,$$

and from equation (**49**) and equation (**69**)

$$d\,n_3 = \frac{d\,\nu}{h_3} = \frac{\lambda\,\mu}{c\,(1 + \nu^2)}\,d\,\nu$$

Substituting these values for $\frac{d\,V}{d\,n_1}$, $d\,n_2$ and $d\,n_3$ in the equation for φ we obtain,

$$\varphi = \int\!\!\int \frac{2\,V_0}{\pi}\cdot\frac{\mu\,d\,\mu\,d\,\nu}{(1 + \nu^2)\sqrt{c^2 - \mu^2}}$$

$$= \frac{4V_0}{\pi}\int_{\nu=\tan 0}^{\nu=\tan 2\pi}\frac{d\,\nu}{1 + \nu^2}\int_{\mu=0}^{\mu=c}\frac{\mu\,d\,\mu}{\sqrt{c^2 - \mu^2}}$$

(Factor 2 is introduced to get flux above and below $X\ Y$ plane.)

$$= \frac{4\,V_0}{\pi}\Big[\text{arc tan}\,\nu\Big]_{\nu=\tan 0}^{\nu=\tan 2\pi}\Big[-\sqrt{c^2 - \mu^2}\Big]_{\mu=0}^{\mu=c}$$

$$= 8\,V_0\,c$$

Since

$$\varphi = 4\,\pi\,Q$$

where Q is the total charge on the disk,

$$4\,\pi\,Q = 8\,V_0\,c$$

or

$$\frac{Q}{V_0} = \frac{2\,c}{\pi} = \text{Capacity of disk.}$$

This is the well known value for the capacity of an isolated circular disk of radius c and can be checked by other methods, for instance, see Berg's "Electrical Engineering, Advance Course" page 201.

Lastly, in connection with the integration for the tubes of flow where the integration for ν is neglected, would it not be more exact to say that the spacing obtained is for any diametral plane and hence independent of ν? That is, the last paragraph on page 1039 shoul dread "and since ν_0 may have any value we may neglect it as far as the spacing *on any diametral plane* is concerned"; and the paragraph in the middle of page 1051 should read, "As far as the spacing of the surfaces of flow *on any diametral* plane is concerned we may.........."

Chester W. Rice: Major Mershon has asked what effect power arcs will have on bushings. I am sorry to say that I have never witnessed such tests but suppose that they will blow the bushings up, as they do almost everything else when they get started.

Mr. Taylor was interested to know why we did not introduce bad distortion into the diagrams by moving the pointer around in our electrolyte, as you would in an electrostatic problem if you tried to obtain the potential distribution by moving the pointer around in space. The beauty of the thing is that we have substituted an electric current for the electrostatic flux. This has two very important advantages, in the first place, we can insulate all but the end of our exploring pointer from the current whereas, there is no insulator for the electrostatic flux, in the second place, we substitute a large conduction current for a small displacement current which allows us to use a reasonable amount of power for operating our instrument without fear of distorting the field. Of course, the insulated pointer does displace a slight amount of liquid and therefore introduces a slight distortion. A method of eliminating this error is pictured in Fig. 5. The energy required to operate a quadrant electrometer is so small compared with the energy flowing through the circuit that distortion due to this cause is considered entirely negligible.

Prof. Gray asked about using the photographic method for three-dimensional problems. I think photographic methods and all of the allied methods, such as little fine particles of glass-wool or mica dust, are well adapted to show the general system of flow lines. I have made some experiments with little glass rods obtained by grinding up glass-wool, and in that way obtained a general picture through any plane, of the desired three-dimensional figure. A very considerable assistance is obtained by this method where one wishes to apply the cut and try method of obtaining a diagram for an electrostatic problem as it gives you something to start guessing with.

Mr. Fortescue has made a correction of the arc-over value of his bushing, which indicates that no surface effect exists in the material which he has used. He gets an arc-over of approximately 207 kilovolts effective, and the calculated arc-over, if my assumptions as to the size and general structure of his test piece are correct, comes out at 205 kilovolts effective. There is therefore no room left for surface effect if his tests are correct.

He also mentioned some further data which have apparently convinced him that there is no such thing as surface effect. For my part, I am confident that there is a large effect of this nature. The difference in opinion shows that further study of this phenomenon is very desirable.

I have never been able to see any very direct bearing between the calculations on the condenser bushing and the bushings as built. In the first place, the calculations assume that the bushing has very large hats on the top and bottom ends (infinite planes); in the second place, the structure is assumed to be symmetrical about the cover of the tank. Neither of these conditions is even approximated to in the bushings of this type which I have seen.

Mr. Farnsworth has asked why the charts were made using thin sheet metal to form the electrodes. This was done in the first place to facilitate the construction and in the second place to make the boundary conditions as simple as possible, so that in case a mathematical solution some day becomes possible the diagrams may be used as a check.

Mr. Haar has asked whether I have made any high-voltage direct-current tests. No, I am sorry to say that I have not had the opportunity.

Mr. Behrend has brought up the question of three-dimensional hydrodynamic problems. If we assume an incompressible and inviscid fluid, the electrodynamic method is of course available for obtaining the solution of any desired problem in hydrodynamics. For example; if we wish to obtain the solution of a torpedo shaped body at rest in an infinite current of fluid, all we have to do is to immerse a non-conducting torpedo shaped body in our tank between large parallel plane electrodes and investigate the potential distribution in the usual manner. This, of course, is also the solution of the analagous electrostatic problem, namely, a torpedo shaped body of zero permittivity in a uniform electrostatic field.

The solution of the type of problem, where a body is moving through a stationary perfect fluid can also be solved by this method. Taking the case of a sphere we could obtain the desired solution by immersing two pointed electrodes at close proximity in the tank and exploring the field in the usual manner. This would result in the well known solution of a sphere moving through an infinite stationary fluid. Of course it is understood that the solution of these problems does not give very interesting data for the naval constructor because actual fluids exert appreciable forces due to viscosity and radiation of compressional waves. A sphere set in motion in a stationary fluid would move on forever at the same speed also a stationary sphere having mass would remain at rest in a uniform stream of perfect fluid. In the ideal fluid theory all bodies are streamline bodies and eddys are non-existent.

Mr. J. B. Morrill's discussion has added some very interesting material, which I believe will be greatly appreciated by all those engineers and students who have occasion to study this subject. I believe that his analysis which shows that the diametral planes can be considered as a degeneration of the hyperboloids of two sheets is new and very illuminating. He is also entirely correct in pointing out the incorrectness of setting

$$\frac{d^2 V}{d\,\alpha^2} = 0, \qquad \frac{d^2 V}{d\,\gamma^2} = 0.$$

as a method of indicating that they are meaningless for the case under consideration and therefore may be struck out of the equation. I think that this slip is often made and therefore it is very well to draw attention to its incorrectness.

The solution of the limiting oblate spheroid is a very appropriate addition.

I am greatly indebted to Mr. Morrill for pointing out by letter an inconsistency in my original equations used in deducing Laplace's equation (pages 1006-1014). On getting into the matter, I found that the trouble arose from considering the top area of the infinitesimal volume equal to the bottom area whereas it is necessary to consider the rate of change of the bottom area as we go towards the top along the normal.

I have therefore taken the liberty of making this correction for publication in the Transactions.

I am further indebted to Mr. Morrill for the corrections of many typographical errors to which he has so kindly drawn my attention.

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ANNUAL LOAD RELIEF MAP, PEAK LOAD AND LOAD FACTOR ANALYSIS

BY WM. LE ROY ROBERTSON

ABSTRACT OF PAPER

Description of the annual load relief map which is a device for visualizing the entire yearly load of central stations.

Review of load characteristics, as exemplified by the annual load relief map.

Notes on daylight saving relative to the decreasing of peak load, and improvement in load factor.

Analysis of load factors as applied to "base" and "top" loads demonstrating the large amount of idle capacity peculiar to central stations.

Annual load relief map valuable in the study and exemplification of all matters having to do with central station loads.

THE familiar daily load diagram is plotted by practically all electric light and power companies. The accumulation of these curves soon becomes a mere record, usually filed away in some manner and always available for reference, and for the study of load conditions. When considering the load throughout the year it is difficult to obtain a comprehensive idea of the whole since it is necessary to glance separately at the greater portion of 365 sheets. Such a record locks up a desired vision in confusion.

The Annual Load Relief Map is a device for visualizing the entire load of the year. Each daily load diagram is marked off on card board and cut out. The cards are stacked up in proper daily sequence, mounted and provided with graduations for kilowatts, hours of the day and months of the year, all properly arranged. The annual load relief map is illustrated in Figs. 1, 2, and 7, showing the Philadelphia load during the year 1916.

Day Load. A distinctive feature brought out by the annual load relief map is the contour of the day load which is consistently uniform throughout the year, always picking up between 7:00 to 8:00 a. m., having a valley at noon and then falling off punctually at about 5:00 p. m. This stands out clearly on the annual load relief map Fig. 2; and especially well, if one will imagine the absence of the night load where it overlaps the day load at

5:00 p. m. The day load corresponds closely to the regular average working day.

Night Load. The night load which picks up rapidly at 8:00 p. m. in mid-summer, and at about 4:30 p. m. to 5:00 p. m. in winter, corresponds closely to the lighting load. It depends absolutely upon the hour of sunset for its beginning and falls off rapidly, shortly after reaching its peak value. After midnight, it settles down to a low value and drops off almost entirely when the street lighting goes off near sunrise. In summer a deep valley will be seen in the morning, (Fig. 2) between the "fall-off" of the night load and the beginning of the day load, while in winter the loads overlap in the morning, filling up this valley.

Peaks. During the summer months there are three distinct peaks—one occurring about 8:00 a. m.; one about 5:00 p.m.; and the third about 8:00 p. m. With the approaching fall and winter months, and as the sun sets earlier each day, the 8:00 p. m. peak moves back toward the 5:00 p. m. peak and near the end of September the two peaks overlap, giving a combined peak greatly exceeding any other peak, which rapidly increases in height until the middle or latter part of December, when it becomes the greatest peak of the year. As the spring months approach the combined peak diminishes and finally near the end of March, disintegrates, forming again the two separate peaks. While the above is a well known fact, the annual load relief map presents the changing condition in a most striking manner.

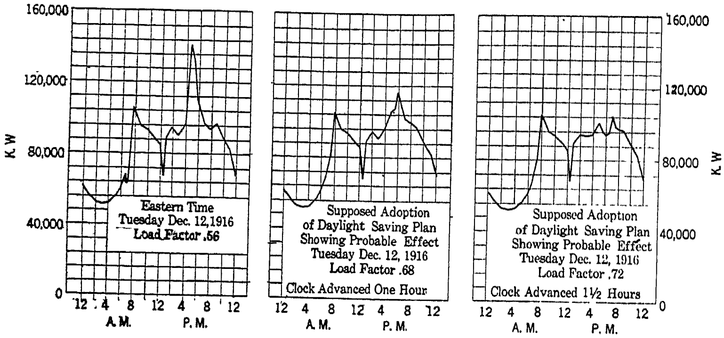

Fig. 3

Daylight Saving. During the past year or more, twelve European countries have adopted the plan of setting the clock one hour ahead during the summer months, in order to utilize a greater amount of daylight. Nova Scotia on this continent

PLATE XLV.
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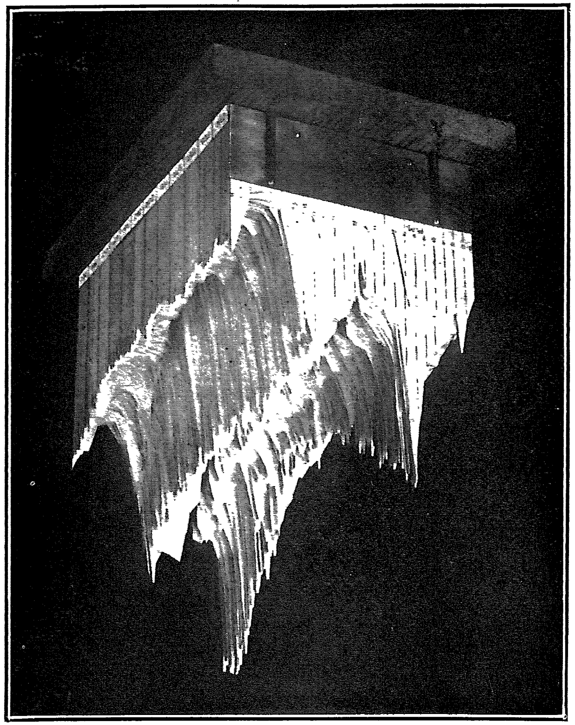

FIG. 1

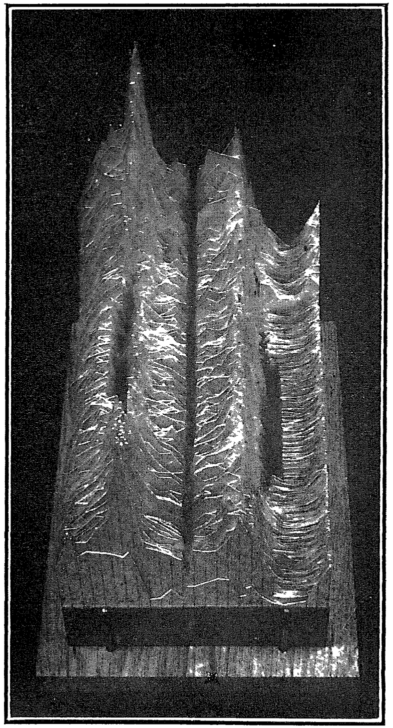

FIG 2 [ROBERTSON]

has also adopted the plan. The cities of Detroit and Cleveland have practically accomplished the same thing by adopting Eastern Standard Time; and, further, the Committee on Daylight Saving of the Chamber of Commerce of the United States, in its report of February 1, 1917, recommended that Congress adopt the plan throughout the United States—advancing the clock one hour throughout the year; and, as an alternative plan, advancing the clock one hour during summer only.

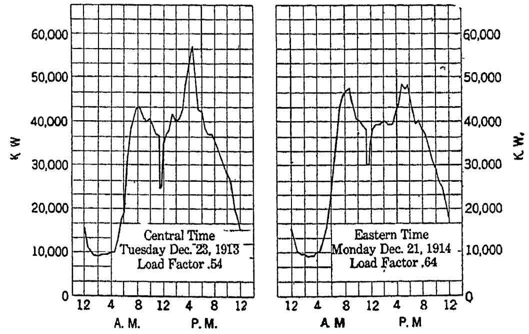

FIG. 4

The annual load relief map will be found useful in conjunction with the study of the effect of daylight saving on peak loads.

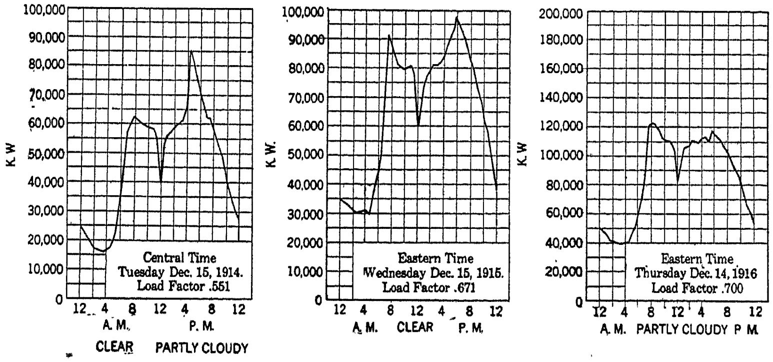

FIG. 5

Advancing the clock will have the same effect as shifting the day load back from the night load. Looking at Fig. 2, it will be seen that by shifting back the day load the valley between the 5:00 p. m. and 8:00 p. m. peaks will extend over a longer period during the summer; and if shifted back sufficiently, this valley will extend over the entire year, thus causing the big peak to disintegrate into the two separate peaks, eliminating entirely the big peak.

In order to entirely eliminate the big peak in Philadelphia, it is estimated that the clock should be set forward one and one-half hours. Fig. 3 illustrates what might have been the effect on the peak load in Philadelphia, had daylight saving been adopted during 1916. It should be understood, however, that these curves are merely a rough prediction, as it would be impossible to attempt to forecast the actual result.

The shifting back of the day load would also cause it to overlap the early morning load to a greater extent. This would increase the morning peak at 8:00 a. m.; but as the morning load is small, the increase would not be great. However, should winter office hours begin before daylight then the morning peak would obtain greater proportion.

Fig. 4 shows the effect of daylight saving on peak loads in Cleveland, while Fig. 5 shows the effect in Detroit. Cleveland's peak has greatly reduced, and it will be noted that the load factor has materially improved. In Detroit, it is understood that the radical change in the peak load has also been effected by other conditions than that of adopting Eastern Standard Time, *i.e.*, the load has greatly increased and all gem lamps have been changed over to Mazda's, during this period. The load curves given in Figs. 4 and 5 were furnished through the courtesy of the Cleveland Electric Illuminating Co. and the Detroit Edison Company, respectively.

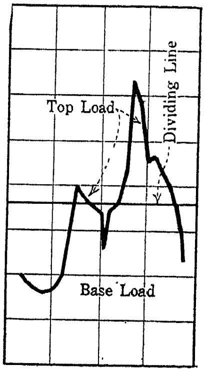

FIG. 6

Base and Top Loads—Load Factors. The total load on any central station may be divided into two sections as follows: Base load; top load. The sum of which equals the annual peak load, and the dividing line being a matter of judgment, as illustrated in Fig. 6.

After assuming any dividing line between the base and the top sections of the load, the annual load factor for the base section and the annual load factor for the top section may be computed from the 365 daily load curves covering the year in question. For a given annual load factor of the total load, the computed load factors of the base and top sections will not vary greatly between various central stations, as all central stations have very similar load diagrams and considerable variation in the individual load diagram may take place before the load factors are affected.

Table I gives load factors of the base and top sections of the load, corresponding to a number of selected dividing lines, and repeated for several annual load factors of the total load. The data in this table are presented here through the courtesy of L. B. Stillwell, Consulting Engineers, New York, from whom the information was obtained.

TABLE I

LOAD FACTORS OF VARIOUS BASE AND TOP LOADS

Dividing line between base and top loads			30% load factor of total load (annual)		40% load factor of total load (annual)	
Base load in per cent.	Top load in per cent.	Total load in per cent.	Annual load factor of the base load	Annual load factor of the top load	Annual load factor of the base load	Annual load factor of the top load
100	0	100	30%	0 %	40%	0 %
80	20	100	37%	¼ %	49%	½ %
60	40	100	50%	1 %	64%	2 %
50	50	100	58%	2 %	74%	6 %
40	60	100	69%	5 %	82%	11 %
20	80	100	91%	14 %	97%	26 %
0	100	100	100%	30 %	100%	40 %

Dividing line between base and top loads			50% load factor of total load (annual		60% load factor of total load (annual)	
Base load in per cent.	Top load in per cent.	Total load in per cent.	Annual load factor of the base load	Annual load factor of the top load	Annual load factor of the base load	Annual load factor of the top load
100	0	100	50%	0%	60%	0%
80	20	100	61%	1%	71%	5%
60	40	100	76%	9%	86%	19%
50	50	100	85%	15%	92%	27%
40	60	100	93%	22%	97%	35%
20	80	100	99%	38%	100%	50%
0	100	100	100%	50%	100%	60%

The results in Table I are most illuminating. Take for instance the case of a central station having an annual load factor of 40 per cent, it will be noted from the table that there is 50 per cent of the load, *i.e.*, top load which has an annual load factor of only 6 per cent, or stating this in a more practical sense—there is equipment in such a plant sufficient in capacity to carry 50 per cent of the maximum load which is idle 94 per cent of the time. Take another case of a company with a 30 per cent annual load factor: there is equipment in this plant sufficient in capacity to carry 60 per cent of the maximum load which is idle 95 per cent of the time. Any one having difficulty in conceiving these facts will find that the annual load relief map will aid in making them clear. Here the "dividing line" becomes a dividing plane, and it will be noted that the top section of the load above the plane for the entire year is indeed very small when the dividing plane is at 50 per cent of the maximum load or above.

By examining the annual load relief map, it will be noted that the empty void between the contour of the load and a horizontal plane placed at the tip of the highest peak represents the business that a central station can theoretically carry along with its present business without increasing its equipment. If the central station load factor is 33 per cent, then this void represents 200 per cent of the business already carried. In cutting out the cards of an annual load relief map, if the upper half of the card is saved and stacked up, the mass of cards will represent the additional business that could be carried in theory by a central station without increasing the capacity. Fig. 7 represents the upper part of the annual load relief map. Here it will be noted that all the Saturdays and Sundays were separated and stacked separately at one end, in order not to hide the effect of the week days.

After the author devised the annual load relief map, it was learned that Mr. H. A. Barre of the Pacific Light & Power Corporation of Los Angeles, California, has been making use of practically the same scheme for the past several years. In connection with certain analysis of the relative values of water power and steam reserve, Mr. Barre found use for such a device in order to explain to the banker's satisfaction how much power would be carried by the water power, and how much by the steam reserve. At first, he experimented with a cube of wood, sawing along the line of the daily and yearly contours of the load which were respectively laid out on the faces of the cube at right angles, and later he stacked up the cards. Fig. 8 is a photograph,

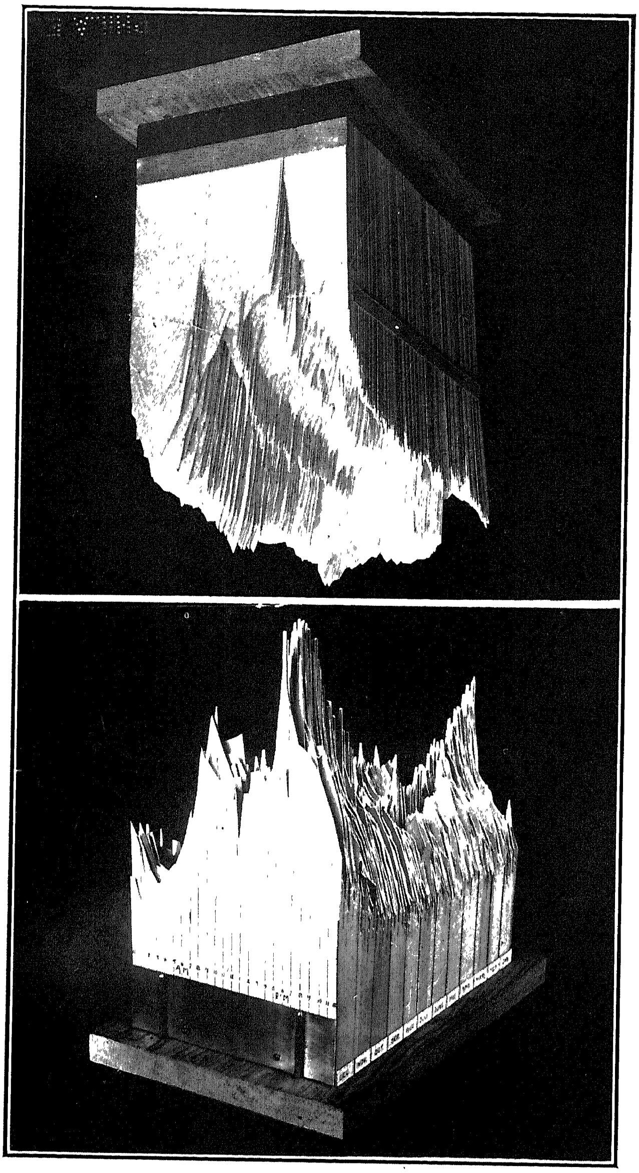

FIG. 7 [ROBERTSON]

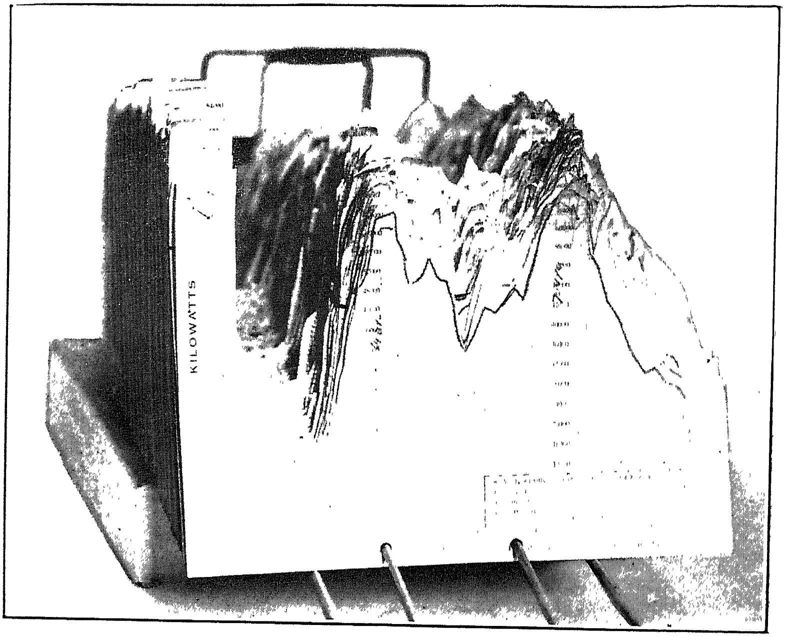

FIG. 8

[ROBERTSON]

which was very kindly furnished by Mr. Barre, showing his plan. Each diagram is cut out in paper and placed in the file—5 years would be 7 to 8 in. (17.7 to 20.3 cm.) thick.

It might be suggested that commercial men may find much interest in following up the load growth from year to year upon the annual load relief map, and again the device may be useful as an aid in explaining various problems in connection with rates and investment before public service commissions.

Presented at the 335th meeting of the American Institute of Electrical Engineers, New York, December 14, 1917.

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PHENOMENA ACCOMPANYING TRANSMISSION WITH SOME TYPES OF STAR TRANSFORMER CONNECTIONS—II*

BY LLOYD N. ROBINSON

Abstract of Paper

1. Hysteresis, as well as variation of permeability, gives rise to harmonics of considerable magnitude in the exciting current of a transformer; the components from the two sources are in quadrature for each harmonic.

2. There is a fallacy in the idea that the harmonics in the exciting current of a transformer are entirely wattless.

3. For transmission lines of modern lengths, the effect of the line is to reduce the third harmonic voltage between neutral and line terminals of a star-star connected bank of transformers having grounded neutral on the line side only.

4. In transmission with star-star transformer banks having grounded neutral on the line side only, the inter-relation of line susceptance from conductor to ground in series with the varying susceptance of the individual transformers may cause the total m.m.f. of one of the transformers to pass through zero at the same instant as the impressed e.m.f.; and thus, give opportunity for a reversal of this transformer due to inertia, thereby producing a periodically reversing leg and abnormal voltages.

5. In a similar system with high quality line insulation, the effect of atmospheric charges on the line conductors may be to cause the transformers of the bank to assume relations with one leg permanently reversed when energized from the station side; thus, imposing unbalanced voltage conditions on the bank and producing, in two of the units, fundamental frequency components of voltage 264 per cent of normal.

6. Combinations and accentuations of the above causes may, through natural requirements, give rise to a double frequency pulsation of the potential of the hypothetical neutral of the line conductors; thus, producing, in the leg voltages, second harmonic components of extremely large dimensions which are limitless so far as is yet known.

7. The above applies also in case a group of star-connected ironclad reactors, such as auto-transformers without tertiary windings, are employed to ground the neutral of an otherwise isolated neutral transmission.

8. Results, obtained in the laboratory with a three-phase star-star connected bank of transformers feeding condensers between line conductors and line side neutral, though not yet sufficiently extensive to confirm all of the above statements, show that the voltages from line terminals to neutral are in considerable measure independent of the primary impressed delta voltages so far as concerns wave shape and frequency.

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*For Part I see TRANS. A. I. E. E., Vol. XXXIV, 1915, p. 2183.

INTRODUCTION

AS A general rule, the use of single-phase transformers connected star-star into a three-phase bank is considered poor electrical engineering practise. There are cases where the connection has been employed with apparently satisfactory results, but they comprise the exceptions rather than the rule. It is doubtful if many of these exceptions would be permitted if a thorough investigation were made of the stability and local voltages involved in the connection, but it is beyond the scope of this thesis to make such an investigation of all the possible cases. It is sufficient to say that where these exceptional connections, star-star, have been employed in transmission practise, special precautions have generally been taken to insure the stability of the system. Where such precautions have not been observed, disastrous results have frequently followed. The precautions have provided a remedy or even a preventive, we might say; but, because of the suddenness and sureness of the undesirable phenomena when such have occurred, it has usually been impossible to observe the actual underlying causes. Theories have attributed the effects to the third harmonic which arises when an ironclad electromagnetic circuit is energized.

Early in the year nineteen hundred and fifteen, the writer was concerned with some tests of star-star connected transformers supplying a three-phase transmission line, that developed unstable conditions and peculiar phenomena which ordinarily would not be discovered in commercial operation or without the use of an oscillograph. It is fairly safe to say that, even in these tests, the insulation of the transformers would have broken down and the windings have been burnt out before the phenomena were discovered had it not been that the impressed voltage applied to the transformers was only approximately one-quarter of the rated voltage.

First, it will be well to show why, in general, the third harmonic of magnetization can not be expected to develop sufficiently high voltages to break down the insulation of a star-star connected bank of transformers connected to a three-phase transmission line provided the windings, leads and bushings are insulated for full rated voltage of the windings, and have been subjected to tests of at least double voltage to case, core and ground and between turns.

Later, the co-ordination of these previously unrecognized phenomena with the general laws of electricity and magnetism will be submitted so far as it has been completed at this time.

HARMONICS OF TRANSFORMER MAGNETIZATION

So much has been said and written concerning the harmonics of magnetization of transformers, that it is hardly essential to mention it more than very briefly here. The indeces of the TRANSACTIONS of the American Institute of Electrical Engineers provide ample bibliography on the subject.

From the general formula of electromagnetic induction, the induced voltage is

$$e_2' = k \frac{d\,\phi}{d\,t}$$

where ϕ is the magnetic flux, t the time, and k is a constant dependent upon the number of turns in the winding, etc.

If

$$\phi = \cos\,(a\,t + \alpha)$$

$$\frac{d\,\phi}{d\,t} = -\,a \sin\,(a\,t + \alpha)$$

whence

$$e_2' = -\,a\,k \sin\,(a\,t + \alpha)$$

That is, we say a cosine wave of flux generates a sine wave of voltage, and because of the inter-relations of the sine and cosine functions, we may say that a sine wave of flux produces a cosine wave of voltage.

With a sine wave of voltage applied at the two terminals of an electric circuit, the wave of counter e.m.f. will ordinarily be a sine wave of equal magnitude but opposite sign when conditions are stable; that is, after the initial transients have disappeared.

Considering only the post-transient conditions: If a voltage, $e = c \sin a\,t$, is impressed on the terminals of an inductive circuit containing resistance (or equivalent resistance), the counter e.m.f. will be

$$e' = -\,c \sin a\,t$$

But e' will have two components; one, the resistance component,

$$e_1' = -\,b \sin\,(a\,t + \beta)$$

and the other, the reactive component,

$$e_2' = -\,a\,k \sin\,(a\,t + \alpha)$$

The current will adjust itself so that

$$c \sin a\,t = b \sin\,(a\,t + \beta) + a\,k \sin\,(a\,t + \alpha)$$

In the circuit having constant inductance, the current will be a sine wave not in phase with the impressed voltage or the counter e.m.f. The phase relation between the current and the impressed voltage depends upon the relative magnitudes of the inductive reactance, $x = 2\pi f L$, and the resistance r. The phase angle is

$$\theta = \tan^{-1}\left(\frac{x}{r}\right)$$

The number of interlinkages of an electric circuit with the lines of magnetic force of the flux produced by unit current in the circuit is called the inductance of the circuit. That is, the inductance is

$$L = m\frac{\phi}{i}$$

where m is a constant, ϕ is the number of lines of flux, and i is the current.

In an ironclad inductive circuit, the ratio $\frac{\phi}{i}$ is not constant, but decreases as the flux density in the iron increases. However, in order to produce a sine wave of counter e.m.f., there must be a cosine wave of flux because

$$D_x(\sin x) = \cos x, \text{ and } D_x(\cos x) = -\sin x$$

Consider, for the present, an iron magnetic circuit without hysteresis*, in order to study alone the effects of what is called the varying permeability by means of the B-H curve. B is the product of the flux multiplied by a constant, and H is the product of the current multiplied by a constant. Since the constants are given for any machine, by change of scale, the *B-H* curve may be translated to a $\phi - i$ curve, and this will be done in the following discussion. In Fig. 1, there is given a typical *B-H* curve for transformer iron. This *B-H* curve translated is the axis of the hysterises loop shown in Fig. 2.

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*Because of the prevalence of a sort of mystery surrounding this word, and the lack of uniformity in its use; the definition from Webster's New International Dictionary is that used and is submitted:

"Hysteresis: (a) A lagging or retardation of the effect, when the forces acting upon a body ar changed, as if from viscosity or internal friction. (b) In a magnetic material, as iron, a lagging in the values of resulting magnetization (denoted by B) due to a changing magnetizing force (denoted by H). A repeated reversal of H causes a changing magnetization B as shown by the hysteresis loop or cycle...."

The meaning of the word bears a marked similarity to that of hesitate; but a search of the derivations shows that hysteresis comes from Sanskrit through the Greek, while hesitate comes through the Latin from an entirely different root.

For a sine wave of flux, say

$$\phi = 10 \sin a\, t$$

curve I of Fig. 3, a wave of wattless magnetizing current, i_m, like curve II of Fig. 3 will be required, based upon the i_m curve of Fig. 2.

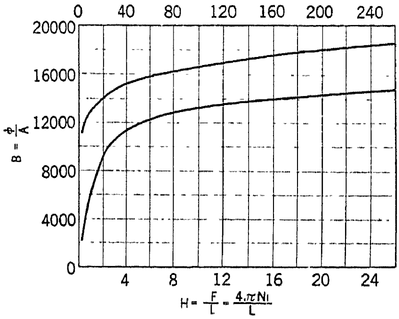

FIG. 1—AVERAGE INDUCTION CURVE—TRANSFORMER SHEETS
Standard Handbook, 1915 Edition, Sect. 4, Fig. 24

Concerned in the magnetization of the iron, there are, besides the wattless magnetizing current, the components of current due to hysteresis and to eddy currents. By the equations for eddy currents, it is possible to eliminate them from the discussion; and now consider combined the wattless component of magnetizing current and the energy component required by hysteresis. The combination is obtained from the hysteresis loop for a given wave of flux, say

$$\phi = 10 \sin at$$

The current corresponding to this flux is obtained from the i_{mh} curve of Fig. 2 and plotted as curve III of Fig. 3.

The co-ordinates of the curves of Fig. 3 are given in Table I, By harmonic analysis, the coefficients of the Fourier series for these three waves are determined and tabulated in Table II.

It is noted that the sine coefficients of the curve III are practically equal to the sine coefficients of curve II. The discrepancies are due to the facts that only eighteen ordinates per half wave were used in the analysis, and that the computations were carried to only three decimal places at most. This agreement between the sine coefficients of II and III is due to the fact that the i_m curve of Fig. 2 is the axis of the i_{mh} curve, so that the components of current in phase with the flux in the two cases are identical.

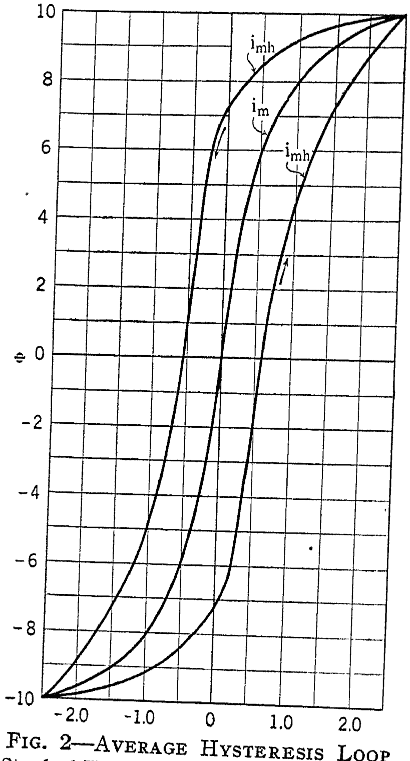

FIG. 2—AVERAGE HYSTERESIS LOOP
Standard Handbook, 1915 Ed., Sect. 4, Fig. 25

It is interesting to note further in curve III that the cosine coefficients are practically one-half the sine coefficients for the fundamental and triple harmonics. In the early discussion of the third harmonic due to transformer magnetization, the harmonics were attributed entirely to hysteresis. Later this was shown to be erroneous and it was fairly generally accepted that the harmonics were almost entirely due to the variation of permeability. One of the favorite arguments supporting the latter is, there is no triple harmonic voltage impressed on the transformer and consequently the triple harmonic component of the exciting current must be entirely wattless. From the coefficients in Table II, it is seen that in this average iron at not abnormal maximum density, hysteresis introduces triple harmonic components of exciting current in quadrature with those introduced by the variation of permeability, and of practically half the magnitude of the latter. Therefore it can not be said that the triple harmonic components in the exciting current of a transformer are due to the hysteresis alone or to the variation of permeability alone. They are produced by the combination of the two.

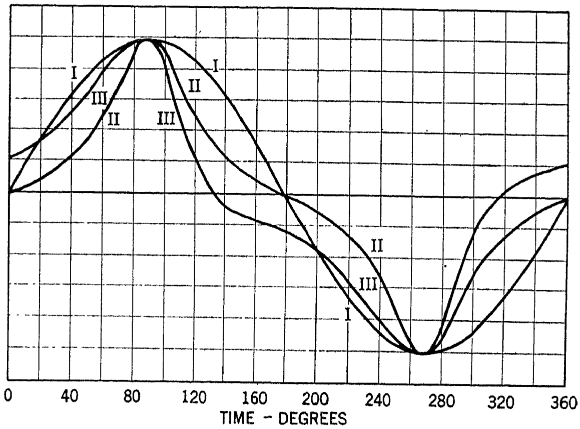

FIG. 3

I—MAGNETIC FLUX
II—WATTLESS MAGNETIZING CURRENT
III—CURRENT REQUIRED BY THE HYSTERESIS LOOP

The components of exciting currents attributed to eddy currents are generally found to contain some small harmonics, but these are present due to the fact that the wave of flux, ultimately produced, is not a pure sine wave, and consequently eddy currents of harmonic frequency are present in the core.

If there is presented a closed circuit for the harmonic components of the exciting current, they will flow in order to adjust the current to the counter e.m.f. requirements. Obviously, these harmonic currents flowing, for example, through the resistance of a generator winding or closed delta of a transformer bank give rise to energy losses. That is, they are not wattless. What really takes place is a conversion of frequency within the transformer.

TABLE I

CO-ORDINATES OF CURVES I, II, III, FIG. 3

Abscissas at deg.	sin at	I $\phi = 10 \sin at$	II current i_m	III current i_{mh}
0	0	0	0	0.57
10	0.174	1.74	0.11	0.70
20	0.342	3.42	0.23	0.84
30	0.500	5.00	0.40	1.07
40	0.643	6.43	0.60	1.35
50	0.766	7.66	0.90	1.63
60	0.866	8.66	1.30	1.96
70	0.940	9.40	1.78	2.25
80	0.985	9.85	2.30	2.44
90	1.000	10.00	2.50	2.50
100	0.985	9.85	2.30	2.10
110	0.940	9.40	1.78	1.26
120	0.866	8.66	1.30	0.63
130	0.766	7.66	0.90	0.17
140	0.643	6.43	0.60	−0.17
150	0.500	5.00	0.40	−0.29
160	0.342	3.42	0.23	−0.38
170	0.174	1.74	0.11	−0.47
180	0	0	0	−0.57
190	−0.174	−1.74	−0.11	−0.70
200	−0.342	−3.42	−0.23	−0.84
etc.	etc.	etc.	etc.	etc.

TABLE II

FOURIER COEFFICIENTS FOR CURVES I, II, III, FIG. 3

Order of Harmonic	I Flux		II i_m		III i_{mh}	
	sine coeff.	cosine coeff.	sine coeff.	cosine coeff.	sine coeff.	cosine coeff.
1	10	0	1.708	0	1.693	0.801
3	0	0	−0.557	0	−0.552	−0.243
5	0	0	0.158	0	0.167	−0.011
7	0	0	−0.053	0	−0.051	0.029
9	0	0	0.018	0	0.027	−0.013
11	0	0	−0.009	0	−0.013	0.025
13	0	0	0.001	0	0.004	−0.022
15	0	0	0.008	0	0.006	0.003

Because the impressed voltage contains no harmonic components is no more ground for saying that the triple harmonic components of transformer exciting current are entirely wattless than it is for concluding that the sixty-cycle currents on the secondary side of a 25- to 60-cycle frequency-changer set must be entirely wattless. In the latter case, we have energy transformation from electrical to magnetic to mechanical to magnetic to electrical; while in the former there is a more direct transformation from electrical to magnetic to electrical.

TRIPLE HARMONICS IN THREE-PHASE BANKS OF TRANSFORMERS.

In a bank of three identical transformers connected star on the primary side and operating under normal balanced conditions on a three-phase system, the voltages between the terminals of the individual transformers, that is, the voltages between the line terminals and neutral, are alike in wave shape, equal in magnitude and displaced from each other by one-third cycle, or 120 time-degrees. Three such voltages are shown in Fig. 4, sine waves of impressed voltage being assumed for simplicity.

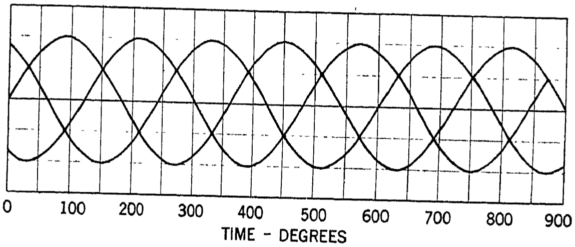

FIG. 4—VOLTAGE RELATIONS—LINES TO NEUTRAL—THREE-PHASE STAR-CONNECTED TRANSFORMER BANK—BALANCED NORMAL CONDITIONS.

It has been shown above that ordinarily, when a transformer is energized, there are produced currents of harmonic frequency in the effort to adjust the counter e.m.f. to the impressed. If three identical transformers have impressed upon them e.m.fs., which are identical in wave shape and magnitude, the exciting currents will be identical in wave shape and magnitude. If the three e.m.fs. impressed upon the three identical transformers, are displaced from each other by 120 time-degrees, the corresponding exciting current waves will be displaced from each other by 120 time-degrees.

Since the three exciting current waves are equal in shape and magnitude and are 120 time-degrees apart, the fundamental components of the exciting currents will be equal in shape and magnitude and 120 time-degrees apart. Since the three exciting current waves are identical in shape and magnitude, harmonics of equal order and magnitude exist in each, and these harmonics bear identical phase relations to their respective fundamental components. That is, if the n th harmonic in the exciting current of transformer No. 1 is $k_n \sin (n\theta + \alpha_n)$, the n th harmonic in the exciting current of transformer No. 2 is $k_n \sin [n (\theta \pm 120°) + \alpha_n]$, and that in transformer No. 3 is $k_n \sin [n (\theta \pm 240°) + \alpha_n]$. The values of the variable parts of these expressions are given in Table III for odd values of n.

TABLE III

n	$\sin (n \theta + \alpha_n)$	$\sin [n (\theta + 120°) + \alpha_n]$	$\sin [n (\theta + 240°) + \alpha_n]$
1	$\sin (\theta + \alpha_1)$	$\sin (\theta + \alpha_1 + 120°)$	$\sin (\theta + \alpha_1 + 240°)$
3	$\sin (3 \theta + \alpha_3)$	$\sin (3 \theta + \alpha_3)$	$\sin (3 \theta + \alpha_3)$
5	$\sin (5 \theta + \alpha_5)$	$\sin (5 \theta + \alpha_5 + 240°)$	$\sin (5 \theta + \alpha_5 + 120°)$
7	$\sin (7 \theta + \alpha_7)$	$\sin (7 \theta + \alpha_7 + 120°)$	$\sin (7 \theta + \alpha_7 + 240°)$
9	$\sin (9 \theta + \alpha_9)$	$\sin (9 \theta + \alpha_9)$	$\sin (9 \theta + \alpha_9)$
11	$\sin (11 \theta + \alpha_{11})$	$\sin (11 \theta + \alpha_{11} + 240°)$	$\sin (11 \theta + \alpha_{11} + 120°)$
13	$\sin (13 \theta + \alpha_{13})$	$\sin (13 \theta + \alpha_{13} + 120°)$	$\sin (13 \theta + \alpha_{13} + 240°)$
15	$\sin (15 \theta + \alpha_{15})$	$\sin (15 \theta + \alpha_{15})$	$\sin (15 \theta + \alpha_{15})$
17	$\sin (17 \theta + \alpha_{17})$	$\sin (17 \theta + \alpha_{17} + 240°)$	$\sin (17 \theta + \alpha_{17} + 120°)$

From Table III and the foregoing, it is seen that the components of third, ninth, fifteenth and of other triple harmonic frequencies in the three exciting currents are respectively identical in phase position as well as in magnitude; that is, the triple harmonics of the same order in the three transformers are displaced from each other by zero time-degrees; while the components of fifth, seventh, eleventh and of other non-triple harmonic frequencies have mutual three-phase displacements of 120 time-degrees.

In general, the magnitudes of the harmonic components in the exciting current of a transformer vary inversely with the orders of the harmonics though not necessarily in inverse proportion. Thus, it is seen that the third harmonic is usually the largest component except the fundamental. For this reason, and because of the above mentioned lack of phase displacement between the triple harmonics in the exciting currents of a three-phase bank of transformers connected star on the primary side; the third harmonic deserves, and has received, the most consideration of any of the harmonics produced in the energization of transformers.

Star-Delta. In a three-phase bank of transformers with primary connected star, and secondary connected delta, as in Fig. 5; the triple harmonic components of exciting current, being in phase in all three transformers, tend to flow, say, from line terminals to neutral at a given instant. The primary neutral is insulated so that these triple harmonic currents can not flow in the primary winding. The lack of these triple harmonics of exciting current distorts the waves of flux in the cores so that the latter contain triple harmonics. Then the voltages, induced in the primary and secondary windings by the variations of the flux, contain triple harmonics, which are in phase in all three of the transformers. These triple harmonic voltages, being in phase with each other, tend to establish currents in the same direction in all three transformers, and these triple harmonic currents actually obtain in the transformer operation, flowing around the closed delta secondary as indicated by the solid arrows in Fig. 5. These triple harmonic currents in the secondary windings establish corresponding triple harmonic fluxes which neutralize the distortion of the flux waves originally caused by the absence of triple harmonic components in the primary exciting current. In effect, the result is, insofar as concerns the transformation of voltage, practically the same as if the triple harmonic currents were present in the primary exciting current. In fact, under such conditions, we say that the triple harmonics of exciting current flow in the secondary delta.

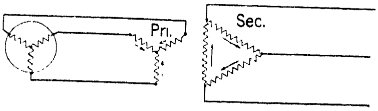

FIG. 5

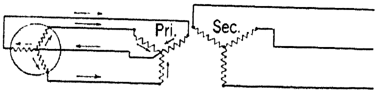

FIG. 6

Star-Star with Four-Wire Primary. In a three-phase bank of transformers connected star-star with the primary neutral connected to the generator neutral, as in Fig. 6; the triple harmonic components of exciting currents are established in the primaries of the transformers with the neutral as a common wire thence in the generator windings and return through the phase wires, as indicated by the arrows in the diagram, Fig. 6.

Star-Star with Isolated Neutrals. In the star-star connected bank of Fig. 7, both the primary and secondary neutrals are insulated, so that no third harmonic components of the exciting current can exist. In such a case the flux waves are badly distorted, and consequently the voltages between lines and neutral contain large triple harmonic components. Since, in a bank of three identical transformers in normal balanced conditions, the respective triple harmonics are equal and in phase in all three transformers, the differences of the triple harmonic components in any two transformers are zero, so that the triple harmonic voltages will not appear between line terminals.

Under average operating flux densities, the third harmonic component in the voltage from line to neutral, in a bank of transformers connected as in Fig. 7, may be 60 per cent of the rated fundamental component *. It is obvious that this will cause a serious extra strain on the insulation if the phase relation between the fundamental and the third harmonic components of voltage in a transformer is such as to result in increasing the maximum value of the total voltage wave by 60 per cent.

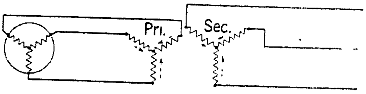

FIG. 7

If, in Fig. 7, the generator neutral is grounded, the triple harmonic components of voltage will be manifested between primary neutral and ground. Also, if the line, to which the secondary is connected, feeds a grounded neutral load, or if the secondary line is so extensive that the admittance between the lines and ground is very large compared to the admittance between the secondary neutral and ground, the triple harmonic voltages will appear between secondary neutral and ground.

Other Types of Connections. The possible three-phase connections and combinations of the above simple cases are innumerable because they include banks made up of dissimilar transformers. But the triple harmonic phenomena concerned in such connections are fundamentally and essentially the same in all.

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*Louis F. Blume, TRANS. A. I. E. E , 1914 p. 751.

One more possible connection will be discussed because it gives rise to rather unique conditions and because upon it the main part of the thesis will dwell.

Star-Star with Grounded Neutral on Line Side Only. In the case of a bank of three identical transformers supplied from an isolated neutral three-phase source, the bank connected star-star with grounded neutral on the line side only and feeding an extensive transmission line, as shown in Fig. 8, there is no path for triple harmonics of exciting current in the primaries, but the admittances between the line conductors and ground complete the circuits so that the triple harmonic exciting currents can be established in the secondary windings through the neutral as a common wire, thence through the admittances between neutral and line conductors and return via the latter as indicated by the heavy arrows in the diagram, Fig. 8.

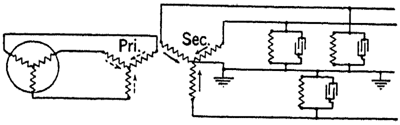

FIG. 8

Since the primary is essentially open-circuited for currents, that are in phase in all three transformers, it is seen that these triple harmonic secondary currents are truly exciting currents, and as such are affected by the exciting reactance (exciting susceptance divided by the square of the exciting admittance) of the transformers.

In the case of modern transmission lines and networks, often involving 100 miles, and sometimes many hundreds of miles of line, it is easily seen that the impedance offered by the line admittance to ground may be very small compared to the open-circuit impedance of a transformer even for currents of fundamental frequency. The reactance of the transformer is inductive and will be much larger at third harmonic frequency than at fundamental. On the other hand, the equivalent reactance of the line is capacitive, and is consequently much less at third harmonic frequency than at fundamental. So that, for the third harmonic exciting currents, the impedances offered between line conductors and ground may be negligible compared to the internal impedances of the transformers. This means that the major part of the third harmonic voltages, which maintains the third harmonic secondary current, is consumed in the transformer winding and that only a very small portion of this third harmonic voltage appears across the terminals of the transformer between line conductor and ground. Essentially, the arrangement is similar to a generator of very high internal impedance, generating a voltage of third harmonic frequency which is applied to a load of comparatively low impedance. It is so like the phenomena of some types of constant current generators that the third harmonic of exciting current is often said to have constant current regulation.

From the above considerations, it is seen that, in cases of long transmission lines connected to star-star transformer banks, having grounded neutral on the line side only, the voltage from line terminal to neutral is not necessarily materially augmented by a third harmonic.

Hence, the theory, that the breakdowns of insulation and burn-outs, which have accrued with this type connection, were due to the third harmonic voltage between line terminals and ground, at least loses weight as a general explanation.* Also the anxiety over triple harmonic electrostatic induction in communication circuits strung in the neighborhood of such power lines is allayed.†

EVEN HARMONIC PHENOMENA

As mentioned before, about two years ago some tests of a star-star connected transformer bank supplying a three-phase transmission line, developed unstable conditions and peculiar phenomena, which had not been observed in any previous tests or operation of such banks so far as general opinion and a survey of the literature on transformers disclosed. The entire absence of any direct-current component in the transformer currents, and the stable operation of the transformers at impressed voltages slightly less than those that gave rise to the phenomena, eliminated the possibility that residual magnetism or direct-current excitation, as ordinarily understood, were responsible for the phenomena.‡ Because of the apparent novelty of the phenomena, the writer became interested in establishing an explanation consistent with the generally accepted fundamental laws of electricity and magnetism.

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*L. N. Robinson, A. I. E E. TRANS. 1915, Vol. XXXIV, Part II, p. 2183-2184 and 2189-2191. (PROC. August 1915, pp. 1675-1676 and 1680-1681).

†H. S. Osborne, A. I. E. E. TRANS. 1915, Vol. XXXIV, Part II, p. 2175-2179. (PROC. March 1916, pp. 423-427).

‡J. B. Taylor, TRANS, A. I. E. E., 1909, pp. 725-732.

The phenomena were accompanied by vibrations and noises issuing from the transformers, and therefore it was first supposed that the trouble might be due to a loose coil or loose iron vibrating in response to the repulsion or attraction of the alternating magnetic fields. The transformers were new from the factory and had never before been in service, so they were tested. The low-tension windings were short-circuited and five times normal current was alternately suddenly applied to, and suddenly removed from, the high-tension windings several times. All the three transformers in the bank were tested thus, and none of them emitted any noise or showed any other signs of distress.

There was available another bank of transformers that had previously been in commercial service several months, and in testing service a year and a half. The transformers of this bank differed from the first in practically every essential even to the name of the manufacturer. This is shown by the following tabular comparison of their ratings:

	Bank No. 1	Bank No. 2
Manufacturer	*A*	*B*
Type	Core	Core
Frequency (cycles)	50	60
Kilovolt-amperes per unit	37.5	50
High-tension volts	15000/14250/12980 12540/7500/7125	22000/21450/20900/20350 19800/11000/10800/9960
Low-tension volts	440/220/110	2400/480

The tests on the first bank had given rise to a new theory in explanation of the phenomena. In several instances, by this theory, it was possible, by estimates based upon quite crude data, to predict, within a few per cent, at what impressed voltage the phenomena would occur in the second bank. In one case in particular, certain phenomena were predicted at a certain impressed voltage and the prediction was fulfilled although, when tried, they did not obtain at an impressed voltage thirteen per cent lower.

While these numerous fulfilled predictions added much weight to, and tended to sustain, the theory, they did not necessarily clearly and conclusively establish it. The explanation of the phenomena, so far as they had been analyzed, was submitted to the American Institute of Electrical Engineers at the September meeting of 1915.* Since the ensuing discussion did not develop a sufficiently definite contradiction of the theory, there was impetus added to further investigation.

Briefly, the original observations involved several different conditions:

"1. A star-star bank, with generator neutral isolated and line-side neutral grounded. The duty was charging a line 37 miles long.

"2. Another bank similarly connected, but composed of units made by a different manufacturer and of different ratings in every respect. The duty of this bank was the same as for that in case 1 but on another day.

"3. A bank of 1 to 1 auto-transformers connected star with grounded neutral, at the sending end of the 37-mile line, which was charged by a delta-delta bank."

"4. A bank of transformers stepping down from isolated neutral star to 'interconnected delta'. In this instance, the current circulating in the 'interconnected delta' contained prominent even harmonics.

"5. A bank of 13,200 to 110-volt potential transformers connected star with grounded neutral to the sending end of the 37-mile line, which was supplied from a delta-star bank with isolated neutral. The secondaries of the potential transformers were open-circuited."

I. "In some cases, when charging the line from transformers connected star-star with grounded neutral on the line side, an undertone of one-half fundamental frequency was present in the leg voltages (voltages from line terminal to neutral) and in the currents on the generator side of the bank."

II. "In some other cases, two of the leg voltages were approximately $\sqrt{7/3}$ times the delta voltage (voltage between line terminals), while the magnitude of the voltage of the third leg was approximately normal; that is, 58 per cent of the delta voltage. The leg voltages had distinctly different wave shapes, and their fundamental components were conspicuously not 120 time-degrees apart in phase." "The currents in the buses on the generator side of the bank were several times normal."

III. "In still other cases, the leg voltages had double frequency components, approximately four times as large as the fundamental components, and the transformers vibrated internally."

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*L. N. Robinson, TRANS, A. I. E. E., 1915, pp. 2183–2195.

IV. "In the case of the 1 to 1 auto-transformers, the delta voltages were affected by the phenomena, becoming nearly twice as great as existed when the auto-transformers were not connected to the line. The delta voltages were of fundamental frequency with no double frequency component, while the leg voltages contained the large double frequency component."

"In all cases in which the second harmonic was prominent in the leg voltages, it was in time-phase in all three legs. This was proved by oscillograms of the simultaneous values of the three leg voltages, by a measurement of their vector sum, and by fact that the second harmonic was not in the waves of delta voltages."

Periodically Reversing Leg. In submitting the previous paper to the Institute, it was said that: "The case (I above) in which the second undertone, or 'one-half' harmonic was present, is explainable by a periodically reversing leg; *.i.e,* one leg of the bank reversed once for each fundamental cycle, so that the fundamental component of current in the leg tended to flow in the same direction through the unit during the entire cycle. In the next cycle the leg reversed again, so that the fundamental component of current tended to flow in the opposite direction from that in the previous cycle."

By this it is meant that the transformers, originally in the relative electrical positions of Fig. 9, reversed at some point in the cycle and assumed the relative positions of Fig. 10; and that at a corresponding point in the next cycle they reversed back to the position of Fig. 9, etc. Because the delta voltages are fixed by the three-phase generator, the vector differences, by pairs, of the fundamental components of the three leg voltages must be equal in magnitude and their vectors must close an equilateral triangle for the conditions of both Fig. 9 and of Fig. 10. This requirement is met by the vector arrangement of the fundamental components of leg voltages shown in Figs. 11 and 12 respectively.

While this gives analytical and graphical illustration of the phenomena, it brings up the question, how can there be a periodically reversing leg in the absence of any pole changing devices?

Causes. As mentioned above, the phenomena were discovered under conditions shown in Fig. 8. In the star-connected, three-phase transformer bank, under normal, stable conditions the fundamental components of the primary impressed voltages may be represented by vectors as in Fig. 13. $\underset{\cdot}{E}_{A}$, $\underset{\cdot}{E}_{B}$ and $\underset{\cdot}{E}_{C}$ represent the voltages impressed upon the bank between line terminals; and $\underset{\cdot}{E}_1$, $\underset{\cdot}{E}_2$ and $\underset{\cdot}{E}_3$ represent the voltages impressed upon the individual transformers Nos. 1, 2 and 3 respectively.

$$\underset{\cdot}{E}_{A} = \underset{\cdot}{E}_1 - \underset{\cdot}{E}_2, \underset{\cdot}{E}_{B} = \underset{\cdot}{E}_2 - \underset{\cdot}{E}_3 \text{ and } \underset{\cdot}{E}_{C} = \underset{\cdot}{E}_3 - \underset{\cdot}{E}_1.$$

If a line in the plane of the diagram, Fig. 13, is rotated about the origin as axis, at a constant angular velocity of $2\,\pi f$ radians per second, where f is the frequency of the fundamental voltages in cycles per second; the magnitudes of the different voltages will be equal to the intercepts upon this line of the circles whose vector diameters represent the maxima of the respective voltage waves. The relative instantaneous values of these six voltages are given in Fig. 15 (*a* to *l*) for each twelfth of one cycle, as indicated.

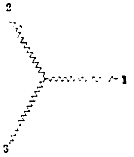

FIG. 9

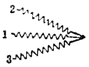

FIG. 10

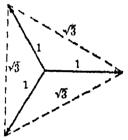

FIG. 11

The fundamental components of the secondary voltages may be represented in similar fashion differing from the respective primary impressed voltages in magnitude according to the ratio of transformation, and in phase by approximately π radians.

For present purposes, the secondaries of any two of the transformers, say Nos. 1 and 2, of the bank may be resolved into the typical circuit shown in Fig. 14. While in the actual case, the generator is not electrically connected to the secondary of the transformer, the generators shown in Fig. 14 serve merely to represent the sources of the fundamental frequency voltages, $\underset{\cdot}{E}_1$ and $\underset{\cdot}{E}_2$, and combined they are the sources of $\underset{\cdot}{E}_{A}$.

These fundamental frequency voltages give rise to fundamental frequency currents in the respective circuits of Fig. 14. These currents involve magnetomotive forces that produce magnetic fluxes in the transformer cores. Because these cores are iron, the ratio, B/H or ϕ/i, varies during each cycle, following a hysteresis loop like that of Fig. 2, Consequently, the inductance of the circuit varies because the inductance is the ratio, ϕ/i, multiplied by a constant.

Under the discussion of "Star-Star Connection with Grounded Neutral on Line Side Only" above, it was shown that the inductive exciting reactance of the transformer might be smaller than, equal to, or greater than, the series capacitive reactance of the line. Since the manner of variation of B/H, or ϕ/i, is dependent upon the maximum value of ϕ, or of i, and since these depend in turn upon the maximum value of the voltage, E_1 or E_2; it is seen that by varying the voltage a value will be reached where the circuit becomes resonant; that is, where the inductive reactance of the transformers and line becomes equal in magnitude and opposite in sign to the capacitive reactance.

This leads directly to the most interesting fact, that the sum of the magnetomotive forces in the transformer, primary and secondary combined,* will pass through zero at the same instant as the impressed voltage, E_1, for some value of this voltage.

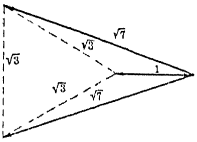

FIG. 12

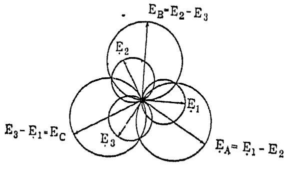

FIG. 13

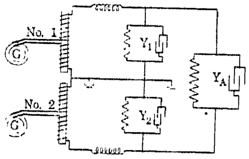

FIG. 14

Consider now the hysteresis loop of Fig. 16. The dashed loop is the one that would obtain under normal stable, or balanced, conditions such as shown in Figs. 9 and 11. Starting from the point, A, the flux will vary along the curve to B as indicated by the arrow. If, as the total magnetomotive force passes through zero at this point, B, the voltage, E_1, also passes through zero; the electrical circuits of transformer No. 1 are collapsed; that is, E_1 is passing through the value shown in Fig. 15 (*d*). In the succeeding instant, as E_1 increases, the transformer No. 1 may assume the position of Fig. 9 or the reversed position of Fig. 10. Which will it select? Nature demands the easiest course, so it will select the reversed relations of Fig. 10 and 12.

At the point, B of Fig. 16, there is energy stored in the core.†

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*C. P. Steinmetz, "Theory and Calculation of Electric Circuits," p. 219, Fig. 106.

†C. P. Steinmetz, "Theory and Calculation of Electric Circuits," p. 56, §36.

Where there is stored energy, there is inertia, either positive or negative. In the magnetic field or moving body, there is positive inertia; that is a tendency to resist alteration of conditions. In the electric field or breech full of gunpowder, there is negative inertia, called a charge; that is, when released, by short-circuiting the condenser or igniting the powder, the stored energy assists alteration with maximum effect at the start. In some cases, both positive and negative inertia exist, *e.g.* in electromagnetic phenomena. A most striking illustration is a carload of gunpowder standing on a track. If a source of motive

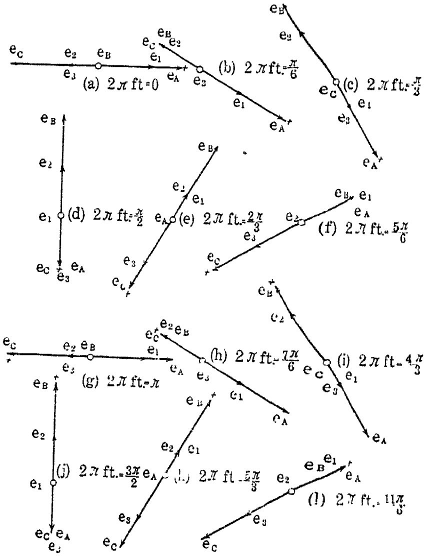

FIG. 15

power is applied, the positive inertia of the car and its load tend to keep the car at rest. If a spark touches the gunpowder, the negative inertia comes into action and an opposite result follows. This distinction of sign of inertia is borne out in consideration of electric and magnetic circuits because the inductance of an electric circuit holds a place in the fundamental equations analogous to the moment of inertia in the equations of mechanics, and the capacitance holds a place analogous to the elasticity of a spring. Of course, elasticity is not synonomous with negative inertia. It is the elastic quality, of the material, that induces storage of energy in such manner as to create negative inertia.

From the standpoint of inertia, when the m.m.f. is altered by a small amount from that at the point, *B* on the loop of Fig. 16, only a slight change in ϕ will be required if the flux proceeds along the solid curve toward *C*; that is, little inertia need be overcome; whereas a large change in ϕ will be required to carry the flux along the dashed curve toward *K*.

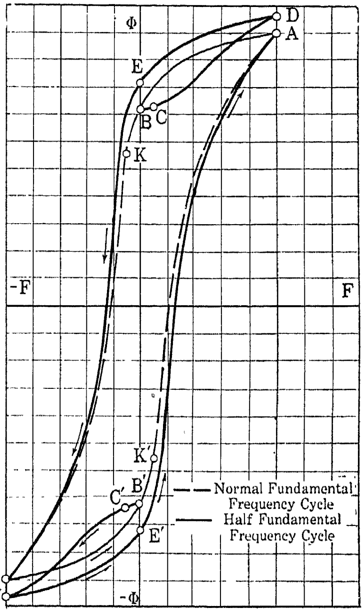

FIG. 16—HYSTERESIS LOOP FOR "REVERSING LEG"

To store more energy or to extract some of the stored energy would require an expenditure of energy proportional to the amount of energy to be stored or extracted. The expenditure of energy results in work being done. To change the amount of stored energy represented by the flux at the point, *B*, to that at the point, *K*, would demand that an amount of work be done which is proportional to the increment of m.m.f. multiplied by the difference of the fluxes; that is, the work would be approximately

$$W_K = \frac{F_K\,(\phi_B - \phi_K)}{2}$$

Whereas, to change the stored energy from that at B to that at C would require approximately

$$W_C = \frac{F_C\,(\phi_C - \phi_B)}{2}$$

for an increment of m.m.f. equal in magnitude to F_K.

It is seen from the diagram, Fig. 16, that $\phi_C - \phi_B$ is less than $\phi_B - \phi_K$ in magnitude,
and since

$$F_K = - F_C$$

$$W_C = \frac{F_C\,(\phi_C - \phi_B)}{2} \text{ is less than } W_K = \frac{F_K\,(\phi_B - \phi_K)}{2}$$

Hence, by both the theories of inertia and of least work, one of the transformers of the bank will reverse under the given conditions.

After the transformer flux has started from the point, B, toward the point, C, it will continue along the solid curve to a point, D, where the maximum m.m.f. is reached, because the impressed voltage is continuously applied. If F_D is equal to, or greater than, F_A, ϕ_D will be greater than ϕ_A because of the energy stored at the point, B, where $F=0$; just as a separately excited generator will build up to higher value of voltage if it has some initial magnetism than it will if the flux is initially zero.

As the m.m.f. decreases, the flux will follow the solid curve from D to E, where ϕ_E is different from ϕ_B; so that, when the curve crosses the axis at E, the magnetomotive force, F, and the impressed voltage, E_1, do not pass through zero together. Consequently another reversal does not take place at the point, E, but the flux proceeds along the solid curve to the point, A', and thence to B', where another reversal takes place, returning the transformers to the relative positions of Figs. 9 and 11; and so on, reversing once in each fundamental cycle.

If the magnetomotive force or voltage waves contain harmonic components, the problem becomes more complicated; but the phenomena of periodic reversal will obtain when the impressed voltage and circuit conditions are such that the sum of the magnetomotive forces in the transformer passes through its zero value at the same instant as the impressed voltage.

The effect of this periodic reversal is to distort the voltages between line terminals and neutral as indicated by Figs. 11 and 12, and to introduce high harmonics because of the enormous magnetic densities to which transformers Nos. 2 and 3 are subjected.

Since the period of a total cycle of a reversing leg is twice as long as the fundamental frequency cycle of the impressed voltage, the phenomena are easily distinguishable by the presence of a second undertone, or "one-half" harmonic, in the voltages between line terminals and neutral.

Tests. It was attempted to simulate the conditions of Fig. 8 in the laboratory by using three similar 10 kv-a. transformers. Unstable conditions were obtained, but almost as soon as they appeared, arcs would develop in the condensers, producing large, very high-frequency effects that made the recording of oscillograms useless. The only condensers, that were available and at all suitable for the purpose, were one microfarad and two microfarad paraffin paper telephone condensers which apparently are not of sufficient dielectric strength. Other methods of obtaining the conditions are under consideration and will be tried as soon as possible.

Permanently Reversed Leg. The previous paper disposed of Case II by saying: "In the case with the stable unbalanced leg voltage condition, the neutral on the generator side of the bank assumed a fixed position with one leg reversed so that the voltages from two of the line conductors to neutral were approximately $\sqrt{7}$, (2.64), times the voltage from the third conductor to neutral.

It is easily demonstrated in the laboratory that one transformer of an energized three-phase star-star connected bank will collapse if a non-reactive load is connected between one line terminal and neutral; that is, the voltage of this transformer will fall to zero, and the other two will operate as in the open-delta connection under no-load.

It is equally simple to demonstrate that, by applying to one transformer of the energized star-star connected three-phase bank an e.m.f. from a single-phase generator, the voltage of this transformer may be made to assume any desired phase relation with respect to the voltage of the other two by merely adjusting the phase relation between the three-phase generator supplying the bank and the auxiliary generator. Among these possibilities is the reversed leg relation of Fig. 10, whose voltage relations are given in Fig. 12.

Of course, it must be noted that in these abnormal relations, the voltages on at least two of the transformers are abnormal because the vector difference of the leg voltages by pairs must always be the voltages between line terminals supplied by the three-phase generator. Hence, unless the tests are conducted at low voltages, dangerous voltages may arise or excessive exciting currents be demanded.

Causes. The question arises; what causes the transformer, under service conditions, to reverse to this abnormal position, Fig. 10, rather than assume the normal position of Fig. 9?

It is to be recalled that the abnormal phenomena arose upon charging the transformers and line together by closing the oil-switch on the generator side on the bank. As stated in the previous paper; "when charging the line the first time, leg *A* might reverse and continue reversed as long as the bank was excited. Switching off and on again might bring either leg *B* or leg *C* in reversed, and the reversed leg would be a stable condition as long as the transformers and line were energized. Occasionally, the three legs would come in without a reversed leg and conditions would be normal."

Subsequent to these tests, large accumulations of atmospheric electricity were observed on the line conductors under favorable seasonal conditions. With the line isolated throughout its length, an electrostatic voltmeter indicated differences of potential between line conductors and ground well up in the hundreds of volts. The electrostatic voltmeter had a range of only 80 volts and by the use of a series capacitance as multiplier the range could be raised to only something like 300 volts, but the potentials went well beyond this range. After grounding the line to drain the charge, the potentials would build up again in a few minutes. When neighboring bodies at or near the same horizontal plane become charged with atmospheric electricity, they all accumulate charges, which, though they may differ among themselves, give all these bodies potentials of the same algebraic sign with respect to the earth potential.

These considerations give foundation to the idea that, shortly after the three-phase alternating-current supply to the lines has been disconnected, the line conductors assume potentials of the same sign, say positive. True, under the conditions where these phenomena were discovered, Fig. 8, the line conductors were connected to grounded neutral transformer windings; but these windings and conductors themselves have resistance, so that any drainage of the charge from conductors to ground necessitates a difference of potential, and all three conductors will continue to have positive potentials, however large or small.

When the oil switch is closed on the generator side of the bank, the inertia of the electric and magnetic fields tends to keep all three of the line conductors and transformer windings at positive potentials with respect to the earth, and the alternating currents start through the transformers in conformity with the initial conditions provided. After once started, the phenomena of the permanently reversed leg must continue unless, at some point in the cycle, the sum of the m.m.fs. in the reversed transformer passes through zero at the same instant as the impressed voltage, and the transformer reverses periodically as discussed above in connection with Fig. 16; or the even multiple frequency discussed below, or other yet undiscovered phenomena intervene and take precedence.

Pulsating Neutral. In the third case presented in the previous paper, the voltages between line conductors and ground contained even multiple frequency components. The original explanation of this phenomenon, though not contradicted, was questioned for want of clearness, and it will therefore be discussed here more fully.

It has been shown above that, when the transformers and line of Fig. 8, are energized by closing a switch on the generator side of the bank, one of the transformers may come in reversed on account of the initial potential of the line conductors (see Fig. 10); and that, if the sum of the magnetomotive forces in the reversed transformer does not later pass through its zero value at an instant when the impressed voltage passes through zero, there will be no opportunity for this transformer to reverse to its normal position, Fig. 9.

With transformer No. 1 reversed and Nos. 2 and 3 in the positions shown in Fig. 10, the fundamental component of voltage in Nos. 2 and 3 is 2.64 times that in No. 1. At low magnetic densities, this unbalance in the voltages apparently does not produce excessive unbalance of energy requirements among the three phases.

But as the voltage is increased, and the hysteresis loop of the transformers, Nos. 2 and 3, bends over, as in the outer loop of Fig. 27, page 52 of Steinmetz' "Theory and Calculation of Electric Circuits," the energy requirement in transformers Nos. 2 and 3 becomes enormous and thus with that of transformer No. 1 produces a gross unbalance.

Such being the case, it appears to be easier for the transformers to assume electrical relations as lateral edges of an inverted isosceles tetrahedron, Fig. 17, and thereby balance their energy requirements rather than assume the plane relations of Fig. 10. The edges of the base of the vector tetrahedron, Fig. 18, corresponding to Fig. 17, are the voltages between line terminals and conform to the limitations set by the generator. The vectors from the center of the base to the corners of the base are the fundamental components of the leg voltages.

Since the line conductors tend to stay at a positive potential with respect to the earth, and since the potential of the earth is constant, zero, the base of the tetrahedron must be lifted from the apex to its maximum position twice in each fundamental cycle. That is, the difference of potential between the apex (neutral of the transformers) and the center of the base (neutral of the line conductors) has two maxima and two minima for each fundamental cycle. Hence, the e.m.f., represented by a vector coincident with the altitude of the tetrahedron, has double frequency. The lateral edges of the tetrahedron do not represent the leg voltages because thus far it is only possible to represent single frequency variables by single vectors; and leg voltages are the vector differences between the double frequency vector (altitude of tetrahedron) and the fundamental frequency leg voltage components (center of base to corners of base).

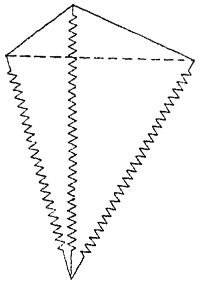

FIG. 17

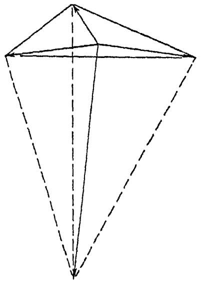

FIG. 18

While this explanation is perhaps still unsatisfactory in completeness, it is as far as it is possible to go at present. To sum it up: it seems that, at higher voltages and consequently higher flux densities, the pulsating neutral phenomena, evidenced by the double-frequency voltages between line terminals and neutral, are preferable to the permanently reversed leg phenomena either because the former produce better balance in the energy requirements of the three units or because they make less total energy demand on the system, or both. The last is probably the actuality because in nature a symmetrical balance is usually requisite to maximum efficiency.

Auto-Transformers. In the fourth case, the high-voltage windings of a bank of transformers were connected star to ground the neutral of an otherwise isolated-neutral transmission line. The low-tension windings of the star-connected transformers were also star-connected with no-load, but neutral grounded to guard against induced potentials. Thus, in effect, these star-connected transformers were 1 to 1 auto-transformers or ironclad reactors connected between line conductors and ground.

The even harmonic phenomena were observed with this arrangement much as in the case of the star-star connected transformers discussed above. Further than this, the voltages between line terminals were approximately doubled by the presence of the auto-transformers.

At this time, nothing more can be said in explanation of the behavior of the auto-transformers than was said in the previous paper, except insofar as the above discussion of star-star connected transformers applies.

Sixth-Harmonic. Under the conditions where the second harmonic prevailed in the voltages between line conductors and ground, there was a large sixth harmonic component in the line and neutral currents, but there was no pronounced sixth harmonic component in the voltages between any line terminals of the transformers and neutral. This sixth harmonic current is, of course, the third harmonic of the second, and is required in the adjustment of the current in the ironclad circuit to meet the second harmonic voltage conditions.

Vibrations—Noises. After a consideration of the tremendous forces, electromotive and magnetomotive, to which the transformers were subjected during the phenomena discussed above, it seems almost like adding insult to injury to wonder why the transformers vibrated and emitted noises evidencing distress.

There can be no doubt that these vibrations and noises were the results of alternate repulsion and relaxation of stresses in the cores or coils, or both, and in their supports and braces, produced by the extremely large fluxes involved.

Laboratory Tests. In order to arrive at a rigorous solution of the phenomena, it is naturally desirable to study them under laboratory conditions where the factors may be segregated and analyzed. With this object in view, it was attempted to simulate the conditions of Fig. 8, in the electrical laboratory here.*

It is obvious that to approach resonance between artificial line and transformers a considerable investment in condensers would be involved unless low-voltage, low-power transformers were used. On this basis, three 60-cycle, 10-watt, 1100/2200-110-volt potential transformers were selected. It was planned to connect them star-star, 1100 to 110-volts, thus putting the artificial line on the 110-volt side because high-voltage condensers were not available.

Meter measurements of the exciting admittances and susceptances from 30 volts up to 110 volts showed approximate equivalent exciting reactances ranging from 150 ohms to 350 ohms. In a rough way, this indicated that 20 microfarads per phase would be required for the tests. Sufficient paraffin paper condensers were provided.

While electrostatic voltmeters would have indicated the effective values, r.m.s., of the abnormal voltages, it is essential that the phenomena be studied by means of an oscillograph. An electrostatic oscillograph was not available, and an electromagnetic oscillograph makes so large an energy demand upon the circuit investigated than an oscillograph vibrator with its series resistance connected between artificial line conductor and neutral would have materially altered the character of the line. So it was decided to investigate the currents in the three transformers and line conductors by the insertion of 1.00-ohm shunts from which drop leads were connected to the oscillograph.

The apparatus was connected as shown in Fig. 8 except that no conductance was introduced other than that inherent in the paraffin condensers. The transformers were in one room near the generator, while, in order to expedite manipulation while operating the oscillograph, the condensers were placed in the oscillograph room about 75 feet from the transformers. This made it possible to insert the 1.00-ohm oscillograph shunts between the condensers and neutral, which latter was grounded for safety.

By an auxiliary potential transformer connected between supply lines, it was possible to keep track of the delta voltage impressed on the transformers.

Because of the excessive leg voltages anticipated, the tests were started at very low impressed voltages, and at each voltage the capacitances between line conductors and ground were varied over the available range, balanced and unbalanced. Of course, the current from line conductors to neutral showed the usual transformer harmonics, but nothing abnormal occurred until 1940, (1120 $\sqrt{3}$), volts between wires was applied to the primary side.

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*University of California.

With 1940 volts applied thus, the waves of line to neutral currents, on the visual tracing table of the oscillograph, began to travel, so to speak. That is, there was introduced a very low frequency component which had not been observed in previous tests. These phenomena came with only six microfarads between each line conductor and neutral. They would not exist at lower voltages with larger capacitance or at this voltage with smaller capacitance. This substantiates the theories that the phenomena depend upon a critical point in the variation of the exciting reactance of the transformer, and upon the inter-relation of the line susceptance and transformer reactance.

The records of these phenomena are submitted in oscillograms, *T*-1, *T*-2 and *T*-3.

Noting the 60-cycle (same frequency as supply) timing wave on oscillogram *T*-3, vibrator 1 and the current from line conductor 3 to neutral, shown on vibrator 3 of *T*-3; it is observed that, for any two successive 1/60-second intervals, the latter wave does not repeat itself but undergoes a consistent alteration. In fact, there appear to be no two identical 1/60-second sections in the whole record, *T*-3, which covers more than 1/6 second. This is consistent with the instability and apparently very low frequency alternation observed on the oscillograph tracing table before the record was taken. Also, the progressive alteration of the wave, compared to the stable timing wave from the same source, indicates conclusively that the currents in the line, and thereby the voltages from line to neutral, are in considerable measure independent of the wave shape and frequency of the impressed supply voltage when the grounded neutral star-star connection is employed in transmission under suitable conditions.

The tests could not be carried materially beyond 1940 volts impressed because the potential transformers were then carrying 1.6 amperes each and were doubtless withstanding excessive voltages. They became so hot that the insulation filler was softened.

Next, series resistances up to approximately 200 ohms each were inserted in the line conductors. These had no observable effect on the production of the phenomena. They probably reduced the magnitudes of the currents somewhat but even this was not noticeable on the oscillograph.

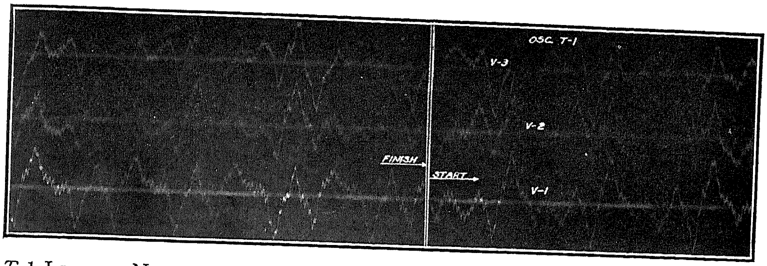
It was suggested that admittances connected directly between line conductors might have some effect. Some different combinations of delta-admittances alone, and of delta- and Y-admittances together were tested. The phenomena depended as before upon the magnitude of the Y-admittances and upon the impressed voltage. The oscillograms, *T*-4, *T*-5 and *T*-6 are the main records. They show substantially the same variation of the line to neutral currents as oscillograms *T*-1, *T*-2 and *T*-3. The conclusion to be drawn is that the admittances connected directly between line conductors, though necessarily affecting the line to neutral currents to some extent, are not essential to the production of the phenomena of instability.

CONCLUSION

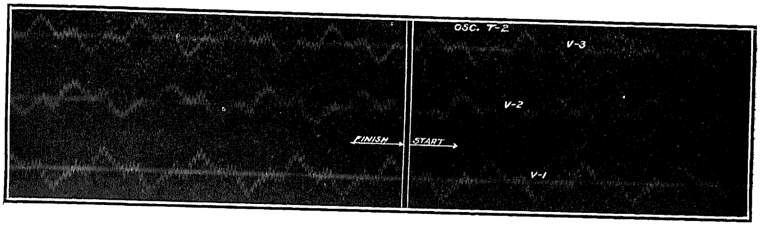
While the foregoing can not be thought of as expressing the conclusive solution of these most interesting phenomena, it is felt that the recent work has resulted in substantial progress. In reviewing the subject, it is gratifying to note that the theory still stands in essence as originally outlined though there have been many additions and adjustments which, however, have served only to make the factors more consistent.

Since the original tests were made, the writer has been told of a case where, in starting up an auxiliary plant, inexplicable vibrations in the transformers were observed under conditions similar to those involved in these experiments. It happened on a grounded neutral 100,000-volt system, where the grounding transformers usually had secondary or tertiary deltas. The bank at this auxiliary station had no such delta. In starting up this time, the line was energized from this station, to be synchronized at a distant station. As soon as the distant station, with its tertiary delta-star grounded neutral bank, was connected to the line, the vibrations in the transformers at the auxiliary station ceased.

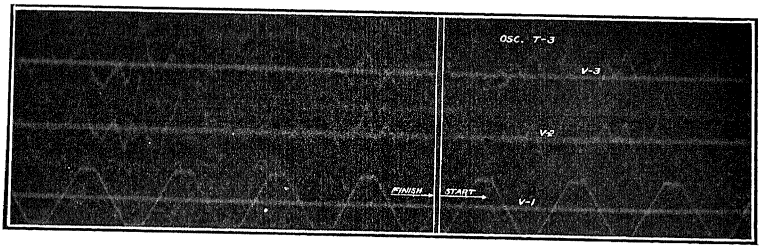
Since then, the writer also noted the remark of Mr. Bernard Price in the latter's recent discussion of the report of the Transmission Committee of the American Institute of Electrical



T-1 LINE TO NEUTRAL CURRENTS IN ALL THREE PHASES, 1940, ($1120\sqrt{3}$) VOLTS BETWEEN LINE TERMINALS IMPRESSED ON TRANSFORMERS,
Approximately 6 microfarads per phase, line to neutral,
V-1 Current of phase 1,
V-2 Current of phase 2,
V-3 Current of phase 3.



T-2 SAME AS *T*-1 EXCEPT 1820, ($1050\sqrt{3}$), VOLTS BETWEEN LINE TERMINALS IMPRESSED ON TRANSFORMERS



T-3 CONDITIONS SAME AS FOR *T*-1

[ROBINSON]

V-1 Fundamental (60-cycle) timing wave,
V-2 Neutral current,
V-3 Current of phase 3.

PLATE XLIX.
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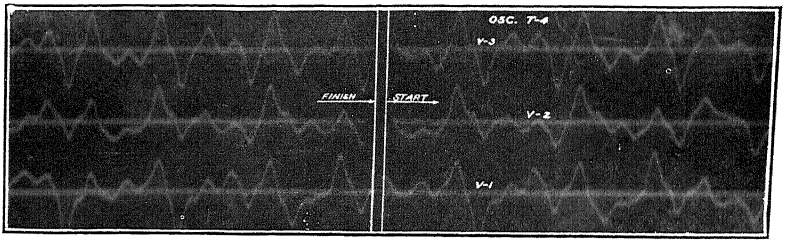
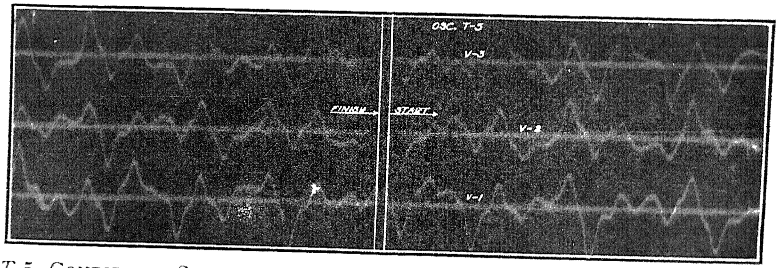


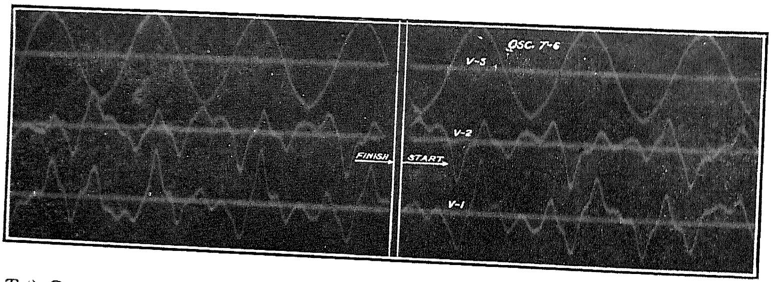
T-4 2130. $(1230\sqrt{3})$, VOLTS BETWEEN LINE TERMINALS IMPRESSED ON TRANSFORMERS
ADMITTANCES BETWEEN LINE CONDUCTORS AND NEUTRAL AND BETWEEN LINE CONDUCTORS

V-1 Current, phase 1 to neutral, 1.59 amperes, r.m.s
V-2 Current, phase 2 to neutral, 1.60 amperes, r.m.s.
V-3 Neutral current, 3.50 amperes, r.m.s.



T-5 CONDITIONS SAME AS FOR *T*-4 EXCEPT 2110 VOLTS BETWEEN LINE TERMINALS IMPRESSED ON TRANSFORMERS

V-1 Current, phase 1 to neutral, 1.50 amperes, r.m.s.
V-2 Current, phase 2 to neutral, 1.50 amperes, r.m.s.
V-3 Current, phase 3 to neutral, 1.50 amperes, r.m.s.



[ROBINSON]

T-6 CONDITIONS SAME AS FOR *T*-4 EXCEPT 2120 VOLTS BETWEEN LINE TERMINALS IMPRESSED ON TRANSFORMERS

V-1 Current, phase 1 to neutral, 1.55 amperes, r.m.s.
V-2 Current, phase 2 to neutral, 1.55 amperes, r.m s.
V-3 Fundamental (60-cycle) timing wave.

NOTE: Line current from each transformer, including currents between lines and currents from line conductors to neutral, ranged from 2.00 to 2.10 amperes, r.m.s., increasing with increased voltage.

Engineers.* In the second from the last paragraph on page 596, Mr. Price says: "With the neutral of the 40,000-volt system earthed at two distant points, we found the presence of a comparatively large amount of even harmonics, the sixth being predominant and over 20 per cent of the fundamental in value and in phase with it." There can be little doubt that, by the use of the oscillograph, he has discovered much the same phenomena as have been here discussed.

An attempt to reach Mr. Price, in South Africa, by letter dated in December 1916 has probably been interrupted by shipping conditions. No reply has been received up to April 24, 1917.

These two cases observed in commercial practise, one in South Africa and one in the United States, coupled with the facts that our original tests were conducted on two dissimilar banks of transformers, and that certain of the associated phenomena were produced in a bank of three 10-watt potential transformers in the laboratory, are, at least, indications that the phenomena were not locally peculiar to special conditions as might be thought at first sight.

The writer believes that a system of notation, similar in its application to that of the present plane vector complex quantity notation used in expression of single-frequency phenomena, would be eminently useful in handling the above phenomena of instability. It hardly seems possible that there is no simple means available for representing in one diagram multiple-frequency variables either separately or combined. In Fig. 18, an attempt has been made toward an extension; but it has not gone very far, and difficulties would obviously be encountered if an attempt were made to include the third, sixth and a few other harmonics. Probably vectors in three-dimensional space will not suffice.

In closing, the writer desires to acknowledge his indebtedness to many engineers who, by their criticisms and suggestions, have encouraged the work; and for the facilities placed at his disposal for conducting the tests and experiments.

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MAGNETIC FLUX DISTRIBUTION IN ANNULAR STEEL LAMINAE

BY A. E. KENNELLY AND P. L. ALGER

Abstract of Paper

The distribution of alternating magnetic flux density in ring laminae is studied experimentally. It is found to differ materially at different radii, not only in root-mean-square magnitude, but also in wave form. The reasons for this distortion are discussed.

IT IS well known that when a circular steel lamina, of the simple geometrical form shown in Figs. 1 and 4, is subjected to a circumferential alternating m.m.f., by the application of a ring winding, the magnetic flux density in the lamina is not constant over the cross section, but varies with the distance from the midplane. The flux density is greatest at the surfaces and is a minimum at the midplane, owing to what is commonly called "skin effect," or magnetic screening due to superficial eddy currents. This variation of flux density may be called "depth variation." The experiments here reported were undertaken to ascertain whether the flux density in the plane of the lamina likewise varied. If "edge effect" were present, the flux density would be greatest at the inner and outer edges, and would reach a minimum at the mid radius. Such variations of flux density may be called "radial variations." The experiments have shown that there is a very marked radial variation of flux density in such laminae, and also a very marked difference in the wave form of alternating magnetic flux density at different radial belts; but these variations are to be attributed, for the most part, to variations in the permeability μ of the metal under the different values of $\mathcal{H}$ at the different radii, the "edge effect" or lateral magnetic screening being small in comparison therewith.

The m.m.f. of the ring winding has of course one and the same value in all of the circumferential belts. The same ampere turns or gilberts act on the outer edge as on the inner edge. The gilberts per cm. are therefore less at the outer than at the inner edge. This excess of magnetic intensity $\mathcal{H}$ towards the inner edge tends to create a greater flux density at that edge. But the permeability varies with $\mathcal{H}$ in such a manner that the crowding at the inner edge is exaggerated for low magnetic densities. Moreover, the wave form of the alternating magnetic flux is considerably distorted at the edges from that of the total flux in the lamina.

These distortions of radial flux density, although very striking, are not entirely new. Kapp has called attention to the fact that the mean flux density in the laminated core of a dynamo armature, obtained by dividing the total flux by the armature cross section, is not the same as the arithmetical mean of the flux densities at the inner and outer radii.[1] Niethammer[2] has also computed the flux densities at different radii of a laminated ring transformer, and has pointed out that at low average densities, the distribution, with a permeability assumed constant, must be markedly different from that with permeability varying with $\mathcal{H}$ in the ordinary way. So far as we are aware, however, the matter has not been investigated experimentally, and the necessarily accompanying distortion in magnetic wave form has not been noticed or referred to.

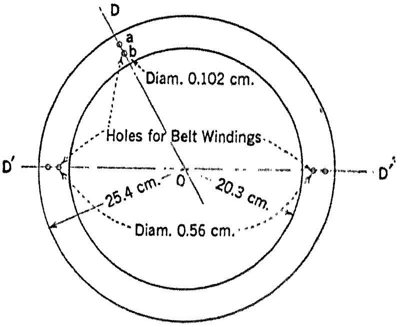

FIG. 1—SILICON STEEL LAMINA

Laminae Tested. The laminae were annular stampings of high-silicon steel. The external diameter of these stampings was 50.8 cm. (20 in.), the internal diameter was 40.6 cm. (16 in.) and their thickness 0.355 mm. (0.014 in.). These were standard blank stampings of best grade transformer steel. The material is characterized by high resistivity (52000 absohm-cm.), with a low temperature coefficient of the same, and a high initial permeability.

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[1] Dynamomaschinen für Gleich-und Wechselstrom, 3rd Edition, p. 254, by G. Kapp.

[2] Sammlung Elektrotechnischer Vorträge, Vol. II, Part 11-12. Magnetism, pp. 10-12, by F. Niethammer.

A laminated core was made up of 25 of the above stampings, using an insulating layer of paper 0.076 mm. (0.003 in.) between laminae. The resulting core had an average thickness of 1.2 cm. (0.47 in.) The weight of steel in the core was 4.78 kg. (10.5 lb.)

As is indicated in Fig. 1, small holes *a*, *b*, were drilled through the core, in a diametral line *O D*, and perpendicular to the plane of the laminae. The diameter of each hole was 1.02 mm. (0.040 in.) Figure 2 gives in greater detail the distances between these holes, and the edges of the core. The object of the holes was to divide the core radially into three concentric annular belts, the magnetic flux in each of which could be separately determined by the aid of suitable windings through the holes. Four fine insulated copper wires were passed through each hole to form a winding of two turns (Fig. 3) around each annular belt. By the use of a Drysdale-Tinsley potentiometer, the voltage induced in these two-turn windings could be measured at all the frequencies employed. In order, however, to obtain oscillograms of the induced alternating-voltage wave forms, it was found necessary to employ windings of more numerous turns, and two sets of larger holes (diam. 5.6 mm. or 0.22 in.) to receive these windings, were drilled on another diametral line $D' D'$, as shown in Fig. 1. Twenty turns were used in these larger belt windings.

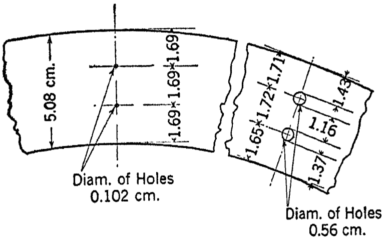

FIG. 2—SHOWING POSITIONS OF HOLES FOR BELT WINDINGS

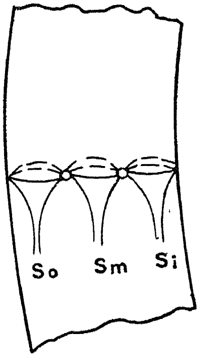

FIG. 3—BELT WINDINGS

A primary single-layer winding was distributed around the core, consisting of 450 turns of d.c.c. copper wire No. 19 A. W. G.

(bare diam. 0.91 mm.) divided into six coils *aa'*, *bb'*, Fig. 4, averaging 75 turns each. In addition, two secondary coils *cc'*, *cc'*, of 75 turns each, were applied at diametrically opposite sectors, around all the laminae forming the core.

The sources of primary impressed voltage were two three-phase alternators, specially designed by Prof. C. A. Adams, to deliver a close approximation to a sinusoidal wave. One of these generators was used up to a frequency of 100 ~, and the other for frequencies from 100 ~ to 800 ~. The electrical connections are indicated in Fig. 4.

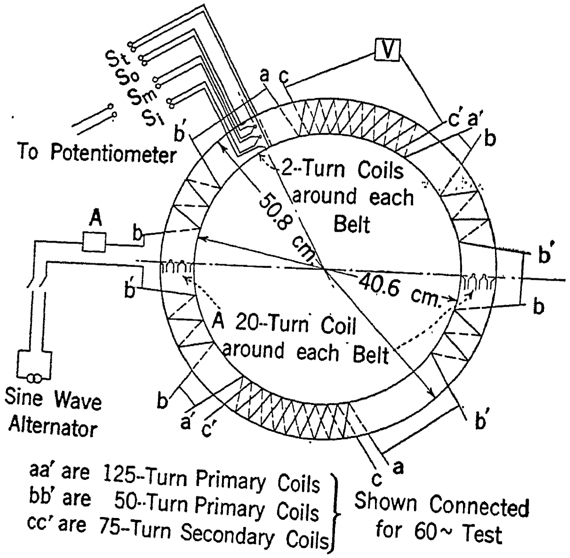
St
So
Sm
Si
To Potentiometer
a c
V
c' a'
b
b'
2-Turn Coils around each Belt
50.8 cm.
40.6 cm.
A
b
b'
b'
b
A 20-Turn Coil around each Belt
Sine Wave Alternator
b
a' c'
b'
c a
aa' are 125-Turn Primary Coils
bb' are 50-Turn Primary Coils
cc' are 75-Turn Secondary Coils
Shown Connected for 60~ Test

FIG. 4

METHODS OF TESTING

A-C. Tests. The method of testing by alternating currents consisted in impressing a sinusoidal e.m.f. on the primary winding *bb'* Fig. 4, of the desired frequency, so as to produce a total flux in the core of sinusoidal wave form. The average maximum cyclic flux density generated in the core was then deduced from observations of the voltmeter, and also from Drysdale potentiometer readings on a single secondary turn S_t around the core. The total flux in the core was found to be substantially sinusoidal, as shown in Fig. 5, which is an oscillogram of the e.m.f. induced in S_t. It was soon found, however, that the fluxes in the three annular belts were non-sinusoidal. The three fundamental components of the inner, middle and outer belt e.m.fs from the inner, middle, and outer secondary coils S_i, S_m and S_o respectively, were measured on the Drysdale potentiometer, as is indicated in Tables I, II, and III for the frequencies of 60, 340 and 696 ~.

Belt Potentiometer Tests. The results arrived at in the foregoing tables show that the maximum fundamental flux densities in the three belt paths are very different, especially towards low densities. The cross sections of the three belt paths, as determined by the positions of the two small holes through the core, were equal within 3 per cent; but the outside belt carried at $B_{max.} = 1000$, 340~, only 24 per cent of the total flux instead of 30.9 per cent on the basis of constant permeability. On the other hand, the inside belt carried 45 per cent. The middle belt carried 31 per cent. The results for three different frequencies are shown at various flux densities averaged over the entire core, in Fig. 6. The curves show that the greatest distortion from uniformity in the three annular belt fluxes occurs at the lowest flux density, and it diminishes as the density increases. At the highest flux density of 13 kilogausses (maximum cyclic averaged over the entire cross section) the three belt flux densities are more nearly in inverse proportion to the lengths of the respective annular flux paths. The reasons for this non-uniform distribution at low flux densities will be considered later on.

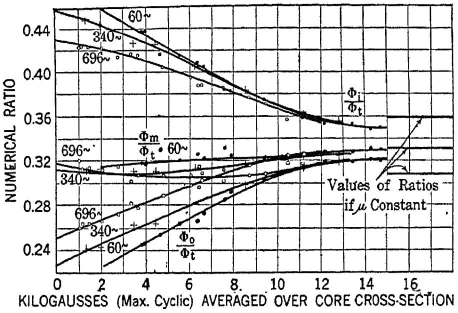

FIG. 6—VARIATION OF FLUX DISTRIBUTION WITH DENSITY

Belt Oscillograms. Figs. 7, 8 and 9 are oscillograms of the secondary e.m.f. induced in windings of 20 turns each on the inner, middle, and outer belts, S_i, S_m and S_o respectively, taken at the same time as Fig. 5. The oscillograph was a Duddell vibration galvanometer tuned to about 2000 ~. This

TABLE I.

RESULTS OF TESTS AT THE FREQUENCY OF 60 CYCLES PER SECOND.

Volts	Potentiometer Readings				$\mathfrak{B}_{max}$									
t_{75}	o_2	m_2	i_2	$t_1/0°$	o	m	i	t_1	t_{75}	t	$\frac{\mathfrak{B}_o}{\Sigma\mathfrak{B}}$	$\frac{\mathfrak{B}_m}{\Sigma\mathfrak{B}}$	$\frac{\mathfrak{B}_i}{\Sigma\mathfrak{B}}$	$\frac{\Sigma\mathfrak{B}}{3\mathfrak{B}_t}$
I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	XIII	XIV	XV
3.61	$0.0234\overline{\backslash 0^\circ.9}$	$0.0302\overline{\backslash 0^\circ.5}$	$0.0418\overline{\backslash 1^\circ.3}$	0.0477	2920	3770	5210	3970	4000	3980	0.246	0.317	0.437	0.998
4.23	$0.0290\overline{\backslash 1^\circ.3}$	$0.0377\underline{/0^\circ.1}$	$0.0474\underline{/1^\circ.4}$	0.0558	3620	4700	5910	4640	4690	4660	0.254	0.330	0.416	1.019
5.65	$0.0405\overline{\backslash 4^\circ.9}$	$0.0497\underline{/0^\circ.3}$	$0.0623\underline{/1^\circ.9}$	0.0771	5050	6200	7760	6420	6260	6370	0.266	0.326	0.408	0.996
7.15	$0.0555\overline{\backslash 3^\circ.5}$	$0.0617\underline{/0^\circ.2}$	$0.0729\underline{/3^\circ.0}$	0.0950	6920	7700	9100	7900	7930	7910	0.292	0.325	0.383	1.000
8.50	$0.0685\overline{\backslash 3^\circ.5}$	$0.0726\underline{/0^\circ.8}$	$0.0844\underline{/2^\circ.1}$	1.139	8540	9050	10530	9470	9430	9460	0.303	0.322	0.375	0.990
10.25	$0.0854\overline{\backslash 4^\circ.5}$	$0.0886\underline{/0^\circ.2}$	$0.0981\underline{/3^\circ.2}$	1.348	10660	11060	12250	11200	11370	11260	0.313	0.326	0.361	1.006
11.1	$0.094\overline{\backslash 3^\circ.7}$	$0.0969\underline{/0^\circ.2}$	$0.1041\underline{/3^\circ.0}$	1.461	11780	12100	13000	12150	12310	12200	0.319	0.328	0.353	1.007
12.27	$0.1015\overline{\backslash 3^\circ.0}$	$0.1045\underline{/0^\circ.5}$	$0.1125\underline{/2^\circ.0}$	1.612	12650	13030	14020	13410	13610	13450	0.319	0.328	0.353	0.985
13.05	$0.1104\overline{\backslash 1^\circ.8}$	$0.1140\underline{/0^\circ.7}$	$0.1199\underline{/2^\circ.4}$	1.709	13780	14220	14950	14210	14480	14250	0.321	0.331	0.348	1.003
13.9	$0.1123\overline{\backslash 2^\circ.0}$	$0.1159\overline{\backslash 1^\circ.9}$	$0.1232\overline{\backslash 1^\circ.4}$	1.765	14010	14460	15380	14680	15400	14700	0.320	0.330	0.350	0.994
3.42	$0.0228\underline{/0^\circ.3}$	$0.0296\underline{/0^\circ.5}$	$0.0406\overline{\backslash 0^\circ.2}$	0.0460	2845	3695	5070	3820	3790	3810	0.245	0.318	0.437	1.017
5.95	$0.0435\overline{\backslash 2^\circ.4}$	$0.0512\overline{\backslash 0^\circ.3}$	$0.0644\underline{/1^\circ.9}$	0.0803	5430	6390	8040	6670	6600	6650	0.273	0.322	0.405	0.998
6.80	$0.0524\overline{\backslash 3^\circ.0}$	$0.0608\overline{\backslash 1^\circ.8}$	$0.0704\underline{/1^\circ.7}$	0.0916	6540	7590	8790	7620	7550	7600	0.285	0.331	0.384	1.006
4.92	$0.0351\overline{\backslash 1^\circ.7}$	$0.0425\underline{/0^\circ.2}$	$0.0551\underline{/1^\circ.4}$	0.0666	4370	5300	6860	5540	5460	5510	0.264	0.321	0.415	1.001

Col. I indicates the induced secondary e.m.f., by voltmeter in the 75-turn over-all winding.
Cols. II, III, IV, give the two-turn belt winding e.m.fs. by a-c. potentiometer, on outer, middle and inner belts respectively.
Col. V gives the single-turn over-all e.m.f. by potentiometer.
Cols. VI, VII, VIII give the corresponding computed values of flux density, max. cyclic gausses, in the outer, middle, and inner belts.
Col. IX gives the computed average flux density by potentiometer, X that by voltmeter in Col. I.
Col. XI gives the accepted average flux density (max. cyc. gausses) allowing weights to IX and X.
Cols. XII, XIII, and XIV give the respective ratios of outer, middle, and inner belt fluxes to the total flux.
Col. XV gives the ratio of the sum of the belt fluxes to the total flux as a check. It should be unity throughout.

TABLE II.

RESULTS OF TESTS AT THE FREQUENCY OF 340 ~

E. m. f. by voltmeter	E. M. F. by Potentiometer				$\mathcal{B}_{max}$						$\frac{\mathcal{B}o}{\Sigma\mathcal{B}}$	$\frac{\mathcal{B}m}{\Sigma\mathcal{B}}$	$\frac{\mathcal{B}i}{\Sigma\mathcal{B}}$	$\frac{\Sigma\mathcal{B}}{3\mathcal{B}_t}$
t_{75}	o_2	m_2	i_2	t_1/0°	o	m	i_1	t_1	t_{75}	t				
6.5	0.0463 /7°.0	0.0594 /2°.0	0.0858 \2°.1	0.0916	1020	1310	1885	1345	1270	1320	0.242	0.311	0.447	1.068
10.0	0.066 /5°.4	0.085 /0°.9	0.121 \3°.2	0.135	1455	1875	2660	1980	1960	1973	0.243	0.313	0.444	1.012
18.0	0.123 /3°.5	0.146 \0°.8	0.200 \2°.7	0.234	2710	3210	4400	3440	3520	3465	0.263	0.311	0.426	0.994
23.0	0.160 /4°.2	0.188 /0°	0.257 \1°.8	0.301	3520	4140	5660	4420	4500	4445	0.264	0.311	0.425	1.000
32.2	0.246 /2°.6	0.275 /1°.0	0.352 /1°.2	0.425	5410	6050	7800	6240	6310	6270	0.281	0.314	0.405	1.023
43.8	0.351 /0°.1	0.371 \3°.0	0.451 /2°.4	0.586	7720	8160	9920	8600	8580	8590	0.299	0.317	0.384	1.001
57.8	0.479 \0°.8	0.494 \3°.0	0.552 /2°.9	0.760	10570	10900	12180	11180	11300	11220	0.314	0.324	0.362	0.999
66.0	0.550 \0°.6	0.560 \2°.9	0.614 /3°.0	0.872	12120	12340	13530	12830	12900	12850	0.319	0.325	0.356	0.985

TABLE III

RESULTS OF TESTS AT THE FREQUENCY OF 696 ~

E. M. F. by voltmeter	E. M. F. by Potentiometer				$\mathfrak{B}_{max}$						$\frac{\mathfrak{B}_o}{\Sigma\mathfrak{B}}$	$\frac{\mathfrak{B}_m}{\Sigma\mathfrak{B}}$	$\mathfrak{B}_i$	$\frac{\Sigma\mathfrak{B}}{3\mathfrak{B}_t}$
t_{75}	o_2	m_2	i_2	$t_1/0°$	o	m	i	t_1	t_{75}	t				
10.7	0.071/2°.8	0.087\0°.7	0.115\3°.0	0.144	765	935	1240	1030	1020	1025	0.260	0.318	0.422	0.956
12.2	0.083/3°.3	0.098\0°.8	0.133\3°.7	0.159	890	1060	1430	1140	1170	1140	0.264	0.313	0.423	0.988
15.3	0.102/3°.2	0.122\1°.0	0.164\4°.4	0.200	1100	1310	1760	1435	1460	1440	0.264	0.314	0.422	0.966
21.4	0.148/2°.3	0.174\1°.1	0.235\3°.1	0.284	1590	1870	2530	2030	2040	2035	0.266	0.312	0.422	0.982
29.0	0.209/2°.3	0.242\1°.5	0.317\2°.6	0.390	2250	2600	3410	2780	2770	2780	0.272	0.315	0.413	0.991
....	0.262/3°.2	0.280\0°.6	0.386\1°.8	0.466	2820	3010	4150	3340	....	3340	0.282	0.302	0.416	0.996
38.4	0.279/2°.9	0.309\0°.9	0.417\2°.1	0.508	3000	3320	4490	3650	3670	3660	0.278	0.307	0.415	0.984
50.4	0.386/2°.4	0.410\1°.2	0.541\1°.4	0.670	4150	4410	5810	4800	4810	4800	0.289	0.307	0.404	0.998
60.9	0.504/1°.4	0.518\1°.2	0.668/0°.2	0.846	5420	5570	7190	6070	5810	6070	0.298	0.307	0.395	0.999
66.4	0.568/0°.8	0.535\2°.0	0.698/1°.0	0.901	6110	5750	7510	6440	6350	6440	0.297	0.315	0.388	1.002
67.7	0.568/1°.0	0.656\2°.0	0.711/0°.8	0.917	6110	5980	7650	6570	6470	6570	0.309	0.303	0.388	1.001
77.6	0.656/0°.9	0.635\2°.7	0.809/2°.2	1.058	7050	6830	8700	7550	7420	7550	0.302	0.312	0.386	0.998
90.6	0.772\0°.1	0.749\2°.2	0.919/2°.1	1.230	8300	8050	9880	8780	8660	8780	0.316	0.307	0.377	0.998
97.2	0.858\0°.2	0.823\4°.6	0.990/3°.1	1.334	9220	8850	10640	9530	9290	9530	0.321	0.308	0.371	1.004
107.8	0.944\0°.6	0.926\3°.6	1.070/3°.2	1.469	10150	9950	11150	10520	10300	10520	0.325	0.318	0.357	0.990
114.7	1.009\0°.5	0.985\4°.3	1.120/3°.6	1.568	10840	10600	12040	11200	10960	11200	0.324	0.317	0.359	0.997

PLATE L.
A. I. E. E.
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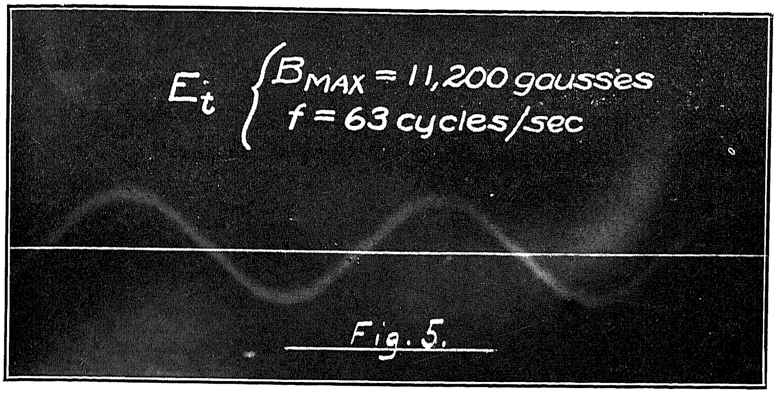

Fig. 5.

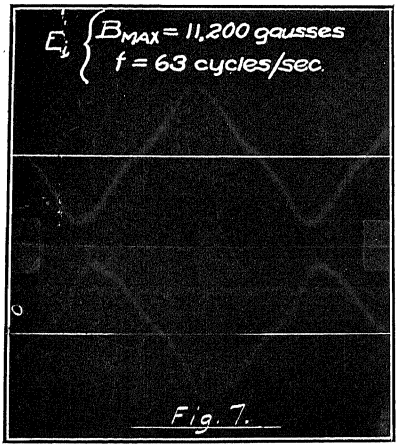

Fig. 7.

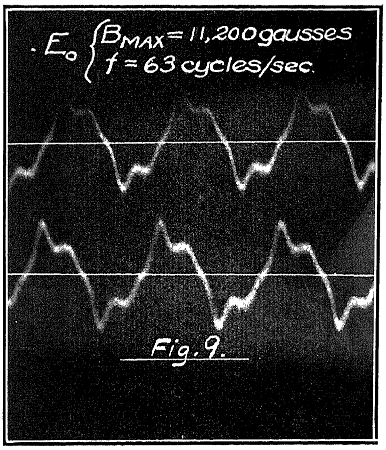

Fig. 9.

[KENNELLY]

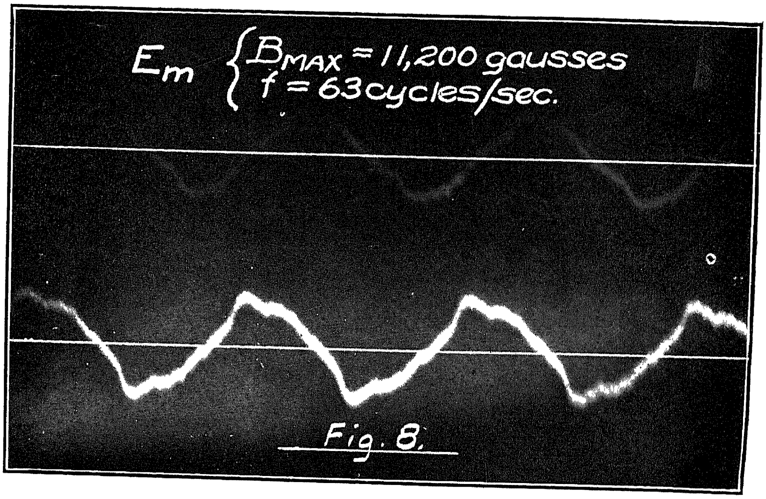
E_m $\begin{cases} B_{MAX} = 11{,}200 \text{ gausses} \\ f = 63 \text{ cycles/sec.} \end{cases}$

Fig. 8.

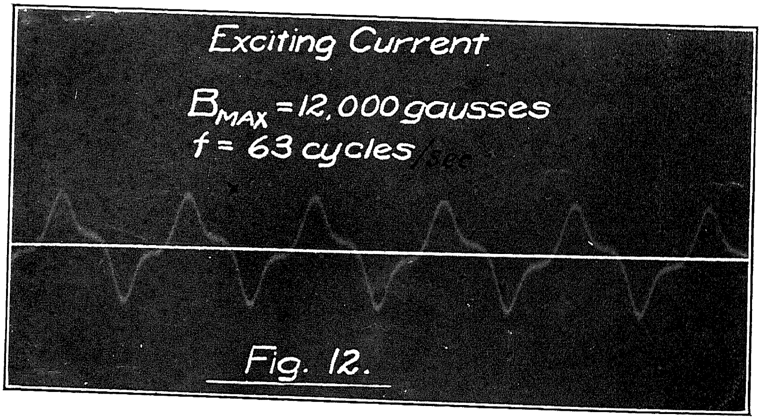
Exciting Current
$B_{MAX} = 12{,}000$ gausses
$f = 63$ cycles/sec

Fig. 12.

instrument was selected for the purpose, instead of the ordinary oil-damped oscillograph because of its sensitiveness, the current required to operate it being only about 10 milliamperes r. m. s. It was desirable to keep the oscillographic current and ampere turns as low as possible, so as not to distort the voltages and flux distributions among the three belts. With 20 secondary belt turns, and 10 milliamperes, this secondary m.m.f. would be only 0.2 r. m. s. ampere-turn per belt. The resistance of the instrument itself was 130 ohms, and an extra resistance of 57 ohms was included in its circuit. The effect of this counter m.m.f. of 0.2 ampere-turn was to distort somewhat the wave shape of flux in each of the three belts. This distortion should however be of the same character for each belt, and if the three belt fluxes were the same, the three belt oscillograms should be substantially the same.

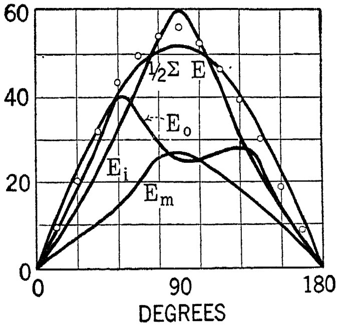

FIG. 10—REPRODUCTION OF BELT OSCILLOGRAMS TO A UNIFORM SCALE AND THE EQUIVALENT SINUSOIDAL TOTAL E.M.F. SUMMATION ORDINATES ARE SHOWN BY THE CIRCLES

Fourier analaysis of flux waves in Fig. 10.

$$E_0 = 34.5 \sin \theta + 5.3 \sin 3\theta - 4.2 \sin 5\theta + 3.1 \cos \theta - 2.9 \cos 3\theta + 0.4 \cos 5\theta +$$

$$E_m = 23.8 \sin \theta - 2.4 \sin 3\theta + 0.9 \sin 5\theta - 1.5 \cos \theta + 1.0 \cos 3\theta + 0.6 \cos 5\theta +$$

$$E_i = 47.7 \sin \theta - 8.6 \sin 3\theta + 2.0 \sin 5\theta + 0.2 \cos \theta + 0.1 \cos 3\theta - 0.4 \cos 5\theta +$$

$$E_0 + E_m + E_i = 106.0 \sin \theta - 5.7 \sin 3\theta - 1.3 \sin 5\theta + 1.8 \cos \theta - 1.8 \cos 3\theta + 0.6 \cos 5\theta +$$

The oscillograms in Figs. 5, 7, 8 and 9 are, however, markedly different, showing that the three belt fluxes differ not only in their maxima, but also in their wave form. Moreover, in order to estimate the amount of distortion in the belt oscillograms due to counter m.m.f., all of the oscillograms were repeated with only one-third of the resistance in the secondary circuit, so as to increase the distorting counter m.m.f. about three times. The upper curves in Figs. 7, 8 and 9 were taken with 0.2 ampere turn of counter m.m.f., while the lower curves were thus taken with 0.6 ampere turn of counter m.m.f. It will be seen that since the differences between the upper and lower curves in each case are not large, the differences between the upper curves and corresponding curves unaffected by counter m.m.f. would be still less.

As a further check on the belt oscillograms, they have been copied to a uniform scale, as shown in Fig. 10, and then combined by addition, as indicated by the small circles. The best representative sine wave is drawn on the same figure for comparison, the total flux being substantially sinusoidal as in Fig. 5. The deviations of the circles from this sine wave are sufficiently small to be attributable to errors in reproducing the oscillograms of Figs. 7, 8 and 9, the amplitudes of which were actually only 1 to 2 cm.

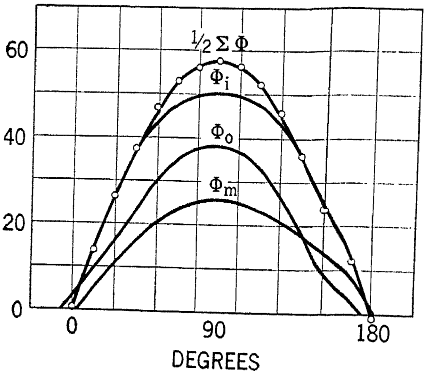

FIG. 11—FLUX WAVES DERIVED FROM INTEGRATION OF THE BELT E.M.F. WAVES IN FIG. 10—ALSO THE SINUSOIDAL TOTAL FLUX INTO WHICH THEY COMBINE.

Fourier analysis of flux waves in Fig. 11.

$\phi_0 = 36.6 \sin \theta - 2.4 \sin 3\,\theta - 0.8 \sin 5\,\theta +$

$+ 3.2 \cos \theta + 0.3 \cos 3\,\theta - 0.4 \cos 5\,\theta +$

$\phi_m = 26.6 \sin \theta + 0.8 \sin 3\,\theta + 0.1 \sin 5\,\theta +$

$- 1.6 \cos \theta - 0.4 \cos 3\,\theta + 0.2 \cos 5\,\theta +$

$\phi_i = 53.2 \sin \theta + 2.6 \sin 3\,\theta + 0.5 \sin 5\,\theta +$

$+ 0.1 \cos \theta - 0.04 \cos 3\,\theta - 0.1 \cos 5\,\theta +$

$\varphi_0 + \varphi_m + \varphi_i = 116.4 \sin \theta + 1.0 \sin 3\,\theta - 0.2 \sin 5\,\theta +$

$+ 1.7 \cos \theta - 0.14 \cos 3\,\theta - 0.3 \cos 5\,\theta +$

The Fourier analysis of the curves E_o, E_m and E_i are indicated in Fig. 10. The harmonics beyond the fifth are negligible throughout. It will be seen that E_o contains a 17 per cent triple and a 12 per cent quintuple component. E_m has a 11 per cent triple component. E_i has an 18 per cent triple component.

In the summation of the three waves, the harmonics almost disappear except for a residual 6 per cent triple component.

On integrating the curves of Fig. 10, so as to reduce the induced e.m.f.s. to their equivalent magnetic fluxes, the curves of Fig. 11 are obtained. As naturally follows such a process of integration, the resulting flux curves are smoother than the original oscillographic e.m.f. curves. The agreement of their sum with a sine wave is accordingly better. The largest belt flux is clearly the inner one ϕ_i. It would be expected that the outer belt flux should be the smallest; but as is shown in Fig. 2, the width of the middle belt was somewhat unduly small. Figs. 10 and 11 are thus in substantial accordance with Fig. 5, in showing that the total flux through the core was sinusoidal, although the individual belt fluxes were far from sinusoidal.

The corresponding Fourier analysis of the curves in Fig. 11 appear beneath it. They are in substantial conformity with the e.m.f. wave analysis of Fig. 10.

The results, appearing in the tables and presented graphically in Fig. 6, also indicate that the numerical sums of the fundamentals of the three belt e.m.fs. are equal to the e.m.fs. of the turns around the entire core. Moreover in Fig. 6 the sums of the ordinates on the belt curves remain equal to unity throughout the entire range of averaged flux density.

REASONS FOR THE DISTORTIONS IN THE BELT FLUX DENSITIES

It is well known that in the primary winding of an excited transformer under no load, the exciting current is non-sinusoidal when the impressed e.m.f. is sinusoidal; firstly because of the varying permeability, and secondly because of hysteresis. The current wave under ordinary conditions is peaked. An oscillogram of the primary current at 63 ~ and 12,000 gausses (max. cyc.) appears in Fig. 12. The peak is to be ascribed to the change of permeability of the iron during the cycle. Assuming a total flux that is sinusoidal, the exciting current at each moment must conform to the $\mathcal{H}$-$\mathcal{B}$ cycle for the entire core. If the various belt paths in the laminae had equal lengths, they would share the flux equally, neglecting skin effects. Actually, however, the inner belt paths are shorter, and these therefore tend to carry a larger share of the flux. This in turn alters the reluctance, because the permeability depends upon the density. At low densities, the tendency is to increase the flux of the inner belt paths yet more, because the permeability at first increases with flux density. At high densities, on the contrary, the tendency is in the opposite direction. Consequently, at low densities, the flux in the inner belts tends to be very different from that in the outer belts, as is shown by Fig. 6. At high densities, however, the permeances of the different belts are more nearly uniform, and the belt fluxes are nearly inversely proportional to the belt lengths.

Effect on Belt-Flux Wave Form

An examination of Fig. 11 shows that the wave form of ϕ_i, the inner belt of flux, is flat by comparison with a sinusoid; whereas ϕ_o is peaked and ϕ_m is intermediate. It is evident that ϕ_i must be relatively flat from the curves of Fig. 6, which show that at low densities the inner belt carries an extra large share of flux, whereas towards higher densities this extra share is reduced. On the other hand, ϕ_o must be relatively peaked because the share of the outer belt increases as the density increases. In Fig. 10 the conditions are inverted, the relatively flat belt flux ϕ_i give rise to a relatively peaked e.m.f. E_i; while the peaked ϕ_o gives rises to the flattened E_o. Moreover, the exciting current has its wave form determined by the total flux in the core, or the sum of all the belt fluxes. At the lower densities existing in the outer belt, this current will be too peaked and distorted to produce a sine wave of flux; so that the outer

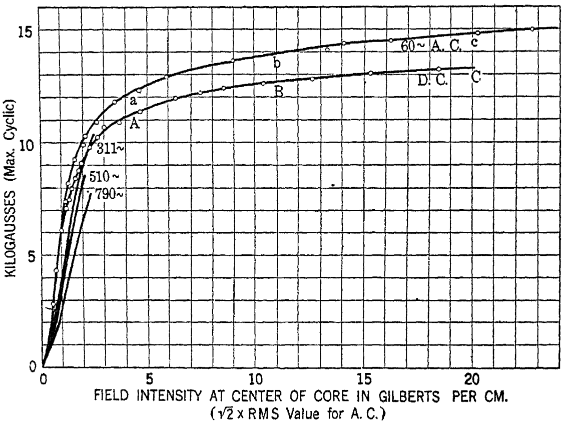

FIG. 13—SATURATION CURVES FOR X 5 SILICON STEEL OBTAINED FROM A LAMINATED ANNULAR CORE—INSIDE DIAMETER 40.6 CM.—OUTSIDE DIAMETER 50.8 CM.—LAMINATIONS 0.0356 CM. THICK

belt flux wave will be correspondingly peaked and distorted. The conditions are just reversed in the inner belt. This accounts for the e.m.f. wave in the outer belt being so much more irregular than in the others. Again, the inner flux, being the largest of the three, has the greatest share in determining the exciting current, and so remains the smoothest.

SATURATION CURVES

Fig. 13 shows the saturation curve *A B C* for the whole core of 25 laminae, as deduced from d-c. observations with a flux meter connected to an enveloping coil of six secondary turns. The upper curve *a b c* represents the corresponding saturation

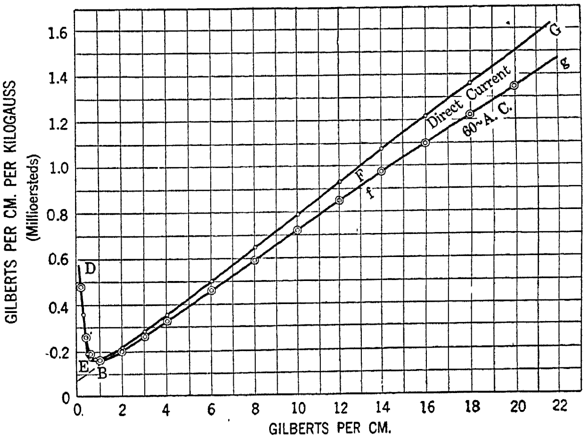

FIG. 14

with 60 ~ alternating current, multiplying the observed r.m.s. primary current by $\sqrt{2}$, to obtain the maximum cyclic magnetic intensity $\mathcal{H}$. The observed e.m.f. per secondary turn was also multiplied by $\sqrt{2}$ to obtain the maximum cyclic kilogausses $\mathcal{B}$. Owing to distortion of the impressed e.m.f. waves from the sinusoidal form at high flux densities, the factor $\sqrt{2}$ is evidently too low for $\mathcal{H}$ and too high for $\mathcal{B}$. It is supposed that if these errors did not exist, and that both $\mathcal{H}$ and $\mathcal{B}$ were sinusoidal quantities throughout, the curve *a b c*, would coincide with the direct-current excitation curve *A B C*.

Similar a-c. saturation curves at higher frequencies are also shown up to 7 and 9 kilogausses. It is supposed that these fall below the low-frequency curve because of skin effect.

The corresponding curves of reluctivity in the laminae, for different values of $\mathcal{H}$, appear in Fig. 14. The reluctivity for direct currents is a rapidly descending line from 0 to 0.5 gilbert per[3] cm. It then becomes a steadily ascending straight line $F\ G$ defined by the equation $\nu = 0.000\ 07 + 0.000\ 0723\ \mathcal{H}$ oersted-cm. The apparent reluctivity for a-c. excitation, as similarly obtained from the saturation curve $a\ b\ c$ of Fig. 13, is the curve $D\ E\ f\ g$. As before, it is supposed that if there were no errors of distortion in impressed wave form, this curve would coincide with $D\ E\ F\ G$.

It must be remembered that the reluctivity straight line $B\ E\ F\ G$ is a mean value obtained for the core as a whole.

Outline Theory of the Radial Variation

If each lamina has internal radius R_1 cm., and external radius R_2 cm. and a thickness of h cm., then the reluctance $\mathcal{R}$ of the lamina will be, in any belt of radius r and width $d\,r$, in terms of the reluctivity ν,

$$\mathcal{R} = \frac{\nu \cdot 2\,\pi\,r}{h \cdot d\,r} \qquad \text{oersteds} \quad (\mathbf{1})$$

Under a total direct-current m.m.f. of $\mathcal{F}$ gilberts, the flux in this belt will be

$$d\,\varphi = \frac{\mathcal{F}}{\mathcal{R}} = \frac{\mathcal{F} \cdot h \cdot dr}{\nu \cdot 2\,\pi r} \qquad \text{maxwells} \quad (\mathbf{2})$$

But

$$\mathcal{F} = 2\,\pi\,r\,\mathcal{H} \qquad \text{gilberts} \quad (\mathbf{3})$$

so that

$$d\,\varphi = \frac{\mathcal{H}}{\nu} \cdot h \cdot d\,r \qquad \text{maxwells} \quad (\mathbf{4})$$

The total flux in the lamina will then be the integral of this, or

$$\phi = h \int_{R_1}^{R_2} \frac{\mathcal{H}}{\nu}\,d\,r \qquad \text{maxwells} \quad (\mathbf{5})$$

---

[3] "Magnetic Reluctance," Trans., A.I.E.E., Vol. 8, Oct. 27th, 1891, pp. 486-533. Also "Investigation of Magnetic Laws for Steel and other Materials," by J. D. Ball, *Journal of the Franklin Institute*, Vol. 181, April, 1916, pp. 459-504.

If ν were directly proportional to $\mathcal{H}$, or if $\nu = b\,\mathcal{H}$ oersted-cm,

then $$\phi = h \int_{R_1}^{R_2} \frac{d\,r}{b} = \frac{h}{b}\,(R_2 - R_1) \qquad \text{maxwells} \quad (6)$$

This would mean uniform flux densities in all of the belt paths and the absence of all radial distortion. But the reluctivity is not directly proportional to $\mathcal{H}$. As we have seen for this particular quality of steel, below $\mathcal{H} = 0.5$, $\nu = 0.0007 - 0.001\,\mathcal{H} = a_1 - b_1\,\mathcal{H}$, and above $\mathcal{H} = 0.5$

$$\nu = 0.000\,07 + 0.000\,0723\,\mathcal{H} = a + b\,\mathcal{H} \qquad \text{oersted-cm.} \quad (7)$$

The effect of this constant term a or a_1 in the reluctivity is therefore to make the belt flux $d\,\varphi_r$ at radius r in terms of the local magnetic intensity $\mathcal{H}_r$ and flux density $\mathcal{B}_R$

$$d\,\varphi_r = \frac{\mathcal{H}_r \cdot h \cdot dr}{a + b\,\mathcal{H}_r} = h \cdot \mathcal{B}_r \cdot d\,r = \frac{\mathcal{F} \cdot h \cdot d\,r}{2\,\pi\,r\,a + b\,\mathcal{F}} \qquad \text{maxwells} \quad (8)$$

The d-c. distribution of flux density for a given value of $\mathcal{F}$, will thus necessarily vary with r, and will tend to be greater on the inside.

We may define the flux-density distortion in a lamina as the difference between the innermost and outermost densities $\mathcal{B}_1 - \mathcal{B}_2$ divided by the flux density $\mathcal{B}_R$ at the geometric mean radius R That is, if R is the geometric mean radius,

$$R = \sqrt{R_1\,R_2} = n\,R_1 = R_2/n \qquad \text{cm.} \quad (9)$$

where n is a numerical ratio, such that $R_2 = n^2\,R_1$
It then follows from (8) that

$$\frac{\mathcal{B}_1 - \mathcal{B}_2}{\mathcal{B}_R} = a\left(n - \frac{1}{n}\right)\frac{a + b\,\mathcal{H}_R}{(a + b\,n\,\mathcal{H}_R)\left(a + \frac{b}{n}\,\mathcal{H}_R\right)}$$

$$= a\left(n - \frac{1}{n}\right)\frac{\nu_R}{\nu_1\,\nu_2} \qquad \text{numeric} \quad (10)$$

Since approximately $$\nu_1\,\nu_2 = \nu^2_R \qquad (\text{oersted-cm.})^2 (11)$$

the distortion is approximately

$$\frac{\mathfrak{B}_1 - \mathfrak{B}_2}{\mathfrak{B}_R} = \frac{a}{\nu_R}\left(n - \frac{1}{n}\right) \qquad \text{numeric } (\mathbf{12})$$

The quantity

$$n - \frac{1}{n} = \frac{R_2 - R_1}{R} \qquad \text{numeric } (\mathbf{13})$$

or is the ratio of the width to the geometric mean radius of the lamina. We may call it the width-radius ratio. In the laminae tested, the width-radius ratio was $\frac{2}{\sqrt{80}} = 0.2236$. Consequently to the degree of approximation represented by (**12**), the flux-density distortion in a lamina subjected to any continuous-current m.m.f. is equal to the width-radius ratio times the ratio of the reluctivity constant a to the total reluctivity at the geometric mean belt R

If $a = 0$, the flux-density distortion disappears, as we have already seen.

If $b = 0$, so that the reluctivity is independent of the excitation, (**12**) becomes $\frac{\mathfrak{B}_1 - \mathfrak{B}_2}{\mathfrak{B}_R} = \left(n - \frac{1}{n}\right)$, or the distortion is numerically equal to the width-radius ratio.

As ν_R increases, with increasing flux density, towards saturation, the ratio a/ν_R becomes very small, and the distortion nearly disappears.

On the ascending straight line of reluctivity, above $\mathfrak{H} = 0.5$, in the case considered, a/ν_R is always less than unity, or the distortion to continuous m.m.f. is always less than the width-radius ratio.

On the initial descending straight line of reluctivity, below $\mathfrak{H} = 0.5$ in the case considered, a/ν_R is likely to exceed unity, so that over this range of $\mathfrak{H}$, the distortion is likely to be greater than the width-radius ratio, and, in particular cases, may be several times greater.

In order to prevent the existence of all distortion, the width-radius ratio must be reduced to zero, which means that the lamina must be cut from a cylinder, instead of from a flat sheet of metal. In the ordinary case of a flat annular stamping, the distortion diminishes as the mean radius is increased. When it is desired to measure the magnetic properties of the steel with considerable precision, the width-radius ratio should be kept as small as possible, otherwise the mean apparent permeability of the steel at any given intensity $\mathcal{H}$, as derived from the joint behavior of all the belts in parallel, will differ from the true permeability in any one belt. The error tends to be greatest at low densities referred to the descending branch of the $\nu - \mathcal{H}$ graph, and diminishes towards high densities.

The authors are indebted to the American Telephone and Telegraph Co., under an appropriation from which the investigation was carried on. They are also indebted to Prof. C. A. Adams and to Mr. R. Eksergian, for valuable suggestions during the work.

CONCLUSIONS

(1) As has already been noted by several writers, the flux density differs in different belts of an annular steel lamina, under a circumferential impressed m.m.f. from a distributed toroidal winding. The phenomenon may be described as radial distortion of flux density.

(2) The distortion occurs both with continuous and alternating m.m.fs. It may be very marked at low densities. At high frequencies, flux-density distortion is complicated by skin effects.

(3) With alternating m.m.fs., a distortion is effected in the alternating magnetic wave form, as well as in the maximum cyclic flux density. Each belt has its own density, and its own wave form. The total flux in the lamina being, say, sinusoidal, the individual belt fluxes will be non-sinusoidal. The external belt fluxes have, in general, the most distorted wave forms.

(4) The magnitude of flux-density distortion may be defined as the difference between the inside and outside flux densities, divided by the flux density at the geometrical mean radius. In the case reported, the observed a-c. flux-density distortion was approximately $\frac{0.45 - 0.24}{0.31} = 0.68$, at low densities.

(5) The magnitude of flux-density distortion depends upon the width-radius ratio of the lamina, or the width of the lamina divided by its geometrical mean radius. In the case reported, this ratio was 0.224. The smaller this ratio, other things being equal, the less the distortion.

(6) For a given width-radius ratio, the flux-density distortion depends, to a first approximation, on the ratio a/ν of the constant

reluctivity term a to the total reluctivity ν in the geometric mean belt. Over the ascending straight range of the reluctivity-intensity $\nu - \mathcal{H}$ graph, this ratio is less than unity, so that, at least in the continuous-current case, the distortion will be less than the width-radius ratio over this range (above $\mathcal{H} = 1$ in the case reported.) On the descending branch of the $\nu - \mathcal{H}$ graph, the ratio a/ν will be greater than unity, so that at low values of $\mathcal{H}$ and $\mathcal{B}$, the distortion is likely to exceed the width-radius ratio (below $\mathcal{H} = 1$, $\mathcal{B} = 6000$ in the case reported).

(7) When the magnetic characteristics of steel laminae are to be measured with precision, small width-radius ratios should be used, in order to avoid errors due to flux-density distortion. Strictly speaking, curves derived from annular laminae are all subject to distortion errors, especially at low densities.

(8) The eddy-current losses occurring in annular laminae under sinusoidal a-c. excitation are exaggerated by the presence of harmonics in the various belt fluxes.

List of Symbols Employed

a, b	Reluctivity constants for a sample of steel.
$\mathcal{B}$	flux density in steel, (gausses, or maxwells per sq. cm.)
$\mathcal{B}_{max}$	maximum cyclic flux density in steel (gausses).
$\mathcal{B}_i$ $\mathcal{B}_m$ $\mathcal{B}_o$	flux densities in inner, middle, and outer belts of a lamina (gausses) and in alternating-current cases maximum cyclic gausses.
$\mathcal{B}_t$	flux density in a lamina averaged over the entire cross section (gausses and in alternating-current cases maximum cyclic gausses).
$\mathcal{B}_1$ $\mathcal{B}_2$ $\mathcal{B}_R$	flux density at inner edge, at outer edge, and at geometric mean belt of a lamina, respectively (gausses, and in a-c. cases, maximum cyclic gausses).
E	electromotive force induced in a winding (volts).
$\mathcal{F}$	magnetomotive force applied to a lamina circumferentially (gilberts or c. g. s. magnetic units).
$\mathcal{H}$	magnetic intensity or magnetic force in steel (gilberts per cm.)
$\mathcal{H}_r$	magnetic intensity in a lamina at belt of radius r (gilberts per cm.)
h	width of a lamina measured along any radius (cm.)

n	ratio of geometric mean radius to inner radius of an annular lamina (numeric).
$\nu = a + b\,\mathcal{H}$	reluctivity of steel (oersted-cm.)
$\nu_1,\ \nu_2,\ \nu_R$	reluctivity at inner, outer, and geometric mean belts of a lamina, respectively (oersted-cm.)
ϕ	total magnetic flux in a lamina (maxwells).
ϕ_r	magnetic flux in a belt of a lamina at radius r (maxwells).
$\phi_i,\ \phi_m,\ \phi_o$	magnetic fluxes in the inner, middle, and outer belts of an annular lamina, respectively (maxwells).
$\mathcal{R}$	reluctance of a belt path (oersteds.)
r	radius of an annular belt (cm.)
$R_1,\ R_2$	internal and external radii of an annular lamina (cm.)
$R = \sqrt{R_1 R_2}$	geometric mean radius of an annular lamina (cm.)
Σ	summation sign, $\angle$ angle sign, $\sim$ sign for cycles per second.

DISCUSSION ON "PHENOMENA ACCOMPANYING TRANSMISSION WITH SOME TYPES OF THE STAR TRANSFORMER CONNECTIONS" (ROBINSON), AND "MAGNETIC FLUX DISTRIBUTION OF ANNULAR STEEL LAMINAE" (KENNELLY AND ALGER), NEW YORK, N. Y., DECEMBER 14, 1917.

B. A. Behrend: My first acquaintance with the phenomenon with which the paper by Prof. Kennelly and Mr. Alger deals was in 1902, on a very large scale, indeed, in designing a large two-pole turbo-generator. If I recall correctly, the inside diameter of that machine was approximately 20 in. (50.8 cm.), and the outside diameter was approximately 60 in. (152.4 cm.). It being a two-pole machine, the difference between the length of the magnetic paths immediately at the bottom of the teeth and near the outside of the punchings was considerable.

Like Prof. Kennelly, we provided exploring coils, using however a voltmeter for the indication of the flux, and not going to the trouble of investigating the wave form of the induced e. m. f. We shaped our punchings in such a manner that an air-gap was formed between two contiguous punchings in the same layer, shaped like an isosceles triangle with its base near the teeth and the apex at the outside boundary of the punchings. Thus, the insertion of a longer path partly in air in the neighborhood of the teeth equalized the difference in length of the paths of the flux through the iron. It was used on all of our machines subsequently, because we found that thus the difference in the e. m. f. between the three exploring coils spanning the same geometric area was comparatively slight, and that it was a great deal less than was to be anticipated, unless the auxiliary "triangular" reluctance had been supplied.

It is interesting to note, to those who like to see the unity of physical phenomena in fields apparently wide apart, how the experiments carried on by the authors on a little toroid in connection with telephone experiments have found their counterpart in the largest and most powerful types of electric machinery, the turbo-alternators.

There is a criticism I have to make on the paper; I refer to the terminology used by the authors, viz., their irrepressible tendency to give names to units, thus introducing difficulties and creating puzzles to the attentive student. The "gauss," the "oersted," the "maxwell," the "gilbert," etc., etc., are these names really needed? The answer to this is an emphatic "no." Our contemporaries in England, leaders and masters of our science as they have been, have recently voiced their protest on this score through Mr. Alexander Russel in the following quotation:

"The author is hampered by his loyal adherence to the nomenclature list published by the Am. Inst. of Electrical Engineers. For instance, he calls the unit of the flux of magnetic induction the maxwell, and the unit of magnetic induction density the gauss. We deduce also that a gauss is both a gilbert per centimetre and a maxwell per square centimetre. It seems to us that there is a quite unnecessary dragging in of the names of great men of science, especially as the definitions are framed on the assumption that permeability is a simple numeric. Clerk Maxwell would not have admitted this assumption. The American gauss is the unit both of magnetic induction and of magnetic force. Many physicists consider that magnetic induction is caused by magnetic force just as strain is caused by stress. The assumption that cause and effect are measured in the same unit is unjustifiable. In our opinion the practise of christening units after the names of men of science should be adopted only very sparingly. The watt and the joule are well named, but we deprecate the growing use of the kelvin for the unit in which electrical energy is bought and sold. Those evil-sounding words, also, the abohm, the abampere, the abfarad, used by Americans are almost libellous to the great men whose memory they are supposed to keep green."

I urge a sympathetic consideration of these remarks of our noted contemporary across the Atlantic.

L. W. Chubb: The paper by Dr. Kennelly and Mr. Alger gives interesting quantitative data for the flux distribution in annular laminae.

For some years we have made similar tests regarding the flux distribution in rectangular cores built with straight and L-shaped punchings. This is, of course, the more important commercial problem and difficult to obtain except by test. The case for the annular punchings treated experimentally by the authors is more simple and neglecting the eddy currents, which damp the flux, has been worked out approximately by graphical methods.

With the straight and L-shaped punchings built into a rectangular core the air gaps at the lap joints complicate the problem and tend to increase the uniformity of the flux especially at the moderate inductions.

The paper by Mr. Robinson discloses novel phenomena and gives an explanation which, to me, is not satisfactory. The author presented a paper on the same subject some time ago. Certain of the data were lacking and I hoped that the present paper would be more satisfactory in clearing up the subject.

The author endeavors to show, in the first part of the paper that the harmonic components of the exciting current are not wattless.

I don't think that it is hard to say what components of current are wattless and which are not, if we assume a simple harmonic variation of flux. Our only conception of a wattless current is that the net work done by it during a complete cycle of the fundamental, is zero. The author might as well say that the fundamental component of the fundamental current is not wattless. It is the larger and its product with the voltage indicates a rate of doing work, at certain points within the cycle, quite comparable with the instantaneous power of the sine component. The difference is that the work done during part of the cycle is stored energy and returned during the rest of the cycle, giving a net work of zero. The author also distinguishes between the sine and cosine components of the harmonic currents saying that only the former are wattless. Both the sine and cosine components are wattless taken for a complete cycle of the fundamental, but both are watted instantaneously and the one which he calls the wattless component is really the one which does the greatest work instantaneously. One distinction can be drawn, however, by saying that, with a sine variation of flux, all of the sine components do a net work of zero between any points of equal induction, while the cosine components, excepting the fundamental, do zero net work only for the half cycle or full cycle.

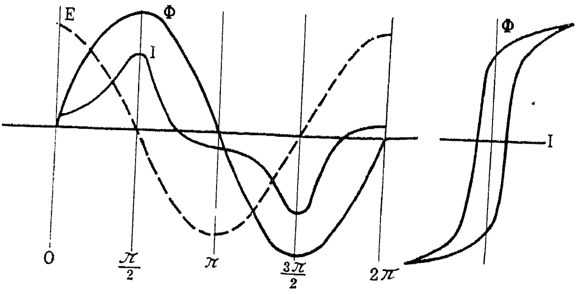

FIG. 1

The diagrams of Figs. 1, 2 and 3 show this fact.

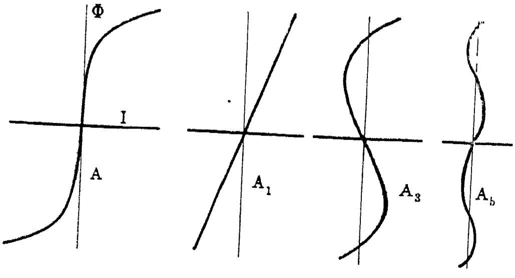

FIG. 2

Fig. 1 shows the time waves of flux ϕ, current I, and voltage E. On the right of the curves the hysteresis loop between flux and current, is plotted with I as abscissae. This lissajous figure can be divided into two similar figures, Fig. 2A and Fig. 3B, containing all of the sine and cosine components respectively, of the current wave. If we assume a sine wave of flux, Fig. 2A can be separated into the lissajous figures, A_1, A_3, A_5, etc., showing respectively the relation between the flux and the 1st, 3rd, 5th, etc. sine components of current.

Fig. 3B can similarly be separated into the components B_1, B_3, B_5, etc. showing the relation between flux and the cosine components of current. In each of the component figures of Fig. 2 it is at once evident that the net work done during a complete cycle is zero because the curve retraces and encloses no area either positive or negative. The component curves of Fig. 3 do enclose area but the net area of all except B_1 is zero. We may say then that the area of the hysteresis loop in Fig. 1 equals the area of the loop of Fig. 3B, which contains all of the cosine components of current, equals the area of the ellipse B_1 in Fig. 3 due to the energy component of the fundamental current alone. It is of course, well known that the net area of such figures is proportional to the work per cycle for

$$\int_0^{2\pi} I\,d\phi = K\int_0^{2\pi} e\,i\,d\,t$$

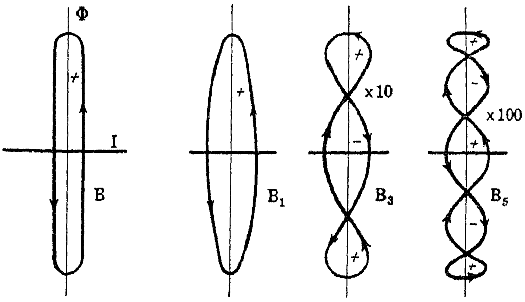

FIG. 3

It may appear from the figures that the author is correct in saying that the sine components represent variations in permeability and the cosine components represent hysteretic lag (and lead). Apparently, this is correct with a sine wave of flux but with a distorted flux the components in Fig. 2 as well as those of Fig. 3 will enclose area, and will or will not have net area depending upon the phase, order and magnitude of harmonic components of flux present.

We cannot say that the medial line of the hysteresis loop, due to the sine components of current, represents the variation in permeability or the sum of the cosine component represents hysteresis until we can prove just what proportion of the instantaneous power at different parts of the loop is going into Joulian heat and what proportion is being stored magnetically. The harmonic variation of temperature rise in cyclically magnetized iron would tell this but because of the relatively high thermal capacity of the iron the cyclic variations of the growth of temperature are too small to be measured accurately by any means which we have considered.

I would like to refer to the resonant condition in the star-star system with the supply side grounded. If in one transformer the total m. m. f. is zero when the voltage is zero and a reversal takes place at the retentivity point of the hysteresis loop of the transformer under question, because it can do so with less expenditure of energy for the instant, why does it continue in this reversed condition, when a little while later, more work will be required in the other transformers to keep it in this condition? We have found in many cases of cyclic magnetization that the flux variations and distribution will adjust themselves so as to require the least expenditure of energy but in the author's explanation more energy is required to excite the bank with one transformer reversed.

From some of our tests in the past on annular punchings, and other shaped cores, conclusions were drawn that the distribution of flux, in the way it crowds at the inside corners of a circuit and spreads out in the straight parts, is not entirely a function of the instantaneous values of permeability but also a function of the energy requirements.

If in Dr. Kennelly's ring punchings there were radial gaps cut so as to equalize the reluctance of the path at all radii at the maximum value, we would expect the amplitude of flux and hence the losses, to be uniform throughout the core. The mean flux would also exist at the mean diameter. Our tests in the past have indicated that this would not be the case, but that the mean flux would be slightly inside of the mean diameter and the losses slightly lowered by the inner parts of the rings (of smaller volume) running at an induction higher than the mean, and the outer half (larger volume) running at an induction lower than the mean. This effect is more than offset in most cases due to eddy currents producing skin flux and higher losses.

I would like to ask Mr. Alger whether the oscillograms, analysis and the curves of Figs. 10 and 11 are made with the proper time axes. It seems to me that time should progress from right to left for in Fig. 11 you will notice that the growth of flux in the outside section is advanced and should be retarded.

Prof. Kennelly spoke of using wide sheets rolled up in cylindrical form, so as to have the ratio of outside to inside radius small, to equalize the flux distribution. Such a sample should be annealed after forming for otherwise the physical strains produced will affect the magnetic quality more than the unequal density in the annular punchings, will apparently affect it. With silicon steel if a punching 0.014 in. (0.035 cm.) thick is bent with a curvature of 6 deg. per inch, the hysteresis loss will be increased 25 per cent and the exciting amperes 100 per cent. Such curvature is not sufficient to pass the elastic limit and the original magnetic properties will be restored if the sheet is allowed to spring back into the straight form. Such deformation of punchings changes the shape of the loop in such a way as to increase all of the harmonic components approximately equally in magnitude and produce a phase shift.

I would like to suggest that Dr. Kennelly repeat the tests without the paper between punchings. In this case the eddies between plates greatly increase the skin flux and instead of just adding a wave of energy current in phase with and of the shape of the voltage wave, as generally supposed, the effect will be approximately a percentage increase in the current wave at all parts of the cycle just as though a greater mass of iron operating at the same induction had been added. We once made such a test on two large transformers of the same design, one of which had unenameled punchings. The losses and exciting current for the unenameled core were increased 100 per cent but the increase of current, instead of being an addition of mostly fundamental current in phase with the voltage, was double throughout the cycle just as though two transformers similar to the enameled one had been tested in parallel.

A study of the three-phase transformer core has been made in a manner similar to that given by the authors. Instead of the flux waves in the partial sections of the core coming out as expected, as in the case of the single-phase core, some very strange flux-time distributions have been found which we have not been able to explain. Strange to say, this is a case in which the flux distribution is not such as to give the minimum loss.

V. M. Montsinger: Mr. Robinson's paper is a very interesting one to me because about three years ago I made tests under very much the same conditions as he describes, although the tests I made were entirely in a laboratory way. The results of these tests were given in a discussion of a paper—Transformer Connections, presented by Mr. L. F. Blume.*

I shall review briefly the conditions under which I made these tests and shall compare some of the results I obtained with those obtained by Mr. Robinson because my results do not bear out some of his conclusions. I have special reference to the statement or conclusion that it is the even and not the odd harmonics that are the cause of all the trouble in practise, when operating transformers connected YY with the line side grounded, and also to the violent internal vibration in the transformers, which he attributed to one leg of the transformer bank being reversed.

Referring to Fig. 4, which shows the connections of the generator and three 5-kv-a. single-phase transformers, for my test it will be seen that the generator and the primary side of the transformers were Y-connected, the secondary side being delta-connected with one corner of the delta open to prevent the flowing of the third harmonic exciting current demanded by the transformers. It is obvious that by connecting the secondary side open-delta, it was possible to observe across this opening the third harmonic voltage or multiples of the third. Oscillograms were taken of both the voltage across the transformer legs and across the opening in the delta under different conditions. These oscillograms showed the presence of marked third but no even harmonic voltages, first without, and second with sufficient capacitance, across either the legs of the transformers or across the opening in the delta, to cause saturation of the transformer iron, consequently intensification of the higher harmonic voltages.

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*TRANS. A. I. E. E. 1914, Vol. XXXIII, Part I, p. 779.

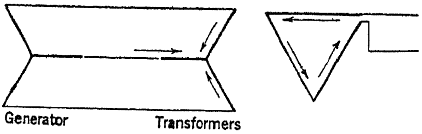

FIG. 4—DIAGRAMMATIC SKETCH OF GENERATOR AND TRANSFORMERS FOR MAKING THIRD-HARMONIC VOLTAGE TESTS. ARROWS SHOW DIRECTION OF TRIPLE-FREQUENCY VOLTS

Fig. 5 shows the third-harmonic voltage in percentage of the fundamental voltage with and without capacitance supplied to the transformers. Without capacitance it will be noted that around the usual working density of the transformer iron, the third-harmonic voltage is from 55 to 60 per cent of the fundamental. With capacitance supplied it is interesting to note that

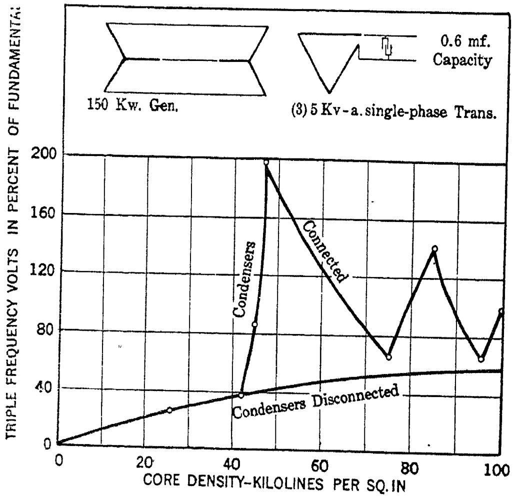

FIG. 5—CURVES SHOWING TRIPLE-FREQUENCY VOLTS OBSERVED ACROSS OPEN DELTA, WITH AND WITHOUT CONDENSERS CONNECTED

at about 46 or 47 kilolines per sq. in. the third-harmonic voltage suddenly increased to about 200 per cent of the fundamental and on further increasing the generator voltage the third-harmonic voltage decreased again to its normal value without the capacitance; and still further increasing the generator voltage, the voltage across the opening in the delta again rose to a peak and decreased again. This happened for the third time. It will be noted that each time the peak voltage occurred across the opening in the delta its percentage of the fundamental decreased. This would be expected because if the iron were fully saturated with fundamental flux it would not be possible to impose a flux of other frequencies. In other words, the lower the normal flux density of the iron, the greater the danger from excessive voltages of higher frequencies, providing of course there is sufficient capacitance present to start building up the voltage.

Now if the core density is exceedingly low it may be impossible to produce by capacitance intensified third harmonics.

Mr. Robinson states that when he had either the 37½ or 50-kv-a. transformer connected to the 37-mile transmission line, he imposed only ¼ excitation voltage on the transformers, which means that the iron density must have been somewhere in the neighborhood of 20 (or less) kilolines per sq. in. The point I wish to emphasize is that Mr. Robinson had a very low core density and a large (relative) capacitance—a condition which is seldom, if ever had in practise. It is possible that had the transformer iron been fairly well saturated before connecting to the transmission lines the effect of the capacitance of the line would have been to produce the third-harmonic voltage that I observed and that there would not have resulted the phenomenon of one leg of the transformer bank being reversed.

While the above phenomenon is no doubt possible under unusual conditions yet I do not think it is usually what happens in practise. In other words the above is probably the exception rather than the rule.

In regard to the internal vibration of the transformers which Mr. Robinson speaks of, I would like to quote one sentence from my discussion in 1914 referred to before.

"At these peaks (referring to Fig. 5) after the first a marked humming sound in the transformers indicated that the frequency was higher than a triple harmonic—probably multiples thereof." This sound was more violent at the third than at the second peak.

Mr. Robinson attributes these violent internal vibrations to the fact that one leg of the transformer bank is reversed, consequently over magnetizing the other two legs. Had one leg of the transformers when connected Y-open-delta been reversed, the voltage across the opening in the delta would have been many times the value shown in Fig. 5; thus it seems that these violent internal vibrations are produced for conditions other than when one leg is reversed.

Unfortunately the oscillograms of the voltage across the opening in the delta did not prove of any value after the first peak and so it is not possible to say at the present time whether these voltages were of triple frequency or multiples of triple frequency. It is hoped to make additional tests to determine more definitely what is happening when these peak voltages occur across the opening in the delta and there is this violent internal vibration.

A. S. McAllister: I ask whether or not the phenomenon disclosed by the tests performed by the author could not be reproduced by the use of three similar stationary transformers wound with a different number of turns connected in series, oscillographs or recording instruments being placed across each transformer?

C. O. Mailloux: In the conclusions, at the end of the paper by Dr. Kennelly and Mr. Alger, the phenomenon is described as "radial distortion of flux-density." I am wondering if the word "distortion" was not intended to mean "distribution." The case is quite analogous to the distribution of currents in a torus, subjected to a constant e. m. f. If we take an ordinary annulus, and suppose an e. m. f. applied to it, and a current passing through it, there will be a condition of inequality of distribution, because the conductor filaments subjected to the e. m. f. differ in length on the inside from those on the outside, and the density of current in them would be different, being greater at the inside than at the outside portion. The distribution of magnetic flux may be looked upon as being substantially the same as the distribution of electric current if we consider instantaneous values. If we take instantaneous values instead of taking cyclic values, in the cases considered in the paper, then, at any instant of time, we may assume the magnetic permeability to be constant, and we have a m. m. f. acting upon a reluctance which may be considered constant, and which produces a certain magnetic flux. The instantaneous value of that magnetic flux would be very similar to the instantaneous value of the electric current-conduction flux through an annulus subjected to a constant e. m. f. We know from the equations, which are very simple, that the resistance of the belts of the annulus and of the latter as a whole, under those conditions will vary, and that in its determination enters the ratio of the diameter of the inner to that of the outer circle of the ring. It seems to me, if the results are analogous and comparable, as I think they are, that it would be better to use the words "distribution of the flux." Now, suppose that, in the annulus, we have a lump, a hole, a flaw, a variation in homogeneity, or in annealing, or in anything that affects the conductivity; then, under those circumstances we can conceive there would be a change in current, and that kind of change, being accidental and local, might be called *distortion* of the current. In the case of magnetic substances, it is difficult to get absolutely uniform permeability throughout, and it is quite conceivable that we might have both of the phenomena together, indeed superimposed. We might have differences of flux-density, or, in other words, differences of distribution, and we might have also differences in flux due to distortion. Theoretically, we know that a torus subject to a m. m. f., due to perfectly distributed windings all over it, should have no external magnetic field. Practically, however, I do not think anybody has ever succeeded in constructing such an apparatus in which there was no external magnetic field; and, in looking about for an explanation of it, we are led to conclude that the explanation is to be sought for in differences of magnetic permeability, or in differences of molecular structure, or other changes of that kind which depart from the ideal conditions to such an extent that there are localized disturbances, localized closed magnetic circuits, so to speak, interfering with the flow of magnetic lines of force through the circuit as a whole. For that reason, I think it would be interesting to make a distinction between distortion, and distribution or density of flux, in the definition of the true phenomena, and then to undertake, by experiment, if possible, to segregate the two things.

A. M. Dudley: Mr. Behrend has asked me what effect, in my judgment, would the flux distribution in the core, as found by Dr. Kennelly and Mr. Alger, have upon the iron loss in induction motors. This with particular reference to high-speed motors having deep cores. In this connection I should like to call attention to some facts brought out by Mr. Lamme in his paper on iron losses at Schenectady a year or so ago. In induction motors in particular the variation in iron loss due to the processes of manufacture are extreme. This is so much the case that two machines built to identical specifications from the same lot of iron and at the same time may vary in iron loss by fifty per cent or more. It has been proven by test that bending a lamination or core plate sheet 0.017 in. thick in an arc of a circle whose curvature is 6 deg. between tangents one inch apart will increase its iron loss by as much as 25 per cent. In calculating iron losses it is customary to calculate the losses in the primary core and the primary teeth and to this to add the so-called pole-face losses due to high-frequency pulsations in the magnetic field caused by the slot openings. To the sum of these various losses is added an item to take care of indeterminate losses from various causes, so that the total iron loss is several times what would be its value calculated from the volume of the iron itself and curves based upon Epstein samples. Since this is true it is very difficult to segregate the different factors with any considerable degree of exactness. These commercial conditions as they exist make it difficult to answer the question asked by Mr. Behrend. I had intended to ask the authors the same question and will do so now. I believe it would be very interesting if Dr. Kennelly would tell us the circumstances that led up to this investigation and give us his ideas as to the effect of the flux distribution described upon the iron loss of a-c. generators and motors.

F. W. Peek, Jr.: I do not believe that Mr. Robinson is right in his conclusion that the greater percentage of the troubles in single-phase Y-Y-connected transformers, ascribed to the third harmonic, are due to some other cause. There is no doubt that when such transformers are operating under normal conditions of load and excitation that third-harmonic voltages do exist in a way that is easily predicted. This is supported by any amount of experimental data. Mr. Robinson's views on the third harmonic are not justified by the data contained in the paper.

The reason for the third harmonic is, of course, well known. If a sine-wave voltage is applied across a coil with an iron core, the magnetizing current will not be a sine wave but a distorted wave. This distortion is principally due to the variation of the reactance, as the permeability changes with the flux density. On account of the inherent characteristics of the iron, the greater part of this distortion is a third harmonic. If some means is taken to prevent the third-harmonic component of the magnetizing current from flowing, the voltage across the coil cannot be a sine wave but must be a peaked wave containing a third-harmonic component. This very condition obtains when these coils are connected in Y to a three-phase circuit, for the following reasons: The three triple-frequency currents flowing to the neutral must be 3 × 120 deg. apart or in phase. Fundamentally, the sum of three currents flowing to a point must be zero. The sum of three currents in phase can only be zero when each current is zero. The Y connections thus suppress the third-harmonic components of the exciting current. The voltage from *neutral* to *line* must thus be peaked. The distortion is not noted by a voltmeter since the effective value is not much changed, but may be measured by a sphere gap. The voltage distortion disappears as soon as the triple-frequency component of magnetizing current is permitted to flow. This component can flow if three single-phase paths are supplied by connecting the neutral of the Y to the neutral of a Y-delta transformer on the same system. If these transformers are grounded, the triple-frequency currents circulate in the ground and cause telephone troubles. If there are secondary windings in the transformers, and these are connected in delta, there is no distortion since the triple-frequency component can flow around the delta and this excitation is supplied from that side. *Three-phase core-type* Y-Y transformers have no appreciable triple-frequency component; it is taken care of in the magnetic circuit.

When the neutral is grounded and the triple-frequency current is suppressed, the maximum voltage between line and neutral is increased about 40 per cent. When certain capacity is connected to the transformer, as for instance, sections of transmission lines, the maximum may be as much as three times normal. The danger may thus be increased if certain values of capacity are connected to the transformers. If the neutral is not grounded, it must wabble, and a voltage appears between neutral and ground. The insulation strains are, however, less than with the grounded neutral.

There is much of the discussion in Mr. Robinson's paper that I cannot follow, and many of his conclusions are not at all substantiated by the experimental data given in the paper. What experimental data aregiven does not apply to transformers operating under normal conditions. The oscillograms were made on 10-watt transformers over-loaded and under-excited. It is not well to attempt to draw broad conclusions from such data.

Mr. Robinson undoubtedly has observed some peculiar phenomena that may occur under abnormal conditions of excitation, load, etc. that are worthy of further experimental investigation. It is, for instance, possible to get a leg reversal in certain transformer connections during a ground or short circuit. The oscillograms on the abnormally operated 10-watt transformers do show an even harmonic; the reason for this should be further investigated before broad conclusions are drawn.

C. O. Mailloux: I would like to ask the authors of the paper if they have made experiments to secure a comparison of the flux density in the inner belt and the outer belt, as a function of the cycle, taking and plotting cyclic time, or angle, and finding out how the flux-density varies at different parts of the cycle. I should expect that the ratio would change greatly as the magnetic flux increases during the cycle. I would like to ask if they have made experiments by the stroboscopic method, which would throw much light on the manner in which the magnetic flux-density increases outward, from the inner to the outer circle, causing the belt of mean flux-density to shift from a point near the inner circle to a point near the middle of the ring.

George B. Thomas: I would like to ask one question in regard to the paper by Dr. Kennelly and Mr. Alger. As to the distortion, as they call it—the distribution of the flux, as I read the paper and have listened to the discussion—it seems to me they only took an oscillogram for one section at a time and I believe it would be easy to use a three-element oscillograph and take the whole three at the same time and compensate for any distortion due to the current in the exploring coil. If that was done it would show definitely whether there was any distortion.

L. W. Chubb: There are two kinds of simple oscillating systems and resonant circuits, one in which the frequency is independent of the amplitude and the other in which the frequency is dependent upon the amplitude. This is illustrated by Figs. 6 and 7. In Fig. 6 the mass *M* is attached to two long helical springs. The stress-strain diagram of the system is shown at the right of the figure. It is well known that a small periodic force of a certain frequency will cause the system to resonate and *M* to vibrate vertically with simple harmonic motion.

Fig. 7 shows another oscillatory system consisting of a closed-end cylinder in which the piston *P* slides and compresses air in the end of the cylinder. The stress-strain diagram in this case is not a straight line but is composed of the compression curves of the air. Such a system has a natural frequency dependent upon the amplitude of vibration and will resonate at any frequency within a given range, provided it is given an amplitude of the proper value to lock it into step.

If we assume that mass corresponds to inductance and the elasticity corresponds to capacitance, the two systems represent pretty well the two kinds of oscillating electric circuits, the first with air-coil inductance having a single frequency and the second with iron inductance having a period dependent upon the amplitude. The analogy is not correct as in Fig. 7 the capacitance decreases with amplitude, with constant inductance, while in the electrical circuit the inductance decreases with amplitude with constant capacitance. Since, however, the product of the two variables gives a similar result, the effect is the same.

The relation to the phenomena noted in the paper is this. When the transformers are thrown on the circuit the unsymmetrical transient flux in the transformers which starts near zero voltage reaches sufficient amplitude to reduce the inductance to a point low enough to resonate with the line capacitance. This resonance then continues with the transformers operating at the high induction, the high voltage being supplied by a phase shift of voltage and possibly the reversal of the third transformer, as indicated in Mr. Robinson's diagrams. We have produced such resonance and taken several oscillograms of the effect on single-phase circuits in which a condenser was connected across a transformer and there was a damping impedance in series with the line. We were not able to obtain stable resonance except with resistance in series with the transformer winding. However, the other transformers, with their losses connected in series, may produce a similar effect in Mr. Robinson's three-phase case.

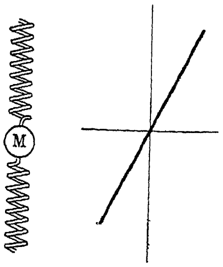

FIG. 6

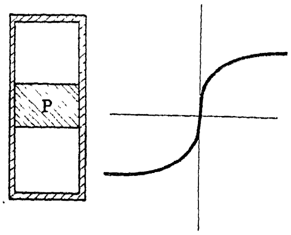

FIG. 7

It is unfortunate that Mr. Robinson did not take more oscillograms and include the voltage waves of the three transformers so that the phase and magnitude of each could be known. The current waves in the type of resonance which I have described took the form of an almost perfect triangle instead of showing many inflections as in Mr. Robinson's case. The difference may be due to the polyphase connection and the presence of saturated iron.

P. L. Alger: It is shown in Table III that nearly all voltages were measured by the potentiometer, and the voltages are all nearly in phase, and only differ by a maximum of a little over 4 deg. We found the angles, not accurately enough to get relative power-loss results, but they do indicate a different power loss due to each flux. It is evident in every case that the nature of the flux determines the loss of phase, being so nearly the same as that of the total as to make no difference.

Mr. McAllister called attention to the fact that three transformers connected in series present the same phenomenon. It is precisely the same phenomenon, and might be used to clear the subject of some of the difficulties. In both the cases of the laminated core and of the three transformers, the current is the same for each component magnetic circuit, and if the flux density is the same, the permeability is also the same, so that the only other factor in the magnetic-circuit equation $B = \frac{4 \pi \mu N I}{e}$ is the ratio of the number of turns divided by the length. In the case of the variations in conditions between sections of the laminated core, the length of the path changes the conditions. In the case of the three transformers in series, changing the number of turns on the three would give us the same variation in the conditions of the three magnetic circuits and I should think that the losses and the permeabilities would follow in the same relation as we observed in the laminae.

I think Mr. Mailloux, when he mentioned the question of the ratio of the flux, raised an interesting point. Fig. 11 shows what I understand he desires to know, for one maximum flux. The flux in each belt is shown at each instant of the cycle, and by giving the ratios of the ordinates in the three cores at each instant, one could see how the flux spreads from the inside to the outside of the arrangement during the progress of a cycle. I mean the ratio of the flux in the strongest to that in the weakest belt is a function of the time of the cycle. That would show how the density of the flux spreads outward. We only experimented with one particular flux density, and in that case, at the beginning of the cycle, the flux in the inner part of the core is much greater than in the outer two parts, so you can see how the flux does change in proportionate distribution as the total density increases. A three-section oscillogram would have shown the lag distortion very well. Such a one might have been used, but no one thought of using that particular scheme, and the method of changing the damping was considered to be sufficient proof that no particular distortion was present.

A. E. Kennelly: Electrical engineers are familiar with even harmonics in the e. m. f. waves of a-c. generators of the induction-coil type. We know that when a wave of e. m. f., as recorded in an oscillograph, is symmetrical with respect to the zero line, so that the positive half-wave is the image of the negative half wave, with respect to that line, there can be no even harmonics. Per contra, the criterion of the existence of even harmonics is a dissymmetry between the + and − half waves.

When a Ruhmkorff induction coil is operated by a voltaic cell, through a vibrating-spring interruptor, we know that the half wave following the break circuit is quite different from the half wave which contains the make circuit, showing that even harmonics are present. In general, whenever magnetic flux is cyclically thrown into and out of an electric circuit, if the process of linking is not symmetrical with the process of removal, there will be dissymmetry between the + and − half waves, showing the presence of even harmonics. An alternator is a machine designed to develop perfectly symmetrical half waves, so as to exclude even harmonics. Nevertheless, since perfect symmetry of mechanical and magnetic conditions is a purely ideal condition, it is to be expected that some traces of even harmonics will present themselves in the e. m. f. wave of an alternator, if we take pains enough to examine the oscillograph with that end in view, as Prof. Jackson has pointed out.

If an alternator impresses upon a circuit an e. m. f. wave with substantial + and − symmetry, and correspondingly negligible even harmonics, the alternating current in the circuit must be expected to be likewise free from even harmonics unless there is either unilateral conductance to destroy the + and − symmetry, or unless there is some synchronous action at work to alter magnetic linkage with the circuit in a dissymmetrical manner. Such conditions must be rare, or we would notice even harmonics in our oscillographs more frequently. Consequently, even harmonics are so unusual on any large scale, that when they are discovered they call for special examination and explanation.

It has been asked what our reasons were for taking up this question of non-uniform magnetic-flux distribution in steel laminae. We recently had occasion to measure in our laboratories the extra electrical resistance of flat copper strips, due to non-uniform electric alternating-current distribution. We found that the ordinary skin-effect formulas, which allow for imperfect electric penetration towards the midplane, were very far from giving correct results with such strips. These formulas took into account "skin effect" or imperfect penetration in depth, but ignored what we have called "edge effect," or imperfect penetration in width. That is, the current density is much greater near the edges of the strip than near the middle. The current tries to avoid the middle of the section, both laterally as well as up and down; so that edge effect is of the same general nature as skin effect, but may be much more noticeable in copper strips.

It occurred to us that since a ring lamina was a magnetic strip conductor, subject to very noticeable skin effect at ordinary power distribution frequencies, there might be a similar magnetic "edge effect." If so, the alternating flux density carried near the edges would be greater than that near the middle of the strip. In order to test this question, windings were applied to the inner, middle and outer thirds of the ring laminae, as described in the paper. Marked differences in flux density were found to exist, but attributable, in the main, to permeability variations. Whatever true edge effect exists was small, by comparison with the permeability variation effect, under the conditions here reported.

The bearing of these results in practise, is that care must be taken, when making magnetic measurements on samples of steel in the form of ring laminae, to avoid errors due to radial distortions in flux density. Moreover, the eddy-current losses in such laminae are likely to be considerably greater than those due to the average flux density assumed as sinusoidal, owing to the complexity of the alternating magnetic flux waves.

When one realizes, from such observations as these here reported, that not only the alternating flux density, but also the alternating flux wave form, is different at each and every radial belt, the entire phenomenon of a-c. magnetic conduction in ring laminae takes an altered aspect.

Waldo V. Lyon: (communicated after adjournment)*: I have only two minor corrections to suggest in the Kennelly and Alger paper. The curves shown in Fig. 6 should not be continued to the left beyond the actually observed points, since in this region of low flux density the magnetic quality of the iron varies in a different manner than at higher flux densities. See Fig. 14. Again under the heading "Outline Theory of the Radial Variation," it is stated that above $H = 0.5$ the reluctivity is $a + b\,H$. It is more correct to place this lower limit of H at 2.0, since in the region between these two values of magnetizing force the reluctivity is not represented by a straight line.

I have found it of interest to carry out some simple mathematical analyses and to compare the results thus obtained with the experimental data.

Eddy Currents. If laminations are placed in a uniform magnetic field that is parallel to their surfaces the effect of the eddy currents can be calculated on the assumption that the permeability is constant. This signifies that there is no loss by hysteresis. It can be shown that the magnetic effect of the eddy currents depend upon $t\sqrt{\dfrac{\mu f}{\rho}}$; in which t is the thickness of the laminations, μ their permeability, ρ their resistivity and f the frequency of the flux variation. The ratio of the flux density at the center of the lamination to that at the surface is

$$\frac{\sqrt{2}}{\left[\cosh 2\,\pi\,t\sqrt{\dfrac{\mu f}{\rho}} + \cos 2\,\pi\,t\sqrt{\dfrac{\mu f}{\rho}}\right]^{\frac{1}{2}}}$$

For this quality of iron, at about $B = 10{,}000$, the ratios are 0.999, 0.975 and 0.90 respectively for frequencies of 60, 340 and

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*The c. g. s. system of units is used throughout this discussion.

696 cycles. Thus even at 696 cycles the eddy currents reduce the flux density but 10 per cent at the center, while at 60 cycles the effect is inappreciable. Since the strength of the eddy currents is proportional to the flux with which they link, they will, in the case of circular laminae, produce the greatest effect at the center of the laminations near the inner circumference. The reduction in the flux density will be progressively less toward the outer circumference.

At the middle of the lamination this reduction has approximately its average value so that the increase in frequency has little effect in modifying the proportional part of the flux that exists in the middle belt. This is shown in Fig. 6. The flux in the inner belt is reduced more than the average while that in the outer belt is reduced less than the average so that as the frequency is increased the proportion of the total flux in the inner belt is decreased while that in the outer belt is increased. This is also shown in Fig. 6. The quantitative analysis, especially if the reluctivity is not assumed to be constant, presents considerable difficulty and has not been done.

Reluctivity and Magnetization Curve. We will neglect the effect of eddy currents and assume that the iron is homogeneous. Since with circular laminae the value of the magnetizing force is not the same for the different flux paths, the curves that are obtained by measuring the *total* flux, even within a narrow belt, give strictly average relations. It is of interest to know how far the average values thus obtained depart from the actual characteristic curves of the material. If we assume that within certain limits the metallic reluctivity is correctly given by

$$\nu = a + b\,H$$

the flux density is actually $B = H + \dfrac{H}{a + b\,H}$. The first component, however, is so small compared with the second, that at all ordinary flux densities, it may reasonably be neglected. See Dr. Kennelly's original paper, TRANS. A. I. E. E., Vol. VIII, page 486. The total flux within a ring of radii R_1 and R_2 and axial thickness W is

$$\phi_0 = \frac{2\,N\,i\,W}{a} \log_e \frac{R_2\,\nu_2}{R_1\,\nu_1};$$

where $N\,i$ is the number of magnetizing ampere turns and ν_1 and ν_2 are reluctivities at the inner and outer radii, R_1 and R_2. The average value of the flux density is

$$B_{av.} = \frac{2\,N\,i}{a\,(R_2 - R_1)} \log \frac{R_2\,\nu_2}{R_1\,\nu_1}$$

The value of the magnetizing force at a *mean* radial distance from the center is

$$H_{av.} = \frac{2\,N\,i}{R_2 + R_1}$$

The expression for the average reluctivity is

$$\nu_{av.} = \frac{2\,a\,(R_2 - R_1)}{R_2 + R_1} \quad \frac{1}{\log_e \dfrac{R_2\,\nu_2}{R_1\,\nu_1}}$$

The only assumption in this calculation is that the relation between the reluctivity and the magnetizing force is a straight line. An approximate value for the average reluctivity is obtained if the $\log_e \dfrac{R_2\,\nu_2}{R_1\,\nu_1}$ is expanded as a series

$$\log_e \frac{R_2\,\nu_2}{R_1\,\nu_1} = \frac{2\,(R_2\,\nu_2 - R_1\,\nu_1)}{(R_2\,\nu_2 + R_1\,\nu_1)} + \ldots\ldots$$

The error made in neglecting the terms of this series beyond the first depends upon the ratio $\dfrac{R_2\,\nu_2}{R_1\,\nu_1}$. A ratio of radii near unity reduces the error to its smallest proportions. The error is always greatest for a maximum density in the ring just below the point that makes the reluctivity a minimum. In this case this occurs at about $H = 0.6$. For this value of H and radii ratios of 2, 1.5 and 1.25 the errors are 18.4, 6.8 and 3.2 per cent respectively. At a moderate value of the magnetizing force of about $H = 5.0$ the error for a radii ratio of 2 is but 0.18 per cent and for the smaller ratios is below 0.05 per cent. With a high value of the magnetizing force of about 20, the errors for any radii ratio below 2 are too small to detect with five-place logarithm tables. In the following analysis I have made use of this approximation on account of its accuracy and simplicity. Making this substitution for $\log_e \dfrac{R_2\,\nu_2}{R_1\,\nu_1}$ in the expression for the average reluctivity gives upon reduction

$$\nu_{av} = a + b\,H_{av}$$

Thus the average value of the reluctivity is essentially equal to its actual value except for large radii ratios and a small value of the magnetizing force.

In Fig. 13 are shown the magnetization curves taken with direct and with alternating current. With the latter the maximum values of the alternating current are computed by multiplying the effective value by $\sqrt{2}$. This is the principal reason why the two curves do not coincide, for, as will be presently shown, the true multiplying factor is much greater. In the following part of the analysis it is assumed that the time variation of the entire flux is sinusoidal. This would be so if the impressed voltage were sinusoidal and the resistance drop in the winding were small. Neglecting the effect of hysteresis the magnetizing current is symmetrical and peaked if the maximum magnetizing force is more than 3 or 4 gilberts per cm. At very low flux densities below $H = 1.0$, the magnetizing current would be flat, due to the fact that the reluctivity decreases as the magnetizing force increases. Hysteresis has no effect on the maximum value of the magnetizing current, and but little effect on its ampere value, except at high flux densities or when the loss is abnormally large. The ratio of the maximum to the ampere value on the assumption of no hysteresis, is thus approximately equal to the actual ratio. This may be calculated if the reluctivity is assumed to vary as Dr. Kennelly indicates, viz. $\nu = a + b\,H$. From this relation it follows that $H = \frac{a\,B}{1 - b\,B}$. It has already been shown that this relation holds, except at very low flux densities, where B is the average flux density in the core and H is the magnetizing force along the mean circumference. The equation is not true, however, for values of B so large that $b\,B$ is equal to or greater than unity. The magnetizing current is

$$i = \frac{R\,a\,B_m}{2\,N}\left(\frac{\sin \omega\,t}{1 - b\,B_m \sin w\,t}\right)$$

Where R is the average radius, N, the number of turns in the magnetizing coil and B_m, the maximum value of the average flux density. The ratio of the maximum current to its effective value reduces to

$$\frac{I \text{ max.}}{I \text{ r. m. s.}} =$$

$$\frac{c}{(1-c)\left[1 + \frac{2\,c}{\pi\,(1 - c^2)} + \frac{2\,c^2 - 1}{(1-c^2)^{3/2}}\left(1 + \frac{2}{\pi}\sin^{-1} c\right)\right]^{\frac{1}{2}}}$$

where $c = b\,B_m$. Dr. Kennelly shows from d-c. measurements that the value of b for this quality of steel is 0.0000723. This calculated ratio should agree with the observed ratio quite closely when the maximum value of the magnetizing force has such a value that the straight line relationship for the reluctivity holds during the major part of the cycle. That is, H_m should either be less than about 0.6 or greater than about 3.0. In the former case the value of b is negative. The following table gives the ratio of maximum to effective magnetizing current for different values of the flux density.

B_m	$I_{max}/I_{r.m.s.}$
10,000	1.77
11,000	1.90
12,000	2.07
12,500	2.23
13,000	2.47

This shows that the effective magnetizing current in a transformer may be about 50 per cent of the maximum value as determined from a d-c. saturation curve instead of 70 per cent, as is usually assumed. If these multiplying factors are used in determining the maximum value of the magnetizing force, the a-c. saturation curve—see Fig. 13—would be determined by the following points. In the third column are given the points on the d-c. curve.

B_m	H_m (60 cycles)	H (d-c.)
10,000	2.50	2.50
11,000	3.56	3.56
12,000	5.56	6.60
12,500	7.73	9.50
13,000	10.8	14.50

The two curves will now coincide up to a flux density of about 12,000. At higher flux densities the resistance drop in the exciting winding reduces and modifies the wave form of the flux an appreciable amount. The effect of hysteresis in altering the ratio of maximum to effective magnetizing force also becomes important. For these reasons the formula can no longer be relied upon to give good results. Furthermore Dr. Kennelly points out that the wave form of the impressed voltage became distorted when the higher values of the flux density were obtained. If the resistance of the exciting winding were low, as is the case in a transformer, the only disturbing element would be the effect of hysteresis.

If the wave form of the flux variation and its maximum value are known, the ratio of the maximum to effective magnetizing current can be determined from the hysteresis loop quite simply by a method I described in the *Electrical World*, Vol. 70, p. 949. A correction may also be made for the effect of eddy currents. It was shown in this paper how a closely approximate analysis of the magnetizing force may be obtained from the hysteresis loop. The effective value of the magnetizing force is equal to the square root of the sum of the squares of the maximum values of the various harmonic components divided by the square root of two. In the numerical example worked out a high value of flux density of 14,000 was used in order to exaggerate the errors of the method. These, however, were shown to be small. Three different voltages were used; a sine wave, a peaked and a flat topped one. The latter two consisted of a fundamental and a 33 per cent third harmonic. The ratios of maximum to effective values are respectively 1.89, 1.48 and 1.98.

Ratios of Belt Fluxes. Due to the fact that above a certain magnetizing force, in this case $H = 1.0$, the reluctivity increases as the magnetizing force increases, the flux density at different radial distances with d-c. excitation is more nearly uniform than if the reluctivity were constant. With a-c. excitation the curves in Fig. 6 apparently indicate that the reverse is true. There are two reasons for this discrepancy. In determining the maximum value of the flux in each belt by a-c. measurement the same relation has been used for each of the three belts; viz. $\phi_m = \frac{E_{\text{r.m.s.}}\ 10^8}{4.44\ N f}$. This assumes a sinusoidal variation of the voltage and flux. This might introduce considerable error in the calculated values of ϕ_m since the oscillograph records show that the wave form of the inner belt voltage is more peaked and of the outer more flat than a sine wave. The form factor of a peaked wave is greater than that of a flat one, so that if this correction were made the flux ratio for the inner belt would be reduced and for the outer belt, increased. For example, a peaked wave consisting of a fundamental and a 10 per cent third harmonic has a form factor 3.3 per cent greater than a sine wave's. A flat wave consisting of the same components has a form factor 3.3 per cent less than a sine wave's. The effect of eddy currents is in the same direction, as has already been indicated, but at 60 cycles it is very small. These corrections do not seem great enough, however, to explain the large deviation of the measured results from the calculated ones.

Hysteresis Loss. It is an experimental fact that the hysteresis loss per cubic centimeter per cycle over a large range of flux density is given by

$$P_h = k\ B^p_m$$

The exponent is usually found to be about 1.6, although it may be greater than 2. Since the flux density in the ring is different at different radial distances, the measurement of the hysteresis gives only the total loss for the particular ring used. From this may be deduced the "average" loss per unit volume. In measuring the reluctivity it was shown that, except at very low flux densities, the average value is essentially the same as the actual value. Will the same be true of the hysteresis loss?

It is not possible to obtain a simple formula that shows the difference between the actual loss per unit volume for the average flux density and the average loss per unit volume. An indication of the magnitude of this difference may be obtained without much difficulty, however. The difference will be greatest with a ring of large radii ratio and at a low flux density. We will consider two general cases, the first for a maximum H of 0.6 and the second for a maximum H of 4.0. The following data give the relative losses at different points in rings having radii ratios of 2 and 1.25. The value of the exponent, p, is taken as 2 in order to simplify the computation and for the reason that it exaggerates the differences above the minimum reluctivity and reduces them below this point. With a radii ratio of 2 and a maximum flux density of 6000 at the inner radius, the relative losses are 36, 1.1 and 2.6 at the inner, outer and average radii. The losses are calculated for rings having the same difference in their radii. The smaller this difference the more accurate are the relative losses. With the same rings and a maximum flux density of 11,100 at the inner radius the relative losses at the inner, outer and average radii are 1.23, 1.69 and 1.55. With a radii ratio of 1.25 and a maximum flux density of 6000 the relative losses are 144, 24 and 46 at the inner, outer and average radii. With this ring and a maximum flux density of 11,100 the relative losses are 49.4, 56.6 and 53.4 at the inner, outer and average radii. Approximate relative values of the average losses are the average losses at the three radial distances. These are 13.2, 1.49, 71 and 53.1 respectively for the four cases mentioned. The relative losses for the average flux density are 2.6, 1.55, 46, 53.4. Thus it follows that the error made in calculating the loss per unit volume on the assumption that the same relation between loss and maximum flux density, viz.: $P_h = k\,B^{p}_{m}$, holds for average values of flux density as for actual values of flux density will be about 80 per cent, 4 per cent, 35 per cent, and 0.6 per cent. This shows that the error made in calculating the hysteresis loss is much greater than that made in calculating the reluctivity. It is much more important to use a ring having a small radii ratio when making hysteresis measurements than in obtaining a magnetization curve. At low flux densities the values of the coefficient and the exponent in the hysteresis equation obtained from a ring of the ordinary dimensions will be different from those obtained from an extremely narrow ring, *i. e.*, from the true values. This no doubt accounts to some extent for the various values of the coefficient and exponent in the equation for the hysteresis loss.

L. N. Robinson: Mr. Chubb inquires why, after a reversal of a transformer has taken place because of less energy requirement at the retentivity point in the hysteresis cycle, does the transformer continue in this reversed condition when, a little later, more work will be required in the bank to maintain the condition? First, I might mention that I was originally led to the deduction, as to how the initial reversal might take place, through a consideration of inertia rather than of minimum energy requirement. That is, at the instant of reversing, the transformer should not be thought of as making a choice that will result in miminum ultimate energy consumption, but rather as taking the course that presents the least obstacle in the next succeeding instant and this, we see, requires minimum energy in the infinitesimal interval of time that immediately follows the reversal.

Besides the inertia of the circuit containing iron-clad inductance, apparently the other prerequisite for the reversal of one transformer of a Y-Y connected bank with grounded neutral on the line side is the coincidence of the zero values of the total m. m. f. and of the voltage impressed on the transformer that reverses.

Consider the case where the impressed voltage is not zero and the total m. m. f. (primary ampere turns plus secondary ampere turns) is passing through a zero value. In the general consideration of this case, the primary and secondary m. m. fs. are not each zero, but are of equal magnitude and of opposite signs. Under the stated conditions, in order for a reversal to take place; that is, for the voltage across one leg of the bank to change instantaneously from one magnitude in one direction to an equal magnitude in the opposite direction; there would be demanded an infinite current, $i = C \frac{d\,e}{d\,t} = \infty$, because $\frac{d\,e}{d\,t} = \infty$ and in any electric circuit C is not zero.

Furthermore, if the leg reversed under these conditions, the primary and secondary m. m. fs. would exchange directions instantaneously; that is, the primary current would be reversed due to the reversal of the primary impressed voltage and the secondary current would reverse in order that the sum of the primary and secondary m. m. fs. be zero. The instantaneous reversal of current means $\frac{d\,i}{d\,t} = \infty$, and it immediately follows that the infinite voltage, $e = L \frac{d\,i}{d\,t} = \infty$, would be required in order to accomplish the reversal. An infinite current or an infinite voltage demands an infinite supply of power which is not available. Hence a reversal cannot take place if the impressed voltage on the transformer is not zero even if the total m. m. f. is zero.

In the second case, if a reversal were to take place when the total m. m. f. were not zero and the impressed voltage were zero; the primary and secondary m. m. fs. would have to exchange values at the instant of reversal and this again demands a voltage,

$$e = L \frac{d\,i}{d\,t} = \infty.$$

From these two cases, it is clear that no reversal can occur if neither the impressed voltage nor the total m. m. f. of one transformer of the bank is zero.

Consequently, the only case, where a reversal may be expected, is that in which the impressed voltage and the total m. m. f. are both zero simultaneously. Then, and only then, can both $\frac{d\,e}{d\,t}$ and $\frac{d\,i}{d\,t}$ be finite during the reversal.

In regard to the vibrations and noises that accompanied the even harmonic *and* reversed leg phenomena, they were hardly to be described as high-frequency hums as inferred by Mr. Montsinger. They were of low frequency and their intensity varied, depending upon the nature of the electrical phenomena. In fact, the vibrations were so intense and of such low frequency that they could be felt by the hand holding a stick against the transformer cases. I attributed the vibrations to excessive magnetization because in-phase currents from line terminals to neutral necessarily flowed through the open-circuit susceptance of the transformers, and in the tests these in-phase currents were of the order of magnitude of the rated current capacity of the transformers.

Before comparing the experiments reported by Mr. Montsinger and those discussed in the present paper, a part of the opening paragraph of Mr. Montsinger's 1914 discussion* should be noted. His was "an investigation of the effect of electrostatic capacity and reactance on *third harmonics* in Y autotransformers." Naturally, in setting out to make a laboratory investigation of third harmonics, the elements of field practise not essential to the investigation are eliminated in order to arrive more directly at the desired object. In none of his tests, were Mr. Montsinger's connections Y-Y with condensers between line-side terminals and neutral. He used an artificial connection that sufficed for his stated purpose. Therefore his tests can not logically be appealed to as embracing all the phenomena of Y-Y operation.

To explain further, without going into details, in Fig. 2*, Mr. Montsinger shows one arrangement where the transformer primaries were connected Y, the secondaries connected delta with one corner of the delta open, and a condenser connected across each secondary leg. Perhaps it would be possible to obtain even harmonics with such a connection provided the condensers were of proper capacitance to suit the transformers, but Mr. Montsinger reports tests of this connection with only one value of capacitance, namely 0.2 microfarad per leg. He did not experiment over an extensive range of capacitance. In his other tests, Figs. 3, 4 and 5,* he had the secondaries connected delta through a single condenser. Regardless of the capacitance of this condenser, Kirchhoff's law as to the continuity of electric current in a circuit shows that this connection does not permit of the instability that is essential to the production of even harmonics in the transformers.

Mr. Montsinger has emphasized the 200 per cent triple-harmonic voltage that appeared across the open corner of the delta in his tests when the third harmonic was intensified to a maximum. It is not evident whether the voltage across the corner of the delta was 200 per cent or 600 per cent of the fundamental component of the leg voltage. The triple-harmonic voltage across the corner of the delta is the *arithmetic* sum of the triple-harmonic voltages across the individual legs. Hence, if the voltage across the open corner of the delta were 200 per cent, the triple-frequency voltage across one leg would be only 67 per cent of the fundamental. Also Mr. Montsinger did not establish the phase relation between the third harmonic and the fundamental leg voltages under the conditions of his experiments. However, assuming the third-harmonic voltage per leg to be 200 per cent of the fundamental leg voltage and that their maxima occur simultaneously, the maximum voltage per leg would be only 300 per cent of the rated fundamental voltage of the transformers. This would be within the limits of safety on transformers insulated to withstand 200 per cent of line (delta) voltage which is 346 per cent of leg voltage. From these and other data, I had felt that we are inclined to be too much concerned about the third harmonic. Consequently, when the even harmonics, especially the second, were discovered with magnitudes transcending those encountered with the third harmonic under the severest conditions, it appeared that the new phenomena might afford a more satisfactory explanation of the perplexing behavior of the Y-Y connection with grounded neutral on the line side.

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*TRANS. A. I. E. E., 1914, Vol. XXXIII, pp. 779-783.

1917 SUPPLEMENT

to the

STANDARDIZATION RULES

of the

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

Except as noted below the Standardization Rules as approved by the Board of Directors on June 28, 1916, and issued in October 1916, and reprinted without change in December 1916, shall be considered in force until further notice.

p. 11—Insert the following:

OTHER APPROVED STANDARDIZATION RULES.

The following Rules were formally presented to the Standards Committee during 1916-17 and are found not to be incompatible with the Standardization Rules.

"Standardization of Service Requirements for Motors", as printed in the 1915 report of the National Electric Light Association.

"Standardization of Sizes, Voltages and Taps for Transformers," as printed in the 1916 report of the Electrical Apparatus Committee of the National Electric Light Association.

"Standard Specifications for Magnetic Tests of Iron and Steel," of the American Society for Testing Materials.

"Report of the Joint Rubber Insulation Committee," published in the April, 1917, PROCEEDINGS of the American Institute of Electrical Engineers.

Accuracy Specifications in Sections IV and V of the Joint Meter Code of the Association of Edison Illuminating Companies and of the National Electric Light Association.

Accuracy Specifications in Section II of Circular 56 of the Bureau of Standards entitled "Standards for Electric Service."

Report of the "Boiler Code Committee" of The American Society of Mechanical Engineers.

"Suggested Safety Rules for Installing and Using Electrical Equipment in Bituminous Coal Mines," issued as Technical Paper 138 by the Bureau of Mines.

p. 18.—The following section is added:

84 **Base Speed of an Adjustable-Speed Motor.**—That speed of the motor obtained with full field under full load with no resistor in the armature circuit.

DEFINITIONS OF TERMS USED IN DESCRIBING ALTERNATING-CURRENT COMMUTATOR MOTORS.

The following sections (§**142** to §**147c**) are added and are tentative only.

142 It is not intended that the use of any one term will exclude, or render unnecessary, the use of any one or more of the other terms. Each term is intended to refer to a certain group of alternating-current commutator-motors. An alternating-current commutator motor logically may be classified under a number of different, but non-conflicting groups.

Classification by Phases of Energy Supply:

143 **Single-Phase Commutator Motor.**—A single-phase commutator motor is one that receives the whole of its energy from only one phase of an alternating-current supply system, without requiring external phase-converting apparatus

143a **Polyphase Commutator Motor.**—A polyphase commutator motor is one that receives its energy from a plurality of phases of an alternating-current supply system, or from a single-phase system through phase-converting apparatus external to the motor.

144 **Classification by Speed Characteristics.**—For convenience, alternating-current commutator motors may be classified with reference to their speed characteristics as (1) constant-speed motors, (2) multi-speed motors, (3) adjustable-speed motors, and (4) varying-speed motors. Definitions of these terms as given in §§**151** to **154** for motors in general, should be adopted for alternating-current commutator motors, in so far as they are applicable.

Classification by Excitation:

145 **Stator-Excited Commutator Motor.**—A stator-excited commutator motor is one in which the torque-producing field* is due to a current in a winding located on the stator.

145a **Rotor-Excited Commutator Motor.**—A rotor-excited commutator motor is one in which the torque-producing field* is due to a current in a winding located on the rotor.

145b **Stator- and Rotor-Excited Commutator Motor.**—A stator- and rotor-excited commutator motor is one in which the torque-producing field* is due to currents in windings located on the stator and on the rotor.

145c **Constant-Field Commutator Motor.**—A constant-field commutator motor is one in which the torque-producing field* remains practically constant, independent of the load.

Alternating-current commutator motors of this class will in general have load-speed characteristics similar to those of the direct-current shunt motor, but not all alternating-current commutator motors having such load-speed characteristics are constant-field machines.

145d **Varying-Field Commutator Motor.**—A varying-field commutator motor is one in which the torque-producing field* varies in some proportion with the current in the armature (which latter is generally the rotor).

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*The term "torque-producing field" is intended to describe that component of the magnetic field which, with the in-phase component of current, produces the torque of the motor.

Such a motor will in general have load-speed characteristics similar to those of the direct-current series motor.

Classification by Neutralization and Compensation:

146 Neutralized Commutator Motor.—A neutralized commutator motor is one in which use is made of a winding for producing a magnetizing force which at each instant and at each point in the air-gap under the pole face is practically equal and opposite to the magnetizing force due to the armature current.

146a Compensated Commutator Motor.—A compensated commutator motor is one in which means, other than a neutralizing winding, are provided within the motor for improving the power-factor.

Classification by Energy Reception:

147 Conduction Commutator Motor.—A conduction commutator motor is one in which the working energy† is supplied to only one of the members, and is conveyed to it by conduction.

147a Transformer Commutator Motor.—A transformer commutator motor is one in which the working energy† is transmitted from one member to the other by transformer action.

A motor in which the energy required by its armature (which is generally the rotor) is conveyed to it by electromagnetic induction or transformer action, may properly be referred to either as an "induction motor" or as a "transformer motor". Although it is equally applicable to a motor having a commutator, the term "induction motor" is usually applied to a motor without a commutator. The term "transformer commutator motor" is therefore recommended for use with motors of the induction, or transformer type, having commutators.

147b Transformer-Conduction Commutator Motor.—A transformer-conduction commutator motor is one in which the energy required by its armature (which is generally the rotor) is conveyed to it by both conduction and electromagnetic induction.

147c Repulsion Commutator Motor.—A repulsion commutator motor is a transformer commutator motor in which use is made of brushes for short-circuiting a number of coils of the commutated winding.

METERS AND INSTRUMENTS

The following sections (§§ **237** to **244**) are added:

237 Period of an Instrument.—The period of an instrument, sometimes called the "periodic time," is the time taken for the pointer to make one complete oscillation (two consecutive swings). A swing is a complete movement in either direction.

In strongly damped instruments, the period is influenced by the amplitude of the movement.

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† The term "working energy" is intended to describe the energy which is directly converted into mechanical energy, and which includes the shaft energy output plus core losses and friction.

238 Damping.—The damping of an instrument shall be expressed in terms of the following quantities, all three of which are essential to a complete description of the behavior:

a. The number of swings taken by the pointer in coming to rest.
b. The time, in seconds, required for the pointer to come to rest.
c. The overshooting, in per cent of the angular displacement due to the disturbance.

These quantities shall be measured by suddenly applying and maintaining a load which will give a steady deflection of one-half full angular scale. The pointer shall be at zero when the load is applied.

239 Rating Limitation of the Circuits in Meters and Instruments.—The rating of any circuit in a meter or instrument shall be no greater than that corresponding to the load to which it may be continuously subjected.

The full scale marking of an instrument does not necessarily correspond to its rating. Where the rating differs from the full scale marking, the rating shall be marked on the instrument.

240 Standard Temperature of Reference for Meter and Instrument Characteristics.—The standard temperature of reference for meter and instrument characteristics shall be 20° C. For purposes of rating, the standard ambient temperature shall be 40°C.

241 Temperature Rise and Ultimate Temperature of Shunts.—This matter is held in abeyance until the permissible temperature rise of shunt terminals can be agreed upon.

242 Temperature Rise of Shunt Terminals.—This matter is being considered by a subcommittee.

243 Marking of Switchboard Shunts.—Marking of switchboard shunts shall include the rating in amperes, the drop in volts at that rating, and, if the shunt is calibrated in connection with a particular instrument, the serial number of such instrument.

When shunts are designed to be used with devices taking sufficient current to be an appreciable portion of the whole, this fact shall be indicated. For example, a 100-ampere device having a drop of 0.050 volts, will have its shunt marked:

Volts 0.050
Amperes 100—10,

indicating that with 90 amperes in the shunt and 10 amperes in the measuring circuit, the drop across the shunt will be 0.050 volts.

244 Grounding of Meters and Instruments.—To avoid errors due to electrostatic action, the covers of meters and instruments which are used with current and potential transformers shall be connected to the secondary circuits of the transformers and grounded.

INSTRUMENT TRANSFORMERS*

[Sections **742** to **744a** below are intended to replace ultimately **§741.** The footnote is added.]

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*The terms "load," "secondary load" and "secondary connected load" have been used in connection with the constants of the circuit connected to the secondary of a current transformer. It is suggested that the term "secondary burden" be used as a general term to designate this external resistance and inductance and shall be expressed quantitatively in terms of ohms and henrys.

742 Instrument Transformer.—An instrument transformer, is a transformer suitable for use with measuring instruments that is, one in which the conditions of phase and of current or potential in the primary circuit, are represented with acceptable accuracy in the secondary circuit. An instrument transformer may be either an instrument current transformer or an instrument potential (voltage) transformer.

743 Instrument Current Transformers on Open Secondary Circuit.—Under conditions of open secondary circuit, current transformers shall be capable of carrying continuously rated primary current without damage to the primary insulation and without interruption of service.

744 Instrument Current Transformers on Closed Secondary Circuit.—Under conditions of closed secondary circuit, current transformers shall withstand 40 times rated current applied for 1 second, without injury.

744a For further definitions relative to instrument transformers, see §§ **205** to **207**. For dielectric tests of instrument potential (voltage) transformers see § **500**. The dielectric tests of instrument current transformers shall be the same as specified in § **509**.

CONTROLLERS, CIRCUIT-BREAKERS, SWITCHES, FUSES AND ACCESSORIES.*

Sections **745** to **759b** inclusive are added:

GENERAL DEFINITIONS

The following definitions are tentative. Criticisms and suggestions, addressed to the Secretary of the Standards Committee, will be welcomed.

745 Conducting Parts.—Those parts designed to carry current or which are conductively connected therewith.

745a Contact.—The surface common to two conducting parts, united by pressure, for the purpose of carrying current.

745b Grounded Parts.—Those parts which may be considered to have the same potential as the earth.

746 "Air" as a Prefix.—The prefix "air" applied to a device which interrupts an electric circuit indicates that the interruption occurs in air.

746a "Oil" as a Prefix.—The prefix "oil" applied to a device which interrupts an electric circuit indicates that the interruption occurs in oil.

747 Fume-Resisting.—Apparatus is designated as fume-resisting when so constructed that it will not be readily injured by the specified fumes.

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*The "National Electrical Code" of the National Fire Protection Association deals with certain capacities of circuit-breakers up to 550 volts and of switches and fuses up to 600 volts.

The question of establishing uniform rules for apparatus coming under this classification is under consideration by the A. I. E. E. Standards Committee and the Electrical Committee of the N. F. P. A.

747a Drip-Proof.—Apparatus is designated as drip-proof when so protected as to exclude falling moisture or dirt. Drip-proof apparatus may be either open or semi-enclosed, if it is provided with suitable protection integral with the apparatus, or so enclosed as to exclude effectively falling solid or liquid material.

747b Dust-Proof.—Apparatus is designated as dust-proof when so constructed or protected that the accumulation of dust within or without the device will not interfere with its successful operation.

747c Dust-Tight.—Apparatus is designated as dust-tight when so constructed that the dust will not enter the enclosing case.

747d Explosion-Proof.—Apparatus is designated as explosion-proof when so constructed that explosions of gas within the casing will not injure it or ignite inflammable gas outside it.

747e Gas-Proof.—Apparatus is designated as gas-proof when so constructed or protected that the specified gas will not interfere with its successful operation.

747f Gas-Tight.—Apparatus is designated as gas-tight when so constructed that the specified gas will not enter the enclosing case.

747g Moisture-Resisting.—Apparatus is designated as moisture-resisting when so constructed or treated that it will not be readily injured by moisture. (Such apparatus shall be capable of operating in a very humid atmosphere, such as found in mines, evaporating rooms, etc.)

747h Splash-Proof.—Apparatus is designated as splash-proof when so constructed or protected that external splashing will not interfere with its successful operation.

747i Submersible.—Apparatus is designated as submersible when so constructed that it will operate successfully when submerged in water under specified conditions of pressure and time.

747j Sleet-Proof.—Apparatus is designated as sleet-proof when so constructed or protected that the accumulation of sleet will not interfere with its successful operation.

748 Under-Voltage or Low-Voltage Release.—A term applied to a device which on the reduction or failure of voltage operates to cause the interruption of power to the main circuit, but which does not prevent the re-establishment of the main circuit on return of voltage.

748a Under-Voltage or Low-Voltage Protection.—A device which, on the reduction or failure of voltage, operates to cause and maintain the interruption of power to the main circuit, is said to provide under-voltage or low-voltage protection.

748b Phase-Failure Protection.—A device which, on the failure of power in one wire of a polyphase circuit, operates to cause and maintain the interruption of power on the remaining circuits, is said to provide phase-failure protection.

748c Phase-Reversal Protection.—A device which, on the reversal of the phase relations in a polyphase circuit, operates to cause and maintain the interruption of power in all of the circuits; is said to provide phase-reversal protection.

749 **Contactor.**—A device for repeatedly establishing and interrupting an electric circuit under normal conditions.

750 **Electric Controller.**—A device, or group of devices, which serves to control in some predetermined manner the operation of the apparatus to which it is connected.

CIRCUIT-BREAKERS.

751 **Circuit-Breaker.**—A circuit-breaker is a device (other than a fuse) constructed primarily for the interruption of a circuit under **infrequent abnormal** conditions. [This section is intended to replace ultimately §**724**].

752 **Rating.**—The rating of a circuit-breaker or switch includes (1) the normal r. m. s. current which it is designed to carry (2) the normal r. m. s. pressure (voltage) of the circuit on which it is intended to operate (3) the normal frequency of the current and (4) the interrupting capacity of the device (see §**753**). [This section is intended to replace ultimately §**725**].

753 **Interrupting Capacity.** By interrupting (breaking or rupturing) capacity is meant the highest r. m. s. current at normal voltage which the device can interrupt under prescribed conditions at stated intervals a specified number of times. [This section is intended to replace ultimately §**728**].

754 **Temperature Tests.**—Rated current at rated frequency shall be applied continuously until the temperature becomes constant. The maximum temperatures of the various parts shall not exceed the following when the ambient temperature of reference is 40° C.:

Contacts in air*	60° C.
Oil and contacts therein	70° C.
Coils (see §§**376** to **379** incl.)	-
Other parts (see §**392**)	

The Institute recognizes the inherent decrease in capacity of switch and circuit-breaker contacts in air, due to oxidization of the contact surfaces. The rating of air switches and circuit-breakers is, therefore, based on sufficient maintenance to keep the temperature within the specified limits. [This section is intended to replace ultimately §**726**].

755 **Dielectric Tests.**—For apparatus of 600 volts or less see §§**482**, **484**, **485**, and **500**. For apparatus above 600 volts see §§**482**, **484**, **485** and **509**. [This section is intended to replace ultimately §**727**].

SWITCHES

756 **Switch.**—A switch is a device for making, breaking or changing the connections in an electric circuit. [This section is intended to replace ultimately §**721**].

756a **Master-Switch.**—A master-switch is a device which serves to govern the operation of contactors and auxiliary devices of an electric controller.

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*Contacts in air may be subjected to an ultimate temperature of 70°C. for periods of short duration.

756b Control Switch.—A control switch is a switch for controlling electrically-operated switches and circuit breakers.

756c Auxiliary Switch.—An auxiliary switch is a switch actuated by some main device, for signalling, interlocking, etc.

757 Rating.—Same as for circuit-breakers; see §§**752** and **753**. [This section is intended to replace ultimately §**722**].

757a Tests.—Same as for circuit-breakers; see §§**754** and **755**. [This section is intended to replace ultimately §**723**].

FUSES

758 Fuse. A fuse is an element designed to melt or dissipate at a predetermined current value, and intended to protect against abnormal conditions of current.

Any terminals, tubes, etc., integral with this element are included as part of the fuse.

Fuses may be divided into two classes:

(A) Those designed to protect the circuit and apparatus both against short-circuit and against definite amounts of overload (*e.g* fuses of the National Electric Code which open on 25 per cent overload.)

(B) Those designed to protect the system only against short-circuits; (*e.g.* expulsion fuses, which blow at several times the current which they are designed to carry continuously). The line separating these two classes is not definitely fixed. [This section is intended to replace ultimately §**729** and the last two paragraphs of §**730**].

758a Continuous Current Carrying-Capacity of a Fuse.—Fuses shall be so constructed that they will carry continuously 110 per cent of their rated current. [This section is intended to replace ultimately the first paragraph of §**730**].

758b Temperature.—The temperature of coils or windings (such as accompany fuses of the magnetic blow-out type) shall not exceed the limits set for machine coils having the same character of insulation. (See §§**376** to **379**). The highest temperature for the fuse proper should not exceed the safe limit for the material employed. [This section is intended to replace ultimately §**731**].

RELAYS

759 Relay.—A relay is a device by means of which contacts in one circuit are operated by change in conditions in the same or other circuits. (See §**956**).

759a Temperature Tests.—Same as § **754**.

759b Dielectric Tests.—Same as § **755**.

765 The following foot-note is added to section **765**:

The nominal rating should be applied only to machinery and apparatus carrying traction loads.

STANDARDS FOR TELEPHONY AND TELEGRAPHY

The following sections are added:

923 **Standard Cable.**—A standard cable is an ideal uniform line in terms of which the attenuation of a line or network may be specified. It is characterized by the following constants: Linear resistance, 88 ohms per loop mile (54.7 ohms per loop km.). Linear capacitance between wires 0.054 microfarad per loop mile (0.03355 microfarad per loop km.). Linear inductance and linear leakance, 0.

940 **Simplex Circuit.**—A simplex circuit in telegraphy is one arranged for operation in one direction at one time.

941 **Duplex Circuit.**—A duplex circuit in telegraphy is one arranged for simultaneous operation in opposite directions.

942 **Diplex Circuit.**—A diplex circuit in telegraphy is one arranged for the simultaneous transmission of two messages in the same direction.

943 **Quadruplex Circuit.**—A quadruplex circuit in telegraphy is one arranged for the simultaneous transmission of two messages in each direction.

944 **Multiplex Circuit.**—A multiplex circuit in telegraphy is one arranged for the simultaneous transmission of several messages in one direction.

REPORT OF THE BOARD OF DIRECTORS
FOR FISCAL YEAR ENDING APRIL 30, 1917

The Board of Directors of the American Institute of Electrical Engineers presents herewith to the membership, its Thirty-third Annual Report for the fiscal year ending April 30, 1917. A General Balance Sheet showing the financial condition of the Institute on April 30, 1917, together with other detailed financial statements is included herein.

The Board has endeavored, as in the past, to keep the membership informed of its proceedings and of the work of the Institute, by publishing monthly in the PROCEEDINGS a resumé of the business transacted at each Directors' meeting. These notices, however, are not a complete report of the work done by the Board at any one meeting, for the reason that important matters are constantly arising which cannot be disposed of at once, and which must therefore be held over for future consideration. Publicity in such cases must frequently be deferred, but eventually full information is given to the members in subsequent issues.

Directors' Meetings.—The Board of Directors held ten meetings during the year. Five of these were held in New York City, and five were held in other cities as follows: June, in Cleveland, Ohio, during the Annual Convention; October, Philadelphia, Pa.; December, Boston, Mass.; January, Pittsburgh, Pa.; March, Chicago, Ill. Heretofore it had been the custom to hold the directors' meetings in New York on the same date as the monthly Institute meeting. When it was decided last fall to hold some of the monthly meetings outside of New York, it was also decided that the Board of Directors should meet in the city in which the Institute meeting was held, and this plan has been carried out.

In addition to the directors' meetings, the Executive Committee held three meetings. Two of these were held on February 6 and 9, 1917, respectively, for the purpose of preparing and issuing to the membership, a circular and data sheet for classifying the availability of Institute members for military or naval service in case of need. The third meeting was held on March 26, 1917. At this meeting the President was authorized to appoint a committee to consider all matters in connection with the Institute's relation to national defense, including coöperation with the various governmental departments.

National Meeting.—The Institute's Annual business meeting, on May 16, 1916, was a truly National meeting, as it was held simultaneously, by means of the long distance telephone, in Boston, New York, Philadelphia, Atlanta, Chicago and San Francisco; each person in each of the six audiences being provided with a receiver and thus being enabled to hear all that was said by the speakers in all six cities. The object was to commemorate the achievements of Institute members in the fields of communication, transportation, light and power. A message of greeting was read from the President of the United States and greetings were exchanged by the various assemblies. The meeting concluded with the adoption of the following resolution:

> RESOLVED, that this National Meeting, of the American Institute of Electrical Engineers in Boston, San Francisco, Atlanta, Chicago,

> Philadelphia and New York now assembled, does hereby express its deep appreciation of the efforts of all those who have coöperated in the holding of such a meeting, which is now for the first time held in any country; and that a record of the proceedings of this meeting, which is made possible by the inventive genius and by the engineering ability of its own membership, be spread upon the minutes of the Institute, where, for generations to come, it will serve as an inspiration to engineers everywhere, and will mark an epoch in the history of American engineering achievement.

The attendance in the cities in which the meeting was held was as follows: Boston 900; New York, 1100; Philadelphia 850; Atlanta 500; Chicago 1000; San Francisco 750.

Annual Convention.—The thirty-third Annual Convention was held in Cleveland, Ohio, June 27-30, 1916. The total registered attendance was 607, of which number 319 were from out of town, twenty-two states being represented. The program consisted of seventeen technical papers on a variety of timely engineering subjects. The usual conferences of the Section Delegates were held, at which all of the Sections were represented, and a conference was also held, for the first time, of representatives of the Student Branches.

Pacific Coast Convention.—The Pacific Coast Convention was held in Seattle, Washington, September 5-9, 1916. Nine technical papers were presented and the total registration numbered 182.

Midwinter Convention, New York.—The fifth Annual Midwinter Convention was held at Institute headquarters in New York, February 14-16, 1917. The total registered attendance was 712. In addition to the eleven technical papers presented, a lecture on Modern Physics was delivered by Professor R. A. Millikan, President of the American Physical Society, and Professor of Physics of the University of Chicago. The convention closed with a subscription dinner-dance at the Hotel Astor on February 16 which was attended by 329 members and guests.

Monthly Institute Meetings.—The present year marked a change in the policy of holding monthly Institute meetings in New York. It had been the custom for many years to hold regular monthly meetings at Institute headquarters, with an occasional special meeting outside of New York under the auspices of one of the Sections, in addition to the regular New York meeting. At the beginning of the present administrative year in August 1916, President Buck expressed the opinion that the time had come to make a change in the policy of holding all of the monthly meetings in New York City, and declared himself in favor of distributing the meetings. This plan was given a trial, with the result that four of the monthly meetings were transferred from New York; one each being held in Philadelphia, Boston, Pittsburgh and Chicago. Another meeting was scheduled to be held in Schenectady in April, but it was found necessary to cancel this meeting. The meetings were largely attended, and the local officers and members were unremitting in their efforts to make them a success. A brief record of the out of town meetings follows:

Philadelphia Meeting.—The Philadelphia meeting was held on October 13, 1916. There were two technical sessions at which four papers were presented and the total registered attendance was 333.

Boston Meeting.—The Boston meeting was held on December 9, 1916. The Institute's technical session, at which two papers were presented, was held in the afternoon. The evening session was devoted wholly to the presentation of the John Fritz Medal to Professor Elihu Thomson. The attendance was about 400.

Pittsburgh Meeting.—The Pittsburgh meeting was held in Pittsburgh, Pa., on January 12, 1917. This meeting was limited to one technical session at which one paper was presented. Four hundred and twenty-five members and guests were present.

Chicago Meeting.—The Chicago meeting was held on March 9, 1917. Two papers were presented at the single technical session, and the attendance numbered 300.

New York Meetings.—A meeting was held in New York at Institute headquarters on November 10, 1916, at which three technical papers were presented. A three-day Midwinter Convention was held in February as referred to elsewhere herein. The Annual Meeting on May 18, 1917, will also be held in New York.

Joint Session A. I. E. E. and A. I. & S. E. E.—A joint session of the A. I. E. E. and the Association of Iron and Steel Electrical Engineers was held during the tenth Annual Convention of the Association in Chicago on September 20, 1916. Two papers on subjects of mutual interest to the two organizations were presented on behalf of the Institute.

President.—President Buck presided at the Institute meetings held during his administration in Philadelphia, New York, Boston Pittsburgh and Chicago. He also attended the Pacific Coast Convention held in Seattle, September 5-9, 1916, the Annual Banquet of the Pittsburgh Section on December 9, 1916, the meeting of the Philadelphia Section held on February 12, 1917, and he represented the Institute at the Annual Dinner of the Providence Engineering Society March 28, 1917.

National Defense.—The Institute has continued to contribute its services to the government in the cause of preparedness and National Defense during the past year, and is now in touch with various governmental agencies through representation on committees and other bodies which are carrying on effective work for the national welfare. A brief summary of the more important activities in which the Institute is interested and in which it has taken a conspicuous part is contained elsewhere in this report.

Early in February 1916, the Executive Committee of the Institute met in special session, at which it was decided to issue to the membership a circular calling attention to the opportunities for patriotic service existing in the Army and Navy of the United States and enclosing therewith a data sheet for the purpose of tabulating the availability of members of the Institute for such service. This circular and data sheet were issued under date of February 8, 1917, and met with a very cordial response, the number of data sheets received in reply exceeding 2,000. A considerable number of the members who have sent in these sheets have been communicated with by the branches most urgently in need of the services available, and the remainder are being held for future reference.

On April 2, 1917, the Institute in cooperation with other engineering societies issued to its membership two pamphlets containing as complete information as could be obtained for members of the engineering profession who contemplated offering their services in the army and navy. These pamphlets have resulted in the filing of many applications by Institute members for commissions in the various ranks of the Officers Reserve Corps of the army, and in the Naval Reserve.

The Executive Committee met again on March 26, 1917, for the purpose of authorizing the President to appoint a Committee on National Defense, to consider all matters which would naturally come within the scope of such a committee, and including continuation of the work formerly carried on by the Institute's representatives upon the Joint Committee on Engineer Officers' Reserve Corps. This committee was subsequently appointed by President Buck and has since been active in carrying on special work relative to National Defense.

Council of National Defense.—As the result of the act of congress of August 29, 1916, the Council on National Defense was organized for the coördination of industries and resources of the United States for the national security and welfare. The Council is made up of the Secretary of War (Chairman), the Secretary of the Navy, the Secretary of the Interior, the Secretary of Agriculture, the Secretary of Commerce and the Secretary of Labor. The act further provided for the appointment by the President, of an Advisory Commission of seven members. The Advisory Commission was subsequently appointed and is now serving the Council in a consulting capacity without compensation. The Institute was invited to appoint two representatives upon the Committee on Science and Research of the Advisory Commission, to which the supervision of engineering matters had been assigned. The Engineering Section of this committee is made up of two respresentatives each of the four national engineering societies and certain other engineering societies.

National Research Council.—The National Research Council was created by the National Academy of Sciences at the request of the President of the United States during the latter part of 1916. The object of the Council is to bring into coöperation, existing governmental, educational and other scientific and research organizations, with the purpose of encouraging investigations of natural phenomena, the increased use of scientific research in the development of American industries, the employment of scientific methods in strengthening the National Defense, and such other applications of science as will promote the national security and welfare. The work of the National Research Council is carried on through the agency of central committees covering each of the physical sciences and chemistry, mathematics, medicine, hygiene, agriculture, and other subjects, including a Committee on Engineering. To this Committee on Engineering each of the National Engineering Societies was invited to appoint two representatives and these representatives, with the engineers who are members of the Council, constitute the Committee on Engineering.

United Engineering Society.—Important changes have taken place in the affairs of the United Engineering Society during the year. On June 23, 1916, the Board of Direction of the American Society of Civil Engineers adopted resolutions accepting an invitation from the United

Engineering Society, to enter into the fraternity of the founder societies and take up their headquarters in the Engineering Societies Building. Active negotiations with the object of bringing about this result had been carried on by the A. S. C. E. and the U. E. S. for about one year prior to the action referred to above. Full details of the arrangement finally agreed upon and a memorandum of the agreement entered into will be found in the September 1916 PROCEEDINGS. As will be seen from this agreement, in order to provide suitable quarters for the A. S. C. E., it was necessary for the United Engineering Society to contract for the addition of three stories to the Engineering Societies Building. This work is now progressing satisfactorily and it is expected that the American Society of Civil Engineers will move its headquarters into the building before the end of the present calendar year, thus bringing under one roof the headquarters of the National Societies of Civil, Mining, Mechanical and Electrical Engineers, with all the obvious advantages for closer coöperation of the entire engineering profession.

Engineering Council.—The Engineering Council is the outcome of several independent movements which took definite shape early during the present year. For a long time individual engineers and the engineering societies have had in mind the desirability of having some properly constituted organization empowered to speak in the name of engineers in general upon subjects of common interest to engineers and to the public. The need of such an organization in the interest of coöperation and uniformity of action on matters relating to the welfare of the engineering profession was brought to the attention of the Board of Directors of the Institute at its meeting held in Philadelphia in October 1916, at which the subject was discussed at considerable length. A resolution was adopted at that meeting authorizing the President to appoint a committee of five members of the Board and to invite the American Society of Civil Engineers, the American Society of Mechanical Engineers, and the American Institute of Mining Engineers, to appoint similar representatives upon a joint committee with a view to formulating some plan of action for organizing such a central body. A series of conferences of the representatives appointed as a result of this action followed, and after considering numerous suggestions the conferees formulated a plan to amend the by-laws of the United Engineering Society so as to provide for a new department of that Society, to be known as The Engineering Council. These amendments were subsequently approved by the four founder societies which constitute the United Engineering Society, and the Council is now being organized. The object of the Council as expressed in the amended by-laws is "to provide for convenient coöperation between the four founder societies, for the proper consideration of questions of general interest to engineers and to the public and to provide the means for united action upon questions of common concern to engineers." The Council will be maintained and conducted independently of the other activities of the United Engineering Society. Under the plan as adopted, each of the four founder societies is entitled to five representatives upon the Council, these representatives to be designated by the governing body of each society, and the United Engineering Society will have four representatives chosen by its Board of Trustees, such representatives to be men not already designated by any of the founder societies. The Council will be empowered to speak authoritatively for all member societies on all public questions of common interest or concern to engineers unless objection be made by a majority of the representatives present of one of the founder societies or one-quarter of the representatives present and voting. The Trustees of the United Engineering Society will have authority to elect to membership in the Engineering Council, other national engineering or technical societies under such rules as the Council prescribes, provided their nomination and the said rules have the unanimous approval of the governing bodies of the four societies.

Representatives.—The Institute has continued its representation upon various national committees and other local and national bodies with which it has been affiliated in past years, and in addition has appointed representatives upon a number of new bodies including several relating to National Defense. A list of these will be found published in each issue of the PROCEEDINGS.

Committees.—There has been very little change in the character of the committees during the year. Last year a committee was appointed to formulate a plan for reorganization of the technical committees. This committee reported in August 1916 recommending a reclassification of the technical committees and the appointment of two new committees; one on electrical machinery, and one on instruments and measurements. When this report came before the Board for final consideration at the October meeting, it was decided to defer action on the reorganization of the committees, but a number of them were renamed in accordance with recommendations contained in the report so as to cover their respective fields more comprehensively.

A change in the manner of reporting upon the work of the technical committees was recently effected upon the recommendation of the Meetings and Papers Committee, through an amendment to the Institute's by-laws, under which all technical committees are now required to submit an annual report of their work at the Annual Convention. Brief reports of the technical committees have previously been included in the Annual Report of the Board of Directors, but in accordance with this change the reports of the technical committees covering the present year will be presented at the Annual Convention in June.

Meetings and Papers Committee.—The Meetings and Papers Committee arranged the programs of Institute meetings and conventions held during the year.

At the suggestion of President Buck, and with the approval of the Board of Directors, the experiment was tried of holding a number of the regular monthly meetings of the Institute in cities outside of New York. Reference to this plan and details regarding the meetings held, may be found elsewhere in this report. The object of this geographical distribution of the regular monthly meetings was to afford members at various central points a better opportunity to be brought into contact with the Institute activities. It is believed that this distribution of meetings has greatly stimulated the Sections where the meetings were held and that the Institute as a whole has been benefited.

Upon the committee's recommendation, a new by-law was adopted by the Board of Directors providing that each technical committee shall present, at the Annual Convention, a report of the work accomplished by the committee during the year, and giving plans and suggestions for future activities. The committee believes that this by-law will be helpful in bringing about a continuity of ideas, and will be the means of increasing the usefulness of the technical committees.

The by-laws as amended in May, 1916, have established a clear distinction between the work of the Meetings and Papers and Editing committees, the former now having jurisdiction over the selection of papers to be presented, and the publication of the PROCEEDINGS, while the editing of papers and the selection of material for the annual TRANSACTIONS is now solely within the province of the Editing Committee.

Owing to unusual and unexpected demands on the finances of the Institute, the amount which could be appropriated for publication of the PROCEEDINGS and TRANSACTIONS has been somewhat reduced. This has made it necessary to exercise unusual care in the selection of material for presentation and publication. It is hoped that the work has been done in such a way that there has been no curtailment in the usefulness of the Institute to the membership.

Sections Committee.—The activities of the Sections reflect a gain over the previous year of about the usual magnitude. In view of the many unusual demands made upon the engineers in this period, this continued progress is gratifying.

No new Sections have been formed this year, but recent agitation in this direction in two or three localities, points to definite developments in the near future. The Toronto Section has resumed its activities in the face of unusual obstacles.

Continued interest manifested in the affiliation of Sections with other technical societies has resulted in the appointment of a special committee to report on such relation at the next delegates' meeting.

BRANCHES.—Perhaps the must notable item of the year is the growth of interest in the work of the Student Branches, which has been recognized by the appointment of a special Committee on Institute Branches, a sub-committee, with its chairman as vice-chairman of the Sections Committee.

This committee has already projected a lecture circuit for the Branches in the middle west, to be inaugurated next fall. The committee hopes to obtain the services of notable engineers without unreasonable demand upon their time. If this plan proves successful, other circuits will be established in localities where Branches are so grouped that such circuits are practicable.

The number of Student Branches has been raised to fifty-nine by the addition, during the year, of those newly organized at Norwich University, Vermont, University of North Dakota, University of Minnesota, Alabama Polytechnic Institute and the Massachusetts Institute of Technology. The activities in the Branches show increases corresponding to those of the Sections, while future growth should be materially favored by the improved organization.

A tabulation showing the activity of the Sections and Branches during the past five years follows:

	For Fiscal Year Ending				
	May 1 1913	May 1 1914	May 1 1915	May 1 1916	May 1 1917
SECTIONS					
Number of Sections.........	29	30	31	32	32
Number of Section meetings held....................	244	233	246	251	265
Total Attendance...........	22,825	22,626	23,507	28,553	31,299
BRANCHES					
Number of Branches........	47	47	52	54	59
Number of Branch meetings held.....................	357	306	328	360	368
Attendance.................	11,808	11,617	12,712	15,166	16,107

Standards Committee.—The 1916 edition of the Standardization Rules was approved by the Board of Directors at the Cleveland Convention, June 28, 1916. This edition does not differ radically from the previous edition, but involves material changes in over 100 sections, with ten new sections.

Of particular value in this revision of the Rules has been the very cordial coöperation of the British Engineering Standards Committee, which was represented at the final meeting of the Standards Committee on May 15 and 16, 1916, by its Secretary, Mr. C. le Maistre, who came from London to New York for this express purpose. The By-Laws of the Standards Committee referred to in the last Annual Report were approved by the Board of Directors on June 28, 1916.

The work of the present committee has been carried on with the aid of 36 sub-committees which report at the monthly meetings of the main committee. The membership of the sub-committees is only partly confined to the main committee although the sub-committee chairmen are always members of the main committee.

The changes and additions to be proposed as a result of this year's work are for the most part not radical, and to avoid the cost of a new edition, will be printed as a supplement. It seems desirable to the committee that a new edition should not be issued oftener than once in two years.

Work is now under way by the sub-committee on "The Form but not the Substance" looking towards a complete rearrangement and standardization of typography for the 1918 Edition.

Each year the need of more effective machinery of coöperation with the standards committees of other societies has become more evident. The invitation issued at the suggestion of the Standards Committee, by the Board of Directors last winter to the A. S. C. E., the A. S. M. E., the A. I. M. E. and the A. S. T. M., each to appoint three representatives on a joint committee with the A. I. E. E., to consider the organization of a National or American Engineering Standards Committee, has met with cordial response, and the organization committee after a winter's work is about ready to report. The chairman of the Standards Committee is chairman of the organization committee above referred to.

Another joint conference committee on standards largely within the electrical field is already in operation; it includes the A. I. E. E., the N. E. L. A., the Electric Power Club, Association of Edison Illuminating Companies., the American Electric Railway Association, the Association of Iron and Steel Electrical Engineers, the Railway Signal Association, Soc. Automobile Engineers, Institute of Radio Engineers, Illuminating Engineering Society, the American Society for Testing Materials, and the Association of Railway Electrical Engineers.

The future of this committee will depend somewhat upon the form of organization of the American Engineering Standards Committee; but whether or not it continues in its present form, it has already served and will continue to serve a very useful purpose until the broader committee is fully organized and ready to undertake its work.

Our Standardization Rules have been translated into French and this translation is now being revised by a committee including several of our French confreres, with Mr. C. O. Mailloux as chairman.

Committee on the Development of Water Power.—The committee has had no opportunity during the past year to obtain action on matters of legislation affecting water power, and has therefore confined its activities to two specific utterances on the general subject.

In the latter part of October 1916, with the approval of the Board of Directors of the Institute, the committee prepared and submitted to the Secretaries of War, Agriculture and the Interior a declaration of principles governing water power.

The second matter referred to was a letter written to the Council on National Defense, dated February 23, 1917, bringing to the Council's attention the importance of legislation in respect to Niagara Falls, and setting forth the intimate and vital relation between the products there manufactured, and national defense. This document the committee believes was of value in aiding the legislation later enacted by congress whereby the diversion of additional water at Niagara Falls was permitted up to the limits of the existing treaty.

U. S. National Committee of the International Electrotechnical Commission.—A general meeting of the U. S. National Committee of the International Electrotechnical Commission with the Standards Committee of the A. I. E. E. was held in New York on May 12, 1916, on the occasion of the visit to the United States of General Secretary C. le Maistre. The question of graphic symbols was discussed at this meeting and referred to a sub-committee of the Standards Committee for examination. Hopes were expressed for the active resumption of the work of the Commission after the world's peace shall have been restored. The Annual Report of the Honorary Secretary of the I. E. C. for the year ending December 31, 1916 was received in January and distributed among the members of the U. S. National Committee. From the report it appears that notwithstanding the continuance of the world war very appreciable contributions to the work of the I. E. C. have been maintained by a number of the individual countries.

Pan American Joint Engineering Committee.—This committee was organized in February 1916 upon the initiative of the Institute with the object of increasing and broadening the influence and usefulness of the national engineering societies of the United States in Central and South American countries. In addition to the Institute, the National Societies of Civil, Mining, and Mechanical Engineers are represented upon the joint committee. Preliminary meetings were held on March 3 and March 24, 1916, and on April 10, 1916, a formal organization was effected. Letters were sent to all South and Central American representatives at Washington and to the representatives of the United States in Central and South America in order to obtain the names of representative engineering organizations in those countries. These names having been obtained, a formal request was sent to the leading engineers and technical associations in each country asking their cooperation with the joint committee. The committee expects to continue this work through correspondence with the view to developing friendly relations and effecting coöperation with the engineers of Central and South America, and later may make an effort to arrange for a public meeting to stimulate interest in the movement among the engineers of the United States.

Code Committee.—The Code Committee has continued to represent the Institute on the Electrical Committee of the National Fire Protection Association. Several sub-committee meetings on various subjects have been held during the year, and the Institute has been represented by one or more members of the Code Committee at each of such meetings.

Editing Committee.—Under the revised Constitution adopted last year, the Editing Committee has entire supervision of the annual TRANSACTIONS of the Institute, and receives an annual appropriation for carrying on this work independent of the monthly PROCEEDINGS, which are now in sole charge of the Meetings and Papers Committee. Under the new regime the Editing Committee has arranged to publish the 1916 TRANSACTIONS early this summer and thereafter to publish a semi-annual volume each fall and spring covering the half-years ending June 30 and December 30 respectively. In other respects the method of handling the publications has been the same as for several years past and appears to meet with general satisfaction.

Board of Examiners.—The Board of Examiners held 13 meetings during the year averaging about three and a half hours each. It considered and referred to the Board of Directors with its recommendations a total of 2006 applications of all kinds. In addition to these, the Board reviewed or reconsidered 47 applications for a second and third time. The result of the Board's work for the year is given in the following statement:

APPLICATIONS FOR ADMISSION.

Recommended for grade of Associate..............	938	
Not recommended for grade of Associate...........	3	941
Recommended for grade of Member...............	67	
Not recommended for the grade of Member.........	33	100
Recommended for grade of Fellow................	2	
Not recommended for Fellow....................	2	4
Recommended for enrolment as students...........	856	856

APPLICATIONS FOR TRANSFER.

Recommended for grade of Member	57	
Not recommended for grade of Member	21	78
Recommended for grade of Fellow	12	
Not recommended for grade of Fellow	15	27
Total number of applications considered		2006
Applications reconsidered		47
Admission and transfer all grades		2053

Membership Committee.—The Membership Committee this year included the chairman of each local Section Membership Committee and six members at large. Its work, therefore, was founded principally upon Section activity. Four meetings were held by the committee during the year.

The membership booklet containing information regarding the object, scope and work of the Institute was revised by rearranging the material, the incorporation of cuts, and including for the convenience of applicants, a tabulation of the critical facts in regard to applications. A booklet relating to Student enrolment was prepared and published.

Following the custom established by the Membership Committee of last year, the present committee cooperated with the New York office in the collection of dues from delinquent members with gratifying success.

The following tabulated statement shows the total number of applications received, also the additions, deductions, and net increase in the membership for the year. A study of these shows the direct effect of the work through the Sections, inasmuch as almost twice as many applications were received from Section territory as from outside, whereas the ratio in the past has generally been about one to one.

	Honorary Member	Fellow	Member	Associate	Total
Membership, April 30, 1916.	5	454	1137	6616	8212
Additions:					
Transferred	....	8	65	....	....
New Members Qualified	....	1	57	895	....
Reinstated	....	....	3	24	....
Deductions:					
Died	1	4	8	35	....
Resigned	....	1	7	118	....
Transferred	....	....	6	67	....
Dropped	....	....	15	293	....
Membership, April 30, 1917	4	458	1226	7022	8710

Net increase in membership during the year.................................. 498

Deaths.—The following deaths have occurred during the year:

Honorary Member.—Dr. Silvanus P. Thompson.

Fellows.—Henry Floy, William Stanley, A. C. Eastwood, Henry G. Stott.

Members.—F. C. Green, Judson H. Boughton, Charles H. Champion, F. W. Myers, Robert E. Noyes, Charles Guckel, A. C. Einstein, Joseph P. Davis.

Associates.—John W. Beyer, E. M. Barton, W. C. Janney, Nathan A. Dreyfus, John H. Wilson, Louis S. Baird, A. Nielsen, R. H. Harrison, Harold Lomas, Porter Eveland, Frank B. McSoley, G. W. Krause, M. Pfatischer, H. P. Collins, N. L. Jennings, Louis E. Reynolds, J. G. M. Connally, J. D. Simpson, J. A. Shepard, John A. Barrett, W. Schlombs; Stewart E. Padfield, Bayse N. Westcott, Arthur J. Howard, Frederic H. Reed, Robert S. Orr, Fred F. Paulsell, William Brophy, P. P. Spaulding, G. Henry Hill, George H. Usher, T. K. Arunachela-Iyer, F. W. Roebling, H. J. Bildhauser, J. H. Steele.

Total deaths, 48.

Finance Committee—The following correspondence and financial statements form a complete summary of the work of the Finance Committee of the year.

New York, May 16, 1917.

Board of Directors,
American Institute of Electrical Engineers.

Gentlemen:

Your Finance Committee respectfully submits the following report for the year ending April 30, 1917.

During the past year the committee has held monthly meetings, has passed upon the expenditures of the Institute for various purposes and otherwise performed the duties prescribed for it in the Constitution and By-laws. Haskins & Sells, certified public accountants, have audited the books, and their certification of the Institute finances follows.

In company with your Secretary, and a representative of the firm of accountants, the committee has examined the securities held by the Institute and finds them to be as stated in the accountants' report.

It is gratifying to note that notwithstanding the unusual conditions that have prevailed during the year, it has been possible to continue all the activities of the Institute without curtailment and, as indicated in the accompanying auditors' report, the financial affairs of the Institute are in a very favorable condition.

Respectfully submitted,
(Signed) J. FRANKLIN STEVENS,
Chairman, Finance Committe.

HASKINS & SELLS
CERTIFIED PUBLIC ACCOUNTANTS
CABLE ADDRESS "HASKSELLS"
30 BROAD STREET
NEW YORK

NEW YORK
CHICAGO
ST. LOUIS
CLEVELAND
BALTIMORE
PITTSBURGH

SAN FRANCISCO
LOS ANGELES
DENVER
ATLANTA
WATERTOWN
LONDON

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

CERTIFICATE

We have audited the books and accounts of the American Institute of Electrical Engineers for the year ended April 30, 1917, and

WE HEREBY CERTIFY that the accompanying General Balance Sheet properly sets forth the financial condition of the Institute on April 30, 1917, that the Statement of Income and Profit and Loss for the year ended that date is correct, and that the books of the Institute are in agreement therewith.

HASKINS & SELLS,
Certified Public Accountants

NEW YORK,
May 12, 1917.

AMERICAN INSTITUTE OF

EXHIBIT A.

GENERAL BALANCE SHEET

ASSETS.

LAND AND BUILDING:			
Interest in United Engineering Society's Real Estate, No. 25 to 33 West 39th Street:			
Land (One-third of Cost)		$180,000.00	
Building		353,346.61	
Total Land and Building			$533 346.61
EQUIPMENT:			
Library—Volumes and Fixtures		$ 39,879.80	
Works of Art, Paintings, etc.		3,001.35	
Office Furniture and Fixtures		11,470.06	
Total		$54,351.21	
Less Reserve for Depreciation		8,104.78	
Remainder—Equipment			$ 46,246 43
INVESTMENTS:			
BONDS—City of Wilmington, Delaware, 4½%, 1934, Par $15,000			$ 15,886.33
WORKING ASSETS:			
Publications entitled "Transactions," etc.		$ 12,884.25	
Paper and Cover Paper		1,173.85	
Badges		647.15	
Total Working Assets			$ 14,705.25
CURRENT ASSETS:			
Cash		$ 11,291.95	
Accounts Receivable:			
Members for Past Dues		9,635.06	
Advertisers		494.20	
Miscellaneous		569.72	
Due from Societies Affiliated in National Defense Movement		653.77	
Accrued Interest on Investments		56.25	
Accrued Interest on Bank Balances		172.34	
Total Current Assets			$ 22,873.29
FUNDS:			
Life Membership Fund:			
Cash	$ 438.67		
Chicago, Burlington & Quincy Railroad Company Bonds, 4%, 1958, Par $5,000	4,868.75		
Accrued Interest	33.33		
		$ 5,340.75	
International Electrical Congress of St. Louis—Library Fund:			
Cash	$ 850.59		
New York City Bonds, 4½%, 1957, Par $2,000.00	2,255.09		
Accrued Interest	45.00	3,150.68	
MAILLOUX FUND:			
Cash	$ 122.35		
New York Telephone Company Bond, 4½%, 1939	1,000.00		
Accrued Interest	22.50	1,144.85	
Midwinter Convention Fund—Cash		95.80	
Total Funds			$ 9,732.08
DUES PAID IN ADVANCE—INTERNATIONAL ELECTROTECHNICAL COMMISSION, LONDON, ENGLAND			750.00
Total			**$643,539.99**

ELECTRICAL ENGINEERS

APRIL 30, 1917.

LIABILITIES.

CURRENT LIABILITIES:		
Accounts Payable—Subject to Approval by the Finance Committee	$ 8,383.29	
Dues Received in Advance	2,097.69	
Entrance Fees and Dues Advanced by Applicants for Membership	310.50	
Total Current Liabilities		$ 10,791.48
FUND RESERVES:		
Life Membership Fund	$ 5,340.75	
International Electrical Congress of St. Louis—Library Fund	3,150.68	
Mailloux Fund	1,144.85	
Midwinter Convention Fund	95.80	
Total Fund Reserves		$ 9,732.08
SURPLUS: Per Exhibit "B"		$623,016.43
Total		$643,539.99

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

STATEMENT OF INCOME AND PROFIT AND LOSS

FOR THE YEAR ENDED APRIL 30, 1917

EXHIBIT B.

REVENUE:			
Entrance Fees		$ 5,207 00	
Dues		92,177.08	
Student's Dues		4,989.00	
Transfer Fees		705.00	
Advertising		7,162.20	
Subscriptions		3,147.55	
Sales of "Transactions," etc.		2,782.33	
Badges Sold	$2,283.00		
Less Cost	1,966 11		
		316.89	
Interest on Investments		675.00	
Interest on Bank Balances		650.33	
Exchange		31.30	
Total			$117,843 68
EXPENSES:			
Meetings and Papers Committee:			
Salaries, Editorial Department		$ 5,944.16	
Paper and Cover Paper		3,665.55	
Engraving Proceedings		1,587.49	
Printing Proceedings		6,715.61	
Binding and Mailing Proceedings		3,880.60	
Envelopes		823.44	
Stationery and Miscellaneous Printing		79.75	
General Expenses		87.18	
Meetings		5,596.98	
National Meeting, May 16, 1916		1,419.89	
Joint Meeting—American Association for the Advancement of Science		169.50	
Volume No. 33. Transactions		15.01	
Volume No. 34. "		14,514.21	
Volume No. 35. "		1,824.84	
Total		$ 46,324.21	
Deduct Increase in Inventory of Publications:			
May 1, 1916	$10,908.50		
April 30, 1917	12,884.25	1,975.75	$ 44,348.46
Executive Department:			
Salaries		$ 17,299.00	
General Expenses		2,550.81	
United Engineering Society—Assessments		4,800.00	
Express		294.54	
Postage		2,047.80	
Advertising		2,149.47	
Stationery and Miscellaneous Printing		3,516.45	
Year Book and Catalogue		3,103.32	$ 35,761.39
FORWARD			$ 80,109.85
REVENUE—(Forward)			$117,843.68

REVENUE—(Forward)		$117,843.68
EXPENSES—(Forward)		$ 80,109.85
Sections Committee:		
Section Meetings	$ 5,938.51	
Branch Meetings	183.10	
Delegates' Convention Expenses	1,436.92	
Salaries, New York Office	2,340.00	
Stationery and Printing, New York Office	672.30	
Express on Advance Copies	26.13	$ 10,596.96
General:		
Library Assessment, United Engineering Society	$ 4,000.00	
Membership Committee	1,051.44	
Finance Committee	150.00	
Standards Committee	778.52	
Code Committee	30.00	
National Defense Committee	803.24	
Pan American Engineering Committee	25.00	
Constitutional Revision Committee	73.35	
International Engineering Congress, 1915	1,800.00	
Annual Dues, International Electrotechnical Commission	250.00	
John Fritz Medal Award	109.20	
President's Special Appropriation	341.92	
Salary and Traveling Expenses, Honorary Secretary	4,421.28	13,833.95
Total		$104,540.76
Add:		
Increase in Accounts Payable—Subject to Approval by the Finance Committee, Expenses Undistributed at:		
May 1, 1916	$ 7,146.64	
April 30, 1917, not including Liability of $708.20 incurred for Account of Societies Affiliated in National Defense Movement	7,675.09	528.45
Total Expenses		$105,069.21
NET REVENUE		$ 12,774.47
PROFIT & LOSS CREDIT—Accessions to Library Volumes and Fixtures		662.50
GROSS SURPLUS FOR THE YEAR		$ 13,436.97
PROFIT & LOSS CHARGES:		
Uncollectible Dues Written Off	$ 3,051.00	
Provision for Depreciation of Furniture and Fixtures	1,329.65	
Amortization of Premium on City of Wilmington, Delaware, 4½% Bonds of 1934	52.50	
Total		4,433.15
NET SURPLUS FOR THE YEAR		$ 9,003.82
SURPLUS, MAY 1, 1916		614,012.61
SURPLUS, APRIL 30, 1917		$623,016.43

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

STATEMENT OF CASH RECEIPTS AND DONATIONS FOR DESIGNATED PURPOSES. ALSO DISBURSEMENTS, FOR THE YEAR ENDED APRIL 30, 1917.

EXHIBIT C.

RECEIPTS:

Life Membership Fund	$365.86
International Electrical Congress of St. Louis Library Fund—Interest and Royalties	94.20
Mailloux Fund—Interest	45.00
Midwinter Convention Fund—Interest	3 16
Total	$508.22

DISBURSEMENTS:

Life Membership Fund	277.08
Mailloux Fund	26.75
Midwinter Convention Fund	66.29
Total	$370.12

RECEIPTS AND DISBURSEMENTS PER YEAR PER MEMBER.

During each fiscal year for the past eight years.

Year ending April 30	**1910**	**1911**	**1912**	**1913**	**1914**	**1915**	**1916**	**1917**
Membership, April 30, each year	6681	7117	7459	7654	7876	8054	8212	8710
Receipts per Member	$13.35	$13.37	$13.19	$13.45	$14.08	$14.06	$13.62	$13.30
Disbursements per Member	12.03	11.03	12.44	15.57	12.86	13 54	13 74	12 75
Credit Balance per Member	$1.32	$2.34	$ .75	*$2.12	$1.22	$ .52	*$ .12	$ .55

*Deficit.

Respectfully submitted for the Board of Directors,

F. L. HUTCHINSON, *Secretary*.

New York, May 18, 1917.

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SYNOPTICAL AND TOPICAL

INDEX

OF

A. I. E. E. TRANSACTIONS

Vol. XXXVI,

The main headings under which these synopses are classified were arrived at by a careful study of all the papers contributed since the organization of the Institute.

The method of making this classification may be called the automatic method, since it is created by sorting the papers themselves into groups and then naming the groups.

Many papers fall naturally into several different groups and in such cases they are inserted under as many different heads as it is thought they rightfully belong.

The classified synopses are designed for those searching for comprehensive information on any given topic, while the subject index is intended for those looking up specific and definite data or information.

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1. EDUCATION

INDUSTRIAL RESEARCH AND THE COLLEGES

A. E. Kennelly **Vol. xxxvi—1917, pp. 833-839**

An outline of the proper relationship that should exist in the industrial research field between the pure science college, the technical college and the industries, themselves.

Discussion incorporated with that of paper by F. B. Jewett on "Industrial Research with Some Notes Concerning its Scope in the Bell Telephone System."

INDUSTRIAL RESEARCH WITH SOME NOTES CONCERNING ITS SCOPE IN THE BELL TELEPHONE SYSTEM

F. B. Jewett **Vol. xxxvi—1917, pp. 841-855**

The interrelation of industrial and "pure science" research and the restrictions which must be imposed to insure mutual existence. A brief outline of the scope and progress of industrial research in the Bell Telephone System.

Discussion (including that of paper by A. E. Kennelly), pages 856-869, by Messrs. V. Karapetoff, A. M. Gray, E. P. Hyde, L. F. Morehouse, C. H. Sharp, W. I. Slichter, H. Pender, H. A. Hornor, R. W. Pope, C. F. Scott, A. E. Kennelly and E. B. Craft.

A general discussion.

INDUSTRIAL RESEARCH AND ITS RELATION TO UNIVERSITY AND GOVERNMENTAL RESEARCH

C. E. Skinner **Vol. xxxvi—1917, pp. 871-882**

1. Introduction emphasizing the great activity in all branches of research at the present time.
2. A division of research activities into three principal classes, university, governmental and industrial.
3. A statement that the principal function of university research should be to train research men and some suggestions as to the type of training required.
4. A brief statement of the function of governmental research which has to do with such things as the development and conservation of our natural resources, the national defense and the promotion of those things which are for the benefit of the whole people.
5. A brief description of the Research Division of the Westinghouse Electric & Manufacturing Company, its rise and its relation to other departments.
6. A statement of the desirability of co-operation among all agencies engaged in research.

Discussion, pages 883-903, by Messrs. B. A. Behrend, V. Karapetoff, C. E. Skinner, H. Pender, O. Smith, L. W. Chubb, C. A. Adams, W. McClellan, A. M. Gray, W. L. R. Emmett and N. S. Diamant.

A general discussion with special emphasis on the research problems involved in the development of the turbo-generator.

2. GENERAL THEORY

OSCILLATING-CURRENT CIRCUITS BY THE METHOD OF GENERALIZED ANGULAR VELOCITIES

V. Bush **Vol. xxxvi—1917, pp. 207-221**

In the same way that alternating-current theorems are obtained, by generalized d-c. theorems, the author obtains oscillating-current theorems by still further generalization. Kirchoff's law is generalized and a simple method of determining generalized angular velocities, frequencies and decrements of free oscillation obtained. Examples are given to show method of procedure. A list of symbols is given at end of paper.

Discussion, pages 222-234, by Messrs. A. E. Kennelly, C. Fortescue, H. Fletcher, T. C. Fry, V. Bush and A. Press.

A general discussion of the application and value of the author's method of solution of oscillating-current circuits.

AN EXPERIMENTAL METHOD OF OBTAINING THE SOLUTION OF ELECTROSTATIC PROBLEMS WITH NOTES ON HIGH-VOLTAGE BUSHING DESIGN

Chester W. Rice **Vol. xxxvi—1917, pp. 905-1051**

The electrodynamic method for obtaining the solution of electrostatic and allied problems is developed to a high degree of accuracy and then applied to the study of high-voltage bushings. An experimental high-air-efficiency bushing was built and tested and also a series of small bushings. An appendix gives unexpurgated mathematical solutions of two flow problems.

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A general discussion of the problems encountered in bushing design.

3. UNITS, MEASUREMENTS AND INSTRUMENTS

INTERNAL TEMPERATURES OF A-C. GENERATORS

Ralph Kelly **Vol. xxxvi—1917, pp. 79-94**

The paper deals with the internal temperatures of a number of typical large a-c. generators; this temperature being measured by a thermo-couple placed between armature coils in the same slot and in the center of the core. The difference between these measured internal temperatures and the corresponding surface temperatures is explained by the aid of the tests and of calculations. A method of calculation for internal temperatures is given based on simple heat laws and on data from tests. By means of the tests and of parallel calculations, the effect on the internal temperature of changes in frequency, core length, thickness and quality of insulation, armature current density and core densities, is explained. The capacity of the end windings to dissipate heat from the center of the core is also discussed.

Discussion, pages 95-104, by Messrs. Henry G. Reist, C. A. Adams, W. F. Dawson, H. M. Hobart, Alexander Gray and Ralph Kelly.

A discussion of the various methods and means of temperature measurement and methods of ventilation of generators, radial vs. axial, etc.

THE ELECTRIC STRENGTH OF AIR—VII

J. B. Whitehead and W. S. Brown **Vol. xxxvi—1917, pp. 169-198**

In view of the variation among the values obtained by different observers, this paper aims to make a careful determination of corona-forming voltages for alternating and for positive and negative continuous voltages in the same apparatus and under the same conditions.

The values with alternating voltage coincide with those of negative continuous voltage. Positive continuous voltage therefore forms corona at the lowest value.

The observations on the negative corona give values higher than any heretofore obtained.

Other experiments are described, giving qualitative indications of the correctness of Townsend's theory of ionization by collision.

Discussion (including that of paper by J. W. Davis and C. S. Breese), pages 199-205, by Messrs. Saul Dushman, J. B. Taylor, L. W. Chubb, F. W. Peek, Jr., and J. B. Whitehead.

A general discussion.

COOLING OF OIL-IMMERSED TRANSFORMER WINDINGS AFTER SHUT-DOWN

V. M. Montsinger **Vol. xxxvi—1917, pp. 711-730**

The 1916 A. I. E. E. Rules require temperature rise of transformer windings be observed by resistance method. This requires time and therefore temperature drop. The author describes three general methods of determining temperature at shut-down. Under "Conclusions" the general advantages and disadvantages of each method are given. In the Appendix are developed certain formulas used in the paper.

Discussion, pages 731-734, by Messrs. A. S. McAllister, V. M. Montsinger, F. F. Brand, L. F. Blume and W. C. Smith.

A general discussion.

MAGNETIC FLUX DISTRIBUTION IN ANNULAR STEEL LAMINAE

A. E. Kennelly and P. L. Alger **Vol. xxxvi—1917, pp. 1113-1131**

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Peek, Jr., G. B. Thomas, P. L. Alger, A. E. Kennelly, W. V. Lyon and L. N. Robinson.

A general discussion.

4. INSULATION AND DIELECTRIC PHENOMENA

CORONA AND RECTIFICATION IN HYDROGEN

J. W. Davis and C. S. Breese **Vol. xxxvi—1917, pp. 153-168**

The results of an investigation of the corona discharge between co-axial cylinders in an atmosphere of hydrogen are given in this paper.

Both direct and alternating electromotive forces were used. The characteristic behavior of the corona is given by means of curves, photographs and oscillograms.

Corona in hydrogen between concentric cylinders is shown to be a practicable method for rectifying high-potential alternating currents.

The apparent evidence of ionization potential gradients at the surface of the tube and the general character of the visual phenomena are discussed.

A brief statement of conclusions is given.

Discussion, incorporated with that of paper by J. B. Whitehead and W. S. Brown on "The Electric Strength of Air-VII".

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A general discussion.

THE INFLUENCE OF DIELECTRIC LOSSES ON THE RATING OF HIGH-TENSION UNDERGROUND CABLES

A. F. Bang and H. C. Louis **Vol. xxxvi—1917, pp. 431-446**

This paper describes a series of tests made to determine the dielectric losses in various kinds of 13,000-volt cable. These losses were found to increase greatly with the temperature and thus become an important factor in the rating of such cables. A method is developed whereby the influence of these losses on the rating can be approximately calculated for duct lines with uniformly loaded cables.

Discussion incorporated with that of paper by W. S. Clark and G. B. Shanklin on "Insulation Characteristics of High-Voltage Cables".

INSULATION CHARACTERISTICS OF HIGH-VOLTAGE CABLES

W. S. Clark and G. B. Shanklin **Vol. xxxvi—1917, pp. 447-493**

The insulation characteristics of different types of single- and three-conductor cable, determined largely by dielectric energy loss measurements, are discussed in detail. Particular attention is given to paper-insulated cable, both new cable and cable that has been in service being considered. Varnished cambric and different grades of rubber insulation, as well as cable compounds are also dealt with briefly.

Discussion (including that of paper by John L. Harper, D. W. Roper, A. F. Bang and H. C. Louis), pages 494-526, by Messrs. W. I. Middleton, C. L. Dawes, Philip Torchio, C. W. Davis, H. R. Woodrow, J. B. Whitehead, D. W. Roper, A. F. Bang, C. A. Adams, W. S. Clark, D. Du Bois, E. B. Meyer, J. L. Harper, C. N. Rakestraw, H. E. Weightman, P. H. Chase, R. W. Atkinson, H. W. Fisher, A. F. Hovey and H. C. Louis.

A general discussion with experiences and data on cables and cable joints acquired in practise.

THE INSULATOR SITUATION

W. D. Peaslee **Vol. xxxvi—1917, pp. 527-533**

This paper gives a résumé of the insulator situation as it exists at the present time.

A statement of the apparent causes of the very rapid deterioration of insulators, even when stored and subject to no electric stress is given, and the conditions necessary to the production of an insulator that will reduce this deterioration cost are discussed.

Microphotographs showing the structure of the porcelain from several insulators are given. Three means are given for improving the insulator situation.

Discussion incorporated with that of paper by A. O. Austin on "Present Practise in the Design and Manufacture of High-Tension Insulators".

EXPANSION EFFECTS AS A CAUSE OF DETERIORATION IN SUSPENSION TYPE INSULATORS

J. A. Brundige **Vol. xxxvi—1917, pp. 535-544**

In seeking causes for the rapid deterioration which has been encountered in suspension type insulators, two leading hypotheses, viz., porosity and mechanical cracking through expansion effects, have been advanced by different groups of investigators. These are briefly outlined in the paper, after which the author presents data in support of the latter. Some of the operating problems attendant upon insulator deterioration are also discussed.

Discussion incorporated with that of paper by A. O. Austin on "Present Practise in the Design and Manufacture of High-Tension Insulators".

PRESENT PRACTISE IN THE DESIGN AND MANUFACTURE OF HIGH-TENSION INSULATORS

A. O. Austin **Vol. xxxvi—1917, pp. 545-562**

Rapid development in the transmission field has rendered the early insulator obsolete. As causes of losses have become evident, means have been found to eliminate them. Study of operating conditions has materially changed the insulator situation. Porosity loss has been reduced to a minimum by improved firing and closer selection. To prevent trouble on old lines it may be necessary to give the insulators a temperature as well as an electrical test. Trouble on modern insulators is prevented by careful attention to temperature gradient, increased mechanical strength and lowering internal stresses by an elastic joint.

Discussion (including that on papers by W. D. Peaslee and J. A. Brundige), pages 563-595, by Messrs. H. J. Ryan, J. C. Clark, P. Torchio, E. E. F. Creighton, E. M. Hewlett, P. H. Thomas, J. B. Taylor, C. F. Harding, J. T. Barron, J. A. Brundige and A. O. Austin.

A general discussion of the insulator situation with much data from practise.

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A general discussion of the problems encountered in bushing design.

5. ELECTRIC CONDUCTORS

PROBLEMS OF OPERATION AND MAINTENANCE OF UNDERGROUND CABLES

John L. Harper **Vol. xxxvi—1917, pp. 417-422**

With a view to increasing transmission capacity the causes of failures of cables are analyzed and conclusion drawn that overheating is most important. Methods of removal of heat from cables are considered and recommendations made for flooding conduits with water.

Discussion incorporated with that of paper by W. S. Clark and G. B. Shanklin on "Insulation Characteristics of High-Voltage Cables".

THE FUNDAMENTALS OF SUCCESSFUL HIGH-TENSION CABLE JOINTS

D. W. Roper **Vol. xxxvi—1917, pp. 423-429**

In this paper the author gives specifications intended to apply in general to joints on cables operating at voltages above 10,000 and in particular to joints operating above 20,000 volts.

Discussion incorporated with that of paper by W. S. Clark and G. B. Shanklin on "Insulation Characteristics of High-Voltage Cables".

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6. MAGNETIC PROPERTIES AND TESTING OF IRON

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A general discussion.

9. ELECTRICAL MACHINERY AND APPARATUS

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Ralph Kelly **Vol. xxxvi—1917, pp. 79-94**

The paper deals with the internal temperatures of a number of typical large a-c. generators; this temperature being measured by a thermo-couple placed between armature coils in the same slot and in the center of the core. The difference between these measured internal temperatures and the corresponding surface temperatures is explained by the aid of the tests and of calculations. A method of calculation for internal temperatures is given based on simple heat laws and on data from tests. By means of the tests and of parallel calculations, the effect on the internal temperature of changes in frequency, core length, thickness and quality of insulation, armature current density and core densities, is explained. The capacity of the end windings to dissipate heat from the center of the core is also discussed.

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A discussion of the various methods and means of temperature measurement and methods of ventilation of generators, radial vs. axial, etc.

INDUSTRIAL CONTROLLERS—WITH PARTICULAR REFERENCE TO THE CONTROL OF DIRECT-CURRENT SHUNT MOTORS

H. D. James **Vol. xxxvi—1917, pp. 253-273**

Stating former limitations as to regulation of starting current from the viewpoint of motor, controller and supply system have been largely removed, the author presents a series of tests on 15 and 20 h.p., 230-volt d-c. shunt motors, started from rest. The motors are of two types, constant and adjustable speed.

From these tests the author concludes, in part; that it is practical with automatic acceleration to use one switch to short-circuit the armature resister used with motors as large as 15 h.p.; that the shunt field of small adjustable-speed motors can be reduced in one step under normal load conditions (this practise can be safely followed up to motors of 50 h.p.); and that adjustable-speed motors can use one step resistance for dynamic braking.

Discussion, incorporated with that of paper by K. L. Hansen on "Analysis of Starting Characteristics of Direct-Current Motors".

ANALYSIS OF STARTING CHARACTERISTICS OF DIRECT-CURRENT MOTORS

K. L. Hansen **Vol. xxxvi—1917, pp. 275-299**

In this paper are derived the mathematical expressions for the current, speed, torque and power at any time during the acceleration period when a shunt, series or compound motor is connected to a supply line.

Discussion, (including that of paper by H. D. James) pages 300-320, by Messrs. C. T. Evans, R. Hellmund, R. G. Widdows, E. J. Murphy, J. H. Albrecht, E. H. Martindale, R. H. McLain, A. A. Gazda, J. B. Fisken, F. W. Gay, A. Gray, J. P. Mallett, H. D. James and R. L. Goetzenberger.

A general discussion presenting arguments from actual practise both for and against the one-step starter. Special emphasis on flashing.

TRANSIENT CONDITIONS IN ASYNCHRONOUS INDUCTION MACHINES AND THEIR RELATION TO CONTROL PROBLEMS

R. E. Hellmund **Vol. xxxvi—1917, pp. 321-337**

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Discussion incorporated with that of paper by A. A. Gazda on "Performance of Polyphase Induction Motors Under Unbalanced Secondary Conditions".

PERFORMANCE OF POLYPHASE INDUCTION MOTORS UNDER UNBALANCED SECONDARY CONDITIONS

A. A. Gazda **Vol. xxxvi—1917, p. 339-348**

Continuous operation of the wound-rotor induction motor, when the internal resistances in the secondary phases are not equal, is shown to be feasible. The effect upon power factor and heating is discussed. Curves showing the performance of polyphase motors with single-phase secondary are presented. The practical advantages of using unbalanced secondary connections are pointed out.

Discussion (including that of paper by R. E. Hellmund) pages 349-359, by Messrs. W. B. Kouwenhoven, A. A. Gazda, R. B. Williamson, H. Maxwell, C. Macmillan, A. E. Averett, R. E. Hellmund, A. Simon and H. D. James.

A general discussion with detailed explanation of the operation of the polyphase motor with single-phase secondary at half speed.

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R. L. Kingsland **Vol. xxxvi—1917, pp. 649-654**

The best form of power to be used in coal mines for drilling, cutting, hauling, pumping, lighting, ventilation. Synchronous converters versus motor-generator sets. Location of generating plant and substations.

Discussion incorporated with that of paper by R. S. Sage on "A New Electric Mine Hoist at Butte, Montana".

A NEW ELECTRIC MINE HOIST AT BUTTE, MONTANA

R. S. Sage **Vol. xxxvi—1917, pp. 655-671**

A description of equipment in connection with a recent Ilgner-Ward Leonard hoist installation at the Elm Orlu Mining Company, Butte, Montana. This installation is one of the two largest of this type in operation in this country.

Complete data on two sets of tests; the first conducted to analyze a typical day's run and the second to determine efficiency while hoisting under known conditions.

Discussion (including that of paper by R. L. Kingsland), pages 672-690, by Messrs. A. S. McAllister, R. L. Kingsland, K. A. Pauly, C. A. Adams, H. H. Clark, E. H. Martindale, R. S. Sage, C. M. Means, A. B. Kiser, G. Bright, F. J. Duffy, L. C. Ilsley and J. B. Morrill.

A general discussion.

COOLING OF OIL-IMMERSED TRANSFORMER WINDINGS AFTER SHUT-DOWN

V. M. Montsinger **Vol. xxxvi—1917, pp. 711-730**

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Discussion, pages 731-734, by Messrs. A. S. McAllister, V. M. Montsinger, F. F. Brand, L. F. Blume and W. C. Smith.

A general discussion.

11. POWER PLANTS AND CENTRAL STATIONS

ECONOMICAL COMBINATION OF WATER POWER AND STEAM PLANTS AND A CONVENIENT METHOD OF SOLUTION

H. St. Clair Putnam **Vol. xxxvi—1917, pp. 691-708**

A determination of the respective economic limits in steam and water power plant developments. Formulas are derived for determining total annual costs of each type of plant and the most economical combination. Certain general conclusions are reached as follows: Practically all water power plants should be developed beyond minimum power available and hence in combination with a steam plant. The economic limit depends largely on the value of money, fuel, labor, the increasing efficiency of steam turbines and the location of the enterprise.

Discussion, pages 709-710, by Messrs. H. R. Summerhayes and H. S. Putnam.

A general discussion.

ANNUAL LOAD RELIEF MAP, PEAK LOAD AND LOAD FACTOR ANALYSIS

Wm. Le Roy Robertson **Vol. xxxvi—1917, pp. 1073-1079**

Description of the annual load relief map which is a device for visualizing the entire yearly load of central stations.

Review of load characteristics, as exemplified by the annual load relief map.

Notes on daylight saving relative to the decreasing of peak load, and improvement in load factor.

Analysis of load factors as applied to "base" and "top" loads demonstrating the large amount of idle capacity peculiar to central stations.

Annual load relief map valuable in the study and exemplification of all matters having to do with central station loads.

No discussion.

13. TRANSMISSION LINES

PROTECTION OF TRANSFORMER NEUTRALS AGAINST DESTRUCTIVE TRANSIENT DISTURBANCES

Max H. Collbohm **Vol. xxxvi—1917, pp. 135-138**

The paper points out the danger to which the neutral point of transformers, connected to long distance transmission lines, are exposed through the building up of excessive potentials at this point under conditions of atmospheric lightning. This is due to the fact that the neutral point in a bank of Y-connected transformers acts as a reflection point for all the waves that are produced by induced lightning. The author recommends the installation of lightning arresters at the neutral point to provide a discharge path for the excess potential, which according to measurement with the spark gap, may reach values of 350,000 volts, and higher.

Discussion, (including that of papers by J. Allen Johnson and P. B. Juhnke) pages 139-152, by Messrs. F. H. Kierstead, H. R. Woodrow, P. B. Juhnke, A. R. Cheyney, L. F. Blume, J. B. Taylor, R. F. Schuchardt, H. R. Summerhayes, C. A. Adams, J. A. Johnson, V. Karapetoff, A. L. Harrington and M. H. Collbohm.

A general discussion of the use of reactors and expressions of opinion on the accuracy of Mr. Collbohm's deductions.

RELAYS FOR HIGH-TENSION LINES

Philip Torchio **Vol. xxxvi—1917, pp. 361-375**

The author describes the various arrangements of relays found desirable with the following transmission systems; radial feeders not in parallel; radial feeders in paralle'; single lines in tandem; two or more parallel lines in tandem; single-line ring systems; parallel-line ring systems; tie lines; Appendices A, B, C describe connections of reverse-power relays, relay testing and a special installation.

Discussion incorporated with that of paper by R. F. Schuchardt on "The Protective Equipment on the System of the Commonwealth Edison Company".

THE PROTECTIVE EQUIPMENT ON THE SYSTEM OF THE COMMONWEALTH EDISON COMPANY

R. F. Schuchardt **Vol. xxxvi—1917, pp. 377-389**

The paper describes briefly the system of the Commonwealth Edison Company of Chicago with special reference to the protective devices installed thereon.

Discussion (including that of paper by Philip Torchio) pages 390-416, by Messrs. H. R. Summerhayes, Wm. H. Cole, E. B. Mayer, L. N. Crichton, J. R. Craighead, O. C. Traver, A. A. Meyer, J. B. Taylor, H. L. Wallau, J. S. Jenks, Philip Torchio, R. F. Schuchardt, N. L. Pollard and Chester Lichtenberg.

A general discussion of the arrangements described and various combinations of relays used on other systems with special emphasis on the split-conductor cable.

PROBLEMS OF OPERATION AND MAINTENANCE OF UNDERGROUND CABLES

John L. Harper **Vol. xxxvi—1917, pp. 417-422**

With a view to increasing transmission capacity the causes of failures of cables are analyzed and conclusion drawn that overheating is most important. Methods of removal of heat from cables are considered and recommendations made for flooding conduits with water.

Discussion incorporated with that of paper by W. S. Clark and G. B. Shanklin on "Insulation Characteristics of High-Voltage Cables".

THE FUNDAMENTALS OF SUCCESSFUL HIGH-TENSION CABLE JOINTS

D. W. Roper **Vol. xxxvi—1917, pp. 423-429**

In this paper the author gives specifications intended to apply in general to joints on cables operating at voltages above 10,000 and in particular to joints operating above 20,000 volts.

Discussion incorporated with that of paper by W. S. Clark and G. B. Shanklin on "Insulation Characteristics of High-Voltage Cables".

THE INFLUENCE OF DIELECTRIC LOSSES ON THE RATING OF HIGH-TENSION UNDERGROUND CABLES

A. F. Bang and H. C. Louis **Vol. xxxvi—1917, pp. 431-446**

This paper describes a series of tests made to determine the dielectric losses in various kinds of 13,000-volt cable. These losses were found to increase greatly with the temperature and thus become an important factor in the rating of such cables. A method is developed whereby the influence of these losses on the rating can be approximately calculated for duct lines with uniformly loaded cables.

Discussion incorporated with that of paper by W. S. Clark and G. B. Shanklin on "Insulation Characteristics of High-Voltage Cables".

INSULATION CHARACTERISTICS OF HIGH-VOLTAGE CABLES

W. S. Clark and G. B. Shanklin **Vol. xxxvi—1917, pp. 447-493**

The insulation characteristics of different types of single- and three-conductor cable, determined largely by dielectric energy loss measurements, are discussed in detail. Particular attention is given to paper-insulated cable, both new cable and cable that has been in service being considered. Varnished cambric and different grades of rubber insulation, as well as cable compounds are also dealt with briefly.

Discussion (including that of paper by John L. Harper, D. W. Roper, A. F. Bang and H. C. Louis), pages 494-526, by Messrs. W. I. Middleton, C. L. Dawes, Philip Torchio, C. W. Davis, H. R. Woodrow, J. B. Whitehead, D. W. Roper, A. F. Bang, C. A. Adams, W. S. Clark, D. Du Bois, E. B. Meyer, J. L. Harper, C. N. Rakestraw, H. E. Weightman, P. H. Chase, R. W. Atkinson, H. W. Fisher, A. F. Hovey and H. C. Louis.

A general discussion with experiences and data on cables and cable joints acquired in practise.

TRANSMISSION LINE DESIGN

F. K. Kirsten **Vol. xxxvi—1917, pp. 735-788**

This paper contains a complete mathematical analysis of the forces which determine the location in space of a cable suspended from points of equal elevation, and gives the designer and constructing engineer of a transmission line some useful working formulas based on this analysis.

Solutions of typical design problems are given in each section of the paper demonstrating the use and manipulation of all formulas derived.

No discussion.

DESIGN, CONSTRUCTION AND TESTS OF AN ARTIFICIAL POWER TRANSMISSION LINE FOR THE TELLURIDE POWER COMPANY OF PROVO, UTAH

George H. Gray **Vol. xxxvi—1917, pp. 789-831**

A description is given of an artificial power-transmission line which duplicates an actual transmission system. Methods are given of calculating the correct distribution of inductance between self and mutual, and of calculating the correct distribution of capacity between wire and wire, and wire and ground. Various conditions of short circuit are described and data presented in the form of oscillograms and vector diagrams giving magnitude and phase relations of current and voltage, also directions of power flow.

No discussion.

14. ELECTRIC SERVICE DISTURBANCES AND PROTECTION

REACTORS IN HYDROELECTRIC STATIONS

J. Allen Johnson **Vol. xxxvi—1917, pp. 105-124**

The advantages and limitations of current limiting reactors in hydroelectric stations. The two beneficial results of reactance, protection and localization, and the two detrimental results of its use, voltage drop and reduction of synchronous stability. The origination of a power surge by sudden load loss and the accompanying hunting oscillation is described. Actual occurrence of a surge described.

Discussion, incorporated with that of paper by Max H. Collbohm on "Protection of Transformer Neutrals Against Destructive Transient Disturbances".

EFFECT OF CURRENT LIMITING REACTORS ON TURBO-GENERATOR SYSTEMS UNDER CONDITIONS OF SHORT CIRCUIT

P. B. Juhnke **Vol. xxxvi—1917, pp. 125-134**

An account of a cable breakdown near one of the generating stations of the Commonwealth Edison Company's system. The breakdown resulted in a second breakdown on same line within the station, which prevented the oil switch from automatically disconnecting the fault. Comparison of maximum stresses encountered in the short circuit as it occurred with what they might have been without reactors present are made, and the conclusion is drawn that without them the damage resulting would have been considerable and the service interruption far more general and serious.

Discussion incorporated with that of paper by Max H. Collbohm on "Protection of Transformer Neutrals Against Destructive Transient Disturbances".

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A general discussion of the arrangements described and various combinations of relays used on other systems with special emphasis on the split-conductor cable.

THE INSULATOR SITUATION

W. D. Peaslee **Vol. xxxvi—1917, pp. 527-533**

This paper gives a résumé of the insulator situation as it exists at the present time.

A statement of the apparent causes of the very rapid deterioration of insulators, even when stored and subject to no electric stress is given, and the conditions necessary to the production of an insulator that will reduce this deterioration cost are discussed.

Microphotographs showing the structure of the porcelain from several insulators are given. Three means are given for improving the insulator situation.

Discussion incorporated with that of paper by A. O. Austin on "Present Practise in the Design and Manufacture of High-Tension Insulators".

EXPANSION EFFECTS AS A CAUSE OF DETERIORATION IN SUSPENSION TYPE INSULATORS

J. A. Brundige **Vol. xxxvi—1917, pp. 535-544**

In seeking causes for the rapid deterioration which has been encountered in suspension type insulators, two leading hypotheses, viz., porosity and mechanical cracking through expansion effects, have been advanced by different groups of investigators. These are briefly outlined in the paper, after which the author presents data in support of the latter. Some of the operating problems attendant upon insulator deterioration are also discussed.

Discussion incorporated with that of paper by A. O. Austin on "Present Practise in the Design and Manufacture of High-Tension Insulators".

PRESENT PRACTISE IN THE DESIGN AND MANUFACTURE OF HIGH-TENSION INSULATORS

A. O. Austin **Vol. xxxvi—1917, pp. 545-562**

Rapid development in the transmission field has rendered the early insulator obsolete. As causes of losses have become evident, means have been found to eliminate them. Study of operating conditions has materially changed the insulator situation. Porosity loss has been reduced to a minimum by improved firing and closer selection. To prevent trouble on old lines it may be necessary to give the insulators a temperature as well as an electrical test. Trouble on modern insulators is prevented by careful attention to temperature gradient, increased mechanical strength and lowering internal stresses by an elastic joint.

Discussion (including that on papers by W. D. Peaslee and J. A. Brundige), pages 563-595, by Messrs. H. J. Ryan, J. C. Clark, P. Torchio, E. E. F. Creighton, E. M. Hewlett, P. H. Thomas, J. B. Taylor, C. F. Harding, J. T. Barron, J. A. Brundige and A. O. Austin.

A general discussion of the insulator situation with much data from practise.

AN EXPERIMENTAL METHOD OF OBTAINING THE SOLUTION OF ELECTROSTATIC PROBLEMS WITH NOTES ON HIGH-VOLTAGE BUSHING DESIGN

Chester W. Rice **Vol. xxxvi—1917, pp. 905-1051**

The electrodynamic method for obtaining the solution of electrostatic and allied problems is developed to a high degree of accuracy and then applied to the study of high-voltage bushings. An experimental high-air-efficiency bushing was built and tested and also a series of small bushings. An appendix gives unexpurgated mathematical solutions of two flow problems, *Discussion*, pages 1052-1072, by Messrs. R. Mershon, C. O. Mailloux, J. B. Taylor, A. M. Gray, C. L. Fortescue, E. D. Eby, F. W. Peek, Jr., M. E. Tressler, H. O. Stevens, S. W. Farnsworth, S. Haar, B. A. Behrend, J. B. Morrill and C. W. Rice.

A general discussion of the problems encountered in bushing design.

PHENOMENA ACCOMPANYING TRANSMISSION WITH SOME TYPES OF STAR TRANSFORMER CONNECTIONS—II

Lloyd N. Robinson **Vol. xxxvi—1917, pp. 1081-1111**

A discussion of exceptional phenomena occurring when single-phase transformers are connected star-star into a three-phase bank. The author offers an explanation of the occurrence of odd and even harmonics, periodically and permanently reversed leg, auto-transformers, vibrations and noises, etc.

Discussion incorporated with that of paper by A. E. Kennelly and P. L. Alger on "Magnetic Flux Distribution in Annular Steel Laminae."

15. DISTRIBUTION SYSTEMS

FORMS OF ELECTRIC POWER BEST SUITED FOR THE VARIOUS LOADS ENCOUNTERED IN THE OPERATION OF BITUMINOUS COAL MINES

R. L. Kingsland **Vol. xxxvi—1917, pp. 649-654**

The best form of power to be used in coal mines for drilling, cutting, hauling, pumping, lighting, ventilation. Synchronous converters versus motor-generator sets. Location of generating plant and substations.

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16. CONTROL, REGULATION AND SWITCHING

REGENERATIVE BRAKING OF ELECTRIC VEHICLES

R. E. Hellmund **Vol. xxxvi—1917, pp. 1-56**

This paper discusses the problems met in applying regenerative control to electric railways. In the introductory section the purposes, advantages, requirements and disadvantages are pointed out. The author describes in detail the three-phase, the phase-converter and the direct-current systems. He also covers the alternating-current commutator-motor system and the possibilities of regenerative control with a vapor converter system.

Discussion, pages 57-78, by Messrs. Robert Lundell, C. A. Adams, E. F. W. Alexanderson, A. J. Hall, R. E. Ferris, W. V. Turner, H. M. Hobart, F. R. Phillips, C. L. Fortescue, K. A. Simon, F. D. Newbury, W. B. Potter, N. W. Storer and R. E. Hellmund.

A general discussion with special emphasis on d-c. systems.

INDUSTRIAL CONTROLLERS—WITH PARTICULAR REFERENCE TO THE CONTROL OF DIRECT-CURRENT SHUNT MOTORS

H. D. James **Vol. xxxvi—1917, pp. 253-273**

Stating former limitations as to regulation of starting current from the viewpoint of motor, controller and supply system have been largely removed, the author presents a series of tests on 15 and 20 h.p., 230-volt d-c. shunt motors, started from rest. The motors are of two types, constant and adjustable speed.

From these tests the author concludes, in part; that it is practical with automatic acceleration to use one switch to short-circuit the armature resister used with motors as large as 15 h.p.; that the shunt field of small adjustable-speed motors can be reduced in one step under normal load conditions (this practise can be safely followed up to motors of 50 h.p.); and that adjustable-speed motors can use one step resistance for dynamic braking.

Discussion, incorporated with that of paper by K. L. Hansen on "Analysis of Starting Characteristics of Direct-Current Motors".

ANALYSIS OF STARTING CHARACTERISTICS OF DIRECT-CURRENT MOTORS

K. L. Hansen **Vol. xxxvi—1917, pp. 275-299**

In this paper are derived the mathematical expressions for the current, speed, torque and power at any time during the acceleration period when a shunt, series or compound motor is connected to a supply line.

Discussion, (including that of paper by H. D. James) pages 300-320, by Messrs. C. T. Evans, R. Hellmund, R. G. Widdows, E. J. Murphy, J. H. Albrecht, E. H. Martindale, R. H. McLain, A. A. Gazda, J. B. Fisken. F. W. Gay, A. Gray, J. P. Mallett, H. D. James and R. L. Goetzenberger.

A general discussion presenting arguments from actual practise both for and against the one-step starter. Special emphasis on flashing.

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A general discussion with special emphasis on d-c. systems.

20. MISCELLANEOUS APPLICATION OF ELECTRICITY

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22. MISCELLANEOUS TOPICS AND INSTITUTE AFFAIRS

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A general discussion with special emphasis on d-c. systems.

MODERN PHYSICS

R. A. Millikan **Vol. xxxvi—1917, pp. 235-252**

A lecture outline of the developments of modern physics.

THE ENGINEER'S DESTINY

President's Address

H. W. Buck **Vol. xxxvi—1917, pp. 597-601**

ADDRESS BY PRESIDENT-ELECT E. W. RICE, JR.

Vol. xxxvi—1917, pp. 603-608

TECHNICAL COMMITTEE REPORTS

Vol. xxxvi—1917, pp. 609-647

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A general discussion.

INDUSTRIAL RESEARCH AND THE COLLEGES

A. E. Kennelly **Vol. xxxvi—1917, pp. 833-839**

An outline of the proper relationship that should exist in the industrial research field between the pure science college, the technical college and the industries, themselves.

Discussion incorporated with that of paper by F. B. Jewett on "Industrial Research with Some Notes Concerning its Scope in the Bell Telephone System."

INDUSTRIAL RESEARCH WITH SOME NOTES CONCERNING ITS SCOPE IN THE BELL TELEPHONE SYSTEM

F. B. Jewett **Vol. xxxvi—1917, pp. 841-855**

The interrelation of industrial and "pure science" research and the restrictions which must be imposed to insure mutual existence. A brief outline of the scope and progress of industrial research in the Bell Telephone System.

Discussion (including that of paper by A. E. Kennelly), pages 856-869, by Messrs. V. Karapetoff, A. M. Gray, E. P. Hyde, L. F. Morehouse, C. H. Sharp, W. I. Slichter, H. Pender, H. A. Hornor, R. W. Pope, C. F. Scott, A. E. Kennelly and E. B. Craft.

A general discussion.

INDUSTRIAL RESEARCH AND ITS RELATION TO UNIVERSITY AND GOVERNMENTAL RESEARCH

C. E. Skinner **Vol. xxxvi—1917, pp. 871-882**

1. Introduction emphasizing the great activity in all branches of research at the present time.
2. A division of research activities into three principal classes, university, governmental and industrial.
3. A statement that the principal function of university research should be to train research men and some suggestions as to the type of training required.
4. A brief statement of the function of governmental research which has to do with such things as the development and conservation of our natural resources, the national defense and the promotion of those things which are for the benefit of the whole people.
5. A brief description of the Research Division of the Westinghouse Electric & Manufacturing Company, its rise and its relation to other departments.
6. A statement of the desirability of co-operation among all agencies engaged in research.

Discussion, pages 883-903, by Messrs. B. A. Behrend, V. Karapetoff, C. E. Skinner, H. Pender, O. Smith, L. W. Chubb, C. A. Adams, W. McClellan, A. M. Gray, W. L. R. Emmett and N. S. Diamant.

A general discussion with special emphasis on the research problems involved in the development of the turbo-generator.

ANNUAL LOAD RELIEF MAP, PEAK LOAD AND LOAD FACTOR ANALYSIS

Wm. Le Roy Robertson **Vol. xxxvi—1917, pp. 1073-1079**

Description of the annual load relief map which is a device for visualizing the entire yearly load of central stations.

Review of load characteristics, as exemplified by the annual load relief map.

Notes on daylight saving relative to the decreasing of peak load, and improvement in load factor.

Analysis of load factors as applied to "base" and "top" loads demonstrating the large amount of idle capacity peculiar to central stations.

Annual load relief map valuable in the study and exemplification of all matters having to do with central station loads.

No discussion.

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